Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.0-a-sin-^m-b-trg-^n

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June 29, 2021 Compiled on June 29, 2021 at 6:29pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [538]. This is test number [65].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (538)	% 0.00 (0)
Mathematica	% 100.00 (538)	% 0.00 (0)
Maple	% 82.16 (442)	% 17.84 (96)
Maxima	% 45.17 (243)	% 54.83 (295)
Fricas	% 53.16 (286)	% 46.84 (252)
Sympy	% 18.40 (99)	% 81.60 (439)
Giac	% 35.50 (191)	% 64.50 (347)
Mupad	% 46.10 (248)	% 53.90 (290)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

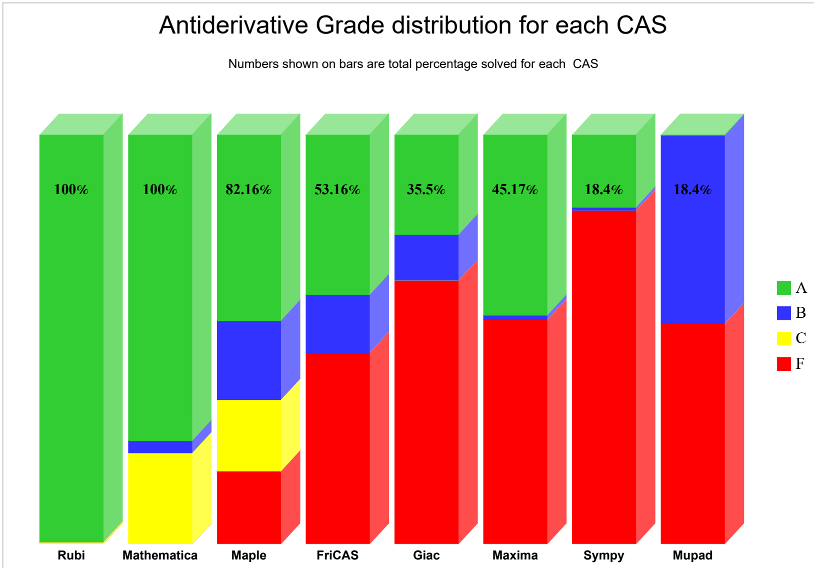
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

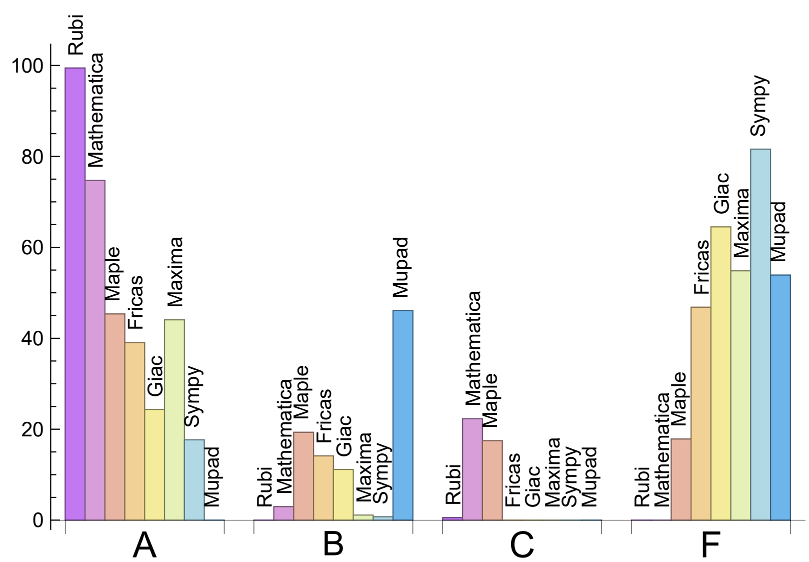
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.44	0.00	0.56	0.00
Mathematica	74.72	2.97	22.30	0.00
Maple	45.35	19.33	17.47	17.84
Maxima	44.05	1.12	0.00	54.83
Fricas	39.03	14.13	0.00	46.84
Sympy	17.66	0.74	0.00	81.60
Giac	24.35	11.15	0.00	64.50
Mupad	0.00	46.10	0.00	53.90

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	96	100.00 %	0.00 %	0.00 %
Maxima	295	98.98 %	1.02 %	0.00 %
Fricas	252	94.84 %	5.16 %	0.00 %
Sympy	439	46.24 %	53.53 %	0.23 %
Giac	347	82.42 %	8.93 %	8.65 %
Mupad	290	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

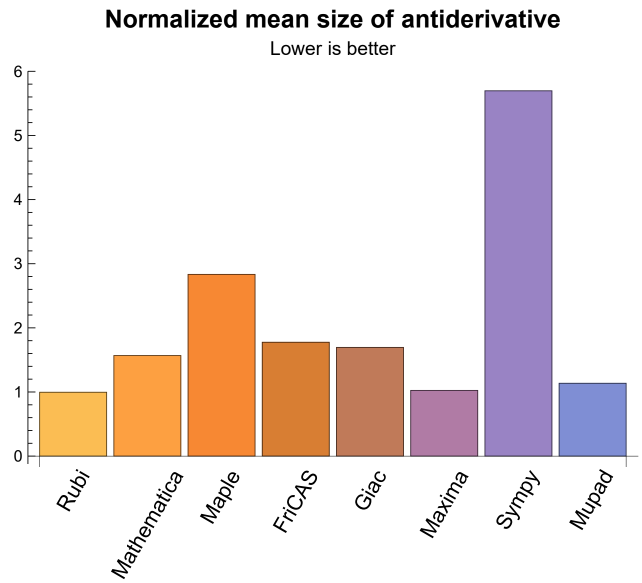
1.3 Performance

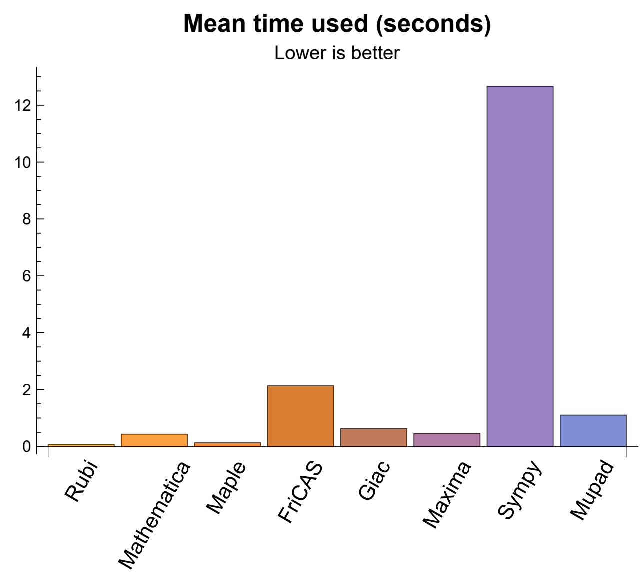
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.07	78.71	1.00	68.00	1.00
Mathematica	0.43	106.35	1.57	56.00	0.88
Maple	0.13	234.90	2.83	109.00	1.69
Maxima	0.45	50.47	1.02	40.00	0.96
Fricas	2.13	166.46	1.77	51.00	1.19
Sympy	12.66	244.30	5.70	65.00	1.64
Giac	0.63	78.71	1.69	48.00	1.15
Mupad	1.10	52.19	1.14	38.50	0.92

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {486, 488, 489, 490, 491, 496, 497, 498, 499, 501, 502}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

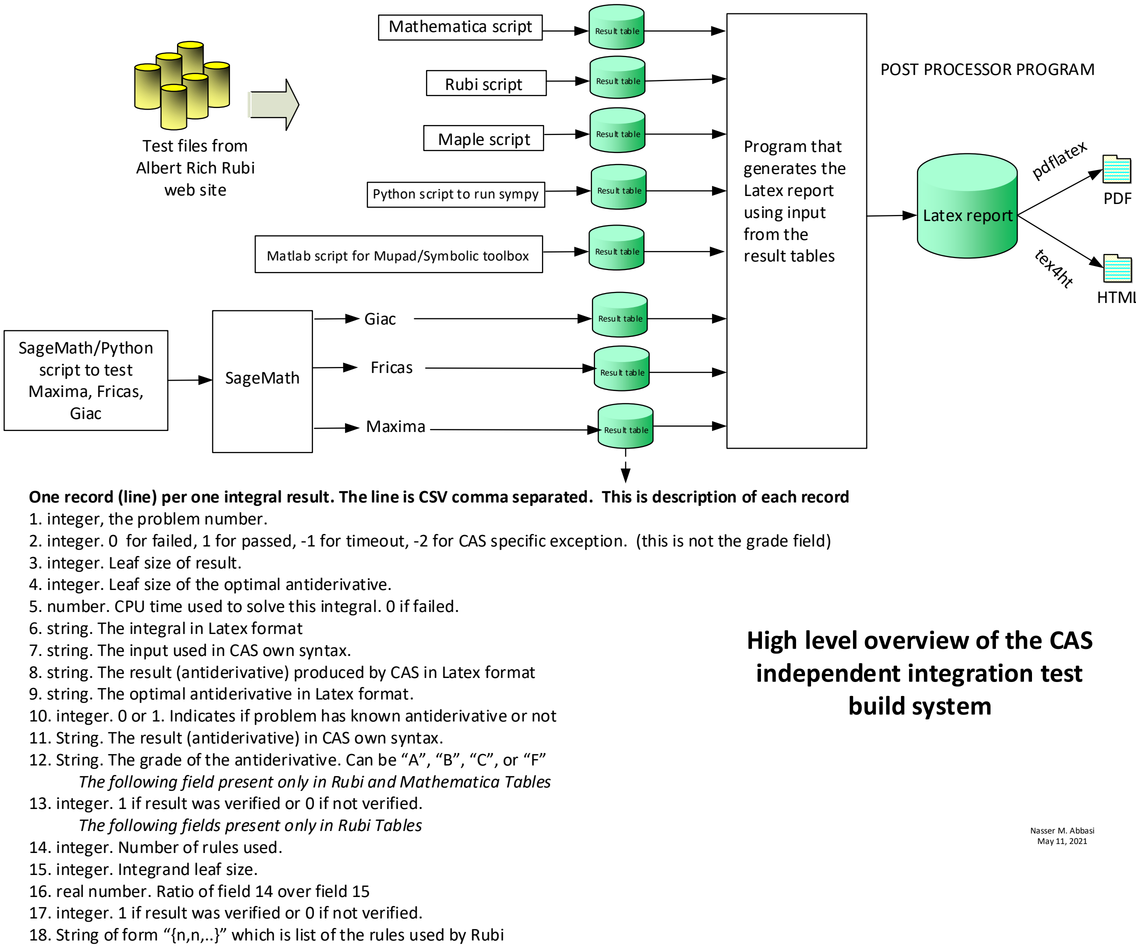
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

B grade: { }

C grade: { 35, 36, 37 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 141, 143, 145, 146, 147, 148, 149, 151, 152, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 177, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 222, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 252, 253, 254, 255, 256, 264, 265, 266, 273, 274, 275, 284, 285, 286, 288, 297, 298, 299, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 453, 454, 455, 466, 467, 468, 469, 470, 471, 476, 483, 484, 485, 487, 492, 493, 494, 495, 500, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

B grade: { 44, 87, 88, 126, 150, 153, 155, 176, 178, 183, 210, 221, 359, 360, 366, 496 }

C grade: { 35, 36, 37, 138, 140, 142, 144, 162, 166, 168, 170, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 228, 229, 230, 231, 243, 245, 246, 247, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 276, 277, 278, 279, 280, 281, 282, 283, 287, 289, 290, 291, 292, 293, 294, 295, 296, 300, 301, 302, 303, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 450, 451, 452, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 486, 488, 489, 490, 491, 497, 498, 499, 501, 502 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 77, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 113, 115, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 200, 204, 206, 211, 212, 213, 214, 215, 216, 217, 232, 234, 239, 240, 252, 253, 254, 255, 256, 264, 265, 266, 267, 268, 269, 270, 273, 274, 275, 284, 285, 286, 291, 292, 294, 295, 297, 298, 299, 342, 357, 374, 388, 400, 410, 411, 412, 413, 424, 425, 426, 427, 437, 438, 439, 440, 453, 454, 455, 456, 457, 469, 470, 471, 476, 477, 478, 479 }

B grade: { 10, 74, 76, 78, 79, 82, 110, 112, 114, 116, 163, 196, 198, 199, 201, 202, 203, 205, 207, 208, 209, 210, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 235, 236, 237, 238, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 257, 258, 259, 260, 261, 276, 277, 278, 279, 280, 281, 288, 293, 371, 372, 373, 375, 376, 377, 385, 386, 387, 389, 390, 397, 398, 399, 401, 402, 403, 414, 415, 416, 428, 429, 430, 441, 442, 443, 458, 459, 460, 461, 462, 463, 464, 465, 468, 480, 481, 482, 494 }

C grade: { 262, 263, 271, 272, 282, 283, 287, 289, 290, 296, 300, 301, 302, 303, 356, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 466, 467, 472, 473, 474, 475, 493, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537 }

F grade: { 33, 34, 35, 36, 37, 38, 39, 40, 41, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 538 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 210, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 340, 341, 342, 355, 356, 357, 371, 372, 373, 374, 375, 376, 377, 385, 386, 387, 388, 389, 390, 397, 398, 399, 400, 401, 402, 403, 410, 411, 412, 413, 414, 415, 416, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 492, 493, 494 }

B grade: { 75, 79, 111, 113, 117, 126 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 155, 157, 158, 159, 160, 161, 163, 165, 167, 169, 170, 171, 172, 173, 174, 175, 183, 187, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 210, 221, 222, 223, 225, 231, 242, 251, 252, 253, 254, 255, 256, 264, 265, 266, 273, 274, 275, 284, 285, 286, 288, 297, 298, 299, 324, 327, 328, 329, 332, 333, 334, 335, 340, 341, 342, 355, 356, 357, 371, 372, 373, 374, 385, 386, 387, 388, 397, 398, 399, 400, 410, 411, 412, 413, 424, 425, 426, 427, 437, 438, 439, 440, 453, 454, 455, 468, 469, 470, 471, 492, 493, 494 }

B grade: { 53, 54, 63, 83, 86, 104, 111, 126, 127, 128, 140, 150, 152, 153, 154, 156, 162, 164, 166, 168, 176, 177, 178, 179, 180, 181, 182, 184, 185, 186, 224, 226, 227, 228, 229, 230, 243, 244, 245, 246, 247, 248, 249, 250, 262, 263, 271, 272, 282, 283, 287, 289, 290, 296, 300, 301, 302, 303, 375, 376, 377, 389, 390, 401, 402, 403, 414, 415, 416, 428, 429, 430, 441, 442, 443, 476 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 257, 258, 259, 260, 261, 267, 268, 269, 270, 276, 277, 278, 279, 280, 281, 291, 292, 293, 294, 295, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 330, 331, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 378, 379, 380, 381, 382, 383, 384, 391, 392, 393, 394, 395, 396, 404, 405, 406, 407, 408, 409, 417, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 49, 50, 51, 52, 58, 59, 60, 61, 66, 67, 68, 69, 70, 80, 81, 82, 83, 89, 90, 91, 92, 98, 99, 100, 101, 102, 103, 104, 120, 121, 122, 123, 124, 125, 133, 134, 135, 136, 137, 138, 139, 145, 146, 147, 148, 149, 150, 151, 157, 158, 159, 160, 161, 162, 163, 164, 165, 171, 172, 173, 174, 175, 176, 177, 178, 179, 186, 187, 188, 189, 190, 191, 204, 205, 206, 207, 252, 254, 340, 341, 342, 355, 356, 357 }

B grade: { 62, 185, 253, 256 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 45, 46, 47, 48, 53, 54, 55, 56, 57, 63, 64, 65, 71, 72, 73, 74, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 97, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 126, 127, 128, 129, 130, 131, 132, 140, 141, 142, 143, 144, 152, 153, 154, 155, 156, 166, 167, 168, 169, 170, 180, 181, 182, 183, 184, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 110, 118, 123, 125, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 149, 151, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 177, 178, 179, 185, 186, 188, 189, 190, 191, 192, 193, 204, 205, 206, 207, 208, 209, 210, 252, 253, 254, 255, 256, 341, 342, 356, 357, 412, 426, 427, 439, 440 }

B grade: { 98, 99, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 124, 126, 127, 128, 129, 130, 131, 132, 138, 145, 146, 147, 148, 150, 152, 153, 154, 155, 156, 162, 171, 172, 173, 174, 175, 176, 180, 181, 182, 183, 184, 226, 227, 228, 229, 230, 231, 247, 248, 249, 250, 251

}

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 187, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 28, 29, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 207, 208, 209, 210, 221, 252, 253, 254, 255, 256, 264, 265, 266, 273, 274, 275, 284, 285, 286, 287, 288, 289, 290, 297, 298, 299, 300, 301, 302, 303, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 340, 341, 342, 355, 356, 357, 374, 381, 388, 399, 400, 413, 427, 440, 453, 454, 455, 468, 469, 470, 471, 476, 492, 493, 494, 511 }

C grade: { }

F grade: { 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 276, 277, 278, 279, 280, 281, 282, 283, 291, 292, 293, 294, 295, 296, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 337, 338, 339, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	22	12	11	11	14	11	11
normalized size	1	1.00	2.00	1.09	1.00	1.00	1.27	1.00	1.00
time (sec)	N/A	0.004	0.076	0.047	0.899	0.429	0.174	0.289	0.020

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	24	23	46	18	18
normalized size	1	1.00	0.92	1.08	0.96	0.92	1.84	0.72	0.72
time (sec)	N/A	0.009	0.031	0.045	0.335	0.426	0.223	0.138	0.430

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	22	22	22	37	25	24
normalized size	1	1.00	1.07	0.81	0.81	0.81	1.37	0.93	0.89
time (sec)	N/A	0.010	0.010	0.120	0.327	0.432	0.475	0.131	0.353

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	38	33	36	95	32	50
normalized size	1	1.00	0.72	0.83	0.72	0.78	2.07	0.70	1.09
time (sec)	N/A	0.020	0.038	0.136	0.316	0.421	1.075	0.141	0.470

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	44	32	34	34	60	38	32
normalized size	1	1.00	1.05	0.76	0.81	0.81	1.43	0.90	0.76
time (sec)	N/A	0.013	0.012	0.103	0.319	0.414	1.879	0.138	0.358

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	48	48	47	139	46	43
normalized size	1	1.00	0.67	0.72	0.72	0.70	2.07	0.69	0.64
time (sec)	N/A	0.033	0.040	0.101	0.318	0.426	3.494	0.116	0.578

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	42	44	44	80	50	43
normalized size	1	1.00	1.09	0.78	0.81	0.81	1.48	0.93	0.80
time (sec)	N/A	0.016	0.008	0.100	0.325	0.432	6.079	0.116	0.375

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	55	58	59	56	184	60	90
normalized size	1	1.00	0.62	0.66	0.67	0.64	2.09	0.68	1.02
time (sec)	N/A	0.048	0.051	0.098	0.337	0.448	10.279	0.134	1.505

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	45	84	0	0	0	0	34
normalized size	1	1.00	0.75	1.40	0.00	0.00	0.00	0.00	0.57
time (sec)	N/A	0.027	0.202	0.171	0.000	0.465	0.000	0.000	0.483

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	35	118	0	0	0	0	34
normalized size	1	1.00	0.85	2.88	0.00	0.00	0.00	0.00	0.83
time (sec)	N/A	0.015	0.036	0.059	0.000	0.460	0.000	0.000	0.414

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	72	0	0	0	0	34
normalized size	1	1.00	0.80	1.76	0.00	0.00	0.00	0.00	0.83
time (sec)	N/A	0.015	0.033	0.053	0.000	0.458	0.000	0.000	0.404

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	77	0	0	0	0	15
normalized size	1	1.00	1.11	4.05	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	0.008	0.022	0.056	0.000	0.469	0.000	0.000	0.366

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	57	0	0	0	0	15
normalized size	1	1.00	1.11	3.00	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	0.008	0.024	0.069	0.000	0.425	0.000	0.000	0.387

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	110	0	0	0	0	34
normalized size	1	1.00	0.86	2.97	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.013	0.048	0.066	0.000	0.445	0.000	0.000	0.473

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	33	72	0	0	0	0	34
normalized size	1	1.00	0.80	1.76	0.00	0.00	0.00	0.00	0.83
time (sec)	N/A	0.015	0.047	0.055	0.000	0.429	0.000	0.000	0.585

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	132	0	0	0	0	34
normalized size	1	1.00	0.85	2.20	0.00	0.00	0.00	0.00	0.57
time (sec)	N/A	0.024	0.047	0.059	0.000	0.447	0.000	0.000	0.560

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	55	104	0	0	0	0	42
normalized size	1	1.00	0.79	1.49	0.00	0.00	0.00	0.00	0.60
time (sec)	N/A	0.029	0.113	0.047	0.000	0.453	0.000	0.000	0.488

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	142	0	0	0	0	42
normalized size	1	1.00	0.94	3.02	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.017	0.081	0.044	0.000	0.458	0.000	0.000	0.450

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	88	0	0	0	0	42
normalized size	1	1.00	0.85	1.87	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.016	0.037	0.043	0.000	0.423	0.000	0.000	0.440

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	91	0	0	0	0	18
normalized size	1	1.00	1.14	4.33	0.00	0.00	0.00	0.00	0.86
time (sec)	N/A	0.008	0.015	0.040	0.000	0.462	0.000	0.000	0.368

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	69	0	0	0	0	18
normalized size	1	1.00	1.14	3.29	0.00	0.00	0.00	0.00	0.86
time (sec)	N/A	0.008	0.018	0.033	0.000	0.418	0.000	0.000	0.377

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	132	0	0	0	0	42
normalized size	1	1.00	0.91	3.07	0.00	0.00	0.00	0.00	0.98
time (sec)	N/A	0.015	0.075	0.044	0.000	0.465	0.000	0.000	0.511

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	88	0	0	0	0	42
normalized size	1	1.00	0.91	1.87	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.016	0.102	0.046	0.000	0.450	0.000	0.000	0.613

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	55	160	0	0	0	0	42
normalized size	1	1.00	0.79	2.29	0.00	0.00	0.00	0.00	0.60
time (sec)	N/A	0.027	0.270	0.051	0.000	0.422	0.000	0.000	0.594

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	80	108	0	0	0	0	-1
normalized size	1	1.00	0.78	1.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.151	0.069	0.000	0.434	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	66	152	0	0	0	0	-1
normalized size	1	1.00	0.88	2.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.098	0.056	0.000	0.447	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	62	97	0	0	0	0	-1
normalized size	1	1.00	0.83	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.050	0.054	0.000	0.468	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	98	0	0	0	0	36
normalized size	1	1.00	0.98	2.28	0.00	0.00	0.00	0.00	0.84
time (sec)	N/A	0.018	0.020	0.053	0.000	0.432	0.000	0.000	0.395

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	74	0	0	0	0	36
normalized size	1	1.00	0.98	1.72	0.00	0.00	0.00	0.00	0.84
time (sec)	N/A	0.018	0.026	0.046	0.000	0.424	0.000	0.000	0.485

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	141	0	0	0	0	-1
normalized size	1	1.00	0.74	1.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.039	0.059	0.000	0.431	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	55	105	0	0	0	0	-1
normalized size	1	1.00	0.71	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.064	0.057	0.000	0.465	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	68	168	0	0	0	0	-1
normalized size	1	1.00	0.65	1.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.148	0.066	0.000	0.455	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.058	0.151	0.000	0.453	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.035	0.171	0.000	0.446	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	58	55	0	0	0	0	0	-1
normalized size	1	0.11	0.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.014	0.033	0.090	0.000	0.481	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	58	55	0	0	0	0	0	-1
normalized size	1	0.23	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.017	0.037	0.072	0.000	0.453	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	56	53	0	0	0	0	0	-1
normalized size	1	0.21	0.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.016	0.037	0.064	0.000	0.450	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.038	0.050	0.000	0.443	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	0	0	0	0	0	54
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.86
time (sec)	N/A	0.016	0.040	0.667	0.000	0.473	0.000	0.000	0.783

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.038	0.587	0.000	0.444	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	76	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.074	1.083	0.000	0.484	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	22	24	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	1.47	1.60	0.87
time (sec)	N/A	0.019	0.004	0.020	0.326	0.418	1.375	0.209	0.056

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	22	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	1.47	0.87	0.87
time (sec)	N/A	0.021	0.004	0.013	0.321	0.429	0.686	0.206	0.037

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	37	14	13	13	19	13	28
normalized size	1	1.00	2.47	0.93	0.87	0.87	1.27	0.87	1.87
time (sec)	N/A	0.011	0.011	0.003	0.326	0.407	0.309	0.166	0.455

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	18	14	0	18	16
normalized size	1	1.00	1.00	1.00	1.50	1.17	0.00	1.50	1.33
time (sec)	N/A	0.004	0.007	0.007	0.373	0.448	0.000	0.212	0.506

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	12	12	0	12	20
normalized size	1	1.00	1.00	1.10	1.20	1.20	0.00	1.20	2.00
time (sec)	N/A	0.011	0.006	0.016	0.319	0.402	0.000	1.176	0.515

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	13	0	13	13
normalized size	1	1.00	1.00	0.93	1.13	0.87	0.00	0.87	0.87
time (sec)	N/A	0.019	0.009	0.018	0.316	0.449	0.000	0.222	0.382

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	0	13	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.00	0.87	0.87
time (sec)	N/A	0.019	0.007	0.017	0.312	0.429	0.000	0.219	0.448

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	60	46	53	88	54	45
normalized size	1	1.00	0.77	0.98	0.75	0.87	1.44	0.89	0.74
time (sec)	N/A	0.043	0.145	0.055	0.307	0.452	19.100	0.202	0.398

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	50	36	43	66	54	36
normalized size	1	1.00	0.80	1.09	0.78	0.93	1.43	1.17	0.78
time (sec)	N/A	0.038	0.085	0.051	0.304	0.439	8.515	0.195	0.399

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	40	26	33	44	26	26
normalized size	1	1.00	0.87	1.29	0.84	1.06	1.42	0.84	0.84
time (sec)	N/A	0.035	0.056	0.049	0.314	0.423	3.020	0.395	0.359

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	21	20	13	13
normalized size	1	1.00	1.00	0.93	0.87	1.40	1.33	0.87	0.87
time (sec)	N/A	0.018	0.003	0.005	0.307	0.442	0.710	0.227	0.023

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	23	19	18	31	0	18	14
normalized size	1	1.00	1.64	1.36	1.29	2.21	0.00	1.29	1.00
time (sec)	N/A	0.008	0.006	0.026	0.426	0.419	0.000	0.191	0.375

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	13	29	0	13	13
normalized size	1	1.00	1.00	1.47	0.87	1.93	0.00	0.87	0.87
time (sec)	N/A	0.030	0.006	0.033	0.319	0.418	0.000	0.231	0.381

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	56	42	26	39	0	26	25
normalized size	1	1.00	1.81	1.35	0.84	1.26	0.00	0.84	0.81
time (sec)	N/A	0.034	0.037	0.035	0.342	0.414	0.000	0.420	0.387

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	77	60	36	51	0	36	35
normalized size	1	1.00	1.67	1.30	0.78	1.11	0.00	0.78	0.76
time (sec)	N/A	0.039	0.036	0.034	0.319	0.419	0.000	0.293	0.407

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	98	78	46	61	0	46	45
normalized size	1	1.00	1.61	1.28	0.75	1.00	0.00	0.75	0.74
time (sec)	N/A	0.040	0.032	0.037	0.315	0.444	0.000	0.416	0.405

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	52	64	48	57	189	60	89
normalized size	1	1.00	0.59	0.73	0.55	0.65	2.15	0.68	1.01
time (sec)	N/A	0.066	0.125	0.052	0.317	0.426	15.099	0.539	1.517

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	40	54	37	47	136	46	43
normalized size	1	1.00	0.60	0.81	0.55	0.70	2.03	0.69	0.64
time (sec)	N/A	0.051	0.076	0.054	0.310	0.436	5.055	0.416	0.558

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	23	43	24	36	92	18	50
normalized size	1	1.00	0.50	0.93	0.52	0.78	2.00	0.39	1.09
time (sec)	N/A	0.040	0.030	0.020	0.312	0.431	1.454	0.175	0.460

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	24	23	46	18	18
normalized size	1	1.00	0.92	1.08	0.96	0.92	1.84	0.72	0.72
time (sec)	N/A	0.009	0.009	0.000	0.313	0.414	0.349	0.210	0.002

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	31	34	36	3160	36	27
normalized size	1	1.00	1.00	1.35	1.48	1.57	137.39	1.57	1.17
time (sec)	N/A	0.015	0.009	0.027	0.321	0.449	55.346	0.251	0.457

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	53	46	61	0	48	69
normalized size	1	1.00	1.00	1.56	1.35	1.79	0.00	1.41	2.03
time (sec)	N/A	0.023	0.014	0.028	0.319	0.424	0.000	0.530	1.225

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	74	65	71	0	82	125
normalized size	1	1.00	1.00	1.35	1.18	1.29	0.00	1.49	2.27
time (sec)	N/A	0.045	0.041	0.031	0.337	0.451	0.000	0.255	6.539

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	95	91	84	0	73	177
normalized size	1	1.00	1.00	1.25	1.20	1.11	0.00	0.96	2.33
time (sec)	N/A	0.059	0.059	0.036	0.322	0.435	0.000	0.270	7.325

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	48	34	36	26	44	27	25
normalized size	1	1.00	1.55	1.10	1.16	0.84	1.42	0.87	0.81
time (sec)	N/A	0.033	0.116	0.021	0.319	0.429	14.211	0.197	0.373

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	34	26	26	46	27	26
normalized size	1	1.00	0.87	1.10	0.84	0.84	1.48	0.87	0.84
time (sec)	N/A	0.034	0.086	0.023	0.314	0.411	8.874	0.188	0.378

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	35	34	26	26	44	26	37
normalized size	1	1.00	1.13	1.10	0.84	0.84	1.42	0.84	1.19
time (sec)	N/A	0.033	0.014	0.020	0.336	0.449	4.327	0.202	0.511

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	34	26	26	46	27	26
normalized size	1	1.00	0.87	1.10	0.84	0.84	1.48	0.87	0.84
time (sec)	N/A	0.032	0.054	0.023	0.321	0.440	2.455	0.181	0.344

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	24	20	13	13
normalized size	1	1.00	1.00	0.93	0.87	1.60	1.33	0.87	0.87
time (sec)	N/A	0.018	0.003	0.006	0.315	0.454	1.319	0.178	0.368

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	27	25	25	0	29	25
normalized size	1	1.00	0.89	0.96	0.89	0.89	0.00	1.04	0.89
time (sec)	N/A	0.021	0.016	0.029	0.311	0.439	0.000	0.508	0.429

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	40	19	22	0	23	20
normalized size	1	1.00	1.00	1.90	0.90	1.05	0.00	1.10	0.95
time (sec)	N/A	0.021	0.020	0.029	0.320	0.426	0.000	0.435	0.486

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	31	34	0	42	27
normalized size	1	1.00	0.93	0.96	1.15	1.26	0.00	1.56	1.00
time (sec)	N/A	0.011	0.023	0.032	0.318	0.455	0.000	0.331	0.383

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	60	25	25	0	25	23
normalized size	1	1.00	1.00	2.22	0.93	0.93	0.00	0.93	0.85
time (sec)	N/A	0.020	0.023	0.033	0.399	0.421	0.000	0.530	0.448

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	39	25	0	25	13
normalized size	1	1.00	1.00	1.47	2.60	1.67	0.00	1.67	0.87
time (sec)	N/A	0.028	0.005	0.035	0.316	0.415	0.000	0.240	0.383

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	78	25	25	0	25	25
normalized size	1	1.00	1.00	2.52	0.81	0.81	0.00	0.81	0.81
time (sec)	N/A	0.032	0.044	0.033	0.330	0.418	0.000	0.244	0.540

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	28	42	49	25	0	25	25
normalized size	1	1.00	0.90	1.35	1.58	0.81	0.00	0.81	0.81
time (sec)	N/A	0.033	0.032	0.036	0.337	0.402	0.000	0.273	0.398

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	96	25	25	0	25	25
normalized size	1	1.00	1.00	3.10	0.81	0.81	0.00	0.81	0.81
time (sec)	N/A	0.032	0.026	0.038	0.324	0.428	0.000	0.234	0.631

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	28	60	59	25	0	25	35
normalized size	1	1.00	0.90	1.94	1.90	0.81	0.00	0.81	1.13
time (sec)	N/A	0.032	0.033	0.037	0.316	0.397	0.000	0.343	0.418

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	78	46	63	88	82	45
normalized size	1	1.00	0.77	1.28	0.75	1.03	1.44	1.34	0.74
time (sec)	N/A	0.040	0.185	0.026	0.322	0.470	49.838	0.239	0.379

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	68	36	53	66	68	36
normalized size	1	1.00	0.80	1.48	0.78	1.15	1.43	1.48	0.78
time (sec)	N/A	0.036	0.108	0.024	0.368	0.417	20.520	0.221	0.383

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	58	26	41	44	26	26
normalized size	1	1.00	0.87	1.87	0.84	1.32	1.42	0.84	0.84
time (sec)	N/A	0.032	0.069	0.024	0.316	0.421	7.363	0.242	0.032

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	31	20	13	13
normalized size	1	1.00	1.00	0.93	0.87	2.07	1.33	0.87	0.87
time (sec)	N/A	0.018	0.003	0.014	0.355	0.429	2.247	0.223	0.359

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	31	54	41	42	0	41	38
normalized size	1	1.00	0.78	1.35	1.02	1.05	0.00	1.02	0.95
time (sec)	N/A	0.039	0.101	0.029	0.430	0.427	0.000	0.362	0.479

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	38	28	29	46	0	29	24
normalized size	1	1.00	1.36	1.00	1.04	1.64	0.00	1.04	0.86
time (sec)	N/A	0.016	0.008	0.033	0.422	0.454	0.000	0.251	0.401

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	13	39	0	13	13
normalized size	1	1.00	1.00	1.47	0.87	2.60	0.00	0.87	0.87
time (sec)	N/A	0.028	0.007	0.035	0.320	0.404	0.000	0.203	0.411

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	77	42	26	49	0	26	25
normalized size	1	1.00	2.48	1.35	0.84	1.58	0.00	0.84	0.81
time (sec)	N/A	0.032	0.030	0.037	0.322	0.420	0.000	0.251	0.398

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	98	60	36	61	0	36	35
normalized size	1	1.00	2.13	1.30	0.78	1.33	0.00	0.78	0.76
time (sec)	N/A	0.036	0.034	0.037	0.454	0.441	0.000	0.253	0.428

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	62	82	48	66	231	74	109
normalized size	1	1.00	0.56	0.74	0.43	0.59	2.08	0.67	0.98
time (sec)	N/A	0.097	0.180	0.025	0.692	0.443	32.101	1.869	1.936

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	33	72	33	56	189	32	90
normalized size	1	1.00	0.37	0.80	0.37	0.62	2.10	0.36	1.00
time (sec)	N/A	0.083	0.044	0.025	0.401	0.549	13.087	1.130	1.504

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	40	61	37	46	136	46	43
normalized size	1	1.00	0.58	0.88	0.54	0.67	1.97	0.67	0.62
time (sec)	N/A	0.068	0.054	0.021	0.456	0.412	4.565	0.323	0.524

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	38	33	36	95	32	50
normalized size	1	1.00	0.72	0.83	0.72	0.78	2.07	0.70	1.09
time (sec)	N/A	0.020	0.009	0.000	0.330	0.436	1.240	0.177	0.424

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	44	46	48	0	48	53
normalized size	1	1.00	1.00	1.16	1.21	1.26	0.00	1.26	1.39
time (sec)	N/A	0.027	0.013	0.027	0.581	0.461	0.000	0.562	0.581

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	40	66	56	74	0	58	98
normalized size	1	1.00	0.82	1.35	1.14	1.51	0.00	1.18	2.00
time (sec)	N/A	0.029	0.090	0.035	0.403	0.452	0.000	0.862	3.969

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	45	87	71	74	0	63	126
normalized size	1	1.00	0.82	1.58	1.29	1.35	0.00	1.15	2.29
time (sec)	N/A	0.042	0.115	0.036	0.511	0.466	0.000	0.392	6.585

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	99	108	91	84	0	73	177
normalized size	1	1.00	1.27	1.38	1.17	1.08	0.00	0.94	2.27
time (sec)	N/A	0.075	0.026	0.039	0.532	0.441	0.000	0.535	7.380

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	64	129	111	94	0	107	229
normalized size	1	1.00	0.65	1.30	1.12	0.95	0.00	1.08	2.31
time (sec)	N/A	0.089	0.290	0.041	0.400	0.471	0.000	1.020	7.442

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	68	52	46	36	65	85	35
normalized size	1	1.00	1.48	1.13	1.00	0.78	1.41	1.85	0.76
time (sec)	N/A	0.041	0.332	0.026	0.386	0.446	71.135	0.257	0.409

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	52	36	36	68	82	36
normalized size	1	1.00	0.80	1.13	0.78	0.78	1.48	1.78	0.78
time (sec)	N/A	0.036	0.265	0.026	0.357	0.482	47.179	0.248	0.409

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	50	52	36	36	65	43	36
normalized size	1	1.00	1.09	1.13	0.78	0.78	1.41	0.93	0.78
time (sec)	N/A	0.040	0.027	0.022	0.306	0.458	30.080	0.223	0.477

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	52	36	36	68	68	36
normalized size	1	1.00	0.80	1.13	0.78	0.78	1.48	1.48	0.78
time (sec)	N/A	0.036	0.116	0.023	0.355	0.432	20.093	0.247	0.376

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	48	52	26	36	65	26	26
normalized size	1	1.00	1.55	1.68	0.84	1.16	2.10	0.84	0.84
time (sec)	N/A	0.032	0.091	0.022	0.376	0.430	12.763	0.520	0.383

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	52	36	36	68	54	36
normalized size	1	1.00	0.80	1.13	0.78	0.78	1.48	1.17	0.78
time (sec)	N/A	0.035	0.079	0.024	0.440	0.438	7.502	0.210	0.370

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	34	20	13	13
normalized size	1	1.00	1.00	0.93	0.87	2.27	1.33	0.87	0.87
time (sec)	N/A	0.017	0.004	0.004	0.331	0.426	4.045	0.175	0.047

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	35	40	37	35	0	226	53
normalized size	1	1.00	0.88	1.00	0.92	0.88	0.00	5.65	1.32
time (sec)	N/A	0.026	0.029	0.029	0.366	0.526	0.000	0.332	0.537

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	39	50	32	33	0	99	31
normalized size	1	1.00	1.05	1.35	0.86	0.89	0.00	2.68	0.84
time (sec)	N/A	0.034	0.025	0.028	0.322	0.491	0.000	0.314	0.495

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	33	60	41	54	0	182	37
normalized size	1	1.00	0.77	1.40	0.95	1.26	0.00	4.23	0.86
time (sec)	N/A	0.038	0.035	0.032	0.316	0.495	0.000	0.298	0.404

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	70	35	35	0	100	35
normalized size	1	1.00	1.00	1.84	0.92	0.92	0.00	2.63	0.92
time (sec)	N/A	0.025	0.022	0.032	0.321	0.424	0.000	0.242	0.535

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	40	54	44	0	226	38
normalized size	1	1.00	0.86	0.93	1.26	1.02	0.00	5.26	0.88
time (sec)	N/A	0.021	0.041	0.035	0.671	0.441	0.000	0.249	0.397

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	88	35	35	0	72	35
normalized size	1	1.00	1.00	2.15	0.85	0.85	0.00	1.76	0.85
time (sec)	N/A	0.023	0.026	0.034	0.302	0.417	0.000	0.233	0.542

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	59	35	0	48	13
normalized size	1	1.00	1.00	1.47	3.93	2.33	0.00	3.20	0.87
time (sec)	N/A	0.027	0.008	0.036	0.370	0.424	0.000	0.367	0.422

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	106	35	35	0	116	35
normalized size	1	1.00	1.00	2.30	0.76	0.76	0.00	2.52	0.76
time (sec)	N/A	0.036	0.029	0.037	0.753	0.431	0.000	0.259	0.594

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	38	42	69	35	0	93	25
normalized size	1	1.00	1.23	1.35	2.23	1.13	0.00	3.00	0.81
time (sec)	N/A	0.031	0.051	0.048	0.355	0.423	0.000	0.307	0.416

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	124	35	35	0	160	35
normalized size	1	1.00	1.00	2.70	0.76	0.76	0.00	3.48	0.76
time (sec)	N/A	0.035	0.031	0.034	0.298	0.415	0.000	0.222	0.767

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	60	79	35	0	139	35
normalized size	1	1.00	0.83	1.30	1.72	0.76	0.00	3.02	0.76
time (sec)	N/A	0.039	0.052	0.040	0.331	0.421	0.000	0.320	0.442

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	142	35	35	0	204	35
normalized size	1	1.00	1.00	3.09	0.76	0.76	0.00	4.43	0.76
time (sec)	N/A	0.035	0.030	0.041	0.452	0.447	0.000	0.297	1.022

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	78	89	35	0	183	45
normalized size	1	1.00	0.83	1.70	1.93	0.76	0.00	3.98	0.98
time (sec)	N/A	0.040	0.126	0.043	0.386	0.426	0.000	0.379	0.420

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	52	79	66	84	0	68	147
normalized size	1	1.00	0.79	1.20	1.00	1.27	0.00	1.03	2.23
time (sec)	N/A	0.047	0.168	0.035	0.480	0.452	0.000	0.342	7.242

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	96	45	45	0	144	50
normalized size	1	1.00	1.00	1.92	0.90	0.90	0.00	2.88	1.00
time (sec)	N/A	0.028	0.029	0.037	0.401	0.437	0.000	0.275	0.540

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	75	58	56	60	1085	145	88
normalized size	1	1.00	1.42	1.09	1.06	1.13	20.47	2.74	1.66
time (sec)	N/A	0.029	0.027	0.033	0.317	0.434	8.156	0.269	5.370

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	35	35	1086	170	66
normalized size	1	1.00	1.00	0.98	0.88	0.88	27.15	4.25	1.65
time (sec)	N/A	0.027	0.013	0.026	0.401	0.451	6.543	0.268	0.504

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	60	45	46	50	473	101	62
normalized size	1	1.00	1.58	1.18	1.21	1.32	12.45	2.66	1.63
time (sec)	N/A	0.026	0.024	0.024	0.436	0.465	2.999	0.185	1.685

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	25	25	369	25	35
normalized size	1	1.00	1.00	0.96	0.93	0.93	13.67	0.93	1.30
time (sec)	N/A	0.021	0.012	0.028	0.540	0.439	2.345	0.568	0.414

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	42	32	34	38	92	57	35
normalized size	1	1.00	1.83	1.39	1.48	1.65	4.00	2.48	1.52
time (sec)	N/A	0.015	0.015	0.024	0.360	0.442	1.308	0.241	0.462

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	19	12	11	13	17	12	26
normalized size	1	1.00	1.73	1.09	1.00	1.18	1.55	1.09	2.36
time (sec)	N/A	0.004	0.008	0.004	0.298	0.423	0.606	0.208	0.406

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	31	12	28	30	0	56	11
normalized size	1	1.00	2.82	1.09	2.55	2.73	0.00	5.09	1.00
time (sec)	N/A	0.010	0.021	0.037	0.379	0.449	0.000	0.243	0.386

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	42	34	36	52	0	55	23
normalized size	1	1.00	1.83	1.48	1.57	2.26	0.00	2.39	1.00
time (sec)	N/A	0.022	0.025	0.038	0.308	0.503	0.000	0.268	0.084

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	36	26	40	56	0	124	35
normalized size	1	1.00	1.33	0.96	1.48	2.07	0.00	4.59	1.30
time (sec)	N/A	0.020	0.029	0.040	0.477	0.518	0.000	0.230	0.096

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	57	47	50	67	0	101	33
normalized size	1	1.00	1.50	1.24	1.32	1.76	0.00	2.66	0.87
time (sec)	N/A	0.026	0.021	0.041	0.310	0.454	0.000	0.212	0.394

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	46	39	65	67	0	170	46
normalized size	1	1.00	1.18	1.00	1.67	1.72	0.00	4.36	1.18
time (sec)	N/A	0.026	0.084	0.040	0.553	0.464	0.000	0.271	0.409

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	72	60	60	77	0	145	45
normalized size	1	1.00	1.36	1.13	1.13	1.45	0.00	2.74	0.85
time (sec)	N/A	0.028	0.021	0.043	0.459	0.504	0.000	0.661	0.403

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	56	52	85	77	0	214	56
normalized size	1	1.00	0.98	0.91	1.49	1.35	0.00	3.75	0.98
time (sec)	N/A	0.031	0.137	0.046	0.400	0.492	0.000	0.342	0.400

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	62	42	43	82	42	43
normalized size	1	1.00	1.00	1.24	0.84	0.86	1.64	0.84	0.86
time (sec)	N/A	0.038	0.023	0.021	0.384	0.420	8.667	0.256	0.475

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	41	66	63	52	119	55	47
normalized size	1	1.00	0.67	1.08	1.03	0.85	1.95	0.90	0.77
time (sec)	N/A	0.046	0.129	0.020	0.585	0.448	5.336	0.200	0.589

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	52	32	33	61	32	35
normalized size	1	1.00	1.00	1.37	0.84	0.87	1.61	0.84	0.92
time (sec)	N/A	0.034	0.016	0.021	0.369	0.414	3.249	0.249	0.448

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	31	56	43	40	75	43	43
normalized size	1	1.00	0.78	1.40	1.08	1.00	1.88	1.08	1.08
time (sec)	N/A	0.037	0.124	0.023	0.849	0.429	1.771	0.171	0.568

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	42	20	22	39	20	23
normalized size	1	1.00	1.00	1.83	0.87	0.96	1.70	0.87	1.00
time (sec)	N/A	0.020	0.011	0.019	0.390	0.423	1.399	0.227	0.446

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	29	21	18	29	29	35	15
normalized size	1	1.00	1.93	1.40	1.20	1.93	1.93	2.33	1.00
time (sec)	N/A	0.008	0.013	0.021	0.634	0.436	1.082	0.184	0.411

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	13	13	20	13	13
normalized size	1	1.00	1.00	1.27	1.18	1.18	1.82	1.18	1.18
time (sec)	N/A	0.009	0.007	0.005	0.440	0.446	1.022	0.588	0.419

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	27	33	36	50	0	38	22
normalized size	1	1.00	1.17	1.43	1.57	2.17	0.00	1.65	0.96
time (sec)	N/A	0.022	0.014	0.029	0.412	0.464	0.000	0.344	0.021

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	13	31	22	33	0	16	14
normalized size	1	1.00	0.59	1.41	1.00	1.50	0.00	0.73	0.64
time (sec)	N/A	0.030	0.009	0.064	0.492	0.411	0.000	0.700	0.393

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	27	55	61	85	0	63	48
normalized size	1	1.00	0.55	1.12	1.24	1.73	0.00	1.29	0.98
time (sec)	N/A	0.043	0.012	0.038	0.418	0.439	0.000	0.349	0.429

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	46	50	32	43	0	32	33
normalized size	1	1.00	1.21	1.32	0.84	1.13	0.00	0.84	0.87
time (sec)	N/A	0.037	0.031	0.038	0.472	0.414	0.000	0.192	0.412

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	27	76	79	95	0	73	67
normalized size	1	1.00	0.39	1.09	1.13	1.36	0.00	1.04	0.96
time (sec)	N/A	0.047	0.013	0.041	0.441	0.450	0.000	0.688	0.094

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	45	74	45	71	1484	231	85
normalized size	1	1.00	0.78	1.28	0.78	1.22	25.59	3.98	1.47
time (sec)	N/A	0.044	0.095	0.021	0.525	0.472	12.973	0.255	0.694

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	103	81	66	93	719	163	129
normalized size	1	1.00	1.56	1.23	1.00	1.41	10.89	2.47	1.95
time (sec)	N/A	0.045	0.035	0.023	0.469	0.449	11.079	0.600	1.346

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	35	61	35	61	614	187	62
normalized size	1	1.00	0.81	1.42	0.81	1.42	14.28	4.35	1.44
time (sec)	N/A	0.039	0.058	0.022	0.536	0.445	4.831	0.273	0.462

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	86	68	56	83	241	140	77
normalized size	1	1.00	1.76	1.39	1.14	1.69	4.92	2.86	1.57
time (sec)	N/A	0.029	0.025	0.025	0.327	0.453	3.925	0.313	0.539

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	34	27	23	41	42	36	36
normalized size	1	1.00	1.21	0.96	0.82	1.46	1.50	1.29	1.29
time (sec)	N/A	0.013	0.076	0.023	0.389	0.456	1.497	0.177	0.429

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	75	55	46	72	58	93	48
normalized size	1	1.00	2.21	1.62	1.35	2.12	1.71	2.74	1.41
time (sec)	N/A	0.023	0.025	0.020	0.581	0.443	1.607	0.224	0.449

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	18	24	13	13
normalized size	1	1.00	1.00	0.93	0.87	1.20	1.60	0.87	0.87
time (sec)	N/A	0.018	0.011	0.006	0.633	1.201	1.460	0.196	0.392

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	34	26	36	65	0	119	34
normalized size	1	1.00	1.26	0.96	1.33	2.41	0.00	4.41	1.26
time (sec)	N/A	0.022	0.036	0.032	0.352	0.437	0.000	0.331	0.050

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	143	57	61	96	0	140	49
normalized size	1	1.00	2.92	1.16	1.24	1.96	0.00	2.86	1.00
time (sec)	N/A	0.043	0.225	0.033	0.294	0.454	0.000	0.395	0.034

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	61	48	64	102	0	188	39
normalized size	1	1.00	1.42	1.12	1.49	2.37	0.00	4.37	0.91
time (sec)	N/A	0.038	0.012	0.038	0.304	0.434	0.000	0.247	0.379

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	205	78	73	112	0	163	60
normalized size	1	1.00	3.11	1.18	1.11	1.70	0.00	2.47	0.91
time (sec)	N/A	0.043	0.390	0.039	0.295	0.442	0.000	0.217	0.458

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	69	82	112	0	232	74
normalized size	1	1.00	0.97	1.19	1.41	1.93	0.00	4.00	1.28
time (sec)	N/A	0.043	0.206	0.038	0.298	0.488	0.000	0.253	0.438

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	90	56	68	105	56	55
normalized size	1	1.00	1.00	1.32	0.82	1.00	1.54	0.82	0.81
time (sec)	N/A	0.044	0.030	0.060	0.359	0.434	22.233	0.216	0.525

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	53	94	75	89	141	68	56
normalized size	1	1.00	0.66	1.18	0.94	1.11	1.76	0.85	0.70
time (sec)	N/A	0.048	0.264	0.056	0.458	0.439	14.834	0.171	1.583

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	80	44	56	82	41	45
normalized size	1	1.00	1.00	1.51	0.83	1.06	1.55	0.77	0.85
time (sec)	N/A	0.040	0.023	0.021	0.323	0.419	9.588	0.212	0.479

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	43	84	55	79	97	55	46
normalized size	1	1.00	0.75	1.47	0.96	1.39	1.70	0.96	0.81
time (sec)	N/A	0.041	0.167	0.022	0.430	0.441	6.722	0.735	0.725

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	68	35	48	63	35	32
normalized size	1	1.00	1.00	1.84	0.95	1.30	1.70	0.95	0.86
time (sec)	N/A	0.025	0.017	0.023	0.424	0.424	3.762	0.209	0.450

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	33	26	34	69	48	62	24
normalized size	1	1.00	1.22	0.96	1.26	2.56	1.78	2.30	0.89
time (sec)	N/A	0.017	0.009	0.023	0.430	0.425	2.537	0.227	0.451

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	60	25	38	42	25	22
normalized size	1	1.00	1.00	2.31	0.96	1.46	1.62	0.96	0.85
time (sec)	N/A	0.021	0.012	0.024	0.300	0.416	2.263	0.180	0.434

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	13	34	71	13	13
normalized size	1	1.00	1.00	1.47	0.87	2.27	4.73	0.87	0.87
time (sec)	N/A	0.029	0.006	0.022	0.435	0.447	2.614	0.228	0.383

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	26	24	13	13
normalized size	1	1.00	1.00	0.93	0.87	1.73	1.60	0.87	0.87
time (sec)	N/A	0.018	0.009	0.005	0.319	0.422	1.573	0.571	0.426

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	31	46	50	94	0	52	32
normalized size	1	1.00	0.82	1.21	1.32	2.47	0.00	1.37	0.84
time (sec)	N/A	0.026	0.013	0.033	0.356	0.423	0.000	0.233	0.025

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	45	50	35	54	0	35	36
normalized size	1	1.00	1.22	1.35	0.95	1.46	0.00	0.95	0.97
time (sec)	N/A	0.035	0.031	0.035	0.532	0.442	0.000	0.219	0.418

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	31	76	73	130	0	72	61
normalized size	1	1.00	0.47	1.15	1.11	1.97	0.00	1.09	0.92
time (sec)	N/A	0.042	0.014	0.040	0.312	0.437	0.000	0.536	0.382

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	68	44	66	0	31	45
normalized size	1	1.00	0.81	1.28	0.83	1.25	0.00	0.58	0.85
time (sec)	N/A	0.038	0.010	0.046	0.321	0.498	0.000	0.263	0.461

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	31	97	91	140	0	85	79
normalized size	1	1.00	0.35	1.09	1.02	1.57	0.00	0.96	0.89
time (sec)	N/A	0.049	0.014	0.049	0.351	0.481	0.000	0.604	0.448

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	55	107	56	100	1664	277	92
normalized size	1	1.00	0.80	1.55	0.81	1.45	24.12	4.01	1.33
time (sec)	N/A	0.048	0.106	0.026	0.448	0.538	27.693	0.436	1.420

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	141	115	89	132	869	209	157
normalized size	1	1.00	1.58	1.29	1.00	1.48	9.76	2.35	1.76
time (sec)	N/A	0.050	0.041	0.023	0.347	0.474	18.450	0.242	0.549

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	95	49	90	733	232	74
normalized size	1	1.00	0.81	1.64	0.84	1.55	12.64	4.00	1.28
time (sec)	N/A	0.043	0.150	0.022	0.481	0.480	11.568	0.289	0.642

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	123	102	79	122	330	164	105
normalized size	1	1.00	1.76	1.46	1.13	1.74	4.71	2.34	1.50
time (sec)	N/A	0.036	0.029	0.023	0.438	0.468	6.829	0.272	0.581

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	46	39	38	70	61	164	52
normalized size	1	1.00	1.10	0.93	0.90	1.67	1.45	3.90	1.24
time (sec)	N/A	0.021	0.100	0.022	0.346	0.431	3.045	0.249	0.422

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	113	89	71	112	92	139	78
normalized size	1	1.00	2.05	1.62	1.29	2.04	1.67	2.53	1.42
time (sec)	N/A	0.042	0.029	0.024	0.352	0.458	4.161	0.594	0.488

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	22	25	39	44	25	25
normalized size	1	1.00	1.00	1.47	1.67	2.60	2.93	1.67	1.67
time (sec)	N/A	0.028	0.005	0.022	0.780	0.439	2.898	0.277	0.394

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	113	76	65	111	58	98	48
normalized size	1	1.00	2.05	1.38	1.18	2.02	1.05	1.78	0.87
time (sec)	N/A	0.045	0.033	0.022	0.341	0.451	3.952	0.310	0.466

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	27	24	13	23
normalized size	1	1.00	1.00	0.93	0.87	1.80	1.60	0.87	1.53
time (sec)	N/A	0.018	0.009	0.006	0.375	0.420	2.679	0.303	0.409

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	44	39	51	105	0	165	79
normalized size	1	1.00	1.10	0.98	1.28	2.62	0.00	4.12	1.98
time (sec)	N/A	0.027	0.109	0.032	0.432	0.438	0.000	0.330	0.442

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	129	78	79	132	0	163	66
normalized size	1	1.00	1.84	1.11	1.13	1.89	0.00	2.33	0.94
time (sec)	N/A	0.043	3.988	0.033	0.342	0.464	0.000	0.225	0.390

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	69	74	138	0	232	82
normalized size	1	1.00	0.93	1.19	1.28	2.38	0.00	4.00	1.41
time (sec)	N/A	0.040	0.302	0.048	0.501	0.449	0.000	0.628	0.469

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	268	99	91	148	0	209	78
normalized size	1	1.00	3.01	1.11	1.02	1.66	0.00	2.35	0.88
time (sec)	N/A	0.046	0.437	0.051	0.367	0.454	0.000	0.376	0.071

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	91	90	92	148	0	278	64
normalized size	1	1.00	1.32	1.30	1.33	2.14	0.00	4.03	0.93
time (sec)	N/A	0.046	0.014	0.051	0.524	0.415	0.000	0.381	0.415

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	27	22	14	33	29	14	19
normalized size	1	1.00	1.59	1.29	0.82	1.94	1.71	0.82	1.12
time (sec)	N/A	0.026	0.021	0.017	0.312	0.414	0.070	0.228	0.084

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	14	30	15	14	13
normalized size	1	1.00	1.00	1.29	0.82	1.76	0.88	0.82	0.76
time (sec)	N/A	0.026	0.008	0.018	0.306	0.426	0.113	0.201	0.439

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	24	34	0	18
normalized size	1	1.00	1.00	0.86	0.82	1.09	1.55	0.00	0.82
time (sec)	N/A	0.025	0.035	0.019	0.538	0.434	52.072	0.000	0.128

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	21	34	18	18
normalized size	1	1.00	1.00	0.86	0.82	0.95	1.55	0.82	0.82
time (sec)	N/A	0.022	0.018	0.014	0.457	0.439	1.667	1.339	0.423

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	32	18	18
normalized size	1	1.00	1.00	0.95	0.90	0.90	1.60	0.90	0.90
time (sec)	N/A	0.023	0.013	0.016	0.402	0.419	1.547	0.189	0.525

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	26	31	18	37
normalized size	1	1.00	1.00	0.95	0.90	1.30	1.55	0.90	1.85
time (sec)	N/A	0.026	0.022	0.013	0.526	0.416	5.956	0.838	0.183

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	32	26	53
normalized size	1	1.00	1.00	0.86	0.82	1.18	1.45	1.18	2.41
time (sec)	N/A	0.027	0.025	0.015	0.535	0.411	56.235	0.814	0.807

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	0	26	65
normalized size	1	1.00	1.00	0.86	0.82	1.18	0.00	1.18	2.95
time (sec)	N/A	0.026	0.039	0.014	0.471	0.428	0.000	0.868	6.709

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	0	26	65
normalized size	1	1.00	1.00	0.86	0.82	1.18	0.00	1.18	2.95
time (sec)	N/A	0.027	0.059	0.014	0.303	0.441	0.000	0.989	4.036

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	60	249	0	0	0	0	-1
normalized size	1	1.00	0.48	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.130	0.137	0.000	0.464	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	60	236	0	0	0	0	-1
normalized size	1	1.00	0.48	1.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.128	0.080	0.000	0.458	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	57	223	0	0	0	0	-1
normalized size	1	1.00	0.58	2.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.049	0.086	0.000	0.448	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	57	208	0	0	0	0	-1
normalized size	1	1.00	0.58	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.064	0.079	0.000	0.466	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	194	0	0	0	0	-1
normalized size	1	1.00	0.84	2.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.086	0.085	0.000	0.450	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	188	0	0	0	0	-1
normalized size	1	1.00	0.84	2.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.100	0.072	0.000	0.432	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	60	168	0	0	0	0	-1
normalized size	1	1.00	0.88	2.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.084	0.093	0.000	0.432	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	242	0	0	0	0	-1
normalized size	1	1.00	0.83	3.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.079	0.104	0.000	0.433	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	59	365	0	0	0	0	-1
normalized size	1	1.00	0.59	3.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.064	0.128	0.000	0.458	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	59	396	0	0	0	0	-1
normalized size	1	1.00	0.59	3.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.062	0.099	0.000	0.428	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	57	63	36	34	65	37	-1
normalized size	1	1.00	1.27	1.40	0.80	0.76	1.44	0.82	-0.02
time (sec)	N/A	0.042	0.305	0.062	0.312	0.429	12.456	21.876	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	57	92	36	28	63	40	-1
normalized size	1	1.00	1.33	2.14	0.84	0.65	1.47	0.93	-0.02
time (sec)	N/A	0.046	0.175	0.051	0.413	0.444	6.172	1.026	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	70	35	36	61	41	-1
normalized size	1	1.00	1.07	1.63	0.81	0.84	1.42	0.95	-0.02
time (sec)	N/A	0.051	0.074	0.119	0.385	0.417	6.510	1.321	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	85	34	38	63	47	66
normalized size	1	1.00	1.12	1.98	0.79	0.88	1.47	1.09	1.53
time (sec)	N/A	0.052	0.095	0.148	0.406	0.436	55.338	1.304	1.064

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	70	98	37	38	0	51	93
normalized size	1	1.00	1.63	2.28	0.86	0.88	0.00	1.19	2.16
time (sec)	N/A	0.051	0.248	0.244	0.456	0.426	0.000	1.361	3.666

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	70	111	37	38	0	51	93
normalized size	1	1.00	1.56	2.47	0.82	0.84	0.00	1.13	2.07
time (sec)	N/A	0.050	0.271	0.250	0.302	0.409	0.000	1.577	3.780

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	94	124	37	38	0	51	279
normalized size	1	1.00	2.09	2.76	0.82	0.84	0.00	1.13	6.20
time (sec)	N/A	0.051	0.528	0.291	0.544	0.433	0.000	1.398	5.330

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	57	275	0	0	0	0	-1
normalized size	1	1.00	0.37	1.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.126	0.119	0.000	0.474	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	57	262	0	0	0	0	-1
normalized size	1	1.00	0.37	1.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.090	0.107	0.000	0.479	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	65	249	0	0	0	0	-1
normalized size	1	1.00	0.51	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.075	0.109	0.000	0.436	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	65	255	0	0	0	0	-1
normalized size	1	1.00	0.51	1.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.108	0.108	0.000	0.426	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	58	221	0	0	0	0	-1
normalized size	1	1.00	0.59	2.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.061	0.102	0.000	0.436	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	58	208	0	0	0	0	-1
normalized size	1	1.00	0.59	2.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.075	0.100	0.000	0.427	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	60	213	0	0	0	0	-1
normalized size	1	1.00	0.60	2.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.066	0.110	0.000	0.420	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	60	286	0	0	0	0	-1
normalized size	1	1.00	0.59	2.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.066	0.109	0.000	0.436	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	65	366	0	0	0	0	-1
normalized size	1	1.00	0.64	3.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.058	0.159	0.000	0.446	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	65	398	0	0	0	0	-1
normalized size	1	1.00	0.64	3.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.064	0.116	0.000	0.411	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	111	103	36	44	0	0	35
normalized size	1	1.00	2.13	1.98	0.69	0.85	0.00	0.00	0.67
time (sec)	N/A	0.035	0.261	0.129	0.311	0.446	0.000	0.000	0.636

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	83	318	98	313	0	0	-1
normalized size	1	1.00	0.83	3.18	0.98	3.13	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.187	0.200	0.434	0.585	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	80	280	98	299	0	0	-1
normalized size	1	1.00	0.81	2.83	0.99	3.02	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.179	0.176	0.433	0.625	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	244	83	281	0	0	-1
normalized size	1	1.00	0.87	3.13	1.06	3.60	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.104	0.181	0.602	0.595	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	65	204	83	259	0	0	-1
normalized size	1	1.00	0.84	2.65	1.08	3.36	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.061	0.171	0.438	0.579	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	179	67	238	0	106	-1
normalized size	1	1.00	0.88	3.09	1.16	4.10	0.00	1.83	-0.02
time (sec)	N/A	0.051	0.036	0.172	0.442	0.514	0.000	1.154	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	50	177	68	246	0	105	-1
normalized size	1	1.00	0.85	3.00	1.15	4.17	0.00	1.78	-0.02
time (sec)	N/A	0.051	0.037	0.198	0.415	0.522	0.000	1.585	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	36	422	79	309	0	156	-1
normalized size	1	1.00	0.46	5.41	1.01	3.96	0.00	2.00	-0.01
time (sec)	N/A	0.067	0.052	0.327	0.553	0.513	0.000	1.119	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	38	624	80	318	0	203	-1
normalized size	1	1.00	0.47	7.70	0.99	3.93	0.00	2.51	-0.01
time (sec)	N/A	0.064	0.048	0.308	0.469	0.514	0.000	1.291	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	38	882	100	342	0	341	-1
normalized size	1	1.00	0.38	8.82	1.00	3.42	0.00	3.41	-0.01
time (sec)	N/A	0.078	0.057	0.320	0.468	0.504	0.000	1.459	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	38	1086	102	342	0	436	-1
normalized size	1	1.00	0.37	10.54	0.99	3.32	0.00	4.23	-0.01
time (sec)	N/A	0.075	0.066	0.372	0.585	0.522	0.000	1.384	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	89	242	0	0	0	0	-1
normalized size	1	1.00	0.72	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.356	0.276	0.000	0.450	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	74	229	0	0	0	0	-1
normalized size	1	1.00	0.77	2.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.236	0.266	0.000	0.491	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	73	216	0	0	0	0	-1
normalized size	1	1.00	0.76	2.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.220	0.245	0.000	0.460	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	58	203	0	0	0	0	-1
normalized size	1	1.00	0.88	3.08	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.063	0.135	0.252	0.000	0.440	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	56	190	0	0	0	0	-1
normalized size	1	1.00	0.85	2.88	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.094	0.264	0.000	0.495	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	203	0	0	0	0	-1
normalized size	1	1.00	0.86	3.12	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.078	0.362	0.000	0.646	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	47	188	0	0	0	0	-1
normalized size	1	1.00	0.73	2.94	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.078	0.261	0.000	0.895	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	65	209	0	0	0	0	-1
normalized size	1	1.00	0.69	2.22	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.168	0.328	0.000	0.566	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	62	190	0	0	0	0	-1
normalized size	1	1.00	0.63	1.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.137	0.377	0.000	0.447	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	82	408	0	0	0	0	-1
normalized size	1	1.00	0.65	3.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.160	0.389	0.000	0.447	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	137	433	133	419	0	0	-1
normalized size	1	1.00	1.01	3.21	0.99	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.089	2.173	0.296	0.582	0.680	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	78	394	118	405	0	0	-1
normalized size	1	1.00	0.69	3.49	1.04	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.635	0.310	0.710	0.652	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	118	327	118	393	0	0	-1
normalized size	1	1.00	1.04	2.89	1.04	3.48	0.00	0.00	-0.01
time (sec)	N/A	0.080	1.159	0.285	2.311	0.623	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	65	300	103	380	0	0	-1
normalized size	1	1.00	0.71	3.30	1.13	4.18	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.286	0.408	0.699	0.529	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	76	294	103	347	0	0	-1
normalized size	1	1.00	0.84	3.23	1.13	3.81	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.178	0.278	0.626	0.518	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	62	289	102	340	0	178	-1
normalized size	1	1.00	0.67	3.11	1.10	3.66	0.00	1.91	-0.01
time (sec)	N/A	0.066	0.252	0.279	0.603	0.496	0.000	1.552	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	69	297	103	334	0	181	-1
normalized size	1	1.00	0.74	3.19	1.11	3.59	0.00	1.95	-0.01
time (sec)	N/A	0.065	0.216	0.283	0.505	0.517	0.000	1.318	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	91	705	117	406	0	361	-1
normalized size	1	1.00	0.79	6.13	1.02	3.53	0.00	3.14	-0.01
time (sec)	N/A	0.083	0.245	0.539	0.874	0.504	0.000	1.177	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	92	909	117	418	0	279	-1
normalized size	1	1.00	0.80	7.90	1.02	3.63	0.00	2.43	-0.01
time (sec)	N/A	0.083	0.363	0.465	0.486	0.544	0.000	1.318	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	102	1165	134	438	0	417	-1
normalized size	1	1.00	0.74	8.50	0.98	3.20	0.00	3.04	-0.01
time (sec)	N/A	0.092	0.472	0.520	0.618	0.542	0.000	1.414	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	21	34	21	18
normalized size	1	1.00	1.00	0.86	0.82	0.95	1.55	0.95	0.82
time (sec)	N/A	0.022	0.020	0.014	1.060	0.435	28.388	1.164	0.097

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	14	167	13	25
normalized size	1	1.00	0.86	0.67	0.62	0.67	7.95	0.62	1.19
time (sec)	N/A	0.024	0.014	0.066	0.316	0.421	37.160	0.310	0.474

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	20	24	13	25
normalized size	1	1.00	0.86	0.67	0.62	0.95	1.14	0.62	1.19
time (sec)	N/A	0.024	0.033	0.055	0.312	0.421	66.230	0.309	0.470

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	20	0	13	25
normalized size	1	1.00	0.86	0.67	0.62	0.95	0.00	0.62	1.19
time (sec)	N/A	0.025	0.012	0.053	0.321	0.447	0.000	0.569	0.447

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	14	13	12	323	13	25
normalized size	1	1.00	0.84	0.74	0.68	0.63	17.00	0.68	1.32
time (sec)	N/A	0.023	0.009	0.059	0.360	0.430	32.714	0.351	0.438

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	70	532	0	0	0	0	-1
normalized size	1	1.00	0.53	4.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.089	0.291	0.000	0.453	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	70	518	0	0	0	0	-1
normalized size	1	1.00	0.74	5.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.084	0.177	0.000	0.452	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	67	505	0	0	0	0	-1
normalized size	1	1.00	1.26	9.53	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.061	0.135	0.000	0.449	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	70	493	0	0	0	0	-1
normalized size	1	1.00	0.75	5.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.102	0.169	0.000	0.416	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	70	528	0	0	0	0	-1
normalized size	1	1.00	0.52	3.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.131	0.186	0.000	0.423	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	70	514	0	2205	0	0	-1
normalized size	1	1.00	0.22	1.61	0.00	6.89	0.00	0.00	-0.00
time (sec)	N/A	0.295	0.111	0.116	0.000	46.859	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	67	271	0	2003	0	0	-1
normalized size	1	1.00	0.24	0.97	0.00	7.15	0.00	0.00	-0.00
time (sec)	N/A	0.189	0.058	0.112	0.000	49.212	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	38	0	42	0	0	50
normalized size	1	1.00	1.00	1.03	0.00	1.14	0.00	0.00	1.35
time (sec)	N/A	0.053	0.077	0.136	0.000	0.456	0.000	0.000	0.987

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	57	50	0	54	0	0	95
normalized size	1	1.00	0.76	0.67	0.00	0.72	0.00	0.00	1.27
time (sec)	N/A	0.113	0.218	0.144	0.000	0.477	0.000	0.000	2.063

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	67	60	0	64	0	0	216
normalized size	1	1.00	0.60	0.54	0.00	0.57	0.00	0.00	1.93
time (sec)	N/A	0.171	0.228	0.213	0.000	0.570	0.000	0.000	6.208

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	71	216	0	0	0	0	-1
normalized size	1	1.00	0.54	1.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.117	0.200	0.000	0.470	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	67	182	0	0	0	0	-1
normalized size	1	1.00	0.72	1.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.080	0.148	0.000	0.458	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	67	186	0	0	0	0	-1
normalized size	1	1.00	0.68	1.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.160	0.123	0.000	0.462	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	70	215	0	0	0	0	-1
normalized size	1	1.00	0.53	1.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.143	0.142	0.000	0.459	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	67	656	0	1868	0	0	-1
normalized size	1	1.00	0.21	2.05	0.00	5.84	0.00	0.00	-0.00
time (sec)	N/A	0.281	0.065	0.112	0.000	27.196	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	67	642	0	1865	0	0	-1
normalized size	1	1.00	0.21	2.05	0.00	5.96	0.00	0.00	-0.00
time (sec)	N/A	0.276	0.153	0.128	0.000	27.414	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	40	38	0	50	0	0	64
normalized size	1	1.00	1.08	1.03	0.00	1.35	0.00	0.00	1.73
time (sec)	N/A	0.060	0.106	0.102	0.000	0.461	0.000	0.000	1.505

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	57	50	0	61	0	0	207
normalized size	1	1.00	0.54	0.47	0.00	0.58	0.00	0.00	1.95
time (sec)	N/A	0.184	0.288	0.106	0.000	0.537	0.000	0.000	6.162

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	67	60	0	73	0	0	193
normalized size	1	1.00	0.48	0.43	0.00	0.52	0.00	0.00	1.37
time (sec)	N/A	0.240	0.309	0.158	0.000	0.672	0.000	0.000	6.674

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	72	545	0	0	0	0	-1
normalized size	1	1.00	0.43	3.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.235	0.170	0.164	0.000	0.513	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	70	532	0	0	0	0	-1
normalized size	1	1.00	0.53	4.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.178	0.112	0.000	0.471	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	67	519	0	0	0	0	-1
normalized size	1	1.00	0.71	5.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.090	0.151	0.000	0.456	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	67	508	0	0	0	0	-1
normalized size	1	1.00	0.71	5.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.120	0.127	0.000	0.476	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	70	531	0	0	0	0	-1
normalized size	1	1.00	0.53	3.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.174	0.171	0.142	0.000	0.474	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	72	544	0	0	0	0	-1
normalized size	1	1.00	0.43	3.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.172	0.119	0.000	0.462	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	67	510	0	2074	0	0	-1
normalized size	1	1.00	0.21	1.59	0.00	6.48	0.00	0.00	-0.00
time (sec)	N/A	0.258	0.117	0.148	0.000	55.064	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	67	532	0	2268	0	0	-1
normalized size	1	1.00	0.21	1.69	0.00	7.20	0.00	0.00	-0.00
time (sec)	N/A	0.266	0.124	0.126	0.000	54.204	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	40	38	0	60	0	0	89
normalized size	1	1.00	1.08	1.03	0.00	1.62	0.00	0.00	2.41
time (sec)	N/A	0.060	0.143	0.098	0.000	0.538	0.000	0.000	1.778

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	57	50	0	74	0	0	176
normalized size	1	1.00	0.54	0.47	0.00	0.70	0.00	0.00	1.66
time (sec)	N/A	0.175	0.286	0.105	0.000	0.616	0.000	0.000	6.335

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	67	60	0	87	0	0	207
normalized size	1	1.00	0.48	0.43	0.00	0.62	0.00	0.00	1.47
time (sec)	N/A	0.235	0.467	0.153	0.000	0.859	0.000	0.000	6.430

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	57	692	0	1282	0	0	44
normalized size	1	1.00	0.25	3.06	0.00	5.67	0.00	0.00	0.19
time (sec)	N/A	0.149	0.055	0.160	0.000	26.088	0.000	0.000	2.048

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	33	0	16	0	0	53
normalized size	1	1.00	1.00	2.06	0.00	1.00	0.00	0.00	3.31
time (sec)	N/A	0.023	0.019	0.063	0.000	0.438	0.000	0.000	0.862

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	38	166	0	447	0	0	25
normalized size	1	1.00	0.31	1.36	0.00	3.66	0.00	0.00	0.20
time (sec)	N/A	0.082	0.013	0.061	0.000	0.809	0.000	0.000	0.721

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	38	2595	0	457	0	0	25
normalized size	1	1.00	0.27	18.15	0.00	3.20	0.00	0.00	0.17
time (sec)	N/A	0.114	0.011	0.149	0.000	0.831	0.000	0.000	0.809

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	70	216	0	0	0	0	-1
normalized size	1	1.00	0.53	1.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.101	0.140	0.000	0.454	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	68	188	0	0	0	0	-1
normalized size	1	1.00	0.74	2.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.107	0.115	0.000	0.512	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	65	151	0	0	0	0	-1
normalized size	1	1.00	1.23	2.85	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.058	0.104	0.000	0.478	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	65	184	0	0	0	0	-1
normalized size	1	1.00	0.67	1.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.099	0.126	0.000	0.484	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	70	212	0	0	0	0	-1
normalized size	1	1.00	0.52	1.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.118	0.164	0.000	0.460	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	65	312	0	1697	0	0	-1
normalized size	1	1.00	0.23	1.11	0.00	6.06	0.00	0.00	-0.00
time (sec)	N/A	0.177	0.058	0.114	0.000	26.976	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	36	38	0	39	0	0	31
normalized size	1	1.00	1.03	1.09	0.00	1.11	0.00	0.00	0.89
time (sec)	N/A	0.054	0.058	0.105	0.000	0.515	0.000	0.000	0.815

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	52	50	0	51	0	0	77
normalized size	1	1.00	0.69	0.67	0.00	0.68	0.00	0.00	1.03
time (sec)	N/A	0.111	0.149	0.113	0.000	0.500	0.000	0.000	1.493

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	67	60	0	61	0	0	123
normalized size	1	1.00	0.60	0.54	0.00	0.54	0.00	0.00	1.10
time (sec)	N/A	0.169	0.209	0.126	0.000	0.598	0.000	0.000	3.826

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	55	292	0	1185	0	0	44
normalized size	1	1.00	0.32	1.68	0.00	6.81	0.00	0.00	0.25
time (sec)	N/A	0.089	0.024	0.120	0.000	25.910	0.000	0.000	1.599

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	55	937	0	1545	0	0	44
normalized size	1	1.00	0.28	4.71	0.00	7.76	0.00	0.00	0.22
time (sec)	N/A	0.114	0.032	0.108	0.000	50.207	0.000	0.000	1.625

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	57	1261	0	1321	0	0	44
normalized size	1	1.00	0.28	6.27	0.00	6.57	0.00	0.00	0.22
time (sec)	N/A	0.115	0.030	0.123	0.000	25.845	0.000	0.000	1.901

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	57	1934	0	1670	0	0	44
normalized size	1	1.00	0.25	8.56	0.00	7.39	0.00	0.00	0.19
time (sec)	N/A	0.139	0.042	0.138	0.000	48.476	0.000	0.000	1.875

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.055	0.227	0.000	0.498	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.044	0.125	0.000	0.501	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.035	0.103	0.000	0.489	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.041	0.097	0.000	0.452	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.038	0.106	0.000	0.468	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.054	0.183	0.000	0.561	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.052	0.122	0.000	0.500	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.042	0.026	0.000	0.472	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.052	0.092	0.000	0.466	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.048	0.107	0.000	0.466	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.053	0.143	0.000	0.468	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.045	0.092	0.000	0.460	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.035	0.084	0.000	0.455	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.041	0.090	0.000	0.505	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.040	0.100	0.000	0.520	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.047	0.138	0.000	0.496	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.044	0.090	0.000	0.476	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.033	0.024	0.000	0.494	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.044	0.089	0.000	0.487	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.039	0.101	0.000	0.476	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	57	0	0	144	0	0	44
normalized size	1	1.00	0.45	0.00	0.00	1.12	0.00	0.00	0.34
time (sec)	N/A	0.151	0.044	0.140	0.000	0.473	0.000	0.000	1.237

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	57	0	0	0	0	0	44
normalized size	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	0.20
time (sec)	N/A	0.329	0.043	0.094	0.000	0.000	0.000	0.000	1.049

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	57	0	0	0	0	0	44
normalized size	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	0.18
time (sec)	N/A	0.348	0.054	0.066	0.000	0.000	0.000	0.000	1.640

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	57	0	0	197	0	0	44
normalized size	1	1.00	0.37	0.00	0.00	1.27	0.00	0.00	0.28
time (sec)	N/A	0.175	0.058	0.072	0.000	0.874	0.000	0.000	1.139

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	57	0	0	195	0	0	44
normalized size	1	1.00	0.37	0.00	0.00	1.26	0.00	0.00	0.28
time (sec)	N/A	0.110	0.053	0.072	0.000	1.116	0.000	0.000	1.644

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	57	0	0	152	0	0	44
normalized size	1	1.00	0.45	0.00	0.00	1.19	0.00	0.00	0.34
time (sec)	N/A	0.082	0.027	0.085	0.000	0.968	0.000	0.000	1.485

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	55	0	0	0	0	0	44
normalized size	1	1.00	0.24	0.00	0.00	0.00	0.00	0.00	0.20
time (sec)	N/A	0.300	0.028	0.084	0.000	0.000	0.000	0.000	1.118

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	55	0	0	0	0	0	44
normalized size	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	0.18
time (sec)	N/A	0.326	0.030	0.067	0.000	0.000	0.000	0.000	1.695

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	57	0	0	189	0	0	44
normalized size	1	1.00	0.37	0.00	0.00	1.22	0.00	0.00	0.28
time (sec)	N/A	0.111	0.033	0.066	0.000	0.973	0.000	0.000	1.200

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	57	0	0	219	0	0	44
normalized size	1	1.00	0.37	0.00	0.00	1.41	0.00	0.00	0.28
time (sec)	N/A	0.134	0.034	0.067	0.000	0.948	0.000	0.000	1.914

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	0	18	0	0	10
normalized size	1	1.00	1.00	0.00	0.00	1.12	0.00	0.00	0.62
time (sec)	N/A	0.022	0.011	0.062	0.000	0.856	0.000	0.000	0.826

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	0	0	10	0	0	94
normalized size	1	1.00	1.00	0.00	0.00	0.62	0.00	0.00	5.88
time (sec)	N/A	0.023	0.016	0.061	0.000	0.818	0.000	0.000	0.822

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	79	0	0	0	0	0	71
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.042	0.105	0.579	0.000	0.909	0.000	0.000	2.346

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.116	0.548	0.000	0.802	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	85	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.074	0.531	0.000	1.020	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	85	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.095	0.567	0.000	0.961	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	55	0	77	70	2050	0	132
normalized size	1	1.00	0.74	0.00	1.04	0.95	27.70	0.00	1.78
time (sec)	N/A	0.070	0.312	1.782	0.563	0.629	66.310	0.000	1.612

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	0	53	46	530	92	62
normalized size	1	1.00	0.96	0.00	1.06	0.92	10.60	1.84	1.24
time (sec)	N/A	0.051	0.091	1.166	0.434	0.659	13.278	3.050	0.879

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	25	25	24	24	58	24	25
normalized size	1	1.00	1.04	1.04	1.00	1.00	2.42	1.00	1.04
time (sec)	N/A	0.025	0.010	0.002	0.468	0.618	2.043	1.928	0.605

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.020	0.385	0.000	0.700	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.023	0.371	0.000	0.796	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.046	1.260	0.000	0.700	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.047	1.114	0.000	0.753	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.039	0.483	0.000	0.797	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.044	0.379	0.000	0.560	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.044	0.380	0.000	0.606	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	78	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.129	0.116	0.000	0.846	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.065	0.106	0.000	0.630	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.070	0.100	0.000	0.769	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	78	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.091	0.096	0.000	0.595	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	78	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.081	0.103	0.000	0.808	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	83	0	78	84	2462	0	132
normalized size	1	1.00	1.09	0.00	1.03	1.11	32.39	0.00	1.74
time (sec)	N/A	0.066	0.307	1.292	0.382	0.627	75.791	0.000	1.544

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	1076	52	50	694	100	65
normalized size	1	1.00	1.00	21.52	1.04	1.00	13.88	2.00	1.30
time (sec)	N/A	0.050	0.123	2.144	0.369	0.755	12.778	1.728	0.912

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	26	26	25	25	61	25	26
normalized size	1	1.00	1.04	1.04	1.00	1.00	2.44	1.00	1.04
time (sec)	N/A	0.022	0.010	0.000	0.331	0.691	1.919	1.922	0.175

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	52	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.032	0.536	0.000	0.745	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	154	0	0	0	0	0	-1
normalized size	1	1.00	3.14	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	2.700	0.561	0.000	0.774	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	244	0	0	0	0	0	-1
normalized size	1	1.00	4.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	4.020	0.303	0.000	0.893	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.113	0.984	0.000	0.701	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.084	1.280	0.000	0.626	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	64	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.044	0.224	0.000	0.722	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	80	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.194	0.407	0.000	0.500	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	82	0	0	0	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.211	0.294	0.000	0.624	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	158	0	0	0	0	0	-1
normalized size	1	1.00	2.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.416	0.123	0.000	0.681	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.159	0.110	0.000	0.757	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	82	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.104	0.115	0.000	0.722	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	82	0	0	0	0	0	-1
normalized size	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.113	0.094	0.000	0.688	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	79	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.133	0.092	0.000	0.806	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	58	517	63	56	0	0	-1
normalized size	1	1.00	0.68	6.08	0.74	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.340	0.382	0.315	0.811	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	48	507	50	46	0	0	-1
normalized size	1	1.00	0.76	8.05	0.79	0.73	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.207	0.192	0.317	0.705	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	36	497	35	34	0	0	-1
normalized size	1	1.00	0.88	12.12	0.85	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.158	0.213	0.389	0.685	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	23	0	0	23
normalized size	1	1.00	1.00	0.94	1.28	1.28	0.00	0.00	1.28
time (sec)	N/A	0.030	0.035	0.033	0.547	0.574	0.000	0.000	0.204

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	73	169	72	247	0	0	-1
normalized size	1	1.00	1.26	2.91	1.24	4.26	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.351	0.185	0.539	0.714	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	95	603	106	354	0	0	-1
normalized size	1	1.00	1.02	6.48	1.14	3.81	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.623	0.221	0.434	0.708	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	107	1089	138	438	0	0	-1
normalized size	1	1.00	0.87	8.85	1.12	3.56	0.00	0.00	-0.01
time (sec)	N/A	0.086	1.010	0.230	0.416	0.927	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	73	165	0	0	0	0	-1
normalized size	1	1.00	0.59	1.34	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.153	0.345	0.000	0.640	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	61	143	0	0	0	0	-1
normalized size	1	1.00	0.64	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.110	0.226	0.000	0.692	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	123	0	0	0	0	-1
normalized size	1	1.00	0.76	1.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.125	0.194	0.000	0.578	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	98	0	0	0	0	35
normalized size	1	1.00	1.00	2.58	0.00	0.00	0.00	0.00	0.92
time (sec)	N/A	0.021	0.038	0.170	0.000	0.645	0.000	0.000	0.563

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	47	184	0	0	0	0	-1
normalized size	1	1.00	0.76	2.97	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.116	0.195	0.000	0.746	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	63	335	0	0	0	0	-1
normalized size	1	1.00	0.66	3.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.242	0.229	0.000	0.715	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	73	485	0	0	0	0	-1
normalized size	1	1.00	0.59	3.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.482	0.249	0.000	0.661	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	52	969	72	54	0	0	-1
normalized size	1	1.00	0.63	11.67	0.87	0.65	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.170	0.321	0.307	0.747	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	42	959	55	43	0	0	-1
normalized size	1	1.00	0.67	15.22	0.87	0.68	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.110	0.227	0.418	0.665	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	30	949	37	31	0	0	-1
normalized size	1	1.00	0.73	23.15	0.90	0.76	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.074	0.192	0.357	0.574	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	18	0	0	18
normalized size	1	1.00	1.00	0.94	1.28	1.00	0.00	0.00	1.00
time (sec)	N/A	0.034	0.033	0.024	0.461	0.689	0.000	0.000	0.478

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	85	235	87	278	0	0	-1
normalized size	1	1.00	1.10	3.05	1.13	3.61	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.959	0.214	0.515	0.801	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	97	644	123	388	0	0	-1
normalized size	1	1.00	0.86	5.70	1.09	3.43	0.00	0.00	-0.01
time (sec)	N/A	0.084	2.327	0.194	0.678	0.561	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	70	330	0	0	0	0	-1
normalized size	1	1.00	0.55	2.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.146	0.287	0.000	0.840	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	60	320	0	0	0	0	-1
normalized size	1	1.00	0.61	3.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.129	0.227	0.000	0.808	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	48	310	0	0	0	0	-1
normalized size	1	1.00	0.73	4.70	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.066	0.083	0.204	0.000	0.666	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	48	322	0	0	0	0	-1
normalized size	1	1.00	0.73	4.88	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.057	0.259	0.000	0.721	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	57	322	0	0	0	0	-1
normalized size	1	1.00	0.63	3.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.152	0.198	0.000	0.711	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	77	622	0	0	0	0	-1
normalized size	1	1.00	0.62	5.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.207	0.222	0.000	0.645	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	52	532	66	70	0	0	-1
normalized size	1	1.00	0.61	6.26	0.78	0.82	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.421	0.210	0.543	0.563	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	42	522	52	57	0	0	-1
normalized size	1	1.00	0.67	8.29	0.83	0.90	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.332	0.204	0.396	0.689	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	32	357	36	42	0	0	50
normalized size	1	1.00	0.78	8.71	0.88	1.02	0.00	0.00	1.22
time (sec)	N/A	0.049	0.197	0.175	0.326	0.584	0.000	0.000	0.908

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	0	39
normalized size	1	1.00	1.00	0.85	1.15	1.40	0.00	0.00	1.95
time (sec)	N/A	0.036	0.036	0.023	0.556	0.631	0.000	0.000	0.624

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	87	237	87	328	0	0	-1
normalized size	1	1.00	1.12	3.04	1.12	4.21	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.180	0.168	0.429	0.673	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	109	699	123	448	0	0	-1
normalized size	1	1.00	0.96	6.19	1.09	3.96	0.00	0.00	-0.01
time (sec)	N/A	0.084	1.896	0.222	0.658	0.738	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	119	1161	155	542	0	0	-1
normalized size	1	1.00	0.83	8.12	1.08	3.79	0.00	0.00	-0.01
time (sec)	N/A	0.098	1.318	0.179	0.429	0.943	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	74	168	0	0	0	0	-1
normalized size	1	1.00	0.57	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.195	0.240	0.000	0.776	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	64	144	0	0	0	0	-1
normalized size	1	1.00	0.64	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.128	0.195	0.000	0.789	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	52	126	0	0	0	0	-1
normalized size	1	1.00	0.74	1.80	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.119	0.176	0.000	0.798	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	51	128	0	0	0	0	-1
normalized size	1	1.00	0.73	1.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.077	0.170	0.000	0.728	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	67	202	0	0	0	0	-1
normalized size	1	1.00	0.68	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.168	0.191	0.000	0.566	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	79	352	0	0	0	0	-1
normalized size	1	1.00	0.64	2.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.394	0.231	0.000	0.753	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	52	56	63	61	0	0	-1
normalized size	1	1.00	0.60	0.64	0.72	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.212	0.217	1.132	0.646	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	42	46	50	51	0	0	-1
normalized size	1	1.00	0.65	0.71	0.77	0.78	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.167	0.172	0.419	0.779	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	32	36	37	41	0	50	-1
normalized size	1	1.00	0.74	0.84	0.86	0.95	0.00	1.16	-0.02
time (sec)	N/A	0.045	0.103	0.164	0.422	0.607	0.000	1.691	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	0	28
normalized size	1	1.00	1.00	0.85	1.15	1.40	0.00	0.00	1.40
time (sec)	N/A	0.031	0.041	0.032	0.475	0.469	0.000	0.000	0.544

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	73	161	73	253	0	0	-1
normalized size	1	1.00	1.24	2.73	1.24	4.29	0.00	0.00	-0.02
time (sec)	N/A	0.042	0.098	0.175	0.454	0.908	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	425	105	361	0	0	-1
normalized size	1	1.00	1.00	4.57	1.13	3.88	0.00	0.00	-0.01
time (sec)	N/A	0.067	1.097	0.211	0.749	0.604	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	107	729	138	450	0	0	-1
normalized size	1	1.00	0.87	5.93	1.12	3.66	0.00	0.00	-0.01
time (sec)	N/A	0.079	1.920	0.217	1.456	0.868	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	73	338	0	0	0	0	-1
normalized size	1	1.00	0.59	2.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.461	0.245	0.000	0.639	0.000	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	63	328	0	0	0	0	-1
normalized size	1	1.00	0.66	3.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.334	0.202	0.000	0.785	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	60	318	0	0	0	0	-1
normalized size	1	1.00	0.90	4.75	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.136	0.227	0.000	0.768	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	306	0	0	0	0	-1
normalized size	1	1.00	1.00	8.05	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.036	0.187	0.000	0.837	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	48	316	0	0	0	0	-1
normalized size	1	1.00	0.76	5.02	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.153	0.192	0.000	0.691	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	74	618	0	0	0	0	-1
normalized size	1	1.00	0.78	6.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.186	0.236	0.000	0.752	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	86	918	0	0	0	0	-1
normalized size	1	1.00	0.70	7.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.134	0.254	0.000	0.604	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	52	56	63	61	0	0	-1
normalized size	1	1.00	0.60	0.64	0.72	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.427	0.214	0.441	0.660	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	42	46	50	51	0	0	-1
normalized size	1	1.00	0.65	0.71	0.77	0.78	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.252	0.165	0.477	0.761	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	32	36	37	41	0	50	-1
normalized size	1	1.00	0.74	0.84	0.86	0.95	0.00	1.16	-0.02
time (sec)	N/A	0.050	0.174	0.146	1.262	0.946	0.000	1.722	0.000

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	30	28
normalized size	1	1.00	1.00	0.85	1.15	1.40	0.00	1.50	1.40
time (sec)	N/A	0.036	0.053	0.025	0.447	0.586	0.000	1.533	0.567

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	89	221	89	314	0	0	-1
normalized size	1	1.00	1.14	2.83	1.14	4.03	0.00	0.00	-0.01
time (sec)	N/A	0.054	1.999	0.170	0.619	1.018	0.000	0.000	0.000

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	98	426	106	364	0	0	-1
normalized size	1	1.00	1.05	4.58	1.14	3.91	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.459	0.180	1.955	0.754	0.000	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	109	729	137	454	0	0	-1
normalized size	1	1.00	0.89	5.93	1.11	3.69	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.627	0.194	0.851	0.782	0.000	0.000	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	81	173	0	0	0	0	-1
normalized size	1	1.00	0.64	1.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.163	0.227	0.000	0.746	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	71	153	0	0	0	0	-1
normalized size	1	1.00	0.72	1.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.132	0.187	0.000	0.585	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	59	131	0	0	0	0	-1
normalized size	1	1.00	0.82	1.82	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.062	0.176	0.000	0.764	0.000	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	58	191	0	0	0	0	-1
normalized size	1	1.00	0.85	2.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.098	0.179	0.000	0.599	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	62	343	0	0	0	0	-1
normalized size	1	1.00	0.61	3.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.220	0.200	0.000	0.576	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	74	493	0	0	0	0	-1
normalized size	1	1.00	0.56	3.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.307	0.240	0.000	0.597	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	62	56	63	61	0	0	-1
normalized size	1	1.00	0.71	0.64	0.72	0.70	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.346	0.225	0.390	0.785	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	52	46	50	51	0	0	-1
normalized size	1	1.00	0.80	0.71	0.77	0.78	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.215	0.169	0.530	0.751	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	36	37	41	0	50	-1
normalized size	1	1.00	0.98	0.84	0.86	0.95	0.00	1.16	-0.02
time (sec)	N/A	0.053	0.158	0.150	0.451	0.732	0.000	1.667	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	23	28	0	30	28
normalized size	1	1.00	1.00	0.85	1.15	1.40	0.00	1.50	1.40
time (sec)	N/A	0.037	0.078	0.024	0.431	0.743	0.000	2.105	0.569

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	90	377	90	319	0	0	-1
normalized size	1	1.00	1.11	4.65	1.11	3.94	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.204	0.185	0.430	1.042	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	98	437	106	370	0	0	-1
normalized size	1	1.00	1.05	4.70	1.14	3.98	0.00	0.00	-0.01
time (sec)	N/A	0.073	2.434	0.186	0.498	0.826	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	110	737	136	458	0	0	-1
normalized size	1	1.00	0.89	5.99	1.11	3.72	0.00	0.00	-0.01
time (sec)	N/A	0.084	2.401	0.197	0.898	0.847	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	83	343	0	0	0	0	-1
normalized size	1	1.00	0.66	2.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.499	0.211	0.000	0.679	0.000	0.000	0.000

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	66	333	0	0	0	0	-1
normalized size	1	1.00	0.67	3.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.395	0.184	0.000	0.686	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	321	0	0	0	0	-1
normalized size	1	1.00	0.83	4.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.088	0.219	0.000	0.643	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	312	0	0	0	0	-1
normalized size	1	1.00	0.75	4.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.156	0.218	0.000	0.586	0.000	0.000	0.000

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	79	623	0	0	0	0	-1
normalized size	1	1.00	0.77	6.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.237	0.228	0.000	0.833	0.000	0.000	0.000

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	87	923	0	0	0	0	-1
normalized size	1	1.00	0.66	6.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.180	0.251	0.000	0.617	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	80	538	0	0	0	0	-1
normalized size	1	1.00	0.18	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	0.370	0.234	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	65	512	0	0	0	0	-1
normalized size	1	1.00	0.16	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.348	0.256	0.185	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	55	273	0	0	0	0	-1
normalized size	1	1.00	0.15	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.265	0.131	0.169	0.000	0.000	0.000	0.000	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	37	40	0	44	0	0	36
normalized size	1	1.00	1.12	1.21	0.00	1.33	0.00	0.00	1.09
time (sec)	N/A	0.052	0.076	0.173	0.000	1.131	0.000	0.000	0.873

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	52	52	0	73	0	0	83
normalized size	1	1.00	0.73	0.73	0.00	1.03	0.00	0.00	1.17
time (sec)	N/A	0.109	0.204	0.180	0.000	1.130	0.000	0.000	1.805

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	65	62	0	96	0	0	169
normalized size	1	1.00	0.61	0.58	0.00	0.91	0.00	0.00	1.59
time (sec)	N/A	0.165	0.213	0.197	0.000	0.975	0.000	0.000	5.695

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	90	212	0	0	0	0	-1
normalized size	1	1.00	0.70	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.215	0.871	0.235	0.000	0.662	0.000	0.000	0.000

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	66	184	0	0	0	0	-1
normalized size	1	1.00	0.73	2.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	1.284	0.189	0.000	0.599	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	66	153	0	0	0	0	-1
normalized size	1	1.00	1.25	2.89	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.099	0.375	0.162	0.000	0.733	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	75	279	0	0	0	0	-1
normalized size	1	1.00	0.79	2.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.435	0.183	0.000	0.731	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	111	532	0	0	0	0	-1
normalized size	1	1.00	0.85	4.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	1.020	0.206	0.000	0.833	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	86	524	0	0	0	0	-1
normalized size	1	1.00	0.75	4.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.533	0.159	0.000	0.796	0.000	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	74	511	0	0	0	0	-1
normalized size	1	1.00	0.87	6.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.300	0.177	0.000	0.659	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	497	0	0	0	0	-1
normalized size	1	1.00	1.18	9.75	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.081	1.102	0.151	0.000	0.699	0.000	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	63	484	0	0	0	0	-1
normalized size	1	1.00	0.78	5.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.251	0.151	0.000	0.707	0.000	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	82	1030	0	0	0	0	-1
normalized size	1	1.00	0.71	8.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	0.471	0.177	0.000	0.582	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	218	646	0	0	0	0	-1
normalized size	1	1.00	0.60	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.268	1.559	0.146	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	166	304	0	0	0	0	-1
normalized size	1	1.00	0.51	0.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.861	0.138	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	70	0	48	0	0	57
normalized size	1	1.00	1.00	2.33	0.00	1.60	0.00	0.00	1.90
time (sec)	N/A	0.038	0.099	0.141	0.000	0.885	0.000	0.000	1.250

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	42	82	0	72	0	0	103
normalized size	1	1.00	0.69	1.34	0.00	1.18	0.00	0.00	1.69
time (sec)	N/A	0.078	0.136	0.146	0.000	0.643	0.000	0.000	2.662

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	52	92	0	95	0	0	163
normalized size	1	1.00	0.57	1.01	0.00	1.04	0.00	0.00	1.79
time (sec)	N/A	0.122	0.211	0.185	0.000	0.500	0.000	0.000	6.239

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	62	102	0	116	0	0	192
normalized size	1	1.00	0.51	0.84	0.00	0.96	0.00	0.00	1.59
time (sec)	N/A	0.162	0.262	0.213	0.000	0.545	0.000	0.000	6.414

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	97	572	0	0	0	0	-1
normalized size	1	1.00	0.20	1.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.553	0.433	0.214	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	82	546	0	0	0	0	-1
normalized size	1	1.00	0.18	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.447	0.324	0.199	0.000	0.000	0.000	0.000	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	76	516	0	0	0	0	-1
normalized size	1	1.00	0.18	1.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.356	0.228	0.214	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	66	957	0	0	0	0	-1
normalized size	1	1.00	0.16	2.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	0.191	0.170	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	45	40	0	68	0	0	84
normalized size	1	1.00	1.29	1.14	0.00	1.94	0.00	0.00	2.40
time (sec)	N/A	0.058	0.108	0.143	0.000	1.098	0.000	0.000	1.862

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	103	246	0	0	0	0	-1
normalized size	1	1.00	0.60	1.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.275	0.847	0.211	0.000	0.817	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	87	218	0	0	0	0	-1
normalized size	1	1.00	0.64	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.405	0.199	0.000	0.945	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	84	190	0	0	0	0	-1
normalized size	1	1.00	0.89	2.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.527	0.168	0.000	1.043	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	78	284	0	0	0	0	-1
normalized size	1	1.00	0.78	2.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.476	0.159	0.000	1.090	0.000	0.000	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	119	540	0	0	0	0	-1
normalized size	1	1.00	0.87	3.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.219	0.839	0.176	0.000	0.841	0.000	0.000	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	131	793	0	0	0	0	-1
normalized size	1	1.00	0.75	4.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.282	1.215	0.218	0.000	1.188	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	96	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	8.678	0.187	0.000	0.945	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	96	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	1.356	0.148	0.000	0.863	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	106	0	0	0	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	1.368	0.158	0.000	0.870	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	289	0	0	0	0	0	-1
normalized size	1	1.00	3.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	1.753	0.148	0.000	0.742	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	116	0	0	0	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	4.035	0.137	0.000	0.722	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	285	0	0	0	0	0	-1
normalized size	1	1.00	3.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	1.466	0.660	0.000	0.638	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	287	0	0	0	0	0	-1
normalized size	1	1.00	3.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.149	0.608	0.000	0.749	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	287	0	0	0	0	0	-1
normalized size	1	1.00	3.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.137	0.615	0.000	0.656	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	289	0	0	0	0	0	-1
normalized size	1	1.00	3.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.132	0.642	0.000	0.743	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	85	85	0	0	134
normalized size	1	1.00	1.00	0.00	1.06	1.06	0.00	0.00	1.68
time (sec)	N/A	0.073	0.365	1.771	0.353	0.617	0.000	0.000	1.601

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	47	1732	59	53	0	0	67
normalized size	1	1.00	0.90	33.31	1.13	1.02	0.00	0.00	1.29
time (sec)	N/A	0.053	0.153	1.855	1.334	0.677	0.000	0.000	0.935

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	120	28	28	0	0	27
normalized size	1	1.00	0.88	4.80	1.12	1.12	0.00	0.00	1.08
time (sec)	N/A	0.033	0.021	0.039	0.362	0.500	0.000	0.000	0.195

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	92	0	0	0	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.331	0.564	0.000	0.689	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	201	0	0	0	0	0	-1
normalized size	1	1.00	4.19	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	4.516	0.556	0.000	0.779	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	8327	0	0	0	0	0	-1
normalized size	1	1.00	114.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	25.459	1.295	0.000	0.721	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	6192	0	0	0	0	0	-1
normalized size	1	1.00	84.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	24.183	1.062	0.000	0.635	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	4143	0	0	0	0	0	-1
normalized size	1	1.00	56.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	18.632	1.335	0.000	0.678	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	61	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.059	0.221	0.000	0.541	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	2638	0	0	0	0	0	-1
normalized size	1	1.00	36.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	15.225	0.441	0.000	0.576	0.000	0.000	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	3833	0	0	0	0	0	-1
normalized size	1	1.00	52.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	17.551	0.306	0.000	0.889	0.000	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	104	0	0	0	0	0	-1
normalized size	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.815	0.186	0.000	0.667	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.123	0.163	0.000	0.629	0.000	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	72	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.139	0.133	0.000	0.734	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	73	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.148	0.126	0.000	0.660	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	67	214	0	0	0	0	-1
normalized size	1	1.00	0.67	2.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.182	0.346	0.000	0.753	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	62	538	0	0	0	0	-1
normalized size	1	1.00	0.83	7.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.124	0.277	0.000	0.736	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	55	187	0	0	0	0	-1
normalized size	1	1.00	0.76	2.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.083	0.184	0.000	0.666	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	525	0	0	0	0	-1
normalized size	1	1.00	0.98	11.93	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.046	0.160	0.000	0.711	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	165	0	0	0	0	63
normalized size	1	1.00	0.98	3.84	0.00	0.00	0.00	0.00	1.47
time (sec)	N/A	0.019	0.037	0.130	0.000	0.650	0.000	0.000	0.619

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	57	514	0	0	0	0	-1
normalized size	1	1.00	0.84	7.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.134	0.158	0.000	0.602	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	55	319	0	0	0	0	-1
normalized size	1	1.00	0.74	4.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.097	0.175	0.000	0.615	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	68	1054	0	0	0	0	-1
normalized size	1	1.00	0.68	10.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.167	0.206	0.000	0.669	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	68	216	0	0	0	0	-1
normalized size	1	1.00	0.66	2.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.137	0.162	0.000	0.689	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	62	545	0	0	0	0	-1
normalized size	1	1.00	0.81	7.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.195	0.181	0.000	0.793	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	56	189	0	0	0	0	-1
normalized size	1	1.00	0.75	2.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.062	0.152	0.000	0.871	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	531	0	0	0	0	-1
normalized size	1	1.00	0.98	11.54	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.031	0.150	0.000	0.630	0.000	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	165	0	0	0	0	-1
normalized size	1	1.00	0.98	3.75	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.018	0.171	0.000	0.819	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	54	520	0	0	0	0	-1
normalized size	1	1.00	0.76	7.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.019	0.138	0.000	0.544	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	58	319	0	0	0	0	-1
normalized size	1	1.00	0.81	4.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.185	0.145	0.000	0.717	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	68	1054	0	0	0	0	-1
normalized size	1	1.00	0.66	10.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.264	0.171	0.000	0.739	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	70	208	0	0	0	0	-1
normalized size	1	1.00	0.69	2.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.108	0.195	0.000	0.735	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	57	547	0	0	0	0	-1
normalized size	1	1.00	0.79	7.60	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.176	0.190	0.000	0.632	0.000	0.000	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	64	181	0	0	0	0	-1
normalized size	1	1.00	0.86	2.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.080	0.162	0.000	0.729	0.000	0.000	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	533	0	0	0	0	-1
normalized size	1	1.00	0.98	12.40	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.016	0.154	0.000	0.810	0.000	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	165	0	0	0	0	-1
normalized size	1	1.00	0.98	3.59	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.017	0.142	0.000	0.509	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	52	522	0	0	0	0	-1
normalized size	1	1.00	0.74	7.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.091	0.161	0.000	0.655	0.000	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	60	318	0	0	0	0	-1
normalized size	1	1.00	0.78	4.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.064	0.175	0.000	0.744	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	70	216	0	0	0	0	-1
normalized size	1	1.00	0.68	2.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.115	0.181	0.000	0.722	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	60	547	0	0	0	0	-1
normalized size	1	1.00	0.81	7.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.033	0.171	0.000	0.768	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	189	0	0	0	0	-1
normalized size	1	1.00	0.82	2.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.018	0.137	0.000	0.875	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	533	0	0	0	0	-1
normalized size	1	1.00	0.98	11.59	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.021	0.030	0.142	0.000	0.606	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	165	0	0	0	0	-1
normalized size	1	1.00	0.98	3.59	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.020	0.142	0.000	0.676	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	522	0	0	0	0	-1
normalized size	1	1.00	0.75	7.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.047	0.159	0.000	0.811	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	60	319	0	0	0	0	-1
normalized size	1	1.00	0.78	4.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.068	0.171	0.000	0.670	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	73	1054	0	0	0	0	-1
normalized size	1	1.00	0.70	10.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.133	0.182	0.000	0.572	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	102	0	0	0	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	10.471	1.067	0.000	0.803	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [290] had the largest ratio of [.6154]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	1	1.00	8	0.125
4	A	3	2	1.00	8	0.250
5	A	2	1	1.00	8	0.125
6	A	4	2	1.00	8	0.250
7	A	2	1	1.00	8	0.125
8	A	5	2	1.00	8	0.250
9	A	3	2	1.00	8	0.250
10	A	2	2	1.00	8	0.250
11	A	2	2	1.00	8	0.250
12	A	1	1	1.00	8	0.125
13	A	1	1	1.00	8	0.125
14	A	2	2	1.00	8	0.250
15	A	2	2	1.00	8	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	3	2	1.00	8	0.250
17	A	3	2	1.00	10	0.200
18	A	2	2	1.00	10	0.200
19	A	2	2	1.00	10	0.200
20	A	1	1	1.00	10	0.100
21	A	1	1	1.00	10	0.100
22	A	2	2	1.00	10	0.200
23	A	2	2	1.00	10	0.200
24	A	3	2	1.00	10	0.200
25	A	4	3	1.00	12	0.250
26	A	3	3	1.00	12	0.250
27	A	3	3	1.00	12	0.250
28	A	2	2	1.00	12	0.167
29	A	2	2	1.00	12	0.167
30	A	3	3	1.00	12	0.250
31	A	3	3	1.00	12	0.250
32	A	4	3	1.00	12	0.250
33	A	1	1	1.00	12	0.083
34	A	1	1	1.00	12	0.083
35	C	1	1	0.11	12	0.083
36	C	1	1	0.23	12	0.083
37	C	1	1	0.21	12	0.083
38	A	1	1	1.00	12	0.083
39	A	1	1	1.00	8	0.125
40	A	1	1	1.00	10	0.100
41	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
42	A	2	2	1.00	15	0.133
43	A	2	2	1.00	15	0.133
44	A	2	2	1.00	13	0.154
45	A	1	1	1.00	6	0.167
46	A	2	2	1.00	13	0.154
47	A	2	2	1.00	15	0.133
48	A	2	2	1.00	15	0.133
49	A	3	2	1.00	17	0.118
50	A	3	2	1.00	17	0.118
51	A	3	2	1.00	17	0.118
52	A	2	2	1.00	15	0.133
53	A	2	2	1.00	8	0.250
54	A	2	2	1.00	17	0.118
55	A	3	2	1.00	17	0.118
56	A	3	2	1.00	17	0.118
57	A	3	2	1.00	17	0.118
58	A	5	3	1.00	17	0.176
59	A	4	3	1.00	17	0.176
60	A	3	3	1.00	17	0.176
61	A	2	2	1.00	8	0.250
62	A	3	3	1.00	13	0.231
63	A	2	2	1.00	15	0.133
64	A	3	3	1.00	17	0.176
65	A	4	3	1.00	17	0.176
66	A	3	2	1.00	17	0.118
67	A	3	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
68	A	3	2	1.00	17	0.118
69	A	3	2	1.00	17	0.118
70	A	2	2	1.00	15	0.133
71	A	3	2	1.00	15	0.133
72	A	3	2	1.00	15	0.133
73	A	2	2	1.00	8	0.250
74	A	2	1	1.00	15	0.067
75	A	2	2	1.00	17	0.118
76	A	3	2	1.00	17	0.118
77	A	3	2	1.00	17	0.118
78	A	3	2	1.00	17	0.118
79	A	3	2	1.00	17	0.118
80	A	3	2	1.00	17	0.118
81	A	3	2	1.00	17	0.118
82	A	3	2	1.00	17	0.118
83	A	2	2	1.00	15	0.133
84	A	4	4	1.00	17	0.235
85	A	3	2	1.00	8	0.250
86	A	2	2	1.00	17	0.118
87	A	3	2	1.00	17	0.118
88	A	3	2	1.00	17	0.118
89	A	6	3	1.00	17	0.176
90	A	5	3	1.00	17	0.176
91	A	4	3	1.00	17	0.176
92	A	3	2	1.00	8	0.250
93	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
94	A	4	4	1.00	15	0.267
95	A	3	2	1.00	15	0.133
96	A	4	3	1.00	17	0.176
97	A	5	3	1.00	17	0.176
98	A	4	3	1.00	17	0.176
99	A	3	2	1.00	17	0.118
100	A	4	3	1.00	17	0.176
101	A	3	2	1.00	17	0.118
102	A	3	2	1.00	17	0.118
103	A	3	2	1.00	17	0.118
104	A	2	2	1.00	15	0.133
105	A	4	3	1.00	15	0.200
106	A	3	2	1.00	17	0.118
107	A	4	3	1.00	17	0.176
108	A	3	2	1.00	15	0.133
109	A	3	2	1.00	8	0.250
110	A	3	2	1.00	15	0.133
111	A	2	2	1.00	17	0.118
112	A	3	2	1.00	17	0.118
113	A	3	2	1.00	17	0.118
114	A	3	2	1.00	17	0.118
115	A	4	3	1.00	17	0.176
116	A	3	2	1.00	17	0.118
117	A	4	3	1.00	17	0.176
118	A	5	4	1.00	17	0.235
119	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	4	3	1.00	15	0.200
121	A	4	3	1.00	15	0.200
122	A	4	3	1.00	15	0.200
123	A	3	2	1.00	15	0.133
124	A	3	3	1.00	13	0.231
125	A	1	1	1.00	6	0.167
126	A	2	2	1.00	13	0.154
127	A	3	3	1.00	15	0.200
128	A	3	2	1.00	15	0.133
129	A	4	3	1.00	15	0.200
130	A	4	3	1.00	15	0.200
131	A	4	3	1.00	15	0.200
132	A	4	3	1.00	15	0.200
133	A	3	2	1.00	17	0.118
134	A	5	4	1.00	17	0.235
135	A	3	2	1.00	17	0.118
136	A	4	4	1.00	17	0.235
137	A	3	2	1.00	15	0.133
138	A	2	2	1.00	8	0.250
139	A	2	2	1.00	13	0.154
140	A	3	3	1.00	15	0.200
141	A	3	2	1.00	17	0.118
142	A	4	4	1.00	17	0.235
143	A	3	2	1.00	17	0.118
144	A	5	4	1.00	17	0.235
145	A	4	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
146	A	5	4	1.00	17	0.235
147	A	4	3	1.00	17	0.176
148	A	4	4	1.00	15	0.267
149	A	2	2	1.00	8	0.250
150	A	2	2	1.00	15	0.133
151	A	2	2	1.00	15	0.133
152	A	3	2	1.00	15	0.133
153	A	4	4	1.00	17	0.235
154	A	4	3	1.00	17	0.176
155	A	5	4	1.00	17	0.235
156	A	4	3	1.00	17	0.176
157	A	3	2	1.00	17	0.118
158	A	6	4	1.00	17	0.235
159	A	3	2	1.00	17	0.118
160	A	5	4	1.00	17	0.235
161	A	3	2	1.00	15	0.133
162	A	3	2	1.00	8	0.250
163	A	2	1	1.00	15	0.067
164	A	2	2	1.00	17	0.118
165	A	2	2	1.00	15	0.133
166	A	4	3	1.00	15	0.200
167	A	3	2	1.00	17	0.118
168	A	5	4	1.00	17	0.235
169	A	3	2	1.00	17	0.118
170	A	6	4	1.00	17	0.235
171	A	4	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	6	4	1.00	17	0.235
173	A	4	3	1.00	17	0.176
174	A	5	4	1.00	15	0.267
175	A	3	2	1.00	8	0.250
176	A	3	2	1.00	15	0.133
177	A	2	2	1.00	17	0.118
178	A	3	3	1.00	17	0.176
179	A	2	2	1.00	15	0.133
180	A	4	3	1.00	15	0.200
181	A	5	4	1.00	17	0.235
182	A	4	3	1.00	17	0.176
183	A	6	4	1.00	17	0.235
184	A	4	3	1.00	17	0.176
185	A	3	2	1.00	9	0.222
186	A	3	2	1.00	9	0.222
187	A	2	2	1.00	19	0.105
188	A	2	2	1.00	19	0.105
189	A	2	2	1.00	19	0.105
190	A	2	2	1.00	19	0.105
191	A	2	2	1.00	19	0.105
192	A	2	2	1.00	19	0.105
193	A	2	2	1.00	19	0.105
194	A	5	4	1.00	21	0.190
195	A	5	4	1.00	21	0.190
196	A	4	4	1.00	21	0.190
197	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
198	A	3	3	1.00	21	0.143
199	A	3	3	1.00	21	0.143
200	A	3	3	1.00	21	0.143
201	A	3	3	1.00	21	0.143
202	A	4	4	1.00	21	0.190
203	A	4	4	1.00	21	0.190
204	A	3	2	1.00	21	0.095
205	A	3	2	1.00	21	0.095
206	A	3	2	1.00	21	0.095
207	A	3	2	1.00	21	0.095
208	A	3	2	1.00	21	0.095
209	A	3	2	1.00	21	0.095
210	A	3	2	1.00	21	0.095
211	A	6	4	1.00	21	0.190
212	A	6	4	1.00	21	0.190
213	A	5	4	1.00	21	0.190
214	A	5	4	1.00	21	0.190
215	A	4	3	1.00	21	0.143
216	A	4	3	1.00	21	0.143
217	A	4	4	1.00	21	0.190
218	A	4	4	1.00	21	0.190
219	A	4	3	1.00	21	0.143
220	A	4	3	1.00	21	0.143
221	A	3	2	1.00	19	0.105
222	A	7	6	1.00	19	0.316
223	A	7	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	6	6	1.00	19	0.316
225	A	6	6	1.00	19	0.316
226	A	5	5	1.00	19	0.263
227	A	5	5	1.00	19	0.263
228	A	6	6	1.00	19	0.316
229	A	6	6	1.00	19	0.316
230	A	7	6	1.00	19	0.316
231	A	7	6	1.00	19	0.316
232	A	5	4	1.00	21	0.190
233	A	4	4	1.00	21	0.190
234	A	4	4	1.00	21	0.190
235	A	3	3	1.00	21	0.143
236	A	3	3	1.00	21	0.143
237	A	3	3	1.00	21	0.143
238	A	3	3	1.00	21	0.143
239	A	4	4	1.00	21	0.190
240	A	4	4	1.00	21	0.190
241	A	5	4	1.00	21	0.190
242	A	8	7	1.00	21	0.333
243	A	7	7	1.00	21	0.333
244	A	7	7	1.00	21	0.333
245	A	6	6	1.00	21	0.286
246	A	6	6	1.00	21	0.286
247	A	6	6	1.00	21	0.286
248	A	6	6	1.00	21	0.286
249	A	7	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
250	A	7	7	1.00	21	0.333
251	A	8	7	1.00	21	0.333
252	A	2	2	1.00	19	0.105
253	A	3	2	1.00	11	0.182
254	A	3	2	1.00	11	0.182
255	A	3	2	1.00	11	0.182
256	A	3	2	1.00	11	0.182
257	A	4	3	1.00	25	0.120
258	A	3	3	1.00	25	0.120
259	A	2	2	1.00	25	0.080
260	A	3	3	1.00	25	0.120
261	A	4	3	1.00	25	0.120
262	A	11	8	1.00	25	0.320
263	A	10	7	1.00	25	0.280
264	A	1	1	1.00	25	0.040
265	A	2	2	1.00	25	0.080
266	A	3	2	1.00	25	0.080
267	A	4	4	1.00	25	0.160
268	A	3	3	1.00	25	0.120
269	A	3	3	1.00	25	0.120
270	A	4	4	1.00	25	0.160
271	A	11	8	1.00	25	0.320
272	A	11	8	1.00	25	0.320
273	A	1	1	1.00	25	0.040
274	A	3	3	1.00	25	0.120
275	A	4	3	1.00	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
276	A	5	4	1.00	25	0.160
277	A	4	4	1.00	25	0.160
278	A	3	3	1.00	25	0.120
279	A	3	3	1.00	25	0.120
280	A	4	4	1.00	25	0.160
281	A	5	4	1.00	25	0.160
282	A	11	8	1.00	25	0.320
283	A	11	8	1.00	25	0.320
284	A	1	1	1.00	25	0.040
285	A	3	3	1.00	25	0.120
286	A	4	3	1.00	25	0.120
287	A	12	8	1.00	21	0.381
288	A	1	1	1.00	13	0.077
289	A	10	7	1.00	13	0.538
290	A	11	8	1.00	13	0.615
291	A	4	3	1.00	25	0.120
292	A	3	3	1.00	25	0.120
293	A	2	2	1.00	25	0.080
294	A	3	3	1.00	25	0.120
295	A	4	3	1.00	25	0.120
296	A	10	7	1.00	25	0.280
297	A	1	1	1.00	25	0.040
298	A	2	2	1.00	25	0.080
299	A	3	2	1.00	25	0.080
300	A	10	7	1.00	21	0.333
301	A	11	8	1.00	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
302	A	11	8	1.00	21	0.381
303	A	12	8	1.00	21	0.381
304	A	1	1	1.00	21	0.048
305	A	1	1	1.00	21	0.048
306	A	1	1	1.00	12	0.083
307	A	1	1	1.00	21	0.048
308	A	1	1	1.00	21	0.048
309	A	1	1	1.00	21	0.048
310	A	1	1	1.00	21	0.048
311	A	1	1	1.00	12	0.083
312	A	1	1	1.00	21	0.048
313	A	1	1	1.00	21	0.048
314	A	1	1	1.00	21	0.048
315	A	1	1	1.00	21	0.048
316	A	1	1	1.00	12	0.083
317	A	1	1	1.00	21	0.048
318	A	1	1	1.00	21	0.048
319	A	1	1	1.00	21	0.048
320	A	1	1	1.00	21	0.048
321	A	1	1	1.00	12	0.083
322	A	1	1	1.00	21	0.048
323	A	1	1	1.00	21	0.048
324	A	8	8	1.00	21	0.381
325	A	11	7	1.00	21	0.333
326	A	12	8	1.00	21	0.381
327	A	9	9	1.00	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
328	A	9	9	1.00	21	0.429
329	A	8	8	1.00	21	0.381
330	A	11	7	1.00	21	0.333
331	A	12	8	1.00	21	0.381
332	A	9	9	1.00	21	0.429
333	A	9	9	1.00	21	0.429
334	A	1	1	1.00	13	0.077
335	A	1	1	1.00	13	0.077
336	A	1	1	1.00	17	0.059
337	A	1	1	1.00	19	0.053
338	A	1	1	1.00	19	0.053
339	A	1	1	1.00	21	0.048
340	A	3	2	1.00	19	0.105
341	A	3	2	1.00	19	0.105
342	A	2	2	1.00	17	0.118
343	A	2	2	1.00	17	0.118
344	A	2	2	1.00	19	0.105
345	A	1	1	1.00	19	0.053
346	A	1	1	1.00	19	0.053
347	A	1	1	1.00	10	0.100
348	A	1	1	1.00	19	0.053
349	A	1	1	1.00	19	0.053
350	A	1	1	1.00	23	0.043
351	A	1	1	1.00	23	0.043
352	A	1	1	1.00	23	0.043
353	A	1	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
354	A	1	1	1.00	23	0.043
355	A	3	2	1.00	19	0.105
356	A	3	2	1.00	19	0.105
357	A	2	2	1.00	17	0.118
358	A	2	2	1.00	17	0.118
359	A	2	2	1.00	19	0.105
360	A	2	2	1.00	19	0.105
361	A	1	1	1.00	19	0.053
362	A	1	1	1.00	19	0.053
363	A	1	1	1.00	10	0.100
364	A	1	1	1.00	19	0.053
365	A	1	1	1.00	19	0.053
366	A	1	1	1.00	23	0.043
367	A	1	1	1.00	23	0.043
368	A	1	1	1.00	23	0.043
369	A	1	1	1.00	23	0.043
370	A	1	1	1.00	23	0.043
371	A	3	2	1.00	21	0.095
372	A	3	2	1.00	21	0.095
373	A	3	2	1.00	21	0.095
374	A	2	2	1.00	19	0.105
375	A	5	5	1.00	19	0.263
376	A	6	6	1.00	21	0.286
377	A	7	6	1.00	21	0.286
378	A	5	3	1.00	21	0.143
379	A	4	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
380	A	3	3	1.00	21	0.143
381	A	2	2	1.00	12	0.167
382	A	3	3	1.00	21	0.143
383	A	4	3	1.00	21	0.143
384	A	5	3	1.00	21	0.143
385	A	3	2	1.00	21	0.095
386	A	3	2	1.00	21	0.095
387	A	3	2	1.00	21	0.095
388	A	2	2	1.00	19	0.105
389	A	6	6	1.00	19	0.316
390	A	7	7	1.00	21	0.333
391	A	5	4	1.00	21	0.190
392	A	4	4	1.00	21	0.190
393	A	3	3	1.00	21	0.143
394	A	3	3	1.00	12	0.250
395	A	4	4	1.00	21	0.190
396	A	5	4	1.00	21	0.190
397	A	3	2	1.00	21	0.095
398	A	3	2	1.00	21	0.095
399	A	3	2	1.00	21	0.095
400	A	2	2	1.00	19	0.105
401	A	6	6	1.00	19	0.316
402	A	7	7	1.00	21	0.333
403	A	8	7	1.00	21	0.333
404	A	5	4	1.00	21	0.190
405	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	3	3	1.00	21	0.143
407	A	3	3	1.00	12	0.250
408	A	4	4	1.00	21	0.190
409	A	5	4	1.00	21	0.190
410	A	3	2	1.00	21	0.095
411	A	3	2	1.00	21	0.095
412	A	3	2	1.00	21	0.095
413	A	2	2	1.00	19	0.105
414	A	5	5	1.00	19	0.263
415	A	6	6	1.00	21	0.286
416	A	7	6	1.00	21	0.286
417	A	5	3	1.00	21	0.143
418	A	4	3	1.00	21	0.143
419	A	3	3	1.00	21	0.143
420	A	2	2	1.00	12	0.167
421	A	3	3	1.00	21	0.143
422	A	4	3	1.00	21	0.143
423	A	5	3	1.00	21	0.143
424	A	3	2	1.00	21	0.095
425	A	3	2	1.00	21	0.095
426	A	3	2	1.00	21	0.095
427	A	2	2	1.00	19	0.105
428	A	6	6	1.00	19	0.316
429	A	6	6	1.00	21	0.286
430	A	7	7	1.00	21	0.333
431	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
432	A	4	4	1.00	21	0.190
433	A	3	3	1.00	12	0.250
434	A	3	3	1.00	21	0.143
435	A	4	4	1.00	21	0.190
436	A	5	4	1.00	21	0.190
437	A	3	2	1.00	21	0.095
438	A	3	2	1.00	21	0.095
439	A	3	2	1.00	21	0.095
440	A	2	2	1.00	19	0.105
441	A	6	6	1.00	19	0.316
442	A	6	6	1.00	21	0.286
443	A	7	7	1.00	21	0.333
444	A	5	4	1.00	21	0.190
445	A	4	4	1.00	21	0.190
446	A	3	3	1.00	12	0.250
447	A	3	3	1.00	21	0.143
448	A	4	4	1.00	21	0.190
449	A	5	4	1.00	21	0.190
450	A	13	9	1.00	25	0.360
451	A	12	9	1.00	25	0.360
452	A	11	8	1.00	25	0.320
453	A	1	1	1.00	25	0.040
454	A	2	2	1.00	25	0.080
455	A	3	2	1.00	25	0.080
456	A	5	4	1.00	25	0.160
457	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
458	A	3	3	1.00	25	0.120
459	A	4	4	1.00	25	0.160
460	A	5	4	1.00	25	0.160
461	A	5	4	1.00	23	0.174
462	A	4	4	1.00	23	0.174
463	A	3	3	1.00	23	0.130
464	A	4	4	1.00	23	0.174
465	A	5	4	1.00	23	0.174
466	A	12	9	1.00	23	0.391
467	A	11	8	1.00	23	0.348
468	A	1	1	1.00	23	0.043
469	A	2	2	1.00	23	0.087
470	A	3	2	1.00	23	0.087
471	A	4	2	1.00	23	0.087
472	A	14	10	1.00	25	0.400
473	A	13	10	1.00	25	0.400
474	A	12	9	1.00	25	0.360
475	A	12	9	1.00	25	0.360
476	A	1	1	1.00	25	0.040
477	A	6	5	1.00	25	0.200
478	A	5	5	1.00	25	0.200
479	A	4	4	1.00	25	0.160
480	A	4	4	1.00	25	0.160
481	A	5	5	1.00	25	0.200
482	A	6	5	1.00	25	0.200
483	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
484	A	2	2	1.00	23	0.087
485	A	2	2	1.00	23	0.087
486	A	2	2	1.00	23	0.087
487	A	2	2	1.00	23	0.087
488	A	2	2	1.00	17	0.118
489	A	2	2	1.00	19	0.105
490	A	2	2	1.00	19	0.105
491	A	2	2	1.00	21	0.095
492	A	3	2	1.00	19	0.105
493	A	3	2	1.00	19	0.105
494	A	2	2	1.00	17	0.118
495	A	2	2	1.00	17	0.118
496	A	2	2	1.00	19	0.105
497	A	2	2	1.00	19	0.105
498	A	2	2	1.00	19	0.105
499	A	2	2	1.00	19	0.105
500	A	2	2	1.00	10	0.200
501	A	2	2	1.00	19	0.105
502	A	2	2	1.00	19	0.105
503	A	2	2	1.00	23	0.087
504	A	2	2	1.00	23	0.087
505	A	2	2	1.00	23	0.087
506	A	2	2	1.00	23	0.087
507	A	5	4	1.00	21	0.190
508	A	4	4	1.00	21	0.190
509	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
510	A	3	3	1.00	19	0.158
511	A	2	2	1.00	12	0.167
512	A	4	4	1.00	19	0.210
513	A	4	4	1.00	21	0.190
514	A	5	4	1.00	21	0.190
515	A	5	4	1.00	21	0.190
516	A	4	4	1.00	21	0.190
517	A	4	4	1.00	21	0.190
518	A	3	3	1.00	21	0.143
519	A	3	3	1.00	19	0.158
520	A	3	3	1.00	12	0.250
521	A	4	4	1.00	19	0.210
522	A	5	4	1.00	21	0.190
523	A	5	4	1.00	21	0.190
524	A	4	4	1.00	21	0.190
525	A	4	4	1.00	19	0.210
526	A	2	2	1.00	12	0.167
527	A	3	3	1.00	19	0.158
528	A	4	4	1.00	21	0.190
529	A	4	4	1.00	21	0.190
530	A	5	4	1.00	21	0.190
531	A	4	4	1.00	19	0.210
532	A	3	3	1.00	12	0.250
533	A	3	3	1.00	19	0.158
534	A	3	3	1.00	21	0.143
535	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
536	A	4	4	1.00	21	0.190
537	A	5	4	1.00	21	0.190
538	A	2	2	1.00	21	0.095

Chapter 3

Listing of integrals

3.1 $\int \sin(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\cos(a+bx)}{b}$$

```
[Out] -cos(b*x+a)/b
```

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2638}

$$-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x],x]
```

```
[Out] -(Cos[a + b*x]/b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(a+bx)\,dx = -\frac{\cos(a+bx)}{b}$$

Mathematica [A] time = 0.08, size = 22, normalized size = 2.00

$$\frac{\sin(a)\sin(bx)}{b} - \frac{\cos(a)\cos(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x],x]
[Out] -((Cos[a]*Cos[b*x])/b) + (Sin[a]*Sin[b*x])/b
```

fricas [A] time = 0.43, size = 11, normalized size = 1.00

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="fricas")
[Out] -cos(b*x + a)/b
```

giac [A] time = 0.29, size = 11, normalized size = 1.00

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="giac")
[Out] -cos(b*x + a)/b
```

maple [A] time = 0.05, size = 12, normalized size = 1.09

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a),x)
[Out] -cos(b*x+a)/b
```

maxima [A] time = 0.90, size = 11, normalized size = 1.00

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -cos(b*x + a)/b
```

mupad [B] time = 0.02, size = 11, normalized size = 1.00

$$-\frac{\cos(a + b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x),x)
```

```
[Out] -cos(a + b*x)/b
```

sympy [A] time = 0.17, size = 14, normalized size = 1.27

$$\begin{cases} -\frac{\cos(a+bx)}{b} & \text{for } b \neq 0 \\ x \sin(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x)
```

```
[Out] Piecewise((-cos(a + b*x)/b, Ne(b, 0)), (x*sin(a), True))
```

3.2 $\int \sin^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

```
[Out] 1/2*x-1/2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2,x]
```

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\,dx &= -\frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.92

$$-\frac{\sin(2(a+bx))-2(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2,x]
```

```
[Out] -1/4*(-2*(a + b*x) + Sin[2*(a + b*x)])/b
```

fricas [A] time = 0.43, size = 23, normalized size = 0.92

$$\frac{bx - \cos\left(bx + a\right)\sin\left(bx + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

giac [A] time = 0.14, size = 18, normalized size = 0.72

$$\frac{1}{2}\,x - \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

maple [A] time = 0.04, size = 27, normalized size = 1.08

$$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

maxima [A] time = 0.33, size = 24, normalized size = 0.96

$$\frac{2\,bx + 2\,a - \sin\left(2\,bx + 2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

mupad [B] time = 0.43, size = 18, normalized size = 0.72

$$\frac{x}{2} - \frac{\sin\left(2\,a + 2\,b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2,x)
```

```
[Out] x/2 - sin(2*a + 2*b*x)/(4*b)
```

sympy [A] time = 0.22, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sin^2\left(a+bx\right)}{2} + \frac{x\cos^2\left(a+bx\right)}{2} - \frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b} & \text{for}\ b \neq 0 \\ x\sin^2\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

3.3 $\int \sin^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

```
[Out] -cos(b*x+a)/b+1/3*cos(b*x+a)^3/b
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3,x]
```

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.07

$$\frac{\cos(3(a+bx))}{12b}-\frac{3\cos(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b)
```

fricas [A] time = 0.43, size = 22, normalized size = 0.81

$$\frac{\cos(bx+a)^3 - 3\cos(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

giac [A] time = 0.13, size = 25, normalized size = 0.93

$$\frac{\cos(bx+a)^3}{3b} - \frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

maple [A] time = 0.12, size = 22, normalized size = 0.81

$$-\frac{\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3,x)
[Out] -1/3/b*(2+sin(b*x+a)^2)*cos(b*x+a)
```

maxima [A] time = 0.33, size = 22, normalized size = 0.81

$$\frac{\cos(bx+a)^3 - 3\cos(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="maxima")
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

mupad [B] time = 0.35, size = 24, normalized size = 0.89

$$-\frac{3\cos(a+bx) - \cos(a+bx)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3,x)
```

```
[Out] -(3*cos(a + b*x) - cos(a + b*x)^3)/(3*b)
```

sympy [A] time = 0.48, size = 37, normalized size = 1.37

$$\begin{cases} -\frac{\sin^{2}(a+bx)\cos(a+bx)}{b} - \frac{2\cos^{3}(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin^{3}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)/b - 2*cos(a + b*x)**3/(3*b), Ne(b,
 0)), (x*sin(a)**3, True))
```

3.4 $\int \sin^4(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}-\frac{3\sin(a+bx)\cos(a+bx)}{8b}+\frac{3x}{8}$$

```
[Out] 3/8*x-3/8*cos(b*x+a)*sin(b*x+a)/b-1/4*cos(b*x+a)*sin(b*x+a)^3/b
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}-\frac{3\sin(a+bx)\cos(a+bx)}{8b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[a + b*x]*Sin[a + b*x])/(8*b) - (Cos[a + b*x]*Sin[a + b*x]^
3)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
\int \sin^4(a+bx)\,dx &= -\frac{\cos(a+bx)\sin^3(a+bx)}{4b}+\frac{3}{4}\int \sin^2(a+bx)\,dx\\
&= -\frac{3\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos(a+bx)\sin^3(a+bx)}{4b}+\frac{3\int 1\,dx}{8}\\
&= \frac{3x}{8}-\frac{3\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos(a+bx)\sin^3(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 0.72

$$\frac{12(a+bx)-8\sin(2(a+bx))+\sin(4(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4,x]
```

```
[Out] (12*(a + b*x) - 8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])/(32*b)
```

fricas [A] time = 0.42, size = 36, normalized size = 0.78

$$\frac{3\,bx+\left(2\,\cos\left(bx+a\right)^{3}-5\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*b*x + (2*cos(b*x + a)^3 - 5*cos(b*x + a))*sin(b*x + a))/b
```

giac [A] time = 0.14, size = 32, normalized size = 0.70

$$\frac{3}{8}\,x+\frac{\sin\left(4\,bx+4\,a\right)}{32\,b}-\frac{\sin\left(2\,bx+2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*b*x + 4*a)/b - 1/4*sin(2*b*x + 2*a)/b
```

maple [A] time = 0.14, size = 38, normalized size = 0.83

$$\frac{-\frac{\left(\sin^{3}(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)
```

maxima [A] time = 0.32, size = 33, normalized size = 0.72

$$\frac{12\,bx+12\,a+\sin\left(4\,bx+4\,a\right)-8\,\sin\left(2\,bx+2\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/32*(12*b*x + 12*a + sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))/b
```

mupad [B] time = 0.47, size = 50, normalized size = 1.09

$$\frac{3\,x}{8}-\frac{\frac{5\,{\tan\left(a+b\,x\right)}^3}{8}+\frac{3\,\tan\left(a+b\,x\right)}{8}}{b\,\left({\tan\left(a+b\,x\right)}^4+2\,{\tan\left(a+b\,x\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4,x)
```

```
[Out] (3*x)/8 - ((3*tan(a + b*x))/8 + (5*tan(a + b*x)^3)/8)/(b*(2*tan(a + b*x)^2
+ tan(a + b*x)^4 + 1))
```

sympy [A] time = 1.08, size = 95, normalized size = 2.07

$$\begin{cases} \frac{3x\sin^4(a+bx)}{8}+\frac{3x\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{3x\cos^4(a+bx)}{8}-\frac{5\sin^3(a+bx)\cos(a+bx)}{8b}-\frac{3\sin(a+bx)\cos^3(a+bx)}{8b} & \text{for } b\neq 0 \\ x\sin^4(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**4/8 + 3*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
3*x*cos(a + b*x)**4/8 - 5*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*sin(a + b*
x)*cos(a + b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**4, True))
```

3.5 $\int \sin^5(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\cos^5(a+bx)}{5b}+\frac{2\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

```
[Out] -cos(b*x+a)/b+2/3*cos(b*x+a)^3/b-1/5*cos(b*x+a)^5/b
```

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$-\frac{\cos^5(a+bx)}{5b}+\frac{2\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^5,x]
[Out] -(Cos[a + b*x]/b) + (2*Cos[a + b*x]^3)/(3*b) - Cos[a + b*x]^5/(5*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^5(a+bx)\,dx&=-\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\cos(a+bx)\right)}{b}\\&=-\frac{\cos(a+bx)}{b}+\frac{2\cos^3(a+bx)}{3b}-\frac{\cos^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 1.05

$$-\frac{5\cos(a+bx)}{8b}+\frac{5\cos(3(a+bx))}{48b}-\frac{\cos(5(a+bx))}{80b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^5,x]
```

```
[Out] (-5*Cos[a + b*x])/(8*b) + (5*Cos[3*(a + b*x)])/(48*b) - Cos[5*(a + b*x)]/(8
0*b)
```

fricas [A] time = 0.41, size = 34, normalized size = 0.81

$$-\frac{3 \cos\left(bx+a\right)^{5}-10 \cos\left(bx+a\right)^{3}+15 \cos\left(bx+a\right)}{15\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(b*x + a)^5 - 10*cos(b*x + a)^3 + 15*cos(b*x + a))/b
```

giac [A] time = 0.14, size = 38, normalized size = 0.90

$$-\frac{\cos\left(bx+a\right)^{5}}{5\, b}+\frac{2 \cos\left(bx+a\right)^{3}}{3\, b}-\frac{\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/5*cos(b*x + a)^5/b + 2/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

maple [A] time = 0.10, size = 32, normalized size = 0.76

$$-\frac{\left(\frac{8}{3}+\sin^{4}\left(bx+a\right)+\frac{4\left(\sin^{2}\left(bx+a\right)\right)}{3}\right)\cos\left(bx+a\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^5,x)
```

```
[Out] -1/5/b*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)
```

maxima [A] time = 0.32, size = 34, normalized size = 0.81

$$-\frac{3 \cos\left(bx+a\right)^{5}-10 \cos\left(bx+a\right)^{3}+15 \cos\left(bx+a\right)}{15\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/15*(3*cos(b*x + a)^5 - 10*cos(b*x + a)^3 + 15*cos(b*x + a))/b
```

mupad [B] time = 0.36, size = 32, normalized size = 0.76

$$-\frac{\frac{\cos(a+b\,x)^5}{5}-\frac{2\cos(a+b\,x)^3}{3}+\cos\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5,x)
[Out] -(cos(a + b*x) - (2*cos(a + b*x)^3)/3 + cos(a + b*x)^5/5)/b
```

sympy [A] time = 1.88, size = 60, normalized size = 1.43

$$\begin{cases} -\frac{\sin^4(a+bx)\cos(a+bx)}{b}-\frac{4\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{8\cos^5(a+bx)}{15b} & \text{for } b\neq 0 \\ x\sin^5(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**5,x)
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)/b - 4*sin(a + b*x)**2*cos(a + b*x)
**3/(3*b) - 8*cos(a + b*x)**5/(15*b), Ne(b, 0)), (x*sin(a)**5, True))
```

3.6 $\int \sin^6(a+bx)\,dx$

Optimal. Leaf size=67

$$-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}-\frac{5\sin^3(a+bx)\cos(a+bx)}{24b}-\frac{5\sin(a+bx)\cos(a+bx)}{16b}+\frac{5x}{16}$$

```
[Out] 5/16*x-5/16*cos(b*x+a)*sin(b*x+a)/b-5/24*cos(b*x+a)*sin(b*x+a)^3/b-1/6*cos(
b*x+a)*sin(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$-\frac{\sin^5(a+bx)\cos(a+bx)}{6b}-\frac{5\sin^3(a+bx)\cos(a+bx)}{24b}-\frac{5\sin(a+bx)\cos(a+bx)}{16b}+\frac{5x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[a + b*x]*Sin[a + b*x])/(16*b) - (5*Cos[a + b*x]*Sin[a + b
*x]^3)/(24*b) - (Cos[a + b*x]*Sin[a + b*x]^5)/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \sin^6(a+bx)\,dx = -\frac{\cos(a+bx)\sin^5(a+bx)}{6b} + \frac{5}{6}\int \sin^4(a+bx)\,dx$$
$$= -\frac{5\cos(a+bx)\sin^3(a+bx)}{24b} - \frac{\cos(a+bx)\sin^5(a+bx)}{6b} + \frac{5}{8}\int \sin^2(a+bx)\,dx$$
$$= -\frac{5\cos(a+bx)\sin(a+bx)}{16b} - \frac{5\cos(a+bx)\sin^3(a+bx)}{24b} - \frac{\cos(a+bx)\sin^5(a+bx)}{6b} + \frac{5\int}{1}$$
$$= \frac{5x}{16} - \frac{5\cos(a+bx)\sin(a+bx)}{16b} - \frac{5\cos(a+bx)\sin^3(a+bx)}{24b} - \frac{\cos(a+bx)\sin^5(a+bx)}{6b}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.67

$$\frac{-45\sin(2(a+bx)) + 9\sin(4(a+bx)) - \sin(6(a+bx)) + 60a + 60bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^6,x]
```

```
[Out] (60*a + 60*b*x - 45*Sin[2*(a + b*x)] + 9*Sin[4*(a + b*x)] - Sin[6*(a + b*x)
])/(192*b)
```

fricas [A] time = 0.43, size = 47, normalized size = 0.70

$$\frac{15\,bx - \left(8\,\cos\left(bx+a\right)^5 - 26\,\cos\left(bx+a\right)^3 + 33\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(15*b*x - (8*cos(b*x + a)^5 - 26*cos(b*x + a)^3 + 33*cos(b*x + a))*sin
(b*x + a))/b
```

giac [A] time = 0.12, size = 46, normalized size = 0.69

$$\frac{5}{16}\,x - \frac{\sin\left(6\,bx + 6\,a\right)}{192\,b} + \frac{3\,\sin\left(4\,bx + 4\,a\right)}{64\,b} - \frac{15\,\sin\left(2\,bx + 2\,a\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 5/16*x - 1/192*sin(6*b*x + 6*a)/b + 3/64*sin(4*b*x + 4*a)/b - 15/64*sin(2*b
*x + 2*a)/b
```

maple [A] time = 0.10, size = 48, normalized size = 0.72

$$\frac{-\frac{\left(\sin^5(bx+a)+\frac{5\left(\sin^3(bx+a)\right)}{4}+\frac{15\sin(bx+a)}{8}\right)\cos(bx+a)}{6}+\frac{5bx}{16}+\frac{5a}{16}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^6,x)
```

```
[Out] 1/b*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b
*x+5/16*a)
```

maxima [A] time = 0.32, size = 48, normalized size = 0.72

$$\frac{4\,\sin\left(2\,bx+2\,a\right)^3+60\,bx+60\,a+9\,\sin\left(4\,bx+4\,a\right)-48\,\sin\left(2\,bx+2\,a\right)}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 1/192*(4*sin(2*b*x + 2*a)^3 + 60*b*x + 60*a + 9*sin(4*b*x + 4*a) - 48*sin(2
*b*x + 2*a))/b
```

mupad [B] time = 0.58, size = 43, normalized size = 0.64

$$\frac{5\,x}{16}-\frac{\frac{15\,\sin(2\,a+2\,b\,x)}{64}-\frac{3\,\sin(4\,a+4\,b\,x)}{64}+\frac{\sin(6\,a+6\,b\,x)}{192}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^6,x)
```

```
[Out] (5*x)/16 - ((15*sin(2*a + 2*b*x))/64 - (3*sin(4*a + 4*b*x))/64 + sin(6*a +
6*b*x)/192)/b
```

sympy [A] time = 3.49, size = 139, normalized size = 2.07

$$\begin{cases}\frac{5x\sin^6(a+bx)}{16}+\frac{15x\sin^4(a+bx)\cos^2(a+bx)}{16}+\frac{15x\sin^2(a+bx)\cos^4(a+bx)}{16}+\frac{5x\cos^6(a+bx)}{16}-\frac{11\sin^5(a+bx)\cos(a+bx)}{16b}-\frac{5\sin^3(a+bx)\cos^3}{6b} \\ x\sin^6(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**6,x)
```

```
[Out] Piecewise((5*x*sin(a + b*x)**6/16 + 15*x*sin(a + b*x)**4*cos(a + b*x)**2/16
 + 15*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + 5*x*cos(a + b*x)**6/16 - 11*si
n(a + b*x)**5*cos(a + b*x)/(16*b) - 5*sin(a + b*x)**3*cos(a + b*x)**3/(6*b)
 - 5*sin(a + b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin(a)**6, True))
```

3.7 $\int \sin^7(a+bx)\,dx$

Optimal. Leaf size=54

$$\frac{\cos^7(a+bx)}{7b}-\frac{3\cos^5(a+bx)}{5b}+\frac{\cos^3(a+bx)}{b}-\frac{\cos(a+bx)}{b}$$

```
[Out] -cos(b*x+a)/b+cos(b*x+a)^3/b-3/5*cos(b*x+a)^5/b+1/7*cos(b*x+a)^7/b
```

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\cos^7(a+bx)}{7b}-\frac{3\cos^5(a+bx)}{5b}+\frac{\cos^3(a+bx)}{b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^7,x]
```

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/b - (3*Cos[a + b*x]^5)/(5*b) + Cos[a + b
*x]^7/(7*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^7(a+bx)\,dx &= -\frac{\text{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{b}-\frac{3\cos^5(a+bx)}{5b}+\frac{\cos^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.09

$$-\frac{35\cos(a+bx)}{64b}+\frac{7\cos(3(a+bx))}{64b}-\frac{7\cos(5(a+bx))}{320b}+\frac{\cos(7(a+bx))}{448b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^7,x]
```

```
[Out] (-35*Cos[a + b*x])/(64*b) + (7*Cos[3*(a + b*x)])/(64*b) - (7*Cos[5*(a + b*x
)])/(320*b) + Cos[7*(a + b*x)]/(448*b)
```

fricas [A] time = 0.43, size = 44, normalized size = 0.81

$$\frac{5 \cos\left(bx+a\right)^{7}-21 \cos\left(bx+a\right)^{5}+35 \cos\left(bx+a\right)^{3}-35 \cos\left(bx+a\right)}{35\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(b*x + a)^7 - 21*cos(b*x + a)^5 + 35*cos(b*x + a)^3 - 35*cos(b*x
 + a))/b
```

giac [A] time = 0.12, size = 50, normalized size = 0.93

$$\frac{\cos\left(bx+a\right)^{7}}{7\, b}-\frac{3 \cos\left(bx+a\right)^{5}}{5\, b}+\frac{\cos\left(bx+a\right)^{3}}{b}-\frac{\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/7*cos(b*x + a)^7/b - 3/5*cos(b*x + a)^5/b + cos(b*x + a)^3/b - cos(b*x +
a)/b
```

maple [A] time = 0.10, size = 42, normalized size = 0.78

$$-\frac{\left(\frac{16}{5}+\sin^{6}\left(bx+a\right)+\frac{6\left(\sin^{4}\left(bx+a\right)\right)}{5}+\frac{8\left(\sin^{2}\left(bx+a\right)\right)}{5}\right)\cos\left(bx+a\right)}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^7,x)
```

```
[Out] -1/7/b*(16/5+sin(b*x+a)^6+6/5*sin(b*x+a)^4+8/5*sin(b*x+a)^2)*cos(b*x+a)
```

maxima [A] time = 0.33, size = 44, normalized size = 0.81

$$\frac{5 \cos\left(bx+a\right)^{7}-21 \cos\left(bx+a\right)^{5}+35 \cos\left(bx+a\right)^{3}-35 \cos\left(bx+a\right)}{35\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/35*(5*cos(b*x + a)^7 - 21*cos(b*x + a)^5 + 35*cos(b*x + a)^3 - 35*cos(b*x
 + a))/b
```

mupad [B] time = 0.38, size = 43, normalized size = 0.80

$$\frac{\cos\left(a+b\,x\right)\,\left(5\cos\left(a+b\,x\right)^6-21\cos\left(a+b\,x\right)^4+35\cos\left(a+b\,x\right)^2-35\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^7,x)
```

```
[Out] (cos(a + b*x)*(35*cos(a + b*x)^2 - 21*cos(a + b*x)^4 + 5*cos(a + b*x)^6 - 3
5))/(35*b)
```

sympy [A] time = 6.08, size = 80, normalized size = 1.48

$$\begin{cases} -\frac{\sin^{6}\left(a+bx\right)\cos\left(a+bx\right)}{b}-\frac{2\sin^{4}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{b}-\frac{8\sin^{2}\left(a+bx\right)\cos^{5}\left(a+bx\right)}{5b}-\frac{16\cos^{7}\left(a+bx\right)}{35b} & \text{for}\ b\neq 0 \\ x\sin^{7}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**7,x)
```

```
[Out] Piecewise((-sin(a + b*x)**6*cos(a + b*x)/b - 2*sin(a + b*x)**4*cos(a + b*x)
**3/b - 8*sin(a + b*x)**2*cos(a + b*x)**5/(5*b) - 16*cos(a + b*x)**7/(35*b)
, Ne(b, 0)), (x*sin(a)**7, True))
```

3.8 $\int \sin^8(a+bx)\,dx$

Optimal. Leaf size=88

$$-\frac{\sin^7(a+bx)\cos(a+bx)}{8b}-\frac{7\sin^5(a+bx)\cos(a+bx)}{48b}-\frac{35\sin^3(a+bx)\cos(a+bx)}{192b}-\frac{35\sin(a+bx)\cos(a+bx)}{128b}$$

```
[Out] 35/128*x-35/128*cos(b*x+a)*sin(b*x+a)/b-35/192*cos(b*x+a)*sin(b*x+a)^3/b-7/
48*cos(b*x+a)*sin(b*x+a)^5/b-1/8*cos(b*x+a)*sin(b*x+a)^7/b
```

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$-\frac{\sin^7(a+bx)\cos(a+bx)}{8b}-\frac{7\sin^5(a+bx)\cos(a+bx)}{48b}-\frac{35\sin^3(a+bx)\cos(a+bx)}{192b}-\frac{35\sin(a+bx)\cos(a+bx)}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^8,x]
```

```
[Out] (35*x)/128 - (35*Cos[a + b*x]*Sin[a + b*x])/(128*b) - (35*Cos[a + b*x]*Sin[
a + b*x]^3)/(192*b) - (7*Cos[a + b*x]*Sin[a + b*x]^5)/(48*b) - (Cos[a + b*x
]*Sin[a + b*x]^7)/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \sin^8(a+bx)\,dx = -\frac{\cos(a+bx)\sin^7(a+bx)}{8b} + \frac{7}{8}\int \sin^6(a+bx)\,dx$$
$$= -\frac{7\cos(a+bx)\sin^5(a+bx)}{48b} - \frac{\cos(a+bx)\sin^7(a+bx)}{8b} + \frac{35}{48}\int \sin^4(a+bx)\,dx$$
$$= -\frac{35\cos(a+bx)\sin^3(a+bx)}{192b} - \frac{7\cos(a+bx)\sin^5(a+bx)}{48b} - \frac{\cos(a+bx)\sin^7(a+bx)}{8b} + \frac{35}{64}$$
$$= -\frac{35\cos(a+bx)\sin(a+bx)}{128b} - \frac{35\cos(a+bx)\sin^3(a+bx)}{192b} - \frac{7\cos(a+bx)\sin^5(a+bx)}{48b} - \mathrm{c}$$
$$= \frac{35x}{128} - \frac{35\cos(a+bx)\sin(a+bx)}{128b} - \frac{35\cos(a+bx)\sin^3(a+bx)}{192b} - \frac{7\cos(a+bx)\sin^5(a+bx}{48b}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.62

$$\frac{-672\sin(2(a+bx)) + 168\sin(4(a+bx)) - 32\sin(6(a+bx)) + 3\sin(8(a+bx)) + 840a + 840bx}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^8,x]
```

```
[Out] (840*a + 840*b*x - 672*Sin[2*(a + b*x)] + 168*Sin[4*(a + b*x)] - 32*Sin[6*(
a + b*x)] + 3*Sin[8*(a + b*x)])/(3072*b)
```

fricas [A] time = 0.45, size = 56, normalized size = 0.64

$$\frac{105\,bx + \left(48\,\cos\left(bx+a\right)^7 - 200\,\cos\left(bx+a\right)^5 + 326\,\cos\left(bx+a\right)^3 - 279\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/384*(105*b*x + (48*cos(b*x + a)^7 - 200*cos(b*x + a)^5 + 326*cos(b*x + a)
^3 - 279*cos(b*x + a))*sin(b*x + a))/b
```

giac [A] time = 0.13, size = 60, normalized size = 0.68

$$\frac{35}{128}\,x + \frac{\sin\left(8\,bx + 8\,a\right)}{1024\,b} - \frac{\sin\left(6\,bx + 6\,a\right)}{96\,b} + \frac{7\,\sin\left(4\,bx + 4\,a\right)}{128\,b} - \frac{7\,\sin\left(2\,bx + 2\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 35/128*x + 1/1024*sin(8*b*x + 8*a)/b - 1/96*sin(6*b*x + 6*a)/b + 7/128*sin(
4*b*x + 4*a)/b - 7/32*sin(2*b*x + 2*a)/b
```

maple [A] time = 0.10, size = 58, normalized size = 0.66

$$\frac{-\frac{\left(\sin^7(bx+a)+\frac{7\left(\sin^5(bx+a)\right)}{6}+\frac{35\left(\sin^3(bx+a)\right)}{24}+\frac{35\sin(bx+a)}{16}\right)\cos(bx+a)}{8}+\frac{35bx}{128}+\frac{35a}{128}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^8,x)
```

```
[Out] 1/b*(-1/8*(sin(b*x+a)^7+7/6*sin(b*x+a)^5+35/24*sin(b*x+a)^3+35/16*sin(b*x+a
))*cos(b*x+a)+35/128*b*x+35/128*a)
```

maxima [A] time = 0.34, size = 59, normalized size = 0.67

$$\frac{128\sin(2\,bx+2\,a)^3+840\,bx+840\,a+3\sin(8\,bx+8\,a)+168\sin(4\,bx+4\,a)-768\sin(2\,bx+2\,a)}{3072\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] 1/3072*(128*sin(2*b*x + 2*a)^3 + 840*b*x + 840*a + 3*sin(8*b*x + 8*a) + 168
*sin(4*b*x + 4*a) - 768*sin(2*b*x + 2*a))/b
```

mupad [B] time = 1.50, size = 90, normalized size = 1.02

$$\frac{35\,x}{128}-\frac{\frac{93\tan(a+b\,x)^7}{128}+\frac{511\tan(a+b\,x)^5}{384}+\frac{385\tan(a+b\,x)^3}{384}+\frac{35\tan(a+b\,x)}{128}}{b\left(\tan(a+b\,x)^8+4\tan(a+b\,x)^6+6\tan(a+b\,x)^4+4\tan(a+b\,x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^8,x)
```

```
[Out] (35*x)/128 - ((35*tan(a + b*x))/128 + (385*tan(a + b*x)^3)/384 + (511*tan(a
 + b*x)^5)/384 + (93*tan(a + b*x)^7)/128)/(b*(4*tan(a + b*x)^2 + 6*tan(a +
b*x)^4 + 4*tan(a + b*x)^6 + tan(a + b*x)^8 + 1))
```

sympy [A] time = 10.28, size = 184, normalized size = 2.09

$$\begin{cases}\frac{35x\sin^8(a+bx)}{128}+\frac{35x\sin^6(a+bx)\cos^2(a+bx)}{32}+\frac{105x\sin^4(a+bx)\cos^4(a+bx)}{64}+\frac{35x\sin^2(a+bx)\cos^6(a+bx)}{32}+\frac{35x\cos^8(a+bx)}{128}-\frac{93\sin^7(a}{} \\ x\sin^8(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**8,x)
```

```
[Out] Piecewise((35*x*sin(a + b*x)**8/128 + 35*x*sin(a + b*x)**6*cos(a + b*x)**2/
32 + 105*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 35*x*sin(a + b*x)**2*cos(a
+ b*x)**6/32 + 35*x*cos(a + b*x)**8/128 - 93*sin(a + b*x)**7*cos(a + b*x)/(
128*b) - 511*sin(a + b*x)**5*cos(a + b*x)**3/(384*b) - 385*sin(a + b*x)**3*
cos(a + b*x)**5/(384*b) - 35*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)
), (x*sin(a)**8, True))
```

3.9 $\int \sin^{\frac{7}{2}}(bx)\,dx$

Optimal. Leaf size=60

$$-\frac{10F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{21b}-\frac{2\sin^{\frac{5}{2}}(bx)\cos(bx)}{7b}-\frac{10\sqrt{\sin(bx)}\,\cos(bx)}{21b}$$

```
[Out] -10/21*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticF(cos(1/4*
Pi+1/2*b*x),2^(1/2))/b-2/7*cos(b*x)*sin(b*x)^(5/2)/b-10/21*cos(b*x)*sin(b*x
)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 2641}

$$-\frac{10F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{21b}-\frac{2\sin^{\frac{5}{2}}(bx)\cos(bx)}{7b}-\frac{10\sqrt{\sin(bx)}\,\cos(bx)}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*x]^(7/2),x]
```

```
[Out] (-10*EllipticF[Pi/4 - (b*x)/2, 2])/(21*b) - (10*Cos[b*x]*Sqrt[Sin[b*x]])/(2
1*b) - (2*Cos[b*x]*Sin[b*x]^(5/2))/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^{\frac{7}{2}}(bx)\,dx &= -\frac{2\cos(bx)\sin^{\frac{5}{2}}(bx)}{7b} + \frac{5}{7}\int \sin^{\frac{3}{2}}(bx)\,dx \\ &= -\frac{10\cos(bx)\sqrt{\sin(bx)}}{21b} - \frac{2\cos(bx)\sin^{\frac{5}{2}}(bx)}{7b} + \frac{5}{21}\int \frac{1}{\sqrt{\sin(bx)}}\,dx \\ &= -\frac{10F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{21b} - \frac{10\cos(bx)\sqrt{\sin(bx)}}{21b} - \frac{2\cos(bx)\sin^{\frac{5}{2}}(bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.20, size = 45, normalized size = 0.75

$$\frac{\sqrt{\sin(bx)}\,(3\cos(3bx)-23\cos(bx))-20F\left(\frac{1}{4}(\pi-2bx)\middle|2\right)}{42b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*x]^(7/2),x]
[Out] (-20*EllipticF[(Pi - 2*b*x)/4, 2] + (-23*Cos[b*x] + 3*Cos[3*b*x])*Sqrt[Sin[
b*x]])/(42*b)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(bx)^2-1\right)\sin(bx)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(7/2),x, algorithm="fricas")
[Out] integral(-(cos(b*x)^2 - 1)*sin(b*x)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(bx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(7/2),x, algorithm="giac")
[Out] integrate(sin(b*x)^(7/2), x)
```

maple [A] time = 0.17, size = 84, normalized size = 1.40

$$\frac{\frac{2\sin(bx)\left(\cos^4(bx)\right)}{7}+\frac{5\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,\operatorname{EllipticF}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)}{21}-\frac{16\left(\cos^2(bx)\right)\sin(bx)}{21}}{\cos(bx)\sqrt{\sin(bx)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x)^(7/2),x)
```

```
[Out] (2/7*sin(b*x)*cos(b*x)^4+5/21*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-si
n(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))-16/21*cos(b*x)^2*si
n(b*x))/cos(b*x)/sin(b*x)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(bx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(7/2), x)
```

mupad [B] time = 0.48, size = 34, normalized size = 0.57

$$-\frac{\cos(b\,x)\,\sin(b\,x)^{9/2}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\left(\sin(b\,x)^2\right)^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x)^(7/2),x)
```

```
[Out] -(cos(b*x)*sin(b*x)^(9/2)*hypergeom([-5/4, 1/2], 3/2, cos(b*x)^2))/(b*(sin(
b*x)^2)^(9/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)**(7/2),x)
```

```
[Out] Timed out
```

3.10 $\int \sin^{\frac{5}{2}}(bx)\,dx$

Optimal. Leaf size=41

$$-\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(bx)\cos(bx)}{5b}$$

```
[Out] -6/5*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticE(cos(1/4*Pi
+1/2*b*x),2^(1/2))/b-2/5*cos(b*x)*sin(b*x)^(3/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 2639}

$$-\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(bx)\cos(bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*x]^(5/2),x]
```

```
[Out] (-6*EllipticE[Pi/4 - (b*x)/2, 2])/(5*b) - (2*Cos[b*x]*Sin[b*x]^(3/2))/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^{\frac{5}{2}}(bx)\,dx &= -\frac{2\cos(bx)\sin^{\frac{3}{2}}(bx)}{5b}+\frac{3}{5}\int\sqrt{\sin(bx)}\,dx\\ &= -\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\cos(bx)\sin^{\frac{3}{2}}(bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 35, normalized size = 0.85

$$-\frac{2\left(3E\left(\frac{1}{4}(\pi-2bx)\middle|2\right)+\sin^{\frac{3}{2}}(bx)\cos(bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*x]^(5/2),x]
```

```
[Out] (-2*(3*EllipticE[(Pi - 2*b*x)/4, 2] + Cos[b*x]*Sin[b*x]^(3/2)))/(5*b)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(bx)^2-1\right)\sqrt{\sin(bx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x)^2 - 1)*sqrt(sin(b*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(bx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x)^(5/2), x)
```

maple [B] time = 0.06, size = 118, normalized size = 2.88

$$\frac{\frac{2\left(\sin^4(bx)\right)}{5}-\frac{2\left(\sin^2(bx)\right)}{5}-\frac{6\sqrt{\sin(bx)+1}\;\sqrt{-2\sin(bx)+2}\;\sqrt{-\sin(bx)}\;\text{EllipticE}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{\sin(bx)+1}\;\sqrt{-2\sin(bx)+2}\;\sqrt{-\sin(bx)}\;\text{E}}{5}}{\cos(bx)\sqrt{\sin(bx)}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x)^(5/2),x)
```

```
[Out] (2/5*sin(b*x)^4-2/5*sin(b*x)^2-6/5*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)
*(-sin(b*x))^(1/2)*EllipticE((sin(b*x)+1)^(1/2),1/2*2^(1/2))+3/5*(sin(b*x)+
1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/
2),1/2*2^(1/2)))/cos(b*x)/sin(b*x)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(bx)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(5/2), x)
```

mupad [B] time = 0.41, size = 34, normalized size = 0.83

$$-\frac{\cos(b\,x)\,\sin(b\,x)^{7/2}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\left(\sin(b\,x)^2\right)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x)^(5/2),x)
```

```
[Out] -(cos(b*x)*sin(b*x)^(7/2)*hypergeom([-3/4, 1/2], 3/2, cos(b*x)^2))/(b*(sin(
b*x)^2)^(7/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{\frac{5}{2}}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)**(5/2),x)
```

```
[Out] Integral(sin(b*x)**(5/2), x)
```

3.11 $\int \sin^{\frac{3}{2}}(bx)\,dx$

Optimal. Leaf size=41

$$-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{3b}-\frac{2\sqrt{\sin(bx)}\,\cos(bx)}{3b}$$

```
[Out] -2/3*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticF(cos(1/4*Pi
+1/2*b*x),2^(1/2))/b-2/3*cos(b*x)*sin(b*x)^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 2641}

$$-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{3b}-\frac{2\sqrt{\sin(bx)}\,\cos(bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*x]^(3/2),x]
```

```
[Out] (-2*EllipticF[Pi/4 - (b*x)/2, 2])/(3*b) - (2*Cos[b*x]*Sqrt[Sin[b*x]])/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^{\frac{3}{2}}(bx)\,dx &= -\frac{2\cos(bx)\sqrt{\sin(bx)}}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\sin(bx)}}\,dx\\ &= -\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{3b}-\frac{2\cos(bx)\sqrt{\sin(bx)}}{3b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.80

$$-\frac{2\left(F\left(\frac{1}{4}(\pi-2bx)\middle|2\right)+\sqrt{\sin(bx)}\cos(bx)\right)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[b*x]^(3/2),x]

[Out] (-2*(EllipticF[(Pi - 2*b*x)/4, 2] + Cos[b*x]*Sqrt[Sin[b*x]]))/(3*b)

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin(bx)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x)^(3/2),x, algorithm="fricas")

[Out] integral(sin(b*x)^(3/2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(bx)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x)^(3/2),x, algorithm="giac")

[Out] integrate(sin(b*x)^(3/2), x)

maple [A] time = 0.05, size = 72, normalized size = 1.76

$$\frac{\frac{\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,\text{EllipticF}\left(\sqrt{\sin(bx)+1},\frac{\sqrt{2}}{2}\right)}{3}-\frac{2\left(\cos^2(bx)\right)\sin(bx)}{3}}{\cos(bx)\sqrt{\sin(bx)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(b*x)^(3/2),x)

[Out] (1/3*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))-2/3*cos(b*x)^2*sin(b*x))/cos(b*x)/sin(b*x)^(1/2)/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(bx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(3/2), x)
```

mupad [B] time = 0.40, size = 34, normalized size = 0.83

$$-\frac{\cos(b\,x)\,\sin(b\,x)^{5/2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\left(\sin(b\,x)^2\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x)^(3/2),x)
```

```
[Out] -(cos(b*x)*sin(b*x)^(5/2)*hypergeom([-1/4, 1/2], 3/2, cos(b*x)^2))/(b*(sin(
b*x)^2)^(5/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{\frac{3}{2}}(bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)**(3/2),x)
```

```
[Out] Integral(sin(b*x)**(3/2), x)
```

3.12 $\int \sqrt{\sin(bx)}\,dx$

Optimal. Leaf size=19

$$-\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

[Out] -2*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticE(cos(1/4*Pi+1/2*b*x),2^(1/2))/b

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2639}

$$-\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sin[b*x]],x]

[Out] (-2*EllipticE[Pi/4 - (b*x)/2, 2])/b

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sin(bx)}\,dx = -\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.11

$$-\frac{2E\left(\frac{1}{2}\left(\frac{\pi}{2}-bx\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sin[b*x]],x]

[Out] (-2*EllipticE[(Pi/2 - b*x)/2, 2])/b

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sin\left(bx\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sin(b*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin\left(bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(b*x)), x)
```

maple [A] time = 0.06, size = 77, normalized size = 4.05

$$-\frac{\sqrt{\sin\left(bx\right)+1}\,\sqrt{-2\sin\left(bx\right)+2}\,\sqrt{-\sin\left(bx\right)}\,\left(2\,\text{EllipticE}\left(\sqrt{\sin\left(bx\right)+1}, \frac{\sqrt{2}}{2}\right)-\text{EllipticF}\left(\sqrt{\sin\left(bx\right)+1}, \frac{\sqrt{2}}{2}\right)\right)}{\cos\left(bx\right)\sqrt{\sin\left(bx\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x)^(1/2),x)
```

```
[Out] -(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*(2*EllipticE((s
in(b*x)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2)))/co
s(b*x)/sin(b*x)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin\left(bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(b*x)), x)
```

mupad [B] time = 0.37, size = 15, normalized size = 0.79

$$-\frac{2\,\mathrm{E}\left(\frac{\pi}{4}-\frac{b\,x}{2}\middle|2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x)^(1/2),x)
```

```
[Out] -(2*ellipticE(pi/4 - (b*x)/2, 2))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin{(bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x)**(1/2),x)
```

```
[Out] Integral(sqrt(sin(b*x)), x)
```

3.13 $\int \frac{1}{\sqrt{\sin(bx)}}\,dx$

Optimal. Leaf size=19

$$-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

```
[Out] -2*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticF(cos(1/4*Pi+1
/2*b*x),2^(1/2))/b
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2641}

$$-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sin[b*x]],x]
```

```
[Out] (-2*EllipticF[Pi/4 - (b*x)/2, 2])/b
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx=-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.11

$$-\frac{2F\left(\frac{1}{2}\left(\frac{\pi}{2}-bx\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sin[b*x]],x]
```

[Out] (-2*EllipticF[(Pi/2 - b*x)/2, 2])/b

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{\sin(bx)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(b*x)^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(sin(b*x)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(b*x)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(sin(b*x)), x)

maple [A] time = 0.07, size = 57, normalized size = 3.00

$$\frac{\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\,\text{EllipticF}\left(\sqrt{\sin(bx)+1}, \frac{\sqrt{2}}{2}\right)}{\cos(bx)\sqrt{\sin(bx)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(b*x)^(1/2),x)

[Out] (sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))/cos(b*x)/sin(b*x)^(1/2)/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin(bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(b*x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(sin(b*x)), x)

mupad [B] time = 0.39, size = 15, normalized size = 0.79

$$-\frac{2\,\mathrm{F}\left(\frac{\pi}{4}-\frac{b\,x}{2}\middle|2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x)^(1/2),x)
[Out] -(2*ellipticF(pi/4 - (b*x)/2, 2))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin\left(bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)**(1/2),x)
[Out] Integral(1/sqrt(sin(b*x)), x)
```

3.14 $\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx$

Optimal. Leaf size=37

$$\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}-\frac{2\cos(bx)}{b\sqrt{\sin(bx)}}$$

```
[Out] 2*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticE(cos(1/4*Pi+1/
2*b*x),2^(1/2))/b-2*cos(b*x)/b/sin(b*x)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2636, 2639}

$$\frac{2E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{b}-\frac{2\cos(bx)}{b\sqrt{\sin(bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*x]^(-3/2),x]
```

```
[Out] (2*EllipticE[Pi/4 - (b*x)/2, 2])/b - (2*Cos[b*x])/(b*Sqrt[Sin[b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx = -\frac{2\cos(bx)}{b\sqrt{\sin(bx)}} - \int \sqrt{\sin(bx)}\,dx$$
$$= \frac{2E\left(\frac{\pi}{4} - \frac{bx}{2}\middle|2\right)}{b} - \frac{2\cos(bx)}{b\sqrt{\sin(bx)}}$$

Mathematica [A] time = 0.05, size = 32, normalized size = 0.86

$$\frac{2\left(E\left(\frac{1}{4}(\pi - 2bx)\middle|2\right) - \frac{\cos(bx)}{\sqrt{\sin(bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*x]^(-3/2),x]
[Out] (2*(EllipticE[(Pi - 2*b*x)/4, 2] - Cos[b*x]/Sqrt[Sin[b*x]]))/b
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{\sin\left(bx\right)}}{\cos\left(bx\right)^{2} - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(3/2),x, algorithm="fricas")
[Out] integral(-sqrt(sin(b*x))/(cos(b*x)^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin\left(bx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(3/2),x, algorithm="giac")
[Out] integrate(sin(b*x)^(-3/2), x)
```

maple [A] time = 0.07, size = 110, normalized size = 2.97

$$\frac{2\sqrt{\sin\left(bx\right)+1}\,\sqrt{-2\sin\left(bx\right)+2}\,\sqrt{-\sin\left(bx\right)}\,\operatorname{EllipticE}\left(\sqrt{\sin\left(bx\right)+1}, \frac{\sqrt{2}}{2}\right) - \sqrt{\sin\left(bx\right)+1}\,\sqrt{-2\sin\left(bx\right)+2}}{\cos\left(bx\right)\sqrt{\sin\left(bx\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x)^(3/2),x)
```

```
[Out] (2*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*EllipticE((si
n(b*x)+1)^(1/2),1/2*2^(1/2))-(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin
(b*x))^(1/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))-2*cos(b*x)^2)/cos(b*
x)/sin(b*x)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(-3/2), x)
```

mupad [B] time = 0.47, size = 34, normalized size = 0.92

$$-\frac{\cos(b\,x)\,\left(\sin(b\,x)^2\right)^{1/4}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\,\sqrt{\sin(b\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x)^(3/2),x)
```

```
[Out] -(cos(b*x)*(sin(b*x)^2)^(1/4)*hypergeom([1/2, 5/4], 3/2, cos(b*x)^2))/(b*si
n(b*x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)**(3/2),x)
```

```
[Out] Integral(sin(b*x)**(-3/2), x)
```

3.15 $\int \frac{1}{\sin^{\frac{5}{2}}(bx)} dx$

Optimal. Leaf size=41

$$-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{3b}-\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}$$

```
[Out] -2/3*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticF(cos(1/4*Pi
+1/2*b*x),2^(1/2))/b-2/3*cos(b*x)/b/sin(b*x)^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2636, 2641}

$$-\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{3b}-\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*x]^(-5/2),x]
```

```
[Out] (-2*EllipticF[Pi/4 - (b*x)/2, 2])/(3*b) - (2*Cos[b*x])/(3*b*Sin[b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx = -\frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)} + \frac{1}{3}\int \frac{1}{\sqrt{\sin(bx)}}\,dx$$
$$= -\frac{2F\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{3b} - \frac{2\cos(bx)}{3b\sin^{\frac{3}{2}}(bx)}$$

Mathematica [A] time = 0.05, size = 33, normalized size = 0.80

$$-\frac{2\left(F\left(\frac{1}{4}(\pi-2bx)\middle|2\right)+\frac{\cos(bx)}{\sin^{\frac{3}{2}}(bx)}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*x]^(-5/2),x]
```

```
[Out] (-2*(EllipticF[(Pi - 2*b*x)/4, 2] + Cos[b*x]/Sin[b*x]^(3/2)))/(3*b)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{1}{\left(\cos(bx)^2-1\right)\sqrt{\sin(bx)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-1/((cos(b*x)^2 - 1)*sqrt(sin(b*x))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x)^(-5/2), x)
```

maple [A] time = 0.06, size = 72, normalized size = 1.76

$$\frac{\sqrt{\sin(bx)+1}\,\sqrt{-2\sin(bx)+2}\,\sqrt{-\sin(bx)}\ \mathrm{EllipticF}\left(\sqrt{\sin(bx)+1}\,,\frac{\sqrt{2}}{2}\right)\sin(bx)-2\left(\cos^2(bx)\right)}{3\sin(bx)^{\frac{3}{2}}\cos(bx)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x)^(5/2),x)
```

```
[Out] 1/3/sin(b*x)^(3/2)*((sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1
/2)*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))*sin(b*x)-2*cos(b*x)^2)/cos(b*
x)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x)^(-5/2), x)
```

mupad [B] time = 0.59, size = 34, normalized size = 0.83

$$-\frac{\cos(b\,x)\,\left(\sin(b\,x)^2\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\sin(b\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x)^(5/2),x)
```

```
[Out] -(cos(b*x)*(sin(b*x)^2)^(3/4)*hypergeom([1/2, 7/4], 3/2, cos(b*x)^2))/(b*si
n(b*x)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin^{\frac{5}{2}}(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)**(5/2),x)
```

```
[Out] Integral(sin(b*x)**(-5/2), x)
```

3.16 $\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx$

Optimal. Leaf size=60

$$\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}-\frac{6\cos(bx)}{5b\sqrt{\sin(bx)}}$$

```
[Out] 6/5*(sin(1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/4*Pi+1/2*b*x)*EllipticE(cos(1/4*Pi+
1/2*b*x),2^(1/2))/b-2/5*cos(b*x)/b/sin(b*x)^(5/2)-6/5*cos(b*x)/b/sin(b*x)^(
1/2)
```

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2636, 2639}

$$\frac{6E\left(\frac{\pi}{4}-\frac{bx}{2}\middle|2\right)}{5b}-\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)}-\frac{6\cos(bx)}{5b\sqrt{\sin(bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[b*x]^(-7/2),x]
```

```
[Out] (6*EllipticE[Pi/4 - (b*x)/2, 2])/(5*b) - (2*Cos[b*x])/(5*b*Sin[b*x]^(5/2))
 - (6*Cos[b*x])/(5*b*Sqrt[Sin[b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx = -\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)} + \frac{3}{5}\int \frac{1}{\sin^{\frac{3}{2}}(bx)}\,dx$$
$$= -\frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)} - \frac{6\cos(bx)}{5b\sqrt{\sin(bx)}} - \frac{3}{5}\int \sqrt{\sin(bx)}\,dx$$
$$= \frac{6E\left(\frac{\pi}{4} - \frac{bx}{2}\middle|2\right)}{5b} - \frac{2\cos(bx)}{5b\sin^{\frac{5}{2}}(bx)} - \frac{6\cos(bx)}{5b\sqrt{\sin(bx)}}$$

Mathematica [A] time = 0.05, size = 51, normalized size = 0.85

$$\frac{-7\cos(bx) + 3\cos(3bx) + 12\sin^{\frac{5}{2}}(bx)E\left(\frac{1}{4}(\pi - 2bx)\middle|2\right)}{10b\sin^{\frac{5}{2}}(bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[b*x]^(-7/2),x]
```

```
[Out] (-7*Cos[b*x] + 3*Cos[3*b*x] + 12*EllipticE[(Pi - 2*b*x)/4, 2]*Sin[b*x]^(5/2
))/(10*b*Sin[b*x]^(5/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{\sin(bx)}}{\cos(bx)^4 - 2\cos(bx)^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sin(b*x))/(cos(b*x)^4 - 2*cos(b*x)^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)^(7/2),x, algorithm="giac")
```

[Out] integrate(sin(b*x)^(-7/2), x)

maple [A] time = 0.06, size = 132, normalized size = 2.20

$$\frac{6\sqrt{\sin(bx)+1}\ \sqrt{-2\sin(bx)+2}\ \sqrt{-\sin(bx)}\ \left(\sin^2(bx)\right)\mathrm{EllipticE}\left(\sqrt{\sin(bx)+1}, \frac{\sqrt{2}}{2}\right) - 3\sqrt{\sin(bx)+1}\ \sqrt{-2\sin}}{5\sin(bx)^{\frac{5}{2}}\cos(b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(b*x)^(7/2),x)

[Out] 1/5/sin(b*x)^(5/2)*(6*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*sin(b*x)^2*EllipticE((sin(b*x)+1)^(1/2),1/2*2^(1/2))-3*(sin(b*x)+1)^(1/2)*(-2*sin(b*x)+2)^(1/2)*(-sin(b*x))^(1/2)*sin(b*x)^2*EllipticF((sin(b*x)+1)^(1/2),1/2*2^(1/2))+6*sin(b*x)^4-4*sin(b*x)^2-2)/cos(b*x)/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(bx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(b*x)^(7/2),x, algorithm="maxima")

[Out] integrate(sin(b*x)^(-7/2), x)

mupad [B] time = 0.56, size = 34, normalized size = 0.57

$$-\frac{\cos(b\,x)\,\left(\sin(b\,x)^2\right)^{5/4}\,{}_2F_1\left(\frac{1}{2},\frac{9}{4};\ \frac{3}{2};\ \cos(b\,x)^2\right)}{b\,\sin(b\,x)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(b*x)^(7/2),x)

[Out] -(cos(b*x)*(sin(b*x)^2)^(5/4)*hypergeom([1/2, 9/4], 3/2, cos(b*x)^2))/(b*sin(b*x)^(5/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin^{\frac{7}{2}}(bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x)**(7/2),x)
[Out] Integral(sin(b*x)**(-7/2), x)
```

3.17 $\int \sin^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{10F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{21b}-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}-\frac{10\sqrt{\sin(a+bx)}\,\cos(a+bx)}{21b}$$

```
[Out] -10/21*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipt
icF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2/7*cos(b*x+a)*sin(b*x+a)^(5/2)/b-
10/21*cos(b*x+a)*sin(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2635, 2641}

$$\frac{10F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{21b}-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}-\frac{10\sqrt{\sin(a+bx)}\,\cos(a+bx)}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(7/2),x]
```

```
[Out] (10*EllipticF[(a - Pi/2 + b*x)/2, 2])/(21*b) - (10*Cos[a + b*x]*Sqrt[Sin[a
+ b*x]])/(21*b) - (2*Cos[a + b*x]*Sin[a + b*x]^(5/2))/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sin^{\frac{7}{2}}(a+bx)\,dx = -\frac{2\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{7b} + \frac{5}{7}\int \sin^{\frac{3}{2}}(a+bx)\,dx$$
$$= -\frac{10\cos(a+bx)\sqrt{\sin(a+bx)}}{21b} - \frac{2\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{7b} + \frac{5}{21}\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx$$
$$= \frac{10F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{21b} - \frac{10\cos(a+bx)\sqrt{\sin(a+bx)}}{21b} - \frac{2\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{7b}$$

Mathematica [A] time = 0.11, size = 55, normalized size = 0.79

$$\frac{\sqrt{\sin(a+bx)}\,(3\cos(3(a+bx)) - 23\cos(a+bx)) - 20F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)}{42b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(7/2),x]
[Out] (-20*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + (-23*Cos[a + b*x] + 3*Cos[3*(a +
 b*x)])*Sqrt[Sin[a + b*x]])/(42*b)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(bx+a\right)^2-1\right)\sin\left(bx+a\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/2),x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(b*x + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^(7/2), x)
```

maple [A] time = 0.05, size = 104, normalized size = 1.49

$$\frac{\frac{2\sin(bx+a)\left(\cos^4(bx+a)\right)}{7}+\frac{5\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{21}-\frac{16\left(\cos^2(bx+a)\right)\sin(bx+a)}{21}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(7/2),x)
```

```
[Out] (2/7*sin(b*x+a)*cos(b*x+a)^4+5/21*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1
/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-16/21*c
os(b*x+a)^2*sin(b*x+a))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(bx+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(7/2), x)
```

mupad [B] time = 0.49, size = 42, normalized size = 0.60

$$-\frac{\cos(a+b\,x)\,\sin(a+b\,x)^{9/2}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\left(\sin(a+b\,x)^2\right)^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^(7/2),x)
```

```
[Out] -(cos(a + b*x)*sin(a + b*x)^(9/2)*hypergeom([-5/4, 1/2], 3/2, cos(a + b*x)^
2))/(b*(sin(a + b*x)^2)^(9/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.18 $\int \sin^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=47

$$\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}$$

```
[Out] -6/5*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Elliptic
E(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2/5*cos(b*x+a)*sin(b*x+a)^(3/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2635, 2639}

$$\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(5/2),x]
```

```
[Out] (6*EllipticE[(a - Pi/2 + b*x)/2, 2])/(5*b) - (2*Cos[a + b*x]*Sin[a + b*x]^(
3/2))/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^{\frac{5}{2}}(a+bx)\,dx &= -\frac{2\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{5b}+\frac{3}{5}\int \sqrt{\sin(a+bx)}\,dx\\ &= \frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{5b}-\frac{2\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 44, normalized size = 0.94

$$-\frac{\sqrt{\sin(a+bx)}\sin(2(a+bx))+6E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(5/2),x]
```

```
[Out] -1/5*(6*EllipticE[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[Sin[a + b*x]]*Sin[2*(a +
 b*x)])/b
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(bx+a)^2-1\right)\sqrt{\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(sin(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(5/2), x)
```

maple [A] time = 0.04, size = 142, normalized size = 3.02

$$\frac{\frac{2\left(\sin^4(bx+a)\right)}{5}-\frac{2\left(\sin^2(bx+a)\right)}{5}-\frac{6\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\text{EllipticE}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx}}{}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(5/2),x)
```

```
[Out] (2/5*sin(b*x+a)^4-2/5*sin(b*x+a)^2-6/5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+
2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+3/
5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Elliptic
F((sin(b*x+a)+1)^(1/2),1/2*2^(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(5/2), x)
```

mupad [B] time = 0.45, size = 42, normalized size = 0.89

$$-\frac{\cos(a+b\,x)\,\sin(a+b\,x)^{7/2}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\left(\sin(a+b\,x)^2\right)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^(5/2),x)
```

```
[Out] -(cos(a + b*x)*sin(a + b*x)^(7/2)*hypergeom([-3/4, 1/2], 3/2, cos(a + b*x)^
2))/(b*(sin(a + b*x)^2)^(7/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{\frac{5}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)**(5/2), x)
```

3.19 $\int \sin^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=47

$$\frac{2F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b}-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}$$

```
[Out] -2/3*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Elliptic
F(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2/3*cos(b*x+a)*sin(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2635, 2641}

$$\frac{2F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b}-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(3/2),x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x]*Sqrt[Sin[a + b
*x]])/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin^{\frac{3}{2}}(a+bx)\,dx &= -\frac{2\cos(a+bx)\sqrt{\sin(a+bx)}}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx\\ &= \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{3b}-\frac{2\cos(a+bx)\sqrt{\sin(a+bx)}}{3b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.85

$$-\frac{2\left(F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+\sqrt{\sin(a+bx)}\cos(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(3/2),x]
```

```
[Out] (-2*(EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Cos[a + b*x]*Sqrt[Sin[a + b*x]])
)/(3*b)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin\left(bx+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sin(b*x + a)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(3/2), x)
```

maple [A] time = 0.04, size = 88, normalized size = 1.87

$$\frac{\frac{\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\mathrm{EllipticF}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{3}-\frac{2\left(\cos^2(bx+a)\right)\sin(bx+a)}{3}}{\cos\left(bx+a\right)\sqrt{\sin\left(bx+a\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(3/2),x)
```

```
[Out] (1/3*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Ellip
ticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2/3*cos(b*x+a)^2*sin(b*x+a))/cos(b*x
+a)/sin(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(3/2), x)
```

mupad [B] time = 0.44, size = 42, normalized size = 0.89

$$-\frac{\cos(a+b\,x)\,\sin(a+b\,x)^{5/2}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\left(\sin(a+b\,x)^2\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^(3/2),x)
```

```
[Out] -(cos(a + b*x)*sin(a + b*x)^(5/2)*hypergeom([-1/4, 1/2], 3/2, cos(a + b*x)^
2))/(b*(sin(a + b*x)^2)^(5/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{\frac{3}{2}}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)**(3/2), x)
```

3.20 $\int \sqrt{\sin(a+bx)}\,dx$

Optimal. Leaf size=21

$$\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

```
[Out] -2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2639}

$$\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\sin(a+bx)}\,dx=\frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.14

$$-\frac{2E\left(\frac{1}{2}\left(-a-bx+\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[a + b*x]],x]
```

```
[Out] (-2*EllipticE[(-a + Pi/2 - b*x)/2, 2])/b
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\sin\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sin(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(b*x + a)), x)
```

maple [A] time = 0.04, size = 91, normalized size = 4.33

$$-\frac{\sqrt{\sin\left(bx+a\right)+1}\,\sqrt{-2\sin\left(bx+a\right)+2}\,\sqrt{-\sin\left(bx+a\right)}\,\left(2\,\text{EllipticE}\left(\sqrt{\sin\left(bx+a\right)+1},\frac{\sqrt{2}}{2}\right)-\text{EllipticF}\left(\sqrt{\sin\right.\right.}{\cos\left(bx+a\right)\sqrt{\sin\left(bx+a\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(1/2),x)
```

```
[Out] -(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*(2*Ellipt
icE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^
(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(b*x + a)), x)
```

mupad [B] time = 0.37, size = 18, normalized size = 0.86

$$\frac{2\,\mathrm{E}\left(\frac{a}{2}-\frac{\pi}{4}+\frac{b\,x}{2}\middle|2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^(1/2),x)
```

```
[Out] (2*ellipticE(a/2 - pi/4 + (b*x)/2, 2))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(sin(a + b*x)), x)
```

3.21 $\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx$

Optimal. Leaf size=21

$$\frac{2F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

```
[Out] -2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b
```

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2641}

$$\frac{2F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sin[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2])/b
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx = \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.14

$$-\frac{2F\left(\frac{1}{2}\left(-a-bx+\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sin[a + b*x]],x]
```

[Out] (-2*EllipticF[(-a + Pi/2 - b*x)/2, 2])/b

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{1}{\sqrt{\sin\left(bx+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(sin(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(sin(b*x + a)), x)
```

maple [A] time = 0.03, size = 69, normalized size = 3.29

$$\frac{\sqrt{\sin\left(bx+a\right)+1}\,\sqrt{-2\sin\left(bx+a\right)+2}\,\sqrt{-\sin\left(bx+a\right)}\,\operatorname{EllipticF}\left(\sqrt{\sin\left(bx+a\right)+1},\frac{\sqrt{2}}{2}\right)}{\cos\left(bx+a\right)\sqrt{\sin\left(bx+a\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x+a)^(1/2),x)
[Out] (sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF(
(sin(b*x+a)+1)^(1/2),1/2*2^(1/2))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(sin(b*x + a)), x)
```

mupad [B] time = 0.38, size = 18, normalized size = 0.86

$$-\frac{2\,\mathrm{F}\left(\frac{\pi}{4}-\frac{a}{2}-\frac{b\,x}{2}\middle|2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*x)^(1/2),x)
```

```
[Out] -(2*ellipticF(pi/4 - a/2 - (b*x)/2, 2))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sin{(a+bx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sin(a + b*x)), x)
```

3.22 $\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=43

$$-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}$$

```
[Out] 2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(c
os(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2*cos(b*x+a)/b/sin(b*x+a)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2636, 2639}

$$-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(-3/2),x]
```

```
[Out] (-2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b - (2*Cos[a + b*x])/(b*Sqrt[Sin[a +
b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}} - \int \sqrt{\sin(a+bx)}\,dx$$
$$= -\frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b} - \frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}$$

Mathematica [A] time = 0.08, size = 39, normalized size = 0.91

$$\frac{2\left(E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)-\frac{\cos(a+bx)}{\sqrt{\sin(a+bx)}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(-3/2),x]
```

```
[Out] (2*(EllipticE[(-2*a + Pi - 2*b*x)/4, 2] - Cos[a + b*x]/Sqrt[Sin[a + b*x]]))
/b
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{\sin(bx+a)}}{\cos(bx+a)^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(sin(b*x + a))/(cos(b*x + a)^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(-3/2), x)
```

maple [A] time = 0.04, size = 132, normalized size = 3.07

$$\frac{2\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\,\text{EllipticE}\left(\sqrt{\sin(bx+a)+1}, \frac{\sqrt{2}}{2}\right) - \sqrt{\sin(bx+a)+1}}{\cos(bx+a)\sqrt{\sin(bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x+a)^(3/2),x)
```

```
[Out] (2*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Ellipti
cE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)
^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*co
s(b*x+a)^2)/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(-3/2), x)
```

mupad [B] time = 0.51, size = 42, normalized size = 0.98

$$-\frac{\cos(a+b\,x)\,\left(\sin(a+b\,x)^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\,\sqrt{\sin(a+b\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*x)^(3/2),x)
```

```
[Out] -(cos(a + b*x)*(sin(a + b*x)^2)^(1/4)*hypergeom([1/2, 5/4], 3/2, cos(a + b*
x)^2))/(b*sin(a + b*x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)**(3/2),x)
```

```
[Out] Integral(sin(a + b*x)**(-3/2), x)
```

3.23 $\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=47

$$\frac{2F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

```
[Out] -2/3*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Elliptic
F(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2/3*cos(b*x+a)/b/sin(b*x+a)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2636, 2641}

$$\frac{2F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(-5/2),x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x])/(3*b*Sin[a +
b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} + \frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx$$
$$= \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{3b} - \frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

Mathematica [A] time = 0.10, size = 43, normalized size = 0.91

$$\frac{2\left(F\left(\frac{1}{4}(2a+2bx-\pi)\middle|2\right)-\frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(-5/2),x]
```

```
[Out] (2*(EllipticF[(2*a - Pi + 2*b*x)/4, 2] - Cos[a + b*x]/Sin[a + b*x]^(3/2)))/
(3*b)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{\left(\cos\left(bx+a\right)^2-1\right)\sqrt{\sin\left(bx+a\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-1/((cos(b*x + a)^2 - 1)*sqrt(sin(b*x + a))), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(-5/2), x)
```

maple [A] time = 0.05, size = 88, normalized size = 1.87

$$\frac{\sqrt{\sin \left(bx+a\right)+1}\,\sqrt{-2\sin \left(bx+a\right)+2}\,\sqrt{-\sin \left(bx+a\right)}\,\ \mathrm{EllipticF}\left(\sqrt{\sin \left(bx+a\right)+1}\,,\frac{\sqrt{2}}{2}\right)\sin \left(bx+a\right)-2\left(\cos^{2}\right.}{3\sin \left(bx+a\right)^{\frac{3}{2}}\cos \left(bx+a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x+a)^(5/2),x)
```

```
[Out] 1/3/sin(b*x+a)^(3/2)*((sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*
x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))*sin(b*x+a)-2*cos(b*
x+a)^2)/cos(b*x+a)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin \left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(-5/2), x)
```

mupad [B] time = 0.61, size = 42, normalized size = 0.89

$$-\frac{\cos \left(a+b\,x\right)\,{\left({\sin \left(a+b\,x\right)}^2\right)}^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ {\cos \left(a+b\,x\right)}^2\right)}{b\,{\sin \left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*x)^(5/2),x)
```

```
[Out] -(cos(a + b*x)*(sin(a + b*x)^2)^(3/4)*hypergeom([1/2, 7/4], 3/2, cos(a + b*
x)^2))/(b*sin(a + b*x)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)**(5/2),x)
```

```
[Out] Integral(sin(a + b*x)**(-5/2), x)
```

3.24 $\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=70

$$-\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}-\frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}}$$

```
[Out] 6/5*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE
(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b-2/5*cos(b*x+a)/b/sin(b*x+a)^(5/2)-6/5
*cos(b*x+a)/b/sin(b*x+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2636, 2639}

$$-\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{5b}-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}-\frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(-7/2),x]
```

```
[Out] (-6*EllipticE[(a - Pi/2 + b*x)/2, 2])/(5*b) - (2*Cos[a + b*x])/(5*b*Sin[a +
 b*x]^(5/2)) - (6*Cos[a + b*x])/(5*b*Sqrt[Sin[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{3}{5}\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$
$$= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}} - \frac{3}{5}\int \sqrt{\sin(a+bx)}\,dx$$
$$= -\frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{5b} - \frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{6\cos(a+bx)}{5b\sqrt{\sin(a+bx)}}$$

Mathematica [A] time = 0.27, size = 55, normalized size = 0.79

$$\frac{2\left(3E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right) - \frac{\left(3\sin^2(a+bx)+1\right)\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(-7/2),x]
```

```
[Out] (2*(3*EllipticE[(-2*a + Pi - 2*b*x)/4, 2] - (Cos[a + b*x]*(1 + 3*Sin[a + b*
x]^2))/Sin[a + b*x]^(5/2)))/(5*b)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{\sin(bx+a)}}{\cos(bx+a)^4 - 2\,\cos(bx+a)^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sin(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(-7/2), x)
```

maple [A] time = 0.05, size = 160, normalized size = 2.29

$$\frac{6\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\left(\sin^2(bx+a)\right)\mathrm{EllipticE}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x+a)^(7/2),x)
```

```
[Out] 1/5/sin(b*x+a)^(5/2)*(6*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(
b*x+a))^(1/2)*sin(b*x+a)^2*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-3*(s
in(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*sin(b*x+a)^2
*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+6*sin(b*x+a)^4-4*sin(b*x+a)^2-
2)/cos(b*x+a)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(-7/2), x)
```

mupad [B] time = 0.59, size = 42, normalized size = 0.60

$$-\frac{\cos(a+b\,x)\,\left(\sin(a+b\,x)^2\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\sin(a+b\,x)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(a + b*x)^(7/2),x)
```

```
[Out] -(cos(a + b*x)*(sin(a + b*x)^2)^(5/4)*hypergeom([1/2, 9/4], 3/2, cos(a + b*
x)^2))/(b*sin(a + b*x)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a)**(7/2),x)
```

```
[Out] Integral(sin(a + b*x)**(-7/2), x)
```

3.25 $\int (c\sin(a+bx))^{7/2}\,dx$

Optimal. Leaf size=103

$$\frac{10c^4\sqrt{\sin(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{21b\sqrt{c\sin(a+bx)}}-\frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

```
[Out] -2/7*c*cos(b*x+a)*(c*sin(b*x+a))^(5/2)/b-10/21*c^4*(sin(1/2*a+1/4*Pi+1/2*b*
x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2
^(1/2))*sin(b*x+a)^(1/2)/b/(c*sin(b*x+a))^(1/2)-10/21*c^3*cos(b*x+a)*(c*sin
(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 2642, 2641}

$$-\frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b}+\frac{10c^4\sqrt{\sin(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{21b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(7/2),x]
```

```
[Out] (10*c^4*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(21*b*Sqrt[c*S
in[a + b*x]]) - (10*c^3*Cos[a + b*x]*Sqrt[c*Sin[a + b*x]])/(21*b) - (2*c*Co
s[a + b*x]*(c*Sin[a + b*x])^(5/2))/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rubi steps

$$\int (c\sin(a+bx))^{7/2}\,dx = -\frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b} + \frac{1}{7}\left(5c^2\right)\int (c\sin(a+bx))^{3/2}\,dx$$

$$= -\frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b} - \frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b} + \frac{1}{21}\left(5c^4\right)\int \frac{\ldots}{\sqrt{c\,\mathrm{si}\ldots}}$$

$$= -\frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b} - \frac{2c\cos(a+bx)(c\sin(a+bx))^{5/2}}{7b} + \frac{\left(5c^4\sqrt{\sin(a+bx}\ldots\right.}{21\sqrt{c\sin\ldots}}$$

$$= \frac{10c^4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{21b\sqrt{c\sin(a+bx)}} - \frac{10c^3\cos(a+bx)\sqrt{c\sin(a+bx)}}{21b} - \frac{2c\cos(a+\ldots}{\ldots}$$

Mathematica [A] time = 0.15, size = 80, normalized size = 0.78

$$\frac{c^3\sqrt{c\sin(a+bx)}\left(\sqrt{\sin(a+bx)}\,(3\cos(3(a+bx))-23\cos(a+bx))-20F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{42b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(7/2),x]
```

```
[Out] (c^3*(-20*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + (-23*Cos[a + b*x] + 3*Cos[3
*(a + b*x)])*Sqrt[Sin[a + b*x]])*Sqrt[c*Sin[a + b*x]])/(42*b*Sqrt[Sin[a + b
*x]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(c^3\cos\left(bx+a\right)^2-c^3\right)\sqrt{c\sin\left(bx+a\right)}\,\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^3*cos(b*x + a)^2 - c^3)*sqrt(c*sin(b*x + a))*sin(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sin\left(bx+a\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sin(b*x+a))^(7/2),x, algorithm="giac")

[Out] integrate((c*sin(b*x + a))^(7/2), x)

maple [A] time = 0.07, size = 108, normalized size = 1.05

$$-\frac{c^4\left(-6\left(\sin^5(bx+a)\right)+5\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sqrt{\sin}\,(bx+a)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(bx+a)}\right.\right.}{21\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*sin(b*x+a))^(7/2),x)

[Out] -1/21*c^4*(-6*sin(b*x+a)^5+5*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-4*sin(b*x+a)^3+10*sin(b*x+a))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(c\sin(bx+a))^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sin(b*x+a))^(7/2),x, algorithm="maxima")

[Out] integrate((c*sin(b*x + a))^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int(c\,\sin(a+b\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*sin(a + b*x))^(7/2),x)

[Out] int((c*sin(a + b*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sin(b*x+a))**(7/2),x)

[Out] Timed out

3.26 $\int (c\sin(a+bx))^{5/2}\,dx$

Optimal. Leaf size=75

$$\frac{6c^2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5b\sqrt{\sin(a+bx)}}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

```
[Out] -2/5*c*cos(b*x+a)*(c*sin(b*x+a))^(3/2)/b-6/5*c^2*(sin(1/2*a+1/4*Pi+1/2*b*x)
^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(
1/2))*(c*sin(b*x+a))^(1/2)/b/sin(b*x+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 2640, 2639}

$$\frac{6c^2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5b\sqrt{\sin(a+bx)}}-\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (6*c^2*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*Sqrt[Sin
[a + b*x]]) - (2*c*Cos[a + b*x]*(c*Sin[a + b*x])^(3/2))/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int (c\sin(a+bx))^{5/2}\,dx = -\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b} + \frac{1}{5}\left(3c^2\right)\int \sqrt{c\sin(a+bx)}\,dx$$
$$= -\frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b} + \frac{\left(3c^2\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(a+bx)}\,dx}{5\sqrt{\sin(a+bx)}}$$
$$= \frac{6c^2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5b\sqrt{\sin(a+bx)}} - \frac{2c\cos(a+bx)(c\sin(a+bx))^{3/2}}{5b}$$

Mathematica [A] time = 0.10, size = 66, normalized size = 0.88

$$-\frac{(c\sin(a+bx))^{5/2}\left(\sqrt{\sin(a+bx)}\,\sin(2(a+bx))+6E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{5b\sin^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2),x]
[Out] -1/5*((c*Sin[a + b*x])^(5/2)*(6*EllipticE[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[
Sin[a + b*x]]*Sin[2*(a + b*x)]))/(b*Sin[a + b*x]^(5/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2\cos(bx+a)^2-c^2\right)\sqrt{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2),x, algorithm="fricas")
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(c*sin(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(5/2), x)
```

maple [A] time = 0.06, size = 152, normalized size = 2.03

$$-\frac{c^3\left(6\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sqrt{\sin}\,(bx+a)\right)\text{EllipticE}\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{-\sin(b}\right.}{5\cos(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2),x)
```

```
[Out] -1/5*c^3*(6*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*E
llipticE((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-3*(-sin(b*x+a)+1)^(1/2)*(2*sin(
b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2
))-2*sin(b*x+a)^4+2*sin(b*x+a)^2)/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,\sin(a+b\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(5/2),x)
```

```
[Out] int((c*sin(a + b*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(a+bx))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**(5/2), x)
```

3.27 $\int (c\sin(a+bx))^{3/2}\,dx$

Optimal. Leaf size=75

$$\frac{2c^2\sqrt{\sin(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

```
[Out] -2/3*c^2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Elli
pticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*sin(b*x+a)^(1/2)/b/(c*sin(b*x+a))^
(1/2)-2/3*c*cos(b*x+a)*(c*sin(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 2642, 2641}

$$\frac{2c^2\sqrt{\sin(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (2*c^2*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b*Sqrt[c*Sin
[a + b*x]]) - (2*c*Cos[a + b*x]*Sqrt[c*Sin[a + b*x]])/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int (c\sin(a+bx))^{3/2}\,dx = -\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}+\frac{1}{3}c^2\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx$$
$$= -\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}+\frac{\left(c^2\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{3\sqrt{c\sin(a+bx)}}$$
$$= \frac{2c^2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{3b\sqrt{c\sin(a+bx)}}-\frac{2c\cos(a+bx)\sqrt{c\sin(a+bx)}}{3b}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.83

$$-\frac{2(c\sin(a+bx))^{3/2}\left(F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+\sqrt{\sin(a+bx)}\cos(a+bx)\right)}{3b\sin^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (-2*(EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Cos[a + b*x]*Sqrt[Sin[a + b*x]])
*(c*Sin[a + b*x])^(3/2))/(3*b*Sin[a + b*x]^(3/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{c\sin(bx+a)}\,c\sin(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*c*sin(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2), x)
```

maple [A] time = 0.05, size = 97, normalized size = 1.29

$$-\frac{c^2\left(\sqrt{-\sin(bx+a)+1}\ \sqrt{2\sin(bx+a)+2}\ \left(\sqrt{\sin}\ (bx+a)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(bx+a)+1}\,,\frac{\sqrt{2}}{2}\right)-2\left(\sin^3(bx\right.\right.}{3\cos(bx+a)\sqrt{c\sin(bx+a)}\ b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2),x)
```

```
[Out] -1/3*c^2*((-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*Ell
ipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*sin(b*x+a)^3+2*sin(b*x+a))/cos(
b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int(c\ \sin(a+b\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(3/2),x)
```

```
[Out] int((c*sin(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(c\sin(a+bx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**(3/2), x)
```

3.28 $\int \sqrt{c \sin(a+bx)}\, dx$

Optimal. Leaf size=43

$$\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

```
[Out] -2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/sin(b*x+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2640, 2639}

$$\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a + b*x]])/(b*Sqrt[Sin[a + b
*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c\sin(a+bx)}\, dx &= \frac{\sqrt{c\sin(a+bx)}\int \sqrt{\sin(a+bx)}\, dx}{\sqrt{\sin(a+bx)}}\\ &= \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.98

$$-\frac{2E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]],x]
[Out] (-2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[c*Sin[a + b*x]])/(b*Sqrt[Sin[a
 + b*x]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c\sin(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*sin(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sin(b*x + a)), x)
```

maple [A] time = 0.05, size = 98, normalized size = 2.28

$$-\frac{c\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sqrt{\sin}\,(bx+a)\right)\left(2\,\text{EllipticE}\left(\sqrt{-\sin(bx+a)+1}, \frac{\sqrt{2}}{2}\right)-\text{EllipticF}\left(\right.\right.}{\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2),x)
[Out] -c*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*(2*Ellipti
cE((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2
^(1/2)))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c \sin(bx + a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a)), x)
```

mupad [B] time = 0.40, size = 36, normalized size = 0.84

$$\frac{2\sqrt{c\ \sin(a+b\,x)}\ \mathrm{E}\left(\frac{a}{2}-\frac{\pi}{4}+\frac{b\,x}{2}\middle|2\right)}{b\,\sqrt{\sin(a+b\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(1/2),x)
```

```
[Out] (2*(c*sin(a + b*x))^(1/2)*ellipticE(a/2 - pi/4 + (b*x)/2, 2))/(b*sin(a + b*
x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c \sin(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sin(a + b*x)), x)
```

3.29 $\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{\sin(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{c\sin(a+bx)}}$$

```
[Out] -2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*sin(b*x+a)^(1/2)/b/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2642, 2641}

$$\frac{2\sqrt{\sin(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[c*Sin[a + b
*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx &= \frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{\sqrt{c\sin(a+bx)}}\\ &= \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.98

$$-\frac{2\sqrt{\sin(a+bx)}F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)}{b\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (-2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]])/(b*Sqrt[c*Sin[a
 + b*x]])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c\sin\left(bx+a\right)}}{c\sin\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))/(c*sin(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*sin(b*x + a)), x)
```

maple [A] time = 0.05, size = 74, normalized size = 1.72

$$-\frac{\sqrt{-\sin\left(bx+a\right)+1}\,\sqrt{2\sin\left(bx+a\right)+2}\,\left(\sqrt{\sin}\,\left(bx+a\right)\right)\text{EllipticF}\left(\sqrt{-\sin\left(bx+a\right)+1},\frac{\sqrt{2}}{2}\right)}{\cos\left(bx+a\right)\sqrt{c\sin\left(bx+a\right)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*EllipticF((-
sin(b*x+a)+1)^(1/2),1/2*2^(1/2))/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*sin(b*x + a)), x)
```

mupad [B] time = 0.48, size = 36, normalized size = 0.84

$$-\frac{2\sqrt{\sin(a+b\,x)}\;\mathrm{F}\left(\frac{\pi}{4}-\frac{a}{2}-\frac{b\,x}{2}\middle|2\right)}{b\,\sqrt{c}\;\sin(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x))^(1/2),x)
```

```
[Out] -(2*sin(a + b*x)^(1/2)*ellipticF(pi/4 - a/2 - (b*x)/2, 2))/(b*(c*sin(a + b*
x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*sin(a + b*x)), x)
```

3.30 $\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{bc^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}$$

```
[Out] -2*cos(b*x+a)/b/c/(c*sin(b*x+a))^(1/2)+2*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2
)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*(c
*sin(b*x+a))^(1/2)/b/c^2/sin(b*x+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2636, 2640, 2639}

$$-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{bc^2\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(-3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(b*c*Sqrt[c*Sin[a + b*x]]) - (2*EllipticE[(a - Pi/2 + b*x
)/2, 2]*Sqrt[c*Sin[a + b*x]])/(b*c^2*Sqrt[Sin[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}} - \frac{\int \sqrt{c\sin(a+bx)}\,dx}{c^2}$$
$$= -\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}} - \frac{\sqrt{c\sin(a+bx)}\int \sqrt{\sin(a+bx)}\,dx}{c^2\sqrt{\sin(a+bx)}}$$
$$= -\frac{2\cos(a+bx)}{bc\sqrt{c\sin(a+bx)}} - \frac{2E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{bc^2\sqrt{\sin(a+bx)}}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.74

$$-\frac{2\left(\cos(a+bx)-\sqrt{\sin(a+bx)}\,E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{bc\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(-3/2),x]
```

```
[Out] (-2*(Cos[a + b*x] - EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]])
)/(b*c*Sqrt[c*Sin[a + b*x]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{c\sin(bx+a)}}{c^2\cos(bx+a)^2-c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*sin(b*x + a))/(c^2*cos(b*x + a)^2 - c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(-3/2), x)
```

maple [A] time = 0.06, size = 141, normalized size = 1.93

$$\frac{2\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sqrt{\sin}\,(bx+a)\right)\mathrm{EllipticE}\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-\sqrt{-\sin(bx+a)}}{c\cos(bx+a)\sqrt{c\sin(bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(3/2),x)
```

```
[Out] 1/c*(2*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(1/2)*Ellipt
icE((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+
2)^(1/2)*sin(b*x+a)^(1/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*co
s(b*x+a)^2)/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\sin(bx+a)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c\,\sin(a+b\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x))^(3/2),x)
```

```
[Out] int(1/(c*sin(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\sin(a+bx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(3/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**(-3/2), x)
```

3.31 $\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{\sin(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3bc^2\sqrt{c\sin(a+bx)}}-\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}$$

```
[Out] -2/3*cos(b*x+a)/b/c/(c*sin(b*x+a))^(3/2)-2/3*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^
(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2)
)*sin(b*x+a)^(1/2)/b/c^2/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2636, 2642, 2641}

$$\frac{2\sqrt{\sin(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3bc^2\sqrt{c\sin(a+bx)}}-\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(-5/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(3*b*c*(c*Sin[a + b*x])^(3/2)) + (2*EllipticF[(a - Pi/2 +
 b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b*c^2*Sqrt[c*Sin[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int \frac{1}{(c\sin(a+bx))^{5/2}}\,dx = -\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}} + \frac{\int \frac{1}{\sqrt{c\sin(a+bx)}}\,dx}{3c^2}$$
$$= -\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}} + \frac{\sqrt{\sin(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{3c^2\sqrt{c\sin(a+bx)}}$$
$$= -\frac{2\cos(a+bx)}{3bc(c\sin(a+bx))^{3/2}} + \frac{2F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{3bc^2\sqrt{c\sin(a+bx)}}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.71

$$-\frac{2\left(\cos(a+bx)+\sin^{\frac{3}{2}}(a+bx)F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{3bc(c\sin(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(-5/2),x]
```

```
[Out] (-2*(Cos[a + b*x] + EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2))
)/(3*b*c*(c*Sin[a + b*x])^(3/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{c\sin(bx+a)}}{\left(c^3\cos(bx+a)^2-c^3\right)\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*sin(b*x + a))/((c^3*cos(b*x + a)^2 - c^3)*sin(b*x + a)), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(-5/2), x)
```

maple [A] time = 0.06, size = 105, normalized size = 1.36

$$-\frac{\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sin^{\frac{5}{2}}(bx+a)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-2\left(\sin^{3}(bx+a)\right.}{3c^{2}\sin(bx+a)^{2}\cos(bx+a)\sqrt{c\sin(bx+a)}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(5/2),x)
```

```
[Out] -1/3/c^2*((-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(5/2)*Ell
ipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*sin(b*x+a)^3+2*sin(b*x+a))/sin(
b*x+a)^2/cos(b*x+a)/(c*sin(b*x+a))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,\sin(a+b\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x))^(5/2),x)
```

```
[Out] int(1/(c*sin(a + b*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(a+bx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(5/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**(-5/2), x)
```

3.32 $\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5bc^4\sqrt{\sin(a+bx)}}-\frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}}-\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}$$

```
[Out] -2/5*cos(b*x+a)/b/c/(c*sin(b*x+a))^(5/2)-6/5*cos(b*x+a)/b/c^3/(c*sin(b*x+a)
)^(1/2)+6/5*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*E
llipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*(c*sin(b*x+a))^(1/2)/b/c^4/sin(
b*x+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2636, 2640, 2639}

$$-\frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}}-\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5bc^4\sqrt{\sin(a+bx)}}-\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(-7/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(5*b*c*(c*Sin[a + b*x])^(5/2)) - (6*Cos[a + b*x])/(5*b*c^
3*Sqrt[c*Sin[a + b*x]]) - (6*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[c*Sin[a
+ b*x]])/(5*b*c^4*Sqrt[Sin[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(c\sin(a+bx))^{7/2}}\,dx &= -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}+\frac{3\int \frac{1}{(c\sin(a+bx))^{3/2}}\,dx}{5c^2}\\
&= -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}-\frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}}-\frac{3\int \sqrt{c\sin(a+bx)}\,dx}{5c^4}\\
&= -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}-\frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}}-\frac{\left(3\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(a+bx)}\,dx}{5c^4\sqrt{\sin(a+bx)}}\\
&= -\frac{2\cos(a+bx)}{5bc(c\sin(a+bx))^{5/2}}-\frac{6\cos(a+bx)}{5bc^3\sqrt{c\sin(a+bx)}}-\frac{6E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{c\sin(a+bx)}}{5bc^4\sqrt{\sin(a+bx)}}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 68, normalized size = 0.65

$$
-\frac{2\left(\frac{3}{2}\sin(2(a+bx))+\cot(a+bx)-3\sin^{\frac{3}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{5bc^2(c\sin(a+bx))^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(-7/2),x]
```

```
[Out] (-2*(Cot[a + b*x] - 3*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2
) + (3*Sin[2*(a + b*x)])/2))/(5*b*c^2*(c*Sin[a + b*x])^(3/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{c\sin(bx+a)}}{c^4\cos(bx+a)^4-2\,c^4\cos(bx+a)^2+c^4},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))/(c^4*cos(b*x + a)^4 - 2*c^4*cos(b*x + a)^2 +
c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{(c\sin(bx+a))^{\frac{7}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(-7/2), x)
```

maple [A] time = 0.07, size = 168, normalized size = 1.60

$$\frac{6\sqrt{-\sin(bx+a)+1}\,\sqrt{2\sin(bx+a)+2}\,\left(\sin^{\frac{7}{2}}(bx+a)\right)\mathrm{EllipticE}\left(\sqrt{-\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{-\sin(bx+a)}}{5c^{3}\sin(bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(7/2),x)
```

```
[Out] 1/5/c^3*(6*(-sin(b*x+a)+1)^(1/2)*(2*sin(b*x+a)+2)^(1/2)*sin(b*x+a)^(7/2)*El
lipticE((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-3*(-sin(b*x+a)+1)^(1/2)*(2*sin(b
*x+a)+2)^(1/2)*sin(b*x+a)^(7/2)*EllipticF((-sin(b*x+a)+1)^(1/2),1/2*2^(1/2)
)+6*sin(b*x+a)^5-4*sin(b*x+a)^3-2*sin(b*x+a))/sin(b*x+a)^3/cos(b*x+a)/(c*si
n(b*x+a))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(-7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,\sin\left(a+b\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x))^(7/2),x)
```

```
[Out] int(1/(c*sin(a + b*x))^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(a+bx))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(7/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**(-7/2), x)
```

3.33 $\int (c\sin(a+bx))^{4/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sin^2(a+bx)\right)}{7bc\sqrt{\cos^2(a+bx)}}$$

```
[Out] 3/7*cos(b*x+a)*hypergeom([1/2, 7/6],[13/6],sin(b*x+a)^2)*(c*sin(b*x+a))^(7/
3)/b/c/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sin^2(a+bx)\right)}{7bc\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(4/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sin[a + b*x]^2]*(c*Sin[a
+ b*x])^(7/3))/(7*b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (c\sin(a+bx))^{4/3}\,dx=\frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sin^2(a+bx)\right)(c\sin(a+bx))^{7/3}}{7bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(a+bx)}\,\tan(a+bx)(c\sin(a+bx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sin^2(a+bx)\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(4/3),x]
```

```
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 7/6, 13/6, Sin[a + b*x]^2]*(
c*Sin[a + b*x])^(4/3)*Tan[a + b*x])/(7*b)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left((c\sin(bx+a))^{\frac{1}{3}}c\sin(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^(1/3)*c*sin(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(c\sin(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(4/3), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int(c\sin(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(4/3),x)
```

```
[Out] int((c*sin(b*x+a))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(c\sin(bx+a))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\ \sin(a + b\,x))^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(4/3),x)
```

```
[Out] int((c*sin(a + b*x))^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(a + bx))^{\frac{4}{3}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(4/3),x)
```

```
[Out] Integral((c*sin(a + b*x))**(4/3), x)
```

3.34 $\int (c\sin(a+bx))^{2/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2(a+bx)\right)}{5bc\sqrt{\cos^2(a+bx)}}$$

```
[Out] 3/5*cos(b*x+a)*hypergeom([1/2, 5/6],[11/6],sin(b*x+a)^2)*(c*sin(b*x+a))^(5/
3)/b/c/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2(a+bx)\right)}{5bc\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(2/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[a + b*x]^2]*(c*Sin[a
+ b*x])^(5/3))/(5*b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (c\sin(a+bx))^{2/3}\,dx=\frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2(a+bx)\right)(c\sin(a+bx))^{5/3}}{5bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(a+bx)}\,\tan(a+bx)(c\sin(a+bx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(2/3),x]
```

```
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[a + b*x]^2]*(
c*Sin[a + b*x])^(2/3)*Tan[a + b*x])/(5*b)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left((c\sin(bx+a))^{\frac{2}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^(2/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(c\sin(bx+a))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(2/3), x)
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int(c\sin(bx+a))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(2/3),x)
```

```
[Out] int((c*sin(b*x+a))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(c\sin(bx+a))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (c\ \sin (a+b\,x))^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(2/3),x)
[Out] int((c*sin(a + b*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin (a+bx))^{\frac{2}{3}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(2/3),x)
[Out] Integral((c*sin(a + b*x))**(2/3), x)
```

3.35 $\int \sqrt[3]{c\sin(a+bx)}\,dx$

Optimal. Leaf size=517

$$\frac{3\left(1-i\sqrt{3}\right)\sqrt{3-i\sqrt{3}}\,\sqrt[3]{c}\,\sec(a+bx)\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}\sqrt{\frac{2(c\sin(a+bx))^{2/3}}{\left(3-i\sqrt{3}\right)c^{2/3}}+\frac{\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2(c\sin(a+bx))^{2/3}}{\left(3+i\sqrt{3}\right)c^{2/3}}+\frac{-\sqrt{3}+i}{-\sqrt{3}+3i}}\,F\left(\sin\right.}{2\sqrt{2}\,b}$$

```
[Out] 3/4*c^(1/3)*EllipticF(2^(1/2)*(1-(c*sin(b*x+a))^(2/3)/c^(2/3))^(1/2)/(3-I*3
^(1/2))^(1/2),((3*I+3^(1/2))/(3*I-3^(1/2)))^(1/2))*sec(b*x+a)*(1-I*3^(1/2))
*(1-(c*sin(b*x+a))^(2/3)/c^(2/3))^(1/2)*((I-3^(1/2))/(3*I-3^(1/2))+2*(c*sin
(b*x+a))^(2/3)/c^(2/3)/(3+I*3^(1/2)))^(1/2)*(3-I*3^(1/2))^(1/2)*(2*(c*sin(b
*x+a))^(2/3)/c^(2/3)/(3-I*3^(1/2))+(3^(1/2)+I)/(3*I+3^(1/2)))^(1/2)/b*2^(1/
2)-3/2*c^(1/3)*EllipticE(2^(1/2)*(1-(c*sin(b*x+a))^(2/3)/c^(2/3))^(1/2)/(3+
I*3^(1/2))^(1/2),((3*I-3^(1/2))/(3*I+3^(1/2)))^(1/2))*sec(b*x+a)*(1-(c*sin(
b*x+a))^(2/3)/c^(2/3))^(1/2)*((I-3^(1/2))/(3*I-3^(1/2))+2*(c*sin(b*x+a))^(2
/3)/c^(2/3)/(3+I*3^(1/2)))^(1/2)*(18-6*I*3^(1/2))^(1/2)*(2*(c*sin(b*x+a))^(
2/3)/c^(2/3)/(3-I*3^(1/2))+(3^(1/2)+I)/(3*I+3^(1/2)))^(1/2)/b
```

Rubi [C] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 0.11, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2643}

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(a+bx)\right)}{4bc\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(1/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[a + b*x]^2]*(c*Sin[a +
 b*x])^(4/3))/(4*b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{c\sin(a+bx)}\, dx = \frac{3\cos(a+bx)\, {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(a+bx)\right)(c\sin(a+bx))^{4/3}}{4bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [C] time = 0.03, size = 55, normalized size = 0.11

$$\frac{3\sqrt{\cos^2(a+bx)}\,\tan(a+bx)\sqrt[3]{c\sin(a+bx)}\, {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[a + b*x]^2]*(c
*Sin[a + b*x])^(1/3)*Tan[a + b*x])/(4*b)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((c\sin(bx+a))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(1/3), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/3),x)
```

```
[Out] int((c*sin(b*x+a))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (c\ \sin(a+b\,x))^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(1/3),x)
```

```
[Out] int((c*sin(a + b*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*sin(a + b*x))**(1/3), x)
```

3.36 $\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=252

$$-\frac{3\sqrt{3-i\sqrt{3}}\,\sec(a+bx)\sqrt{1-\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}}\sqrt{\frac{2(c\sin(a+bx))^{2/3}}{\left(3-i\sqrt{3}\right)c^{2/3}}+\frac{\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2(c\sin(a+bx))^{2/3}}{\left(3+i\sqrt{3}\right)c^{2/3}}+\frac{-\sqrt{3}+i}{-\sqrt{3}+3i}}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{1-\cdots}}{\sqrt{\cdots}}\right.\right.}{\sqrt{2}\,b\sqrt[3]{c}}$$

```
[Out] -3/2*EllipticF(2^(1/2)*(1-(c*sin(b*x+a))^(2/3)/c^(2/3))^(1/2)/(3-I*3^(1/2))
^(1/2),((3*I+3^(1/2))/(3*I-3^(1/2)))^(1/2))*sec(b*x+a)*(1-(c*sin(b*x+a))^(2
/3)/c^(2/3))^(1/2)*((I-3^(1/2))/(3*I-3^(1/2))+2*(c*sin(b*x+a))^(2/3)/c^(2/3
)/(3+I*3^(1/2)))^(1/2)*(3-I*3^(1/2))^(1/2)*(2*(c*sin(b*x+a))^(2/3)/c^(2/3)/
(3-I*3^(1/2))+(3^(1/2)+I)/(3*I+3^(1/2)))^(1/2)/b/c^(1/3)*2^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 0.23, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(a+bx)(c\sin(a+bx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(a+bx)\right)}{2bc\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(-1/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[a + b*x]^2]*(c*Sin[a +
 b*x])^(2/3))/(2*b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{c\sin(a+bx)}}\,dx=\frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(a+bx)\right)(c\sin(a+bx))^{2/3}}{2bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [C] time = 0.04, size = 55, normalized size = 0.22

$$\frac{3\sqrt{\cos^2(a+bx)}\,\tan(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(a+bx)\right)}{2b\sqrt[3]{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[a + b*x]^2]*Ta
n[a + b*x])/(2*b*(c*Sin[a + b*x])^(1/3))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(c\sin(bx+a))^{\frac{2}{3}}}{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^(2/3)/(c*sin(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(c\sin(bx+a))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(-1/3), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{1}{(c\sin(bx+a))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(1/3),x)
```

```
[Out] int(1/(c*sin(b*x+a))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \sin\left(bx+a\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(1/3),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c\,\sin\left(a+b\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x))^(1/3),x)
[Out] int(1/(c*sin(a + b*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{c \sin\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(1/3),x)
[Out] Integral((c*sin(a + b*x))**(-1/3), x)
```

3.37 $\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx$

Optimal. Leaf size=271

$$\frac{3^{3/4}\sec(a+bx)\sqrt[3]{c\sin(a+bx)}\,\left(c^{2/3}-(c\sin(a+bx))^{2/3}\right)\sqrt{\frac{c^{4/3}\left(\frac{(c\sin(a+bx))^{4/3}}{c^{4/3}}+\frac{(c\sin(a+bx))^{2/3}}{c^{2/3}}+1\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)(c\sin(a+bx))^{2/3}\right)^2}}\,F\left(\cos^{-1}\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)(c\sin}{c^{2/3}-\left(1+\sqrt{3}\right)(c\sin}\right.\right.}{2bc^{5/3}\sqrt{-\frac{(c\sin(a+bx))^{2/3}\left(c^{2/3}-(c\sin(a+bx))^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)(c\sin(a+bx))^{2/3}\right)^2}}}$$

```
[Out] 1/2*3^(3/4)*((c^(2/3)-(c*sin(b*x+a))^(2/3)*(1-3^(1/2)))^2/(c^(2/3)-(c*sin(b
*x+a))^(2/3)*(1+3^(1/2)))^2)^(1/2)/(c^(2/3)-(c*sin(b*x+a))^(2/3)*(1-3^(1/2)
))*(c^(2/3)-(c*sin(b*x+a))^(2/3)*(1+3^(1/2)))*EllipticF((1-(c^(2/3)-(c*sin(
b*x+a))^(2/3)*(1-3^(1/2)))^2/(c^(2/3)-(c*sin(b*x+a))^(2/3)*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*sec(b*x+a)*(c*sin(b*x+a))^(1/3)*(c^(2/3)-(c*
sin(b*x+a))^(2/3))*(c^(4/3)*(1+(c*sin(b*x+a))^(2/3)/c^(2/3)+(c*sin(b*x+a))^
(4/3)/c^(4/3))/(c^(2/3)-(c*sin(b*x+a))^(2/3)*(1+3^(1/2)))^2)^(1/2)/b/c^(5/3
)/(-(c*sin(b*x+a))^(2/3)*(c^(2/3)-(c*sin(b*x+a))^(2/3))/(c^(2/3)-(c*sin(b*x
+a))^(2/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 0.21, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(a+bx)\sqrt[3]{c\sin(a+bx)}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(-2/3),x]
```

```
[Out] (3*Cos[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[a + b*x]^2]*(c*Sin[a +
 b*x])^(1/3))/(b*c*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(c\sin(a+bx))^{2/3}}\,dx = \frac{3\cos(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(a+bx)\right)\sqrt[3]{c\sin(a+bx)}}{bc\sqrt{\cos^2(a+bx)}}$$

Mathematica [C] time = 0.04, size = 53, normalized size = 0.20

$$\frac{3\sqrt{\cos^2(a+bx)}\,\tan(a+bx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2(a+bx)\right)}{b(c\sin(a+bx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(-2/3),x]
[Out] (3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[a + b*x]^2]*Ta
n[a + b*x])/(b*(c*Sin[a + b*x])^(2/3))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(c\sin(bx+a))^{\frac{1}{3}}}{c\sin(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(2/3),x, algorithm="fricas")
[Out] integral((c*sin(b*x + a))^(1/3)/(c*sin(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(bx+a))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(2/3),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^(-2/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(bx+a))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(2/3),x)
```

```
[Out] int(1/(c*sin(b*x+a))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(bx+a))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(-2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(c\,\sin(a+b\,x))^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x))^(2/3),x)
```

```
[Out] int(1/(c*sin(a + b*x))^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(a+bx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*sin(a + b*x))**(-2/3), x)
```

3.38 $\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\cos(a+bx)\,_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}\,\sqrt[3]{c\sin(a+bx)}}$$

```
[Out] -3*cos(b*x+a)*hypergeom([-1/6, 1/2],[5/6],sin(b*x+a)^2)/b/c/(c*sin(b*x+a))^
(1/3)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\cos(a+bx)\,_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}\,\sqrt[3]{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(-4/3),x]
```

```
[Out] (-3*Cos[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sin[a + b*x]^2])/(b*c*Sq
rt[Cos[a + b*x]^2]*(c*Sin[a + b*x])^(1/3))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(c\sin(a+bx))^{4/3}}\,dx = -\frac{3\cos(a+bx)\,_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sin^2(a+bx)\right)}{bc\sqrt{\cos^2(a+bx)}\,\sqrt[3]{c\sin(a+bx)}}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.95

$$-\frac{3\sqrt{\cos^2(a+bx)}\,\tan(a+bx)\,_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sin^2(a+bx)\right)}{b(c\sin(a+bx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(-4/3),x]
```

```
[Out] (-3*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sin[a + b*x]^2]*
Tan[a + b*x])/(b*(c*Sin[a + b*x])^(4/3))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(c\sin(bx+a))^{\frac{2}{3}}}{c^2\cos(bx+a)^2-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral(-(c*sin(b*x + a))^(2/3)/(c^2*cos(b*x + a)^2 - c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(bx+a))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(-4/3), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(bx+a))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a))^(4/3),x)
```

```
[Out] int(1/(c*sin(b*x+a))^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(bx+a))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(-4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,\sin(a+b\,x))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x))^(4/3),x)
```

```
[Out] int(1/(c*sin(a + b*x))^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\sin(a+bx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a))**(4/3),x)
```

```
[Out] Integral((c*sin(a + b*x))**(-4/3), x)
```

3.39 $\int \sin^n(a+bx)\,dx$

Optimal. Leaf size=63

$$\frac{\cos(a+bx)\sin^{n+1}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{b(n+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(b*x+a)^2)*sin(b*x+a)^
(1+n)/b/(1+n)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2643}

$$\frac{\cos(a+bx)\sin^{n+1}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{b(n+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^n,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*
Sin[a + b*x]^(1 + n))/(b*(1 + n)*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sin^n(a+bx)\,dx=\frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)\sin^{1+n}(a+bx)}{b(1+n)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 1.00

$$\frac{\sqrt{\cos^2(a+bx)}\,\sec(a+bx)\sin^{n+1}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^n,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a +
b*x]^2]*Sec[a + b*x]*Sin[a + b*x]^(1 + n))/(b*(1 + n))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\sin\left(bx+a\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(sin(b*x + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^n, x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \sin^{n}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^n,x)
```

```
[Out] int(sin(b*x+a)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^n, x)
```

mupad [B] time = 0.78, size = 54, normalized size = 0.86

$$-\frac{\cos(a+b\,x)\,\sin(a+b\,x)^{n+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{1}{2}-\frac{n}{2};\ \frac{3}{2};\ \cos(a+b\,x)^2\right)}{b\left(\sin(a+b\,x)^2\right)^{\frac{n}{2}+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^n,x)
```

```
[Out] -(cos(a + b*x)*sin(a + b*x)^(n + 1)*hypergeom([1/2, 1/2 - n/2], 3/2, cos(a
+ b*x)^2))/(b*(sin(a + b*x)^2)^(n/2 + 1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^n(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**n,x)
```

```
[Out] Integral(sin(a + b*x)**n, x)
```

3.40 $\int (c\sin(a+bx))^n\,dx$

Optimal. Leaf size=68

$$\frac{\cos(a+bx)(c\sin(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bc(n+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(b*x+a)^2)*(c*sin(b*x+
a))^(1+n)/b/c/(1+n)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2643}

$$\frac{\cos(a+bx)(c\sin(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{bc(n+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^n,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(1 + n))/(b*c*(1 + n)*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (c\sin(a+bx))^n\,dx=\frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+n}}{bc(1+n)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.93

$$\frac{\sqrt{\cos^2(a+bx)}\,\tan(a+bx)(c\sin(a+bx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\sin^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^n,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x])^n*Tan[a + b*x])/(b*(1 + n))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((c\sin(bx+a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^n, x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^n,x)
```

```
[Out] int((c*sin(b*x+a))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\ \sin(a+b\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^n,x)
```

```
[Out] int((c*sin(a + b*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(a+bx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**n,x)
```

```
[Out] Integral((c*sin(a + b*x))**n, x)
```

3.41 $\int (a\sin(e+fx))^m (b\sin(e+fx))^n\,dx$

Optimal. Leaf size=81

$$\frac{\cos(e+fx)(a\sin(e+fx))^{m+1}(b\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(e+fx)\right)}{af(m+n+1)\sqrt{\cos^2(e+fx)}}$$

```
[Out] cos(f*x+e)*hypergeom([1/2, 1/2+1/2*m+1/2*n],[3/2+1/2*m+1/2*n],sin(f*x+e)^2)
*(a*sin(f*x+e))^(1+m)*(b*sin(f*x+e))^n/a/f/(1+m+n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2643}

$$\frac{\cos(e+fx)(a\sin(e+fx))^{m+1}(b\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(e+fx)\right)}{af(m+n+1)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^m*(b*Sin[e + f*x])^n,x]
```

```
[Out] (Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Sin[e +
f*x]^2]*(a*Sin[e + f*x])^(1 + m)*(b*Sin[e + f*x])^n)/(a*f*(1 + m + n)*Sqrt[
Cos[e + f*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (a\sin(e+fx))^m(b\sin(e+fx))^n\,dx = \left((a\sin(e+fx))^{-n}(b\sin(e+fx))^n\right)\int (a\sin(e+fx))^{m+n}\,dx$$
$$= \frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);\sin^2(e+fx)\right)(a\sin(e+}{af(1+m+n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.07, size = 76, normalized size = 0.94

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)(a\sin(e+fx))^m(b\sin(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(e+fx)\right)}{f(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^m*(b*Sin[e + f*x])^n,x]
```

```
[Out] (Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2,
Sin[e + f*x]^2]*(a*Sin[e + f*x])^m*(b*Sin[e + f*x])^n*Tan[e + f*x])/(f*(1 +
 m + n))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\sin\left(fx+e\right)\right)^m\left(b\sin\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*(b*sin(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)\right)^m\left(b\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*(b*sin(f*x + e))^n, x)
```

maple [F] time = 1.08, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)\right)^m\left(b\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x)
```

```
[Out] int((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)\right)^{m}\left(b\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^m*(b*sin(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^m*(b*sin(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(b\,\sin\left(e+f\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^m*(b*sin(e + f*x))^n,x)
```

```
[Out] int((a*sin(e + f*x))^m*(b*sin(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(e+fx\right)\right)^{m}\left(b\sin\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**m*(b*sin(f*x+e))**n,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*(b*sin(e + f*x))**n, x)
```

3.42 $\int \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cos^4(a+bx)}{4b}$$

```
[Out] -1/4*cos(b*x+a)^4/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2565, 30}

$$-\frac{\cos^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -Cos[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^3\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{\cos^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/4*Cos[a + b*x]^4/b
```

fricas [A] time = 0.42, size = 13, normalized size = 0.87

$$-\frac{\cos\left(bx+a\right)^{4}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*cos(b*x + a)^4/b
```

giac [A] time = 0.21, size = 24, normalized size = 1.60

$$-\frac{\sin\left(bx+a\right)^{4}-2\,\sin\left(bx+a\right)^{2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(sin(b*x + a)^4 - 2*sin(b*x + a)^2)/b
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$-\frac{\cos^{4}\left(bx+a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] -1/4*cos(b*x+a)^4/b
```

maxima [A] time = 0.33, size = 13, normalized size = 0.87

$$-\frac{\cos\left(bx+a\right)^{4}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*cos(b*x + a)^4/b
```

mupad [B] time = 0.06, size = 13, normalized size = 0.87

$$-\frac{\cos\left(a+b\,x\right)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x),x)
```

```
[Out] -cos(a + b*x)^4/(4*b)
```

sympy [A] time = 1.37, size = 22, normalized size = 1.47

$$\begin{cases} -\frac{\cos^4\left(a+bx\right)}{4b} & \text{for}\ b \neq 0 \\ x\sin\left(a\right)\cos^3\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-cos(a + b*x)**4/(4*b), Ne(b, 0)), (x*sin(a)*cos(a)**3, True))
```

3.43 $\int \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cos^3(a+bx)}{3b}$$

[Out] -1/3*cos(b*x+a)^3/b

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2565, 30}

$$-\frac{\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]^2*Sin[a + b*x],x]

[Out] -Cos[a + b*x]^3/(3*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^2\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x],x]
[Out] -1/3*Cos[a + b*x]^3/b
```

fricas [A] time = 0.43, size = 13, normalized size = 0.87

$$-\frac{\cos(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
[Out] -1/3*cos(b*x + a)^3/b
```

giac [A] time = 0.21, size = 13, normalized size = 0.87

$$-\frac{\cos(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
[Out] -1/3*cos(b*x + a)^3/b
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$-\frac{\cos^3(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a),x)
[Out] -1/3*cos(b*x+a)^3/b
```

maxima [A] time = 0.32, size = 13, normalized size = 0.87

$$-\frac{\cos(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
[Out] -1/3*cos(b*x + a)^3/b
```

mupad [B] time = 0.04, size = 13, normalized size = 0.87

$$-\frac{\cos\left(a+b\,x\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x),x)
```

```
[Out] -cos(a + b*x)^3/(3*b)
```

sympy [A] time = 0.69, size = 22, normalized size = 1.47

$$\begin{cases} -\frac{\cos^3\left(a+bx\right)}{3b} & \text{for}\ b \neq 0 \\ x\sin\left(a\right)\cos^2\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-cos(a + b*x)**3/(3*b), Ne(b, 0)), (x*sin(a)*cos(a)**2, True))
```

3.44 $\int \cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^2(a+bx)}{2b}$$

```
[Out] 1/2*sin(b*x+a)^2/b
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2564, 30}

$$\frac{\sin^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] Sin[a + b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin(a+bx)\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^2(a+bx)}{2b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 2.47

$$\frac{1}{2}\left(\frac{\sin(2a)\sin(2bx)}{2b}-\frac{\cos(2a)\cos(2bx)}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x],x]
[Out] (-1/2*(Cos[2*a]*Cos[2*b*x])/b + (Sin[2*a]*Sin[2*b*x])/(2*b))/2
```

fricas [A] time = 0.41, size = 13, normalized size = 0.87

$$-\frac{\cos\left(bx+a\right)^{2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
[Out] -1/2*cos(b*x + a)^2/b
```

giac [A] time = 0.17, size = 13, normalized size = 0.87

$$\frac{\sin\left(bx+a\right)^{2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
[Out] 1/2*sin(b*x + a)^2/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\sin^{2}\left(bx+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a),x)
[Out] 1/2*sin(b*x+a)^2/b
```

maxima [A] time = 0.33, size = 13, normalized size = 0.87

$$-\frac{\cos\left(bx+a\right)^{2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
[Out] -1/2*cos(b*x + a)^2/b
```

mupad [B] time = 0.45, size = 28, normalized size = 1.87

$$\begin{cases} \frac{x \sin(2a)}{2} & \text{if } b = 0 \\ -\frac{\cos(2a+2bx)}{4b} & \text{if } b \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x),x)
[Out] piecewise(b == 0, (x*sin(2*a))/2, b ~= 0, -cos(2*a + 2*b*x)/(4*b))
```

sympy [A] time = 0.31, size = 19, normalized size = 1.27

$$\begin{cases} \frac{\sin^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x \sin(a) \cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a),x)
[Out] Piecewise((sin(a + b*x)**2/(2*b), Ne(b, 0)), (x*sin(a)*cos(a), True))
```

3.45 $\int \tan(a+bx)\,dx$

Optimal. Leaf size=12

$$-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] -ln(cos(b*x+a))/b
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x],x]
```

```
[Out] -(Log[Cos[a + b*x]]/b)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x],x]
```

```
[Out] -(Log[Cos[a + b*x]]/b)
```

fricas [A] time = 0.45, size = 14, normalized size = 1.17

$$-\frac{\log\left(-\cos\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -log(-cos(b*x + a))/b
```

giac [A] time = 0.21, size = 18, normalized size = 1.50

$$-\frac{\log\left(\frac{|\cos(bx+a)|}{|b|}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -log(abs(cos(b*x + a))/abs(b))/b
```

maple [A] time = 0.01, size = 12, normalized size = 1.00

$$\frac{\ln\left(\sec\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*ln(sec(b*x+a))
```

maxima [A] time = 0.37, size = 18, normalized size = 1.50

$$-\frac{\log\left(-\sin\left(bx+a\right)^{2}+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*log(-sin(b*x + a)^2 + 1)/b
```

mupad [B] time = 0.51, size = 16, normalized size = 1.33

$$\frac{\ln\left(\tan\left(a+b\,x\right)^{2}+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(a + b*x),x)
```

```
[Out] log(tan(a + b*x)^2 + 1)/(2*b)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*sec(a + b*x), x)
```

3.46 $\int \sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\sec(a+bx)}{b}$$

```
[Out] sec(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 8}

$$\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)\tan(a+bx)\,dx &= \frac{\text{Subst}(\int 1\,dx, x, \sec(a+bx))}{b}\\ &= \frac{\sec(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x],x]
[Out] Sec[a + b*x]/b
```

fricas [A] time = 0.40, size = 12, normalized size = 1.20

$$\frac{1}{b \cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
[Out] 1/(b*cos(b*x + a))
```

giac [A] time = 1.18, size = 12, normalized size = 1.20

$$\frac{1}{b \cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
[Out] 1/(b*cos(b*x + a))
```

maple [A] time = 0.02, size = 11, normalized size = 1.10

$$\frac{\sec(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(b*x+a),x)
[Out] sec(b*x+a)/b
```

maxima [A] time = 0.32, size = 12, normalized size = 1.20

$$\frac{1}{b \cos(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
[Out] 1/(b*cos(b*x + a))
```

mupad [B] time = 0.52, size = 20, normalized size = 2.00

$$-\frac{2}{b\,\left({\mathrm{tan}\left(\frac{a}{2}+\frac{b\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(a + b*x)^2,x)
[Out] -2/(b*(tan(a/2 + (b*x)/2)^2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin{\left(a+bx \right)} \sec^{2}{\left(a+bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a),x)
[Out] Integral(sin(a + b*x)*sec(a + b*x)**2, x)
```

3.47 $\int \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sec^2(a+bx)}{2b}$$

```
[Out] 1/2*sec(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$\frac{\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\tan(a+bx)\,dx &= \frac{\text{Subst}\left(\int x\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\sec^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]^2/(2*b)
```

fricas [A] time = 0.45, size = 13, normalized size = 0.87

$$\frac{1}{2\,b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2/(b*cos(b*x + a)^2)
```

giac [A] time = 0.22, size = 13, normalized size = 0.87

$$\frac{1}{2\,b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2/(b*cos(b*x + a)^2)
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$\frac{\sec^{2}\left(bx+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/2*sec(b*x+a)^2/b
```

maxima [A] time = 0.32, size = 17, normalized size = 1.13

$$-\frac{1}{2\left(\sin\left(bx+a\right)^{2}-1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2/((sin(b*x + a)^2 - 1)*b)
```

mupad [B] time = 0.38, size = 13, normalized size = 0.87

$$\frac{\tan\left(a+b\,x\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(a + b*x)^3,x)
```

```
[Out] tan(a + b*x)^2/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a+bx\right)\sec^3\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*sec(a + b*x)**3, x)
```

3.48 $\int \sec^3(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sec^3(a+bx)}{3b}$$

```
[Out] 1/3*sec(b*x+a)^3/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$\frac{\sec^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^3(a+bx)\tan(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\sec^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sec^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x],x]
```

```
[Out] Sec[a + b*x]^3/(3*b)
```

fricas [A] time = 0.43, size = 13, normalized size = 0.87

$$\frac{1}{3\,b\cos\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3/(b*cos(b*x + a)^3)
```

giac [A] time = 0.22, size = 13, normalized size = 0.87

$$\frac{1}{3\,b\cos\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3/(b*cos(b*x + a)^3)
```

maple [A] time = 0.02, size = 14, normalized size = 0.93

$$\frac{\sec^3\left(bx+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*sin(b*x+a),x)
```

```
[Out] 1/3*sec(b*x+a)^3/b
```

maxima [A] time = 0.31, size = 13, normalized size = 0.87

$$\frac{1}{3\,b\cos\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3/(b*cos(b*x + a)^3)
```

mupad [B] time = 0.45, size = 13, normalized size = 0.87

$$\frac{1}{3\,b\cos\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(a + b*x)^4,x)
[Out] 1/(3*b*cos(a + b*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a+bx\right)\sec^4\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a),x)
[Out] Integral(sin(a + b*x)*sec(a + b*x)**4, x)
```

3.49 $\int \cos^7(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=61

$$-\frac{\sin^9(a+bx)}{9b}+\frac{3\sin^7(a+bx)}{7b}-\frac{3\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{3b}$$

```
[Out] 1/3*sin(b*x+a)^3/b-3/5*sin(b*x+a)^5/b+3/7*sin(b*x+a)^7/b-1/9*sin(b*x+a)^9/b
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 270}

$$-\frac{\sin^9(a+bx)}{9b}+\frac{3\sin^7(a+bx)}{7b}-\frac{3\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^7*Sin[a + b*x]^2,x]
```

```
[Out] Sin[a + b*x]^3/(3*b) - (3*Sin[a + b*x]^5)/(5*b) + (3*Sin[a + b*x]^7)/(7*b)
- Sin[a + b*x]^9/(9*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int \cos^7(a+bx)\sin^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^3\,dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(x^2-3x^4+3x^6-x^8\right)dx,x,\sin(a+bx)\right)}{b}\\
&= \frac{\sin^3(a+bx)}{3b}-\frac{3\sin^5(a+bx)}{5b}+\frac{3\sin^7(a+bx)}{7b}-\frac{\sin^9(a+bx)}{9b}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 47, normalized size = 0.77

$$\frac{\sin^3(a+bx)(1389\cos(2(a+bx))+330\cos(4(a+bx))+35\cos(6(a+bx))+1606)}{10080b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^7*Sin[a + b*x]^2,x]
```

```
[Out] ((1606 + 1389*Cos[2*(a + b*x)] + 330*Cos[4*(a + b*x)] + 35*Cos[6*(a + b*x)]
)*Sin[a + b*x]^3)/(10080*b)
```

fricas [A] time = 0.45, size = 53, normalized size = 0.87

$$-\frac{\left(35\,\cos\left(bx+a\right)^{8}-5\,\cos\left(bx+a\right)^{6}-6\,\cos\left(bx+a\right)^{4}-8\,\cos\left(bx+a\right)^{2}-16\right)\sin\left(bx+a\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/315*(35*cos(b*x + a)^8 - 5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x
 + a)^2 - 16)*sin(b*x + a)/b
```

giac [A] time = 0.20, size = 54, normalized size = 0.89

$$-\frac{\sin\left(9\,bx+9\,a\right)}{2304\,b}-\frac{5\,\sin\left(7\,bx+7\,a\right)}{1792\,b}-\frac{\sin\left(5\,bx+5\,a\right)}{160\,b}+\frac{7\,\sin\left(bx+a\right)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2304*sin(9*b*x + 9*a)/b - 5/1792*sin(7*b*x + 7*a)/b - 1/160*sin(5*b*x +
5*a)/b + 7/128*sin(b*x + a)/b
```

maple [A] time = 0.06, size = 60, normalized size = 0.98

$$\frac{-\frac{\sin(bx+a)\left(\cos^{8}(bx+a)\right)}{9}+\frac{\left(\frac{16}{5}+\cos^{6}(bx+a)+\frac{6\left(\cos^{4}(bx+a)\right)}{5}+\frac{8\left(\cos^{2}(bx+a)\right)}{5}\right)\sin(bx+a)}{63}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/9*sin(b*x+a)*cos(b*x+a)^8+1/63*(16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+
8/5*cos(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.31, size = 46, normalized size = 0.75

$$-\frac{35 \sin(bx+a)^9 - 135 \sin(bx+a)^7 + 189 \sin(bx+a)^5 - 105 \sin(bx+a)^3}{315\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/315*(35*sin(b*x + a)^9 - 135*sin(b*x + a)^7 + 189*sin(b*x + a)^5 - 105*s
in(b*x + a)^3)/b
```

mupad [B] time = 0.40, size = 45, normalized size = 0.74

$$\frac{-\frac{\sin(a+b\,x)^9}{9}+\frac{3\sin(a+b\,x)^7}{7}-\frac{3\sin(a+b\,x)^5}{5}+\frac{\sin(a+b\,x)^3}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^7*sin(a + b*x)^2,x)
```

```
[Out] (sin(a + b*x)^3/3 - (3*sin(a + b*x)^5)/5 + (3*sin(a + b*x)^7)/7 - sin(a + b
*x)^9/9)/b
```

sympy [A] time = 19.10, size = 88, normalized size = 1.44

$$\begin{cases} \frac{16\sin^9(a+bx)}{315b} + \frac{8\sin^7(a+bx)\cos^2(a+bx)}{35b} + \frac{2\sin^5(a+bx)\cos^4(a+bx)}{5b} + \frac{\sin^3(a+bx)\cos^6(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin^2(a)\cos^7(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((16*sin(a + b*x)**9/(315*b) + 8*sin(a + b*x)**7*cos(a + b*x)**2/(
35*b) + 2*sin(a + b*x)**5*cos(a + b*x)**4/(5*b) + sin(a + b*x)**3*cos(a + b
*x)**6/(3*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**7, True))
```

3.50 $\int \cos^5(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sin^7(a+bx)}{7b}-\frac{2\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{3b}$$

```
[Out] 1/3*sin(b*x+a)^3/b-2/5*sin(b*x+a)^5/b+1/7*sin(b*x+a)^7/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 270}

$$\frac{\sin^7(a+bx)}{7b}-\frac{2\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Sin[a + b*x]^2,x]
```

```
[Out] Sin[a + b*x]^3/(3*b) - (2*Sin[a + b*x]^5)/(5*b) + Sin[a + b*x]^7/(7*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^5(a+bx)\sin^2(a+bx)\,dx &= \frac{\operatorname{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\operatorname{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\sin^3(a+bx)}{3b}-\frac{2\sin^5(a+bx)}{5b}+\frac{\sin^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 37, normalized size = 0.80

$$\frac{\sin^3(a+bx)(108\cos(2(a+bx))+15\cos(4(a+bx))+157)}{840b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Sin[a + b*x]^2,x]
```

```
[Out] ((157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(840*b)
```

fricas [A] time = 0.44, size = 43, normalized size = 0.93

$$-\frac{\left(15\,\cos\left(bx+a\right)^{6}-3\,\cos\left(bx+a\right)^{4}-4\,\cos\left(bx+a\right)^{2}-8\right)\sin\left(bx+a\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b*
x + a)/b
```

giac [A] time = 0.20, size = 54, normalized size = 1.17

$$-\frac{\sin\left(7\,bx+7\,a\right)}{448\,b}-\frac{3\,\sin\left(5\,bx+5\,a\right)}{320\,b}-\frac{\sin\left(3\,bx+3\,a\right)}{192\,b}+\frac{5\,\sin\left(bx+a\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/448*sin(7*b*x + 7*a)/b - 3/320*sin(5*b*x + 5*a)/b - 1/192*sin(3*b*x + 3*
a)/b + 5/64*sin(b*x + a)/b
```

maple [A] time = 0.05, size = 50, normalized size = 1.09

$$\frac{-\frac{\sin(bx+a)\left(\cos^6(bx+a)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(bx+a)}{35}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/7*sin(b*x+a)*cos(b*x+a)^6+1/35*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*
sin(b*x+a))
```

maxima [A] time = 0.30, size = 36, normalized size = 0.78

$$\frac{15\,\sin\left(bx+a\right)^{7}-42\,\sin\left(bx+a\right)^{5}+35\,\sin\left(bx+a\right)^{3}}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*sin(b*x + a)^7 - 42*sin(b*x + a)^5 + 35*sin(b*x + a)^3)/b
```

mupad [B] time = 0.40, size = 36, normalized size = 0.78

$$\frac{15\sin\left(a+b\,x\right)^{7}-42\sin\left(a+b\,x\right)^{5}+35\sin\left(a+b\,x\right)^{3}}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5*sin(a + b*x)^2,x)
```

```
[Out] (35*sin(a + b*x)^3 - 42*sin(a + b*x)^5 + 15*sin(a + b*x)^7)/(105*b)
```

sympy [A] time = 8.52, size = 66, normalized size = 1.43

$$\begin{cases} \frac{8\sin^{7}\left(a+bx\right)}{105b}+\frac{4\sin^{5}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{15b}+\frac{\sin^{3}\left(a+bx\right)\cos^{4}\left(a+bx\right)}{3b} & \text{for}\ b\neq 0\\ x\sin^{2}\left(a\right)\cos^{5}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((8*sin(a + b*x)**7/(105*b) + 4*sin(a + b*x)**5*cos(a + b*x)**2/(1
5*b) + sin(a + b*x)**3*cos(a + b*x)**4/(3*b), Ne(b, 0)), (x*sin(a)**2*cos(a
)**5, True))
```

3.51 $\int \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sin^3(a+bx)}{3b}-\frac{\sin^5(a+bx)}{5b}$$

[Out] 1/3*sin(b*x+a)^3/b-1/5*sin(b*x+a)^5/b

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sin^3(a+bx)}{3b}-\frac{\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]^3*Sin[a + b*x]^2,x]

[Out] Sin[a + b*x]^3/(3*b) - Sin[a + b*x]^5/(5*b)

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\sin^3(a+bx)}{3b}-\frac{\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 27, normalized size = 0.87

$$\frac{\sin^3(a+bx)(3\cos(2(a+bx))+7)}{30b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x]^2,x]
[Out] ((7 + 3*Cos[2*(a + b*x)])*Sin[a + b*x]^3)/(30*b)
```

fricas [A] time = 0.42, size = 33, normalized size = 1.06

$$-\frac{\left(3\,\cos\left(bx+a\right)^4-\cos\left(bx+a\right)^2-2\right)\sin\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
[Out] -1/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

giac [A] time = 0.39, size = 26, normalized size = 0.84

$$-\frac{3\,\sin\left(bx+a\right)^5-5\,\sin\left(bx+a\right)^3}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
[Out] -1/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

maple [A] time = 0.05, size = 40, normalized size = 1.29

$$\frac{-\frac{\sin(bx+a)\left(\cos^4(bx+a)\right)}{5}+\frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{15}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2,x)
[Out] 1/b*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.31, size = 26, normalized size = 0.84

$$-\frac{3\,\sin\left(bx+a\right)^5-5\,\sin\left(bx+a\right)^3}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

mupad [B] time = 0.36, size = 26, normalized size = 0.84

$$\frac{5 \sin (a + b\,x)^3 - 3 \sin (a + b\,x)^5}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2,x)
```

```
[Out] (5*sin(a + b*x)^3 - 3*sin(a + b*x)^5)/(15*b)
```

sympy [A] time = 3.02, size = 44, normalized size = 1.42

$$\begin{cases} \frac{2 \sin^{5}{\left (a+bx \right)}}{15b} + \frac{\sin^{3}{\left (a+bx \right)} \cos^{2}{\left (a+bx \right)}}{3b} & \text{for}\: b \neq 0 \\ x \sin^{2}{\left (a \right)} \cos^{3}{\left (a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*sin(a + b*x)**5/(15*b) + sin(a + b*x)**3*cos(a + b*x)**2/(3*b)
, Ne(b, 0)), (x*sin(a)**2*cos(a)**3, True))
```

3.52 $\int \cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^3(a+bx)}{3b}$$

[Out] 1/3*sin(b*x+a)^3/b

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]*Sin[a + b*x]^2,x]

[Out] Sin[a + b*x]^3/(3*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] Sin[a + b*x]^3/(3*b)
```

fricas [A] time = 0.44, size = 21, normalized size = 1.40

$$-\frac{\left(\cos\left(bx+a\right)^2-1\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/b
```

giac [A] time = 0.23, size = 13, normalized size = 0.87

$$\frac{\sin\left(bx+a\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*sin(b*x + a)^3/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\sin^3\left(bx+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/3*sin(b*x+a)^3/b
```

maxima [A] time = 0.31, size = 13, normalized size = 0.87

$$\frac{\sin\left(bx+a\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sin(b*x + a)^3/b
```

mupad [B] time = 0.02, size = 13, normalized size = 0.87

$$\frac{\sin\left(a+b\,x\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2,x)
```

```
[Out] sin(a + b*x)^3/(3*b)
```

sympy [A] time = 0.71, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sin^{3}{\left(a+bx \right)}}{3b} & \text{for}\: b \neq 0 \\ x \sin^{2}{\left(a \right)} \cos{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((sin(a + b*x)**3/(3*b), Ne(b, 0)), (x*sin(a)**2*cos(a), True))
```

3.53 $\int \tan^2(a+bx)\,dx$

Optimal. Leaf size=14

$$\frac{\tan(a+bx)}{b}-x$$

[Out] -x+tan(b*x+a)/b

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 8}

$$\frac{\tan(a+bx)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x]^2,x]
```

```
[Out] -x + Tan[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^2(a+bx)\,dx &= \frac{\tan(a+bx)}{b}-\int 1\,dx\\ &= -x+\frac{\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.64

$$\frac{\tan(a+bx)}{b}-\frac{\tan^{-1}(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x]^2,x]
```

```
[Out] -(ArcTan[Tan[a + b*x]]/b) + Tan[a + b*x]/b
```

fricas [B] time = 0.42, size = 31, normalized size = 2.21

$$-\frac{bx\cos(bx+a)-\sin(bx+a)}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b*x*cos(b*x + a) - sin(b*x + a))/(b*cos(b*x + a))
```

giac [A] time = 0.19, size = 18, normalized size = 1.29

$$-\frac{bx+a-\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(b*x + a - tan(b*x + a))/b
```

maple [A] time = 0.03, size = 19, normalized size = 1.36

$$\frac{\tan(bx+a)-bx-a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(tan(b*x+a)-b*x-a)
```

maxima [A] time = 0.43, size = 18, normalized size = 1.29

$$-\frac{bx+a-\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b*x + a - tan(b*x + a))/b
```

mupad [B] time = 0.38, size = 14, normalized size = 1.00

$$\frac{\tan(a+b\,x)}{b}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^2,x)
```

```
[Out] tan(a + b*x)/b - x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^2(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)**2, x)
```

3.54 $\int \sec^2(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\tan^3(a+bx)}{3b}$$

[Out] 1/3*tan(b*x+a)^3/b

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$\frac{\tan^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b*x]^2*Tan[a + b*x]^2,x]

[Out] Tan[a + b*x]^3/(3*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\tan^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\tan^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tan^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b)
```

fricas [B] time = 0.42, size = 29, normalized size = 1.93

$$-\frac{\left(\cos\left(bx+a\right)^{2}-1\right)\sin\left(bx+a\right)}{3\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/(b*cos(b*x + a)^3)
```

giac [A] time = 0.23, size = 13, normalized size = 0.87

$$\frac{\tan\left(bx+a\right)^{3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*tan(b*x + a)^3/b
```

maple [A] time = 0.03, size = 22, normalized size = 1.47

$$\frac{\sin^{3}\left(bx+a\right)}{3b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*sin(b*x+a)^2,x)
```

```
[Out] 1/3/b*sin(b*x+a)^3/cos(b*x+a)^3
```

maxima [A] time = 0.32, size = 13, normalized size = 0.87

$$\frac{\tan\left(bx+a\right)^{3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*tan(b*x + a)^3/b
```

mupad [B] time = 0.38, size = 13, normalized size = 0.87

$$\frac{\tan(a+b\,x)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^4,x)
```

```
[Out] tan(a + b*x)^3/(3*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^2(a+bx)\sec^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)**4, x)
```

3.55 $\int \sec^4(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

```
[Out] 1/3*tan(b*x+a)^3/b+1/5*tan(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 14}

$$\frac{\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b) + Tan[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^4(a+bx)\tan^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\tan^3(a+bx)}{3b}+\frac{\tan^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 1.81

$$-\frac{2\tan(a+bx)}{15b}+\frac{\tan(a+bx)\sec^4(a+bx)}{5b}-\frac{\tan(a+bx)\sec^2(a+bx)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*Tan[a + b*x]^2,x]
```

```
[Out] (-2*Tan[a + b*x])/(15*b) - (Sec[a + b*x]^2*Tan[a + b*x])/(15*b) + (Sec[a +
b*x]^4*Tan[a + b*x])/(5*b)
```

fricas [A] time = 0.41, size = 39, normalized size = 1.26

$$-\frac{\left(2\cos(bx+a)^4+\cos(bx+a)^2-3\right)\sin(bx+a)}{15\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(b*x + a)^4 + cos(b*x + a)^2 - 3)*sin(b*x + a)/(b*cos(b*x + a)^
5)
```

giac [A] time = 0.42, size = 26, normalized size = 0.84

$$\frac{3\tan(bx+a)^5+5\tan(bx+a)^3}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(b*x + a)^5 + 5*tan(b*x + a)^3)/b
```

maple [A] time = 0.04, size = 42, normalized size = 1.35

$$\frac{\frac{\sin^3(bx+a)}{5\cos(bx+a)^5}+\frac{2\left(\sin^3(bx+a)\right)}{15\cos(bx+a)^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/5*sin(b*x+a)^3/cos(b*x+a)^5+2/15*sin(b*x+a)^3/cos(b*x+a)^3)
```

maxima [A] time = 0.34, size = 26, normalized size = 0.84

$$\frac{3\tan\left(bx+a\right)^{5}+5\tan\left(bx+a\right)^{3}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(b*x + a)^5 + 5*tan(b*x + a)^3)/b
```

mupad [B] time = 0.39, size = 25, normalized size = 0.81

$$\frac{\tan\left(a+b\,x\right)^{3}\,\left(3\tan\left(a+b\,x\right)^{2}+5\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^6,x)
```

```
[Out] (tan(a + b*x)^3*(3*tan(a + b*x)^2 + 5))/(15*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.56 $\int \sec^6(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\tan^7(a+bx)}{7b}+\frac{2\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

```
[Out] 1/3*tan(b*x+a)^3/b+2/5*tan(b*x+a)^5/b+1/7*tan(b*x+a)^7/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 270}

$$\frac{\tan^7(a+bx)}{7b}+\frac{2\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b) + (2*Tan[a + b*x]^5)/(5*b) + Tan[a + b*x]^7/(7*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^6(a+bx)\tan^2(a+bx)\,dx&=\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^2\,dx,x,\tan(a+bx)\right)}{b}\\&=\frac{\text{Subst}\left(\int\left(x^2+2x^4+x^6\right)\,dx,x,\tan(a+bx)\right)}{b}\\&=\frac{\tan^3(a+bx)}{3b}+\frac{2\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A]   time = 0.04, size = 77, normalized size = 1.67

$$-\frac{8\tan(a+bx)}{105b}+\frac{\tan(a+bx)\sec^6(a+bx)}{7b}-\frac{\tan(a+bx)\sec^4(a+bx)}{35b}-\frac{4\tan(a+bx)\sec^2(a+bx)}{105b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6*Tan[a + b*x]^2,x]
```

```
[Out] (-8*Tan[a + b*x])/(105*b) - (4*Sec[a + b*x]^2*Tan[a + b*x])/(105*b) - (Sec[
a + b*x]^4*Tan[a + b*x])/(35*b) + (Sec[a + b*x]^6*Tan[a + b*x])/(7*b)
```

fricas [A]   time = 0.42, size = 51, normalized size = 1.11

$$-\frac{\left(8\cos\left(bx+a\right)^6+4\cos\left(bx+a\right)^4+3\cos\left(bx+a\right)^2-15\right)\sin\left(bx+a\right)}{105\,b\cos\left(bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/105*(8*cos(b*x + a)^6 + 4*cos(b*x + a)^4 + 3*cos(b*x + a)^2 - 15)*sin(b*
x + a)/(b*cos(b*x + a)^7)
```

giac [A]   time = 0.29, size = 36, normalized size = 0.78

$$\frac{15\tan\left(bx+a\right)^7+42\tan\left(bx+a\right)^5+35\tan\left(bx+a\right)^3}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*tan(b*x + a)^7 + 42*tan(b*x + a)^5 + 35*tan(b*x + a)^3)/b
```

maple [A]   time = 0.03, size = 60, normalized size = 1.30

$$\frac{\frac{\sin^3(bx+a)}{7\cos(bx+a)^7}+\frac{4\left(\sin^3(bx+a)\right)}{35\cos(bx+a)^5}+\frac{8\left(\sin^3(bx+a)\right)}{105\cos(bx+a)^3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^8*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/7*sin(b*x+a)^3/cos(b*x+a)^7+4/35*sin(b*x+a)^3/cos(b*x+a)^5+8/105*sin
(b*x+a)^3/cos(b*x+a)^3)
```

maxima [A] time = 0.32, size = 36, normalized size = 0.78

$$\frac{15 \tan\left(bx + a\right)^{7} + 42 \tan\left(bx + a\right)^{5} + 35 \tan\left(bx + a\right)^{3}}{105 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^2,x, algorithm="maxima")
[Out] 1/105*(15*tan(b*x + a)^7 + 42*tan(b*x + a)^5 + 35*tan(b*x + a)^3)/b
```

mupad [B] time = 0.41, size = 35, normalized size = 0.76

$$\frac{\tan\left(a+b\,x\right)^3\,\left(15\,\tan\left(a+b\,x\right)^4+42\,\tan\left(a+b\,x\right)^2+35\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^8,x)
[Out] (tan(a + b*x)^3*(42*tan(a + b*x)^2 + 15*tan(a + b*x)^4 + 35))/(105*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**8*sin(b*x+a)**2,x)
[Out] Timed out
```

3.57 $\int \sec^8(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=61

$$\frac{\tan^9(a+bx)}{9b}+\frac{3\tan^7(a+bx)}{7b}+\frac{3\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

```
[Out] 1/3*tan(b*x+a)^3/b+3/5*tan(b*x+a)^5/b+3/7*tan(b*x+a)^7/b+1/9*tan(b*x+a)^9/b
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 270}

$$\frac{\tan^9(a+bx)}{9b}+\frac{3\tan^7(a+bx)}{7b}+\frac{3\tan^5(a+bx)}{5b}+\frac{\tan^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^8*Tan[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]^3/(3*b) + (3*Tan[a + b*x]^5)/(5*b) + (3*Tan[a + b*x]^7)/(7*b)
+ Tan[a + b*x]^9/(9*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^8(a+bx)\tan^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^3\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^2+3x^4+3x^6+x^8\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\tan^3(a+bx)}{3b}+\frac{3\tan^5(a+bx)}{5b}+\frac{3\tan^7(a+bx)}{7b}+\frac{\tan^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 98, normalized size = 1.61

$$-\frac{16\tan(a+bx)}{315b}+\frac{\tan(a+bx)\sec^8(a+bx)}{9b}-\frac{\tan(a+bx)\sec^6(a+bx)}{63b}-\frac{2\tan(a+bx)\sec^4(a+bx)}{105b}-\frac{8\tan(a+bx}{31}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^8*Tan[a + b*x]^2,x]
```

```
[Out] (-16*Tan[a + b*x])/(315*b) - (8*Sec[a + b*x]^2*Tan[a + b*x])/(315*b) - (2*S
ec[a + b*x]^4*Tan[a + b*x])/(105*b) - (Sec[a + b*x]^6*Tan[a + b*x])/(63*b)
+ (Sec[a + b*x]^8*Tan[a + b*x])/(9*b)
```

fricas [A] time = 0.44, size = 61, normalized size = 1.00

$$-\frac{\left(16\cos\left(bx+a\right)^{8}+8\cos\left(bx+a\right)^{6}+6\cos\left(bx+a\right)^{4}+5\cos\left(bx+a\right)^{2}-35\right)\sin\left(bx+a\right)}{315\,b\cos\left(bx+a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/315*(16*cos(b*x + a)^8 + 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 5*cos(b*x
 + a)^2 - 35)*sin(b*x + a)/(b*cos(b*x + a)^9)
```

giac [A] time = 0.42, size = 46, normalized size = 0.75

$$\frac{35\tan\left(bx+a\right)^{9}+135\tan\left(bx+a\right)^{7}+189\tan\left(bx+a\right)^{5}+105\tan\left(bx+a\right)^{3}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*tan(b*x + a)^9 + 135*tan(b*x + a)^7 + 189*tan(b*x + a)^5 + 105*ta
n(b*x + a)^3)/b
```

maple [A] time = 0.04, size = 78, normalized size = 1.28

$$\frac{\frac{\sin^{3}(bx+a)}{9\cos(bx+a)^{9}}+\frac{2\left(\sin^{3}(bx+a)\right)}{21\cos(bx+a)^{7}}+\frac{8\left(\sin^{3}(bx+a)\right)}{105\cos(bx+a)^{5}}+\frac{16\left(\sin^{3}(bx+a)\right)}{315\cos(bx+a)^{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^10*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/9*sin(b*x+a)^3/cos(b*x+a)^9+2/21*sin(b*x+a)^3/cos(b*x+a)^7+8/105*sin
(b*x+a)^3/cos(b*x+a)^5+16/315*sin(b*x+a)^3/cos(b*x+a)^3)
```

maxima [A] time = 0.31, size = 46, normalized size = 0.75

$$\frac{35\,\tan\left(bx+a\right)^{9}+135\,\tan\left(bx+a\right)^{7}+189\,\tan\left(bx+a\right)^{5}+105\,\tan\left(bx+a\right)^{3}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*tan(b*x + a)^9 + 135*tan(b*x + a)^7 + 189*tan(b*x + a)^5 + 105*ta
n(b*x + a)^3)/b
```

mupad [B] time = 0.41, size = 45, normalized size = 0.74

$$\frac{\frac{\tan(a+b\,x)^9}{9}+\frac{3\tan(a+b\,x)^7}{7}+\frac{3\tan(a+b\,x)^5}{5}+\frac{\tan(a+b\,x)^3}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^10,x)
```

```
[Out] (tan(a + b*x)^3/3 + (3*tan(a + b*x)^5)/5 + (3*tan(a + b*x)^7)/7 + tan(a + b
*x)^9/9)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**10*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.58 $\int \cos^6(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=88

$$-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}+\frac{\sin(a+bx)\cos^5(a+bx)}{48b}+\frac{5\sin(a+bx)\cos^3(a+bx)}{192b}+\frac{5\sin(a+bx)\cos(a+bx)}{128b}+\frac{5}{12}$$

```
[Out] 5/128*x+5/128*cos(b*x+a)*sin(b*x+a)/b+5/192*cos(b*x+a)^3*sin(b*x+a)/b+1/48*
cos(b*x+a)^5*sin(b*x+a)/b-1/8*cos(b*x+a)^7*sin(b*x+a)/b
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$-\frac{\sin(a+bx)\cos^7(a+bx)}{8b}+\frac{\sin(a+bx)\cos^5(a+bx)}{48b}+\frac{5\sin(a+bx)\cos^3(a+bx)}{192b}+\frac{5\sin(a+bx)\cos(a+bx)}{128b}+\frac{5}{12}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^6*Sin[a + b*x]^2,x]
```

```
[Out] (5*x)/128 + (5*Cos[a + b*x]*Sin[a + b*x])/(128*b) + (5*Cos[a + b*x]^3*Sin[a
 + b*x])/(192*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(48*b) - (Cos[a + b*x]^7*S
in[a + b*x])/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \cos^6(a+bx)\sin^2(a+bx)\,dx &= -\frac{\cos^7(a+bx)\sin(a+bx)}{8b} + \frac{1}{8}\int \cos^6(a+bx)\,dx \\ &= \frac{\cos^5(a+bx)\sin(a+bx)}{48b} - \frac{\cos^7(a+bx)\sin(a+bx)}{8b} + \frac{5}{48}\int \cos^4(a+bx)\,dx \\ &= \frac{5\cos^3(a+bx)\sin(a+bx)}{192b} + \frac{\cos^5(a+bx)\sin(a+bx)}{48b} - \frac{\cos^7(a+bx)\sin(a+b}{8b} \\ &= \frac{5\cos(a+bx)\sin(a+bx)}{128b} + \frac{5\cos^3(a+bx)\sin(a+bx)}{192b} + \frac{\cos^5(a+bx)\sin(a+}{48b} \\ &= \frac{5x}{128} + \frac{5\cos(a+bx)\sin(a+bx)}{128b} + \frac{5\cos^3(a+bx)\sin(a+bx)}{192b} + \frac{\cos^5(a+bx)\,\mathrm{s}}{48b}\end{aligned}$$

Mathematica [A] time = 0.13, size = 52, normalized size = 0.59

$$\frac{48\sin(2(a+bx)) - 24\sin(4(a+bx)) - 16\sin(6(a+bx)) - 3\sin(8(a+bx)) + 120bx}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^6*Sin[a + b*x]^2,x]
```

```
[Out] (120*b*x + 48*Sin[2*(a + b*x)] - 24*Sin[4*(a + b*x)] - 16*Sin[6*(a + b*x)]
- 3*Sin[8*(a + b*x)])/(3072*b)
```

fricas [A] time = 0.43, size = 57, normalized size = 0.65

$$\frac{15\,bx - \left(48\,\cos(bx+a)^7 - 8\,\cos(bx+a)^5 - 10\,\cos(bx+a)^3 - 15\,\cos(bx+a)\right)\sin(bx+a)}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(15*b*x - (48*cos(b*x + a)^7 - 8*cos(b*x + a)^5 - 10*cos(b*x + a)^3 -
 15*cos(b*x + a))*sin(b*x + a))/b
```

giac [A] time = 0.54, size = 60, normalized size = 0.68

$$\frac{5}{128}\,x - \frac{\sin(8\,bx + 8\,a)}{1024\,b} - \frac{\sin(6\,bx + 6\,a)}{192\,b} - \frac{\sin(4\,bx + 4\,a)}{128\,b} + \frac{\sin(2\,bx + 2\,a)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5/128*x - 1/1024*sin(8*b*x + 8*a)/b - 1/192*sin(6*b*x + 6*a)/b - 1/128*sin(
4*b*x + 4*a)/b + 1/64*sin(2*b*x + 2*a)/b
```

maple [A] time = 0.05, size = 64, normalized size = 0.73

$$\frac{-\frac{\sin(bx+a)\left(\cos^7(bx+a)\right)}{8}+\frac{\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{48}+\frac{5bx}{128}+\frac{5a}{128}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/8*sin(b*x+a)*cos(b*x+a)^7+1/48*(cos(b*x+a)^5+5/4*cos(b*x+a)^3+15/8*
cos(b*x+a))*sin(b*x+a)+5/128*b*x+5/128*a)
```

maxima [A] time = 0.32, size = 48, normalized size = 0.55

$$\frac{64\sin(2\,bx+2\,a)^3+120\,bx+120\,a-3\,\sin(8\,bx+8\,a)-24\,\sin(4\,bx+4\,a)}{3072\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3072*(64*sin(2*b*x + 2*a)^3 + 120*b*x + 120*a - 3*sin(8*b*x + 8*a) - 24*s
in(4*b*x + 4*a))/b
```

mupad [B] time = 1.52, size = 89, normalized size = 1.01

$$\frac{5\,x}{128}+\frac{\frac{5\tan(a+b\,x)^7}{128}+\frac{55\tan(a+b\,x)^5}{384}+\frac{73\tan(a+b\,x)^3}{384}-\frac{5\tan(a+b\,x)}{128}}{b\,\left(\tan\left(a+b\,x\right)^8+4\tan\left(a+b\,x\right)^6+6\tan\left(a+b\,x\right)^4+4\tan\left(a+b\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^6*sin(a + b*x)^2,x)
```

```
[Out] (5*x)/128 + ((73*tan(a + b*x)^3)/384 - (5*tan(a + b*x))/128 + (55*tan(a + b
*x)^5)/384 + (5*tan(a + b*x)^7)/128)/(b*(4*tan(a + b*x)^2 + 6*tan(a + b*x)^
4 + 4*tan(a + b*x)^6 + tan(a + b*x)^8 + 1))
```

sympy [A] time = 15.10, size = 189, normalized size = 2.15

$$\begin{cases}\frac{5x\sin^8(a+bx)}{128}+\frac{5x\sin^6(a+bx)\cos^2(a+bx)}{32}+\frac{15x\sin^4(a+bx)\cos^4(a+bx)}{64}+\frac{5x\sin^2(a+bx)\cos^6(a+bx)}{32}+\frac{5x\cos^8(a+bx)}{128}+\frac{5\sin^7(a+bx)\text{co}}{128b}\\ x\sin^2(a)\cos^6(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((5*x*sin(a + b*x)**8/128 + 5*x*sin(a + b*x)**6*cos(a + b*x)**2/32
 + 15*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 5*x*sin(a + b*x)**2*cos(a + b*
x)**6/32 + 5*x*cos(a + b*x)**8/128 + 5*sin(a + b*x)**7*cos(a + b*x)/(128*b)
 + 55*sin(a + b*x)**5*cos(a + b*x)**3/(384*b) + 73*sin(a + b*x)**3*cos(a +
b*x)**5/(384*b) - 5*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)), (x*sin
(a)**2*cos(a)**6, True))
```

3.59 $\int \cos^4(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=67

$$-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{\sin(a+bx)\cos^3(a+bx)}{24b}+\frac{\sin(a+bx)\cos(a+bx)}{16b}+\frac{x}{16}$$

```
[Out] 1/16*x+1/16*cos(b*x+a)*sin(b*x+a)/b+1/24*cos(b*x+a)^3*sin(b*x+a)/b-1/6*cos(
b*x+a)^5*sin(b*x+a)/b
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$-\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{\sin(a+bx)\cos^3(a+bx)}{24b}+\frac{\sin(a+bx)\cos(a+bx)}{16b}+\frac{x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Sin[a + b*x]^2,x]
```

```
[Out] x/16 + (Cos[a + b*x]*Sin[a + b*x])/(16*b) + (Cos[a + b*x]^3*Sin[a + b*x])/(
24*b) - (Cos[a + b*x]^5*Sin[a + b*x])/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cos^4(a+bx)\sin^2(a+bx)\,dx = -\frac{\cos^5(a+bx)\sin(a+bx)}{6b} + \frac{1}{6}\int \cos^4(a+bx)\,dx$$
$$= \frac{\cos^3(a+bx)\sin(a+bx)}{24b} - \frac{\cos^5(a+bx)\sin(a+bx)}{6b} + \frac{1}{8}\int \cos^2(a+bx)\,dx$$
$$= \frac{\cos(a+bx)\sin(a+bx)}{16b} + \frac{\cos^3(a+bx)\sin(a+bx)}{24b} - \frac{\cos^5(a+bx)\sin(a+bx)}{6b}$$
$$= \frac{x}{16} + \frac{\cos(a+bx)\sin(a+bx)}{16b} + \frac{\cos^3(a+bx)\sin(a+bx)}{24b} - \frac{\cos^5(a+bx)\sin(a}{6b}$$

Mathematica [**A**] time = 0.08, size = 40, normalized size = 0.60

$$-\frac{-3\sin(2(a+bx)) + 3\sin(4(a+bx)) + \sin(6(a+bx)) - 12bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Sin[a + b*x]^2,x]
```

```
[Out] -1/192*(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)
])/b
```

fricas [**A**] time = 0.44, size = 47, normalized size = 0.70

$$\frac{3\,bx - \left(8\,\cos\left(bx+a\right)^5 - 2\,\cos\left(bx+a\right)^3 - 3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b*
x + a))/b
```

giac [**A**] time = 0.42, size = 46, normalized size = 0.69

$$\frac{1}{16}\,x - \frac{\sin\left(6\,bx + 6\,a\right)}{192\,b} - \frac{\sin\left(4\,bx + 4\,a\right)}{64\,b} + \frac{\sin\left(2\,bx + 2\,a\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*x - 1/192*sin(6*b*x + 6*a)/b - 1/64*sin(4*b*x + 4*a)/b + 1/64*sin(2*b*
x + 2*a)/b
```

maple [A] time = 0.05, size = 54, normalized size = 0.81

$$\frac{-\frac{\sin(bx+a)\left(\cos^5(bx+a)\right)}{6}+\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{24}+\frac{bx}{16}+\frac{a}{16}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/6*sin(b*x+a)*cos(b*x+a)^5+1/24*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*
x+a)+1/16*b*x+1/16*a)
```

maxima [A] time = 0.31, size = 37, normalized size = 0.55

$$\frac{4\sin(2bx+2a)^3+12bx+12a-3\sin(4bx+4a)}{192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/192*(4*sin(2*b*x + 2*a)^3 + 12*b*x + 12*a - 3*sin(4*b*x + 4*a))/b
```

mupad [B] time = 0.56, size = 43, normalized size = 0.64

$$\frac{x}{16}-\frac{\frac{\sin(4a+4bx)}{64}-\frac{\sin(2a+2bx)}{64}+\frac{\sin(6a+6bx)}{192}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4*sin(a + b*x)^2,x)
```

```
[Out] x/16 - (sin(4*a + 4*b*x)/64 - sin(2*a + 2*b*x)/64 + sin(6*a + 6*b*x)/192)/b
```

sympy [A] time = 5.06, size = 136, normalized size = 2.03

$$\begin{cases} \frac{x\sin^6(a+bx)}{16}+\frac{3x\sin^4(a+bx)\cos^2(a+bx)}{16}+\frac{3x\sin^2(a+bx)\cos^4(a+bx)}{16}+\frac{x\cos^6(a+bx)}{16}+\frac{\sin^5(a+bx)\cos(a+bx)}{16b}+\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b} \\ x\sin^2(a)\cos^4(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**6/16 + 3*x*sin(a + b*x)**4*cos(a + b*x)**2/16 +
3*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + x*cos(a + b*x)**6/16 + sin(a + b*x
)**5*cos(a + b*x)/(16*b) + sin(a + b*x)**3*cos(a + b*x)**3/(6*b) - sin(a +
b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**4, True))
```

3.60 $\int \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{\sin(a+bx)\cos(a+bx)}{8b}+\frac{x}{8}$$

```
[Out] 1/8*x+1/8*cos(b*x+a)*sin(b*x+a)/b-1/4*cos(b*x+a)^3*sin(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$-\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{\sin(a+bx)\cos(a+bx)}{8b}+\frac{x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] x/8 + (Cos[a + b*x]*Sin[a + b*x])/(8*b) - (Cos[a + b*x]^3*Sin[a + b*x])/(4*
b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^2(a+bx)\,dx = -\frac{\cos^3(a+bx)\sin(a+bx)}{4b} + \frac{1}{4}\int \cos^2(a+bx)\,dx$$
$$= \frac{\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin(a+bx)}{4b} + \frac{\int 1\,dx}{8}$$
$$= \frac{x}{8} + \frac{\cos(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin(a+bx)}{4b}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 0.50

$$-\frac{\sin(4(a+bx)) - 4(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^2,x]
[Out] -1/32*(-4*(a + b*x) + Sin[4*(a + b*x)])/b
```

fricas [A] time = 0.43, size = 36, normalized size = 0.78

$$\frac{bx - \left(2\,\cos\left(bx+a\right)^3 - \cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/8*(b*x - (2*cos(b*x + a)^3 - cos(b*x + a))*sin(b*x + a))/b
```

giac [A] time = 0.18, size = 18, normalized size = 0.39

$$\frac{1}{8}\,x - \frac{\sin\left(4\,bx + 4\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/8*x - 1/32*sin(4*b*x + 4*a)/b
```

maple [A] time = 0.02, size = 43, normalized size = 0.93

$$\frac{-\frac{\left(\cos^3(bx+a)\right)\sin(bx+a)}{4} + \frac{\cos(bx+a)\sin(bx+a)}{8} + \frac{bx}{8} + \frac{a}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)
```

maxima [A] time = 0.31, size = 24, normalized size = 0.52

$$\frac{4\,bx+4\,a-\sin\left(4\,bx+4\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(4*b*x + 4*a - sin(4*b*x + 4*a))/b
```

mupad [B] time = 0.46, size = 50, normalized size = 1.09

$$\frac{x}{8}-\frac{\frac{\tan(a+b\,x)}{8}-\frac{\tan(a+b\,x)^3}{8}}{b\,\left(\tan\left(a+b\,x\right)^4+2\,\tan\left(a+b\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2,x)
```

```
[Out] x/8 - (tan(a + b*x)/8 - tan(a + b*x)^3/8)/(b*(2*tan(a + b*x)^2 + tan(a + b*
x)^4 + 1))
```

sympy [A] time = 1.45, size = 92, normalized size = 2.00

$$\begin{cases} \frac{x\sin^4(a+bx)}{8}+\frac{x\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{x\cos^4(a+bx)}{8}+\frac{\sin^3(a+bx)\cos(a+bx)}{8b}-\frac{\sin(a+bx)\cos^3(a+bx)}{8b} & \text{for } b\neq 0 \\ x\sin^2(a)\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**4/8 + x*sin(a + b*x)**2*cos(a + b*x)**2/4 + x*co
s(a + b*x)**4/8 + sin(a + b*x)**3*cos(a + b*x)/(8*b) - sin(a + b*x)*cos(a +
 b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**2*cos(a)**2, True))
```

3.61 $\int \sin^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

```
[Out] 1/2*x-1/2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2,x]
```

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\,dx &= -\frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$-\frac{\sin(2(a+bx))-2(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2,x]
[Out] -1/4*(-2*(a + b*x) + Sin[2*(a + b*x)])/b
```

fricas [A] time = 0.41, size = 23, normalized size = 0.92

$$\frac{bx - \cos(bx + a)\sin(bx + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

giac [A] time = 0.21, size = 18, normalized size = 0.72

$$\frac{1}{2}x - \frac{\sin(2bx + 2a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

maple [A] time = 0.00, size = 27, normalized size = 1.08

$$\frac{-\frac{\cos(bx+a)\sin(bx+a)}{2} + \frac{bx}{2} + \frac{a}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2,x)
[Out] 1/b*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

maxima [A] time = 0.31, size = 24, normalized size = 0.96

$$\frac{2bx + 2a - \sin(2bx + 2a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="maxima")
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

mupad [B] time = 0.00, size = 18, normalized size = 0.72

$$\frac{x}{2}-\frac{\sin\left(2\,a+2\,b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2,x)
```

```
[Out] x/2 - sin(2*a + 2*b*x)/(4*b)
```

sympy [A] time = 0.35, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sin^{2}(a+bx)}{2}+\frac{x\cos^{2}(a+bx)}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b\neq 0 \\ x\sin^{2}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

3.62 $\int \sin(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] arctanh(sin(b*x+a))/b-sin(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2592, 321, 206}

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \sin(a+bx)\tan(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= -\frac{\sin(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{b} - \frac{\sin(a+bx)}{b}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(a+bx))}{b} - \frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x],x]
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b
```

fricas [A] time = 0.45, size = 36, normalized size = 1.57

$$\frac{\log\left(\sin\left(bx+a\right)+1\right) - \log\left(-\sin\left(bx+a\right)+1\right) - 2\,\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1) - 2*sin(b*x + a))/b
```

giac [A] time = 0.25, size = 36, normalized size = 1.57

$$\frac{\log\left(\left|\sin\left(bx+a\right)+1\right|\right) - \log\left(\left|\sin\left(bx+a\right)-1\right|\right) - 2\,\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/2*(log(abs(sin(b*x + a) + 1)) - log(abs(sin(b*x + a) - 1)) - 2*sin(b*x +
a))/b
```

maple [A] time = 0.03, size = 31, normalized size = 1.35

$$-\frac{\sin\left(bx+a\right)}{b} + \frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] -sin(b*x+a)/b+1/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.32, size = 34, normalized size = 1.48

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(\sin\left(bx+a\right)-1\right)-2\,\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))/b
```

mupad [B] time = 0.46, size = 27, normalized size = 1.17

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}-\frac{\sin\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/cos(a + b*x),x)
```

```
[Out] (2*atanh(tan(a/2 + (b*x)/2)))/b - sin(a + b*x)/b
```

sympy [B] time = 55.35, size = 3160, normalized size = 137.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((log(tan(a + b*x) + sec(a + b*x))/b, Ne(b, 0)), (x*(tan(a)*sec(a)
 + sec(a)**2)/(tan(a) + sec(a)), True))/2 + 2*Piecewise((0, Eq(b, 0)), (sin
(b*x)/b, Eq(a, -pi/2)), (-sin(b*x)/b, Eq(a, pi/2)), (-2*log(tan(b*x/2) - ta
n(a/2)/(tan(a/2) - 1) - 1/(tan(a/2) - 1))*tan(a/2)**3*tan(b*x/2)**2/(b*tan(
a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b
*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - tan(a/2)/(tan(a/2)
 - 1) - 1/(tan(a/2) - 1))*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(
a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2
 + b) + 2*log(tan(b*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/(tan(a/2) - 1))*tan(
a/2)*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a
/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b
```

```
*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/(tan(a/2) - 1))*tan(a/2)/(b*tan(a/2)**4
*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/
2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) + tan(a/2)/(tan(a/2) + 1) -
 1/(tan(a/2) + 1))*tan(a/2)**3*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 +
 b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*
x/2)**2 + b) + 2*log(tan(b*x/2) + tan(a/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1
))*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)*
*2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2
) + tan(a/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1))*tan(a/2)*tan(b*x/2)**2/(b*t
an(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 +
2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + tan(a/2)/(tan(a
/2) + 1) - 1/(tan(a/2) + 1))*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(
a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2
 + b) + 2*tan(a/2)**4/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*ta
n(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*tan(a/
2)**3*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2
)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*tan(a/2)*ta
n(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan
(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2/(b*tan(a/2)**4*tan(
b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2
 + b*tan(b*x/2)**2 + b), True))*sin(a)*cos(a) + Piecewise((log(tan(b*x/2))/
b, Eq(a, -pi/2)), (-log(tan(b*x/2))/b, Eq(a, pi/2)), (x/cos(a), Eq(b, 0)),
(log(tan(b*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/(tan(a/2) - 1))/b - log(tan(b
*x/2) + tan(a/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1))/b, True))*cos(a)**2 - P
iecewise((log(tan(b*x/2))/b, Eq(a, -pi/2)), (-log(tan(b*x/2))/b, Eq(a, pi/2
)), (x/cos(a), Eq(b, 0)), (log(tan(b*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/(ta
n(a/2) - 1))/b - log(tan(b*x/2) + tan(a/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1
))/b, True))/2 - 2*Piecewise((x/cos(a), Eq(b, 0)), (log(tan(b*x/2))*tan(b*x
/2)**2/(b*tan(b*x/2)**2 + b) + log(tan(b*x/2))/(b*tan(b*x/2)**2 + b) + 2/(b
*tan(b*x/2)**2 + b), Eq(a, -pi/2)), (-log(tan(b*x/2))*tan(b*x/2)**2/(b*tan(
b*x/2)**2 + b) - log(tan(b*x/2))/(b*tan(b*x/2)**2 + b) - 2/(b*tan(b*x/2)**2
 + b), Eq(a, pi/2)), (4*log(tan(b*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/(tan(a
/2) - 1))*tan(a/2)**2*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/
2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 +
 b) + 4*log(tan(b*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/(tan(a/2) - 1))*tan(a/
2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*
x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*log(tan(b*x/2) + tan(a
/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1))*tan(a/2)**2*tan(b*x/2)**2/(b*tan(a/2
)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*ta
n(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*log(tan(b*x/2) + tan(a/2)/(tan(a/2) +
1) - 1/(tan(a/2) + 1))*tan(a/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2
)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 +
b) - 2*tan(a/2)**4*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4
+ 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) -
4*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**
```

```
2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*tan(a/2)/(b*ta
n(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2
*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x
/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 +
b*tan(b*x/2)**2 + b), True))*cos(a)**2 + Piecewise((x/cos(a), Eq(b, 0)), (l
og(tan(b*x/2))*tan(b*x/2)**2/(b*tan(b*x/2)**2 + b) + log(tan(b*x/2))/(b*tan
(b*x/2)**2 + b) + 2/(b*tan(b*x/2)**2 + b), Eq(a, -pi/2)), (-log(tan(b*x/2))
*tan(b*x/2)**2/(b*tan(b*x/2)**2 + b) - log(tan(b*x/2))/(b*tan(b*x/2)**2 + b
) - 2/(b*tan(b*x/2)**2 + b), Eq(a, pi/2)), (4*log(tan(b*x/2) - tan(a/2)/(ta
n(a/2) - 1) - 1/(tan(a/2) - 1))*tan(a/2)**2*tan(b*x/2)**2/(b*tan(a/2)**4*ta
n(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)*
*2 + b*tan(b*x/2)**2 + b) + 4*log(tan(b*x/2) - tan(a/2)/(tan(a/2) - 1) - 1/
(tan(a/2) - 1))*tan(a/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 +
2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*
log(tan(b*x/2) + tan(a/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1))*tan(a/2)**2*ta
n(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*
tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*log(tan(b*x/2) +
 tan(a/2)/(tan(a/2) + 1) - 1/(tan(a/2) + 1))*tan(a/2)**2/(b*tan(a/2)**4*tan
(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**
2 + b*tan(b*x/2)**2 + b) - 2*tan(a/2)**4*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/
2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b
*tan(b*x/2)**2 + b) - 4*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/
2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 +
 b) - 4*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2
)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*tan(b*x/2)/
(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**
2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b), True))
```

3.63 $\int \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

```
[Out] -1/2*arctanh(sin(b*x+a))/b+1/2*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)\tan^2(a+bx)\,dx &= \frac{\sec(a+bx)\tan(a+bx)}{2b}-\frac{1}{2}\int \sec(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\sec(a+bx)\tan(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{\tan(a+bx)\sec(a+bx)}{2b}-\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -1/2*ArcTanh[Sin[a + b*x]]/b + (Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

fricas [B] time = 0.42, size = 61, normalized size = 1.79

$$-\frac{\cos\left(bx+a\right)^{2}\log\left(\sin\left(bx+a\right)+1\right)-\cos\left(bx+a\right)^{2}\log\left(-\sin\left(bx+a\right)+1\right)-2\,\sin\left(bx+a\right)}{4\,b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x +
a) + 1) - 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

giac [A] time = 0.53, size = 48, normalized size = 1.41

$$-\frac{\frac{2\,\sin(bx+a)}{\sin(bx+a)^{2}-1}+\log\left(\left|\sin\left(bx+a\right)+1\right|\right)-\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(abs(sin(b*x + a) + 1)) - lo
g(abs(sin(b*x + a) - 1)))/b
```

maple [A] time = 0.03, size = 53, normalized size = 1.56

$$\frac{\sin^{3}\left(bx+a\right)}{2b\cos\left(bx+a\right)^{2}}+\frac{\sin\left(bx+a\right)}{2b}-\frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/2/b*sin(b*x+a)^3/cos(b*x+a)^2+1/2*sin(b*x+a)/b-1/2/b*ln(sec(b*x+a)+tan(b*
x+a))
```

maxima [A] time = 0.32, size = 46, normalized size = 1.35

$$-\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1}+\log(\sin(bx+a)+1)-\log(\sin(bx+a)-1)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin
(b*x + a) - 1))/b
```

mupad [B] time = 1.22, size = 69, normalized size = 2.03

$$\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{b\,\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^3,x)
```

```
[Out] (tan(a/2 + (b*x)/2) + tan(a/2 + (b*x)/2)^3)/(b*(tan(a/2 + (b*x)/2)^4 - 2*ta
n(a/2 + (b*x)/2)^2 + 1)) - atanh(tan(a/2 + (b*x)/2))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^2(a+bx)\sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)**3, x)
```

3.64 $\int \sec^3(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}-\frac{\tan(a+bx)\sec(a+bx)}{8b}$$

```
[Out] -1/8*arctanh(sin(b*x+a))/b-1/8*sec(b*x+a)*tan(b*x+a)/b+1/4*sec(b*x+a)^3*tan
(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$-\frac{\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\tan(a+bx)\sec^3(a+bx)}{4b}-\frac{\tan(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x]^2,x]
```

```
[Out] -ArcTanh[Sin[a + b*x]]/(8*b) - (Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Sec[a +
 b*x]^3*Tan[a + b*x])/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(a+bx)\tan^2(a+bx)\,dx = \frac{\sec^3(a+bx)\tan(a+bx)}{4b} - \frac{1}{4}\int \sec^3(a+bx)\,dx$$
$$= -\frac{\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan(a+bx)}{4b} - \frac{1}{8}\int \sec(a+bx)\,dx$$
$$= -\frac{\tanh^{-1}(\sin(a+bx))}{8b} - \frac{\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan(a+bx)}{4b}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 1.00

$$-\frac{\tanh^{-1}(\sin(a+bx))}{8b} + \frac{\tan(a+bx)\sec^3(a+bx)}{4b} - \frac{\tan(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x]^2,x]
```

```
[Out] -1/8*ArcTanh[Sin[a + b*x]]/b - (Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Sec[a +
 b*x]^3*Tan[a + b*x])/(4*b)
```

fricas [A] time = 0.45, size = 71, normalized size = 1.29

$$-\frac{\cos(bx+a)^4\log(\sin(bx+a)+1) - \cos(bx+a)^4\log(-\sin(bx+a)+1) + 2\left(\cos(bx+a)^2 - 2\right)\sin(bx+a)}{16\,b\cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(cos(b*x + a)^4*log(sin(b*x + a) + 1) - cos(b*x + a)^4*log(-sin(b*x +
 a) + 1) + 2*(cos(b*x + a)^2 - 2)*sin(b*x + a))/(b*cos(b*x + a)^4)
```

giac [A] time = 0.25, size = 82, normalized size = 1.49

$$\frac{\frac{4\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)}{\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^2-4} - \log\left(\left|\frac{1}{\sin(bx+a)} + \sin(bx+a) + 2\right|\right) + \log\left(\left|\frac{1}{\sin(bx+a)} + \sin(bx+a) - 2\right|\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*(4*(1/sin(b*x + a) + sin(b*x + a))/((1/sin(b*x + a) + sin(b*x + a))^2
- 4) - log(abs(1/sin(b*x + a) + sin(b*x + a) + 2)) + log(abs(1/sin(b*x + a)
 + sin(b*x + a) - 2)))/b
```

maple [A] time = 0.03, size = 74, normalized size = 1.35

$$\frac{\sin^3(bx+a)}{4b\cos(bx+a)^4}+\frac{\sin^3(bx+a)}{8b\cos(bx+a)^2}+\frac{\sin(bx+a)}{8b}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5*sin(b*x+a)^2,x)
```

```
[Out] 1/4/b*sin(b*x+a)^3/cos(b*x+a)^4+1/8/b*sin(b*x+a)^3/cos(b*x+a)^2+1/8*sin(b*x
+a)/b-1/8/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.34, size = 65, normalized size = 1.18

$$\frac{\frac{2\left(\sin(bx+a)^3+\sin(bx+a)\right)}{\sin(bx+a)^4-2\sin(bx+a)^2+1}-\log(\sin(bx+a)+1)+\log(\sin(bx+a)-1)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(sin(b*x + a)^3 + sin(b*x + a))/(sin(b*x + a)^4 - 2*sin(b*x + a)^2
+ 1) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b
```

mupad [B] time = 6.54, size = 125, normalized size = 2.27

$$\frac{\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^7}{4}+\frac{7\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^5}{4}+\frac{7\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3}{4}+\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{4}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8-4\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6+6\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4-4\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^5,x)
```

```
[Out] (tan(a/2 + (b*x)/2)/4 + (7*tan(a/2 + (b*x)/2)^3)/4 + (7*tan(a/2 + (b*x)/2)^
5)/4 + tan(a/2 + (b*x)/2)^7/4)/(b*(6*tan(a/2 + (b*x)/2)^4 - 4*tan(a/2 + (b*
x)/2)^2 - 4*tan(a/2 + (b*x)/2)^6 + tan(a/2 + (b*x)/2)^8 + 1)) - atanh(tan(a
/2 + (b*x)/2))/(4*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^2(a+bx)\sec^5(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5*sin(b*x+a)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)**5, x)
```

3.65 $\int \sec^5(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=76

$$-\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}-\frac{\tan(a+bx)\sec^3(a+bx)}{24b}-\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

```
[Out] -1/16*arctanh(sin(b*x+a))/b-1/16*sec(b*x+a)*tan(b*x+a)/b-1/24*sec(b*x+a)^3*
tan(b*x+a)/b+1/6*sec(b*x+a)^5*tan(b*x+a)/b
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$-\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}-\frac{\tan(a+bx)\sec^3(a+bx)}{24b}-\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5*Tan[a + b*x]^2,x]
```

```
[Out] -ArcTanh[Sin[a + b*x]]/(16*b) - (Sec[a + b*x]*Tan[a + b*x])/(16*b) - (Sec[a
 + b*x]^3*Tan[a + b*x])/(24*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(6*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(a+bx)\tan^2(a+bx)\,dx = \frac{\sec^5(a+bx)\tan(a+bx)}{6b} - \frac{1}{6}\int \sec^5(a+bx)\,dx$$
$$= -\frac{\sec^3(a+bx)\tan(a+bx)}{24b} + \frac{\sec^5(a+bx)\tan(a+bx)}{6b} - \frac{1}{8}\int \sec^3(a+bx)\,dx$$
$$= -\frac{\sec(a+bx)\tan(a+bx)}{16b} - \frac{\sec^3(a+bx)\tan(a+bx)}{24b} + \frac{\sec^5(a+bx)\tan(a+$$
$$= -\frac{\tanh^{-1}(\sin(a+bx))}{16b} - \frac{\sec(a+bx)\tan(a+bx)}{16b} - \frac{\sec^3(a+bx)\tan(a+bx)}{24b}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 1.00

$$-\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec^5(a+bx)}{6b}-\frac{\tan(a+bx)\sec^3(a+bx)}{24b}-\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5*Tan[a + b*x]^2,x]
```

```
[Out] -1/16*ArcTanh[Sin[a + b*x]]/b - (Sec[a + b*x]*Tan[a + b*x])/(16*b) - (Sec[a
 + b*x]^3*Tan[a + b*x])/(24*b) + (Sec[a + b*x]^5*Tan[a + b*x])/(6*b)
```

fricas [A] time = 0.43, size = 84, normalized size = 1.11

$$-\frac{3\cos(bx+a)^6\log(\sin(bx+a)+1)-3\cos(bx+a)^6\log(-\sin(bx+a)+1)+2\left(3\cos(bx+a)^4+2\cos(b\right.}{96\,b\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/96*(3*cos(b*x + a)^6*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^6*log(-sin(b
*x + a) + 1) + 2*(3*cos(b*x + a)^4 + 2*cos(b*x + a)^2 - 8)*sin(b*x + a))/(b
*cos(b*x + a)^6)
```

giac [A] time = 0.27, size = 73, normalized size = 0.96

$$\frac{\frac{2\left(3\sin(bx+a)^5-8\sin(bx+a)^3-3\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^3}-3\log(|\sin(bx+a)+1|)+3\log(|\sin(bx+a)-1|)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/96*(2*(3*sin(b*x + a)^5 - 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a
)^2 - 1)^3 - 3*log(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/b
```

maple [A] time = 0.04, size = 95, normalized size = 1.25

$$\frac{\sin^3(bx+a)}{6b\cos(bx+a)^6}+\frac{\sin^3(bx+a)}{8b\cos(bx+a)^4}+\frac{\sin^3(bx+a)}{16b\cos(bx+a)^2}+\frac{\sin(bx+a)}{16b}-\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^7*sin(b*x+a)^2,x)
```

```
[Out] 1/6/b*sin(b*x+a)^3/cos(b*x+a)^6+1/8/b*sin(b*x+a)^3/cos(b*x+a)^4+1/16/b*sin(
b*x+a)^3/cos(b*x+a)^2+1/16*sin(b*x+a)/b-1/16/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.32, size = 91, normalized size = 1.20

$$\frac{\frac{2\left(3\sin(bx+a)^5-8\sin(bx+a)^3-3\sin(bx+a)\right)}{\sin(bx+a)^6-3\sin(bx+a)^4+3\sin(bx+a)^2-1}-3\log(\sin(bx+a)+1)+3\log(\sin(bx+a)-1)}{96b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(2*(3*sin(b*x + a)^5 - 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a
)^6 - 3*sin(b*x + a)^4 + 3*sin(b*x + a)^2 - 1) - 3*log(sin(b*x + a) + 1) +
3*log(sin(b*x + a) - 1))/b
```

mupad [B] time = 7.33, size = 177, normalized size = 2.33

$$\frac{\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{11}}{8}+\frac{47\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{9}}{24}+\frac{13\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{7}}{4}+\frac{13\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{5}}{4}+\frac{47\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{3}}{24}+\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{8}}{b\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{12}-6\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{10}+15\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{8}-20\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{6}+15\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{4}-6\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/cos(a + b*x)^7,x)
```

```
[Out] (tan(a/2 + (b*x)/2)/8 + (47*tan(a/2 + (b*x)/2)^3)/24 + (13*tan(a/2 + (b*x)/
2)^5)/4 + (13*tan(a/2 + (b*x)/2)^7)/4 + (47*tan(a/2 + (b*x)/2)^9)/24 + tan(
a/2 + (b*x)/2)^11/8)/(b*(15*tan(a/2 + (b*x)/2)^4 - 6*tan(a/2 + (b*x)/2)^2 -
 20*tan(a/2 + (b*x)/2)^6 + 15*tan(a/2 + (b*x)/2)^8 - 6*tan(a/2 + (b*x)/2)^1
0 + tan(a/2 + (b*x)/2)^12 + 1)) - atanh(tan(a/2 + (b*x)/2))/(8*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**7*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.66 $\int \cos^5(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\cos^8(a+bx)}{8b}-\frac{\cos^6(a+bx)}{6b}$$

```
[Out] -1/6*cos(b*x+a)^6/b+1/8*cos(b*x+a)^8/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 14}

$$\frac{\cos^8(a+bx)}{8b}-\frac{\cos^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Sin[a + b*x]^3,x]
```

```
[Out] -Cos[a + b*x]^6/(6*b) + Cos[a + b*x]^8/(8*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^5(a+bx)\sin^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^5\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(x^5-x^7\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\cos^6(a+bx)}{6b}+\frac{\cos^8(a+bx)}{8b}\end{aligned}$$

Mathematica [A] time = 0.12, size = 48, normalized size = 1.55

$$\frac{-72\cos(2(a+bx))-12\cos(4(a+bx))+8\cos(6(a+bx))+3\cos(8(a+bx))}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Sin[a + b*x]^3,x]
[Out] (-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8*
(a + b*x)])/(3072*b)
```

fricas [A] time = 0.43, size = 26, normalized size = 0.84

$$\frac{3\,\cos\left(bx+a\right)^{8}-4\,\cos\left(bx+a\right)^{6}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/24*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

giac [A] time = 0.20, size = 27, normalized size = 0.87

$$\frac{\cos\left(bx+a\right)^{8}}{8\,b}-\frac{\cos\left(bx+a\right)^{6}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/8*cos(b*x + a)^8/b - 1/6*cos(b*x + a)^6/b
```

maple [A] time = 0.02, size = 34, normalized size = 1.10

$$\frac{-\frac{\left(\cos^{6}(bx+a)\right)\left(\sin^{2}(bx+a)\right)}{8}-\frac{\left(\cos^{6}(bx+a)\right)}{24}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5*sin(b*x+a)^3,x)
[Out] 1/b*(-1/8*cos(b*x+a)^6*sin(b*x+a)^2-1/24*cos(b*x+a)^6)
```

maxima [A] time = 0.32, size = 36, normalized size = 1.16

$$\frac{3\,\sin\left(bx+a\right)^{8}-8\,\sin\left(bx+a\right)^{6}+6\,\sin\left(bx+a\right)^{4}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*sin(b*x + a)^8 - 8*sin(b*x + a)^6 + 6*sin(b*x + a)^4)/b
```

mupad [B] time = 0.37, size = 25, normalized size = 0.81

$$\frac{\cos\left(a+b\,x\right)^6\,\left(3\,\cos\left(a+b\,x\right)^2-4\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5*sin(a + b*x)^3,x)
```

```
[Out] (cos(a + b*x)^6*(3*cos(a + b*x)^2 - 4))/(24*b)
```

sympy [A] time = 14.21, size = 44, normalized size = 1.42

$$\begin{cases} -\frac{\sin^{2}{\left(a+bx \right)}\cos^{6}{\left(a+bx \right)}}{6b} - \frac{\cos^{8}{\left(a+bx \right)}}{24b} & \text{for}\: b \neq 0 \\ x \sin^{3}{\left(a \right)} \cos^{5}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)**6/(6*b) - cos(a + b*x)**8/(24*b),
 Ne(b, 0)), (x*sin(a)**3*cos(a)**5, True))
```

3.67 $\int \cos^4(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\cos^7(a+bx)}{7b}-\frac{\cos^5(a+bx)}{5b}$$

```
[Out] -1/5*cos(b*x+a)^5/b+1/7*cos(b*x+a)^7/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 14}

$$\frac{\cos^7(a+bx)}{7b}-\frac{\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Sin[a + b*x]^3,x]
```

```
[Out] -Cos[a + b*x]^5/(5*b) + Cos[a + b*x]^7/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\int \cos^4(a+bx)\sin^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(x^4-x^6\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\cos^5(a+bx)}{5b}+\frac{\cos^7(a+bx)}{7b}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 27, normalized size = 0.87

$$\frac{\cos^5(a+bx)(5\cos(2(a+bx))-9)}{70b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Sin[a + b*x]^3,x]
[Out] (Cos[a + b*x]^5*(-9 + 5*Cos[2*(a + b*x)]))/(70*b)
```

fricas [A] time = 0.41, size = 26, normalized size = 0.84

$$\frac{5\cos\left(bx+a\right)^7-7\cos\left(bx+a\right)^5}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

giac [A] time = 0.19, size = 27, normalized size = 0.87

$$\frac{\cos\left(bx+a\right)^7}{7\,b}-\frac{\cos\left(bx+a\right)^5}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/7*cos(b*x + a)^7/b - 1/5*cos(b*x + a)^5/b
```

maple [A] time = 0.02, size = 34, normalized size = 1.10

$$\frac{-\frac{\left(\cos^5(bx+a)\right)\left(\sin^2(bx+a)\right)}{7}-\frac{2\left(\cos^5(bx+a)\right)}{35}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*sin(b*x+a)^3,x)
[Out] 1/b*(-1/7*cos(b*x+a)^5*sin(b*x+a)^2-2/35*cos(b*x+a)^5)
```

maxima [A] time = 0.31, size = 26, normalized size = 0.84

$$\frac{5\cos\left(bx+a\right)^7-7\cos\left(bx+a\right)^5}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

mupad [B] time = 0.38, size = 26, normalized size = 0.84

$$-\frac{7\cos\left(a+b\,x\right)^5-5\cos\left(a+b\,x\right)^7}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4*sin(a + b*x)^3,x)
```

```
[Out] -(7*cos(a + b*x)^5 - 5*cos(a + b*x)^7)/(35*b)
```

sympy [A] time = 8.87, size = 46, normalized size = 1.48

$$\begin{cases} -\frac{\sin^{2}\left(a+bx\right)\cos^{5}\left(a+bx\right)}{5b} - \frac{2\cos^{7}\left(a+bx\right)}{35b} & \text{for}\: b \neq 0 \\ x\sin^{3}\left(a\right)\cos^{4}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)**5/(5*b) - 2*cos(a + b*x)**7/(35*b
), Ne(b, 0)), (x*sin(a)**3*cos(a)**4, True))
```

3.68 $\int \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sin^4(a+bx)}{4b}-\frac{\sin^6(a+bx)}{6b}$$

```
[Out] 1/4*sin(b*x+a)^4/b-1/6*sin(b*x+a)^6/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sin^4(a+bx)}{4b}-\frac{\sin^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] Sin[a + b*x]^4/(4*b) - Sin[a + b*x]^6/(6*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^3\left(1-x^2\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^3-x^5\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\sin^4(a+bx)}{4b}-\frac{\sin^6(a+bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.13

$$\frac{1}{8}\left(\frac{\cos(6(a+bx))}{24b}-\frac{3\cos(2(a+bx))}{8b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x]^3,x]
[Out] ((-3*Cos[2*(a + b*x)])/(8*b) + Cos[6*(a + b*x)]/(24*b))/8
```

fricas [A] time = 0.45, size = 26, normalized size = 0.84

$$\frac{2\cos\left(bx+a\right)^{6}-3\cos\left(bx+a\right)^{4}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/12*(2*cos(b*x + a)^6 - 3*cos(b*x + a)^4)/b
```

giac [A] time = 0.20, size = 26, normalized size = 0.84

$$-\frac{2\sin\left(bx+a\right)^{6}-3\sin\left(bx+a\right)^{4}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/12*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)/b
```

maple [A] time = 0.02, size = 34, normalized size = 1.10

$$\frac{-\frac{\left(\cos^{4}(bx+a)\right)\left(\sin^{2}(bx+a)\right)}{6}-\frac{\left(\cos^{4}(bx+a)\right)}{12}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3,x)
[Out] 1/b*(-1/6*cos(b*x+a)^4*sin(b*x+a)^2-1/12*cos(b*x+a)^4)
```

maxima [A] time = 0.34, size = 26, normalized size = 0.84

$$-\frac{2\sin\left(bx+a\right)^{6}-3\sin\left(bx+a\right)^{4}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)/b
```

mupad [B] time = 0.51, size = 37, normalized size = 1.19

$$\frac{\cos\left(a+b\,x\right)^4\,\left(\cos\left(a+b\,x\right)^2-1\right)}{4\,b}-\frac{\cos\left(a+b\,x\right)^6}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3,x)
```

```
[Out] (cos(a + b*x)^4*(cos(a + b*x)^2 - 1))/(4*b) - cos(a + b*x)^6/(12*b)
```

sympy [A] time = 4.33, size = 44, normalized size = 1.42

$$\begin{cases} -\frac{\sin^{2}{\left(a+bx \right)}\cos^{4}{\left(a+bx \right)}}{4b} - \frac{\cos^{6}{\left(a+bx \right)}}{12b} & \text{for}\: b \neq 0 \\ x\sin^{3}{\left(a \right)}\cos^{3}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - cos(a + b*x)**6/(12*b),
 Ne(b, 0)), (x*sin(a)**3*cos(a)**3, True))
```

3.69 $\int \cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\cos^5(a+bx)}{5b}-\frac{\cos^3(a+bx)}{3b}$$

```
[Out] -1/3*cos(b*x+a)^3/b+1/5*cos(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 14}

$$\frac{\cos^5(a+bx)}{5b}-\frac{\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -Cos[a + b*x]^3/(3*b) + Cos[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\cos^3(a+bx)}{3b}+\frac{\cos^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.05, size = 27, normalized size = 0.87

$$\frac{\cos^3(a+bx)(3\cos(2(a+bx))-7)}{30b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^3,x]
[Out] (Cos[a + b*x]^3*(-7 + 3*Cos[2*(a + b*x)]))/(30*b)
```

fricas [A] time = 0.44, size = 26, normalized size = 0.84

$$\frac{3\,\cos\left(bx+a\right)^5-5\,\cos\left(bx+a\right)^3}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

giac [A] time = 0.18, size = 27, normalized size = 0.87

$$\frac{\cos\left(bx+a\right)^5}{5\,b}-\frac{\cos\left(bx+a\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/5*cos(b*x + a)^5/b - 1/3*cos(b*x + a)^3/b
```

maple [A] time = 0.02, size = 34, normalized size = 1.10

$$\frac{-\frac{\left(\cos^3(bx+a)\right)\left(\sin^2(bx+a)\right)}{5}-\frac{2\left(\cos^3(bx+a)\right)}{15}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3,x)
[Out] 1/b*(-1/5*cos(b*x+a)^3*sin(b*x+a)^2-2/15*cos(b*x+a)^3)
```

maxima [A] time = 0.32, size = 26, normalized size = 0.84

$$\frac{3\,\cos\left(bx+a\right)^5-5\,\cos\left(bx+a\right)^3}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

mupad [**B**] time = 0.34, size = 26, normalized size = 0.84

$$-\frac{5\cos\left(a+b\,x\right)^3-3\cos\left(a+b\,x\right)^5}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3,x)
```

```
[Out] -(5*cos(a + b*x)^3 - 3*cos(a + b*x)^5)/(15*b)
```

sympy [**A**] time = 2.46, size = 46, normalized size = 1.48

$$\begin{cases} -\frac{\sin^{2}{\left(a+bx \right)}\cos^{3}{\left(a+bx \right)}}{3b} - \frac{2\cos^{5}{\left(a+bx \right)}}{15b} & \text{for}\: b \neq 0 \\ x\sin^{3}{\left(a \right)}\cos^{2}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*cos(a + b*x)**5/(15*b
), Ne(b, 0)), (x*sin(a)**3*cos(a)**2, True))
```

3.70 $\int \cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^4(a+bx)}{4b}$$

```
[Out] 1/4*sin(b*x+a)^4/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sin^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] Sin[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^3\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sin^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] Sin[a + b*x]^4/(4*b)
```

fricas [A] time = 0.45, size = 24, normalized size = 1.60

$$\frac{\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(cos(b*x + a)^4 - 2*cos(b*x + a)^2)/b
```

giac [A] time = 0.18, size = 13, normalized size = 0.87

$$\frac{\sin\left(bx+a\right)^{4}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*sin(b*x + a)^4/b
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$\frac{\sin^{4}\left(bx+a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/4*sin(b*x+a)^4/b
```

maxima [A] time = 0.32, size = 13, normalized size = 0.87

$$\frac{\sin\left(bx+a\right)^{4}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*sin(b*x + a)^4/b
```

mupad [B] time = 0.37, size = 13, normalized size = 0.87

$$\frac{\sin\left(a+b\,x\right)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3,x)
```

```
[Out] sin(a + b*x)^4/(4*b)
```

sympy [A] time = 1.32, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sin^4(a+bx)}{4b} & \text{for}\ b \neq 0 \\ x\sin^3(a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((sin(a + b*x)**4/(4*b), Ne(b, 0)), (x*sin(a)**3*cos(a), True))
```

3.71 $\int \sin^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{\cos^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] 1/2*cos(b*x+a)^2/b-ln(cos(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\cos^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Cos[a + b*x]^2/(2*b) - Log[Cos[a + b*x]]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(a+bx)\tan(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)dx, x, \cos(a+bx)\right)}{b}\\
&= \frac{\cos^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.89

$$-\frac{\log(\cos(a+bx))-\frac{1}{2}\cos^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Tan[a + b*x],x]
[Out] -((-1/2*Cos[a + b*x]^2 + Log[Cos[a + b*x]])/b)
```

fricas [A] time = 0.44, size = 25, normalized size = 0.89

$$\frac{\cos\left(bx+a\right)^2-2\,\log\left(-\cos\left(bx+a\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(cos(b*x + a)^2 - 2*log(-cos(b*x + a)))/b
```

giac [A] time = 0.51, size = 29, normalized size = 1.04

$$\frac{\cos\left(bx+a\right)^2-\log\left(\frac{\cos(bx+a)^2}{b^2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
[Out] 1/2*(cos(b*x + a)^2 - log(cos(b*x + a)^2/b^2))/b
```

maple [A] time = 0.03, size = 27, normalized size = 0.96

$$-\frac{\sin^2\left(bx+a\right)}{2b}-\frac{\ln\left(\cos\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^3,x)
[Out] -1/2*sin(b*x+a)^2/b-ln(cos(b*x+a))/b
```

maxima [A] time = 0.31, size = 25, normalized size = 0.89

$$-\frac{\sin\left(bx+a\right)^2+\log\left(\sin\left(bx+a\right)^2-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))/b
```

mupad [B] time = 0.43, size = 25, normalized size = 0.89

$$\frac{\cos(a+b\,x)^2+\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/cos(a + b*x),x)
[Out] (log(tan(a + b*x)^2 + 1) + cos(a + b*x)^2)/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^3(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**3,x)
[Out] Integral(sin(a + b*x)**3*sec(a + b*x), x)
```

3.72 $\int \sin(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

```
[Out] cos(b*x+a)/b+sec(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] Cos[a + b*x]/b + Sec[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\tan^2(a+bx)\,dx&=-\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(a+bx)\right)}{b}\\&=-\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,x,\cos(a+bx)\right)}{b}\\&=\frac{\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x]^2,x]
[Out] Cos[a + b*x]/b + Sec[a + b*x]/b
```

fricas [A] time = 0.43, size = 22, normalized size = 1.05

$$\frac{\cos\left(bx+a\right)^{2}+1}{b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
[Out] (cos(b*x + a)^2 + 1)/(b*cos(b*x + a))
```

giac [A] time = 0.43, size = 23, normalized size = 1.10

$$\frac{\cos\left(bx+a\right)}{b}+\frac{1}{b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
[Out] cos(b*x + a)/b + 1/(b*cos(b*x + a))
```

maple [A] time = 0.03, size = 40, normalized size = 1.90

$$\frac{\frac{\sin^{4}(bx+a)}{\cos(bx+a)}+\left(2+\sin^{2}\left(bx+a\right)\right)\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(b*x+a)^3,x)
[Out] 1/b*(sin(b*x+a)^4/cos(b*x+a)+(2+sin(b*x+a)^2)*cos(b*x+a))
```

maxima [A] time = 0.32, size = 19, normalized size = 0.90

$$\frac{\frac{1}{\cos(bx+a)}+\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (1/cos(b*x + a) + cos(b*x + a))/b
```

mupad [B] time = 0.49, size = 20, normalized size = 0.95

$$-\frac{4}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^2,x)
```

```
[Out] -4/(b*(tan(a/2 + (b*x)/2)^4 - 1))
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.73 $\int \tan^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\tan^2(a+bx)}{2b}+\frac{\log(\cos(a+bx))}{b}$$

```
[Out] ln(cos(b*x+a))/b+1/2*tan(b*x+a)^2/b
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 3475}

$$\frac{\tan^2(a+bx)}{2b}+\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x]^3,x]
```

```
[Out] Log[Cos[a + b*x]]/b + Tan[a + b*x]^2/(2*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^3(a+bx)\,dx &= \frac{\tan^2(a+bx)}{2b}-\int \tan(a+bx)\,dx\\ &= \frac{\log(\cos(a+bx))}{b}+\frac{\tan^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.93

$$\frac{\tan^2(a+bx)+2\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x]^3,x]
```

```
[Out] (2*Log[Cos[a + b*x]] + Tan[a + b*x]^2)/(2*b)
```

fricas [A] time = 0.46, size = 34, normalized size = 1.26

$$\frac{2 \cos(bx+a)^2 \log(-\cos(bx+a)) + 1}{2 b \cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + a)^2*log(-cos(b*x + a)) + 1)/(b*cos(b*x + a)^2)
```

giac [A] time = 0.33, size = 42, normalized size = 1.56

$$\frac{\log\left(\frac{\cos(bx+a)^2}{b^2}\right)}{2 b} - \frac{\cos(bx+a)^2 - 1}{2 b \cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*log(cos(b*x + a)^2/b^2)/b - 1/2*(cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^2)
```

maple [A] time = 0.03, size = 26, normalized size = 0.96

$$\frac{\ln(\cos(bx+a))}{b} + \frac{\tan^2(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] ln(cos(b*x+a))/b+1/2*tan(b*x+a)^2/b
```

maxima [A] time = 0.32, size = 31, normalized size = 1.15

$$-\frac{\frac{1}{\sin(bx+a)^2 - 1} - \log\left(\sin(bx+a)^2 - 1\right)}{2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b
```

mupad [B] time = 0.38, size = 27, normalized size = 1.00

$$-\frac{\ln\left(\tan\left(a+b\,x\right)^2+1\right)-\tan\left(a+b\,x\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^3,x)
[Out] -(log(tan(a + b*x)^2 + 1) - tan(a + b*x)^2)/(2*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**3,x)
[Out] Timed out
```

3.74 $\int \sec(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\sec^3(a+bx)}{3b}-\frac{\sec(a+bx)}{b}$$

```
[Out] -sec(b*x+a)/b+1/3*sec(b*x+a)^3/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2606}

$$\frac{\sec^3(a+bx)}{3b}-\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Tan[a + b*x]^3,x]
```

```
[Out] -(Sec[a + b*x]/b) + Sec[a + b*x]^3/(3*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)\tan^3(a+bx)\,dx&=\frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\sec(a+bx)\right)}{b}\\&=-\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$\frac{\sec^3(a+bx)}{3b}-\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x]^3,x]
```

```
[Out] -(Sec[a + b*x]/b) + Sec[a + b*x]^3/(3*b)
```

fricas [A] time = 0.42, size = 25, normalized size = 0.93

$$-\frac{3\cos\left(bx+a\right)^{2}-1}{3\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

giac [A] time = 0.53, size = 25, normalized size = 0.93

$$-\frac{3\cos\left(bx+a\right)^{2}-1}{3\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

maple [B] time = 0.03, size = 60, normalized size = 2.22

$$\frac{\frac{\sin^{4}(bx+a)}{3\cos(bx+a)^{3}}-\frac{\sin^{4}(bx+a)}{3\cos(bx+a)}-\frac{\left(2+\sin^{2}(bx+a)\right)\cos(bx+a)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/3*sin(b*x+a)^4/cos(b*x+a)^3-1/3*sin(b*x+a)^4/cos(b*x+a)-1/3*(2+sin(b
*x+a)^2)*cos(b*x+a))
```

maxima [A] time = 0.40, size = 25, normalized size = 0.93

$$-\frac{3\cos\left(bx+a\right)^{2}-1}{3\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3*(3*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

mupad [B] time = 0.45, size = 23, normalized size = 0.85

$$-\frac{\cos\left(a+b\,x\right)^2-\frac{1}{3}}{b\cos\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^4,x)
[Out] -(cos(a + b*x)^2 - 1/3)/(b*cos(a + b*x)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a)**3,x)
[Out] Timed out
```

3.75 $\int \sec^2(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\tan^4(a+bx)}{4b}$$

```
[Out] 1/4*tan(b*x+a)^4/b
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$\frac{\tan^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x]^3,x]
```

```
[Out] Tan[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\tan^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^3\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\tan^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tan^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x]^3,x]
```

```
[Out] Tan[a + b*x]^4/(4*b)
```

fricas [A] time = 0.41, size = 25, normalized size = 1.67

$$-\frac{2\,\cos\left(bx+a\right)^{2}-1}{4\,b\cos\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4)
```

giac [A] time = 0.24, size = 25, normalized size = 1.67

$$-\frac{2\,\cos\left(bx+a\right)^{2}-1}{4\,b\cos\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4)
```

maple [A] time = 0.04, size = 22, normalized size = 1.47

$$\frac{\sin^{4}\left(bx+a\right)}{4b\cos\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5*sin(b*x+a)^3,x)
```

```
[Out] 1/4/b*sin(b*x+a)^4/cos(b*x+a)^4
```

maxima [B] time = 0.32, size = 39, normalized size = 2.60

$$\frac{2\,\sin\left(bx+a\right)^{2}-1}{4\left(\sin\left(bx+a\right)^{4}-2\,\sin\left(bx+a\right)^{2}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sin(b*x + a)^2 - 1)/((sin(b*x + a)^4 - 2*sin(b*x + a)^2 + 1)*b)
```

mupad [B] time = 0.38, size = 13, normalized size = 0.87

$$\frac{\tan(a+b\,x)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^5,x)
[Out] tan(a + b*x)^4/(4*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5*sin(b*x+a)**3,x)
[Out] Timed out
```

3.76 $\int \sec^3(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{3b}$$

```
[Out] -1/3*sec(b*x+a)^3/b+1/5*sec(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 14}

$$\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x]^3,x]
```

```
[Out] -Sec[a + b*x]^3/(3*b) + Sec[a + b*x]^5/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^3(a+bx)\tan^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= -\frac{\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.00

$$\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x]^3,x]
```

```
[Out] -1/3*Sec[a + b*x]^3/b + Sec[a + b*x]^5/(5*b)
```

fricas [A] time = 0.42, size = 25, normalized size = 0.81

$$-\frac{5\cos(bx+a)^2-3}{15\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^5)
```

giac [A] time = 0.24, size = 25, normalized size = 0.81

$$-\frac{5\cos(bx+a)^2-3}{15\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/15*(5*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^5)
```

maple [B] time = 0.03, size = 78, normalized size = 2.52

$$\frac{\frac{\sin^4(bx+a)}{5\cos(bx+a)^5}+\frac{\sin^4(bx+a)}{15\cos(bx+a)^3}-\frac{\sin^4(bx+a)}{15\cos(bx+a)}-\frac{\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{15}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/5*sin(b*x+a)^4/cos(b*x+a)^5+1/15*sin(b*x+a)^4/cos(b*x+a)^3-1/15*sin(
b*x+a)^4/cos(b*x+a)-1/15*(2+sin(b*x+a)^2)*cos(b*x+a))
```

maxima [A] time = 0.33, size = 25, normalized size = 0.81

$$-\frac{5\,\cos\left(bx+a\right)^{2}-3}{15\,b\cos\left(bx+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(5*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^5)
```

mupad [B] time = 0.54, size = 25, normalized size = 0.81

$$-\frac{\frac{\cos(a+b\,x)^2}{3}-\frac{1}{5}}{b\cos\left(a+b\,x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^6,x)
```

```
[Out] -(cos(a + b*x)^2/3 - 1/5)/(b*cos(a + b*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.77 $\int \sec^4(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sec^6(a+bx)}{6b}-\frac{\sec^4(a+bx)}{4b}$$

```
[Out] -1/4*sec(b*x+a)^4/b+1/6*sec(b*x+a)^6/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 14}

$$\frac{\sec^6(a+bx)}{6b}-\frac{\sec^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*Tan[a + b*x]^3,x]
```

```
[Out] -Sec[a + b*x]^4/(4*b) + Sec[a + b*x]^6/(6*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^4(a+bx)\tan^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-x^3+x^5\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= -\frac{\sec^4(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.90

$$-\frac{3\sec^4(a+bx)-2\sec^6(a+bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*Tan[a + b*x]^3,x]
```

```
[Out] -1/12*(3*Sec[a + b*x]^4 - 2*Sec[a + b*x]^6)/b
```

fricas [A] time = 0.40, size = 25, normalized size = 0.81

$$-\frac{3\cos(bx+a)^2-2}{12\,b\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^6)
```

giac [A] time = 0.27, size = 25, normalized size = 0.81

$$-\frac{3\cos(bx+a)^2-2}{12\,b\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^6)
```

maple [A] time = 0.04, size = 42, normalized size = 1.35

$$\frac{\frac{\sin^4(bx+a)}{6\cos(bx+a)^6}+\frac{\sin^4(bx+a)}{12\cos(bx+a)^4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^7*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/6*sin(b*x+a)^4/cos(b*x+a)^6+1/12*sin(b*x+a)^4/cos(b*x+a)^4)
```

maxima [A] time = 0.34, size = 49, normalized size = 1.58

$$-\frac{3\sin(bx+a)^2-1}{12\left(\sin(bx+a)^6-3\sin(bx+a)^4+3\sin(bx+a)^2-1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*sin(b*x + a)^2 - 1)/((sin(b*x + a)^6 - 3*sin(b*x + a)^4 + 3*sin(b*
x + a)^2 - 1)*b)
```

mupad [**B**] time = 0.40, size = 25, normalized size = 0.81

$$\frac{\tan\left(a+b\,x\right)^4\,\left(2\,\tan\left(a+b\,x\right)^2+3\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^7,x)
```

```
[Out] (tan(a + b*x)^4*(2*tan(a + b*x)^2 + 3))/(12*b)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**7*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.78 $\int \sec^5(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sec^7(a+bx)}{7b}-\frac{\sec^5(a+bx)}{5b}$$

```
[Out] -1/5*sec(b*x+a)^5/b+1/7*sec(b*x+a)^7/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 14}

$$\frac{\sec^7(a+bx)}{7b}-\frac{\sec^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5*Tan[a + b*x]^3,x]
```

```
[Out] -Sec[a + b*x]^5/(5*b) + Sec[a + b*x]^7/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^5(a+bx)\tan^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-x^4+x^6\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= -\frac{\sec^5(a+bx)}{5b}+\frac{\sec^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 1.00

$$\frac{\sec^7(a+bx)}{7b}-\frac{\sec^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5*Tan[a + b*x]^3,x]
[Out] -1/5*Sec[a + b*x]^5/b + Sec[a + b*x]^7/(7*b)
```

fricas [A] time = 0.43, size = 25, normalized size = 0.81

$$-\frac{7\,\cos\left(bx+a\right)^2-5}{35\,b\cos\left(bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^3,x, algorithm="fricas")
[Out] -1/35*(7*cos(b*x + a)^2 - 5)/(b*cos(b*x + a)^7)
```

giac [A] time = 0.23, size = 25, normalized size = 0.81

$$-\frac{7\,\cos\left(bx+a\right)^2-5}{35\,b\cos\left(bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^3,x, algorithm="giac")
[Out] -1/35*(7*cos(b*x + a)^2 - 5)/(b*cos(b*x + a)^7)
```

maple [B] time = 0.04, size = 96, normalized size = 3.10

$$\frac{\frac{\sin^4(bx+a)}{7\cos(bx+a)^7}+\frac{3\left(\sin^4(bx+a)\right)}{35\cos(bx+a)^5}+\frac{\sin^4(bx+a)}{35\cos(bx+a)^3}-\frac{\sin^4(bx+a)}{35\cos(bx+a)}-\frac{\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{35}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^8*sin(b*x+a)^3,x)
[Out] 1/b*(1/7*sin(b*x+a)^4/cos(b*x+a)^7+3/35*sin(b*x+a)^4/cos(b*x+a)^5+1/35*sin(
b*x+a)^4/cos(b*x+a)^3-1/35*sin(b*x+a)^4/cos(b*x+a)-1/35*(2+sin(b*x+a)^2)*co
s(b*x+a))
```

maxima [A] time = 0.32, size = 25, normalized size = 0.81

$$-\frac{7\,\cos\left(bx+a\right)^2-5}{35\,b\cos\left(bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/35*(7*cos(b*x + a)^2 - 5)/(b*cos(b*x + a)^7)
```

mupad [B] time = 0.63, size = 25, normalized size = 0.81

$$-\frac{7\cos\left(a+b\,x\right)^2-5}{35\,b\cos\left(a+b\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^8,x)
[Out] -(7*cos(a + b*x)^2 - 5)/(35*b*cos(a + b*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**8*sin(b*x+a)**3,x)
[Out] Timed out
```

3.79 $\int \sec^6(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sec^8(a+bx)}{8b}-\frac{\sec^6(a+bx)}{6b}$$

```
[Out] -1/6*sec(b*x+a)^6/b+1/8*sec(b*x+a)^8/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 14}

$$\frac{\sec^8(a+bx)}{8b}-\frac{\sec^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6*Tan[a + b*x]^3,x]
```

```
[Out] -Sec[a + b*x]^6/(6*b) + Sec[a + b*x]^8/(8*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^6(a+bx)\tan^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^5\left(-1+x^2\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-x^5+x^7\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= -\frac{\sec^6(a+bx)}{6b}+\frac{\sec^8(a+bx)}{8b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.90

$$-\frac{4\sec^6(a+bx)-3\sec^8(a+bx)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6*Tan[a + b*x]^3,x]
```

```
[Out] -1/24*(4*Sec[a + b*x]^6 - 3*Sec[a + b*x]^8)/b
```

fricas [A] time = 0.40, size = 25, normalized size = 0.81

$$-\frac{4\cos(bx+a)^2-3}{24\,b\cos(bx+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(4*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^8)
```

giac [A] time = 0.34, size = 25, normalized size = 0.81

$$-\frac{4\cos(bx+a)^2-3}{24\,b\cos(bx+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/24*(4*cos(b*x + a)^2 - 3)/(b*cos(b*x + a)^8)
```

maple [B] time = 0.04, size = 60, normalized size = 1.94

$$\frac{\frac{\sin^4(bx+a)}{8\cos(bx+a)^8}+\frac{\sin^4(bx+a)}{12\cos(bx+a)^6}+\frac{\sin^4(bx+a)}{24\cos(bx+a)^4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^9*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/8*sin(b*x+a)^4/cos(b*x+a)^8+1/12*sin(b*x+a)^4/cos(b*x+a)^6+1/24*sin(
b*x+a)^4/cos(b*x+a)^4)
```

maxima [B] time = 0.32, size = 59, normalized size = 1.90

$$\frac{4\,\sin\left(bx+a\right)^{2}-1}{24\left(\sin\left(bx+a\right)^{8}-4\,\sin\left(bx+a\right)^{6}+6\,\sin\left(bx+a\right)^{4}-4\,\sin\left(bx+a\right)^{2}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(4*sin(b*x + a)^2 - 1)/((sin(b*x + a)^8 - 4*sin(b*x + a)^6 + 6*sin(b*x
 + a)^4 - 4*sin(b*x + a)^2 + 1)*b)
```

mupad [B] time = 0.42, size = 35, normalized size = 1.13

$$\frac{\tan\left(a+b\,x\right)^4\,\left(3\,\tan\left(a+b\,x\right)^4+8\,\tan\left(a+b\,x\right)^2+6\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/cos(a + b*x)^9,x)
```

```
[Out] (tan(a + b*x)^4*(8*tan(a + b*x)^2 + 3*tan(a + b*x)^4 + 6))/(24*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**9*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.80 $\int \cos^7(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=61

$$-\frac{\sin^{11}(a+bx)}{11b}+\frac{\sin^9(a+bx)}{3b}-\frac{3\sin^7(a+bx)}{7b}+\frac{\sin^5(a+bx)}{5b}$$

```
[Out] 1/5*sin(b*x+a)^5/b-3/7*sin(b*x+a)^7/b+1/3*sin(b*x+a)^9/b-1/11*sin(b*x+a)^11
/b
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 270}

$$-\frac{\sin^{11}(a+bx)}{11b}+\frac{\sin^9(a+bx)}{3b}-\frac{3\sin^7(a+bx)}{7b}+\frac{\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^7*Sin[a + b*x]^4,x]
```

```
[Out] Sin[a + b*x]^5/(5*b) - (3*Sin[a + b*x]^7)/(7*b) + Sin[a + b*x]^9/(3*b) - Si
n[a + b*x]^11/(11*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \cos^7(a+bx)\sin^4(a+bx)\,dx = \frac{\text{Subst}\left(\int x^4\left(1-x^2\right)^3\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(x^4-3x^6+3x^8-x^{10}\right)\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{\sin^5(a+bx)}{5b} - \frac{3\sin^7(a+bx)}{7b} + \frac{\sin^9(a+bx)}{3b} - \frac{\sin^{11}(a+bx)}{11b}$$

Mathematica [A] time = 0.18, size = 47, normalized size = 0.77

$$\frac{\sin^5(a+bx)(3335\cos(2(a+bx))+910\cos(4(a+bx))+105\cos(6(a+bx))+3042)}{36960b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^7*Sin[a + b*x]^4,x]
```

```
[Out] ((3042 + 3335*Cos[2*(a + b*x)] + 910*Cos[4*(a + b*x)] + 105*Cos[6*(a + b*x)
])*Sin[a + b*x]^5)/(36960*b)
```

fricas [A] time = 0.47, size = 63, normalized size = 1.03

$$\frac{\left(105\,\cos\left(bx+a\right)^{10}-140\,\cos\left(bx+a\right)^{8}+5\,\cos\left(bx+a\right)^{6}+6\,\cos\left(bx+a\right)^{4}+8\,\cos\left(bx+a\right)^{2}+16\right)\sin\left(bx+a\right)}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*cos
(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

giac [A] time = 0.24, size = 82, normalized size = 1.34

$$\frac{\sin\left(11\,bx+11\,a\right)}{11264\,b}+\frac{\sin\left(9\,bx+9\,a\right)}{3072\,b}-\frac{\sin\left(7\,bx+7\,a\right)}{7168\,b}-\frac{11\,\sin\left(5\,bx+5\,a\right)}{5120\,b}-\frac{\sin\left(3\,bx+3\,a\right)}{512\,b}+\frac{7\,\sin\left(bx+a\right)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/11264*sin(11*b*x + 11*a)/b + 1/3072*sin(9*b*x + 9*a)/b - 1/7168*sin(7*b*x
 + 7*a)/b - 11/5120*sin(5*b*x + 5*a)/b - 1/512*sin(3*b*x + 3*a)/b + 7/512*s
in(b*x + a)/b
```

maple [A] time = 0.03, size = 78, normalized size = 1.28

$$\frac{-\frac{\left(\sin^3(bx+a)\right)\left(\cos^8(bx+a)\right)}{11}-\frac{\sin(bx+a)\left(\cos^8(bx+a)\right)}{33}+\frac{\left(\frac{16}{5}+\cos^6(bx+a)+\frac{6\left(\cos^4(bx+a)\right)}{5}+\frac{8\left(\cos^2(bx+a)\right)}{5}\right)\sin(bx+a)}{231}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/11*sin(b*x+a)^3*cos(b*x+a)^8-1/33*sin(b*x+a)*cos(b*x+a)^8+1/231*(16
/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.32, size = 46, normalized size = 0.75

$$-\frac{105\,\sin\left(bx+a\right)^{11}-385\,\sin\left(bx+a\right)^{9}+495\,\sin\left(bx+a\right)^{7}-231\,\sin\left(bx+a\right)^{5}}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/1155*(105*sin(b*x + a)^11 - 385*sin(b*x + a)^9 + 495*sin(b*x + a)^7 - 23
1*sin(b*x + a)^5)/b
```

mupad [B] time = 0.38, size = 45, normalized size = 0.74

$$\frac{-\frac{\sin(a+b\,x)^{11}}{11}+\frac{\sin(a+b\,x)^9}{3}-\frac{3\sin(a+b\,x)^7}{7}+\frac{\sin(a+b\,x)^5}{5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^7*sin(a + b*x)^4,x)
```

```
[Out] (sin(a + b*x)^5/5 - (3*sin(a + b*x)^7)/7 + sin(a + b*x)^9/3 - sin(a + b*x)^
11/11)/b
```

sympy [A] time = 49.84, size = 88, normalized size = 1.44

$$\begin{cases} \frac{16\sin^{11}(a+bx)}{1155b}+\frac{8\sin^{9}(a+bx)\cos^{2}(a+bx)}{105b}+\frac{6\sin^{7}(a+bx)\cos^{4}(a+bx)}{35b}+\frac{\sin^{5}(a+bx)\cos^{6}(a+bx)}{5b} & \text{for } b\neq 0\\ x\sin^{4}(a)\cos^{7}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((16*sin(a + b*x)**11/(1155*b) + 8*sin(a + b*x)**9*cos(a + b*x)**2
/(105*b) + 6*sin(a + b*x)**7*cos(a + b*x)**4/(35*b) + sin(a + b*x)**5*cos(a
 + b*x)**6/(5*b), Ne(b, 0)), (x*sin(a)**4*cos(a)**7, True))
```

3.81 $\int \cos^5(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sin^9(a+bx)}{9b}-\frac{2\sin^7(a+bx)}{7b}+\frac{\sin^5(a+bx)}{5b}$$

```
[Out] 1/5*sin(b*x+a)^5/b-2/7*sin(b*x+a)^7/b+1/9*sin(b*x+a)^9/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 270}

$$\frac{\sin^9(a+bx)}{9b}-\frac{2\sin^7(a+bx)}{7b}+\frac{\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Sin[a + b*x]^4,x]
```

```
[Out] Sin[a + b*x]^5/(5*b) - (2*Sin[a + b*x]^7)/(7*b) + Sin[a + b*x]^9/(9*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^5(a+bx)\sin^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\sin^5(a+bx)}{5b}-\frac{2\sin^7(a+bx)}{7b}+\frac{\sin^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.11, size = 37, normalized size = 0.80

$$\frac{\sin^5(a+bx)(220\cos(2(a+bx))+35\cos(4(a+bx))+249)}{2520b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Sin[a + b*x]^4,x]
```

```
[Out] ((249 + 220*Cos[2*(a + b*x)] + 35*Cos[4*(a + b*x)])*Sin[a + b*x]^5)/(2520*b
)
```

fricas [A] time = 0.42, size = 53, normalized size = 1.15

$$\frac{\left(35\,\cos\left(bx+a\right)^{8}-50\,\cos\left(bx+a\right)^{6}+3\,\cos\left(bx+a\right)^{4}+4\,\cos\left(bx+a\right)^{2}+8\right)\sin\left(bx+a\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*x
 + a)^2 + 8)*sin(b*x + a)/b
```

giac [A] time = 0.22, size = 68, normalized size = 1.48

$$\frac{\sin\left(9\,bx+9\,a\right)}{2304\,b}+\frac{\sin\left(7\,bx+7\,a\right)}{1792\,b}-\frac{\sin\left(5\,bx+5\,a\right)}{320\,b}-\frac{\sin\left(3\,bx+3\,a\right)}{192\,b}+\frac{3\,\sin\left(bx+a\right)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/2304*sin(9*b*x + 9*a)/b + 1/1792*sin(7*b*x + 7*a)/b - 1/320*sin(5*b*x + 5
*a)/b - 1/192*sin(3*b*x + 3*a)/b + 3/128*sin(b*x + a)/b
```

maple [A] time = 0.02, size = 68, normalized size = 1.48

$$\frac{-\frac{\left(\sin^{3}(bx+a)\right)\left(\cos^{6}(bx+a)\right)}{9}-\frac{\sin(bx+a)\left(\cos^{6}(bx+a)\right)}{21}+\frac{\left(\frac{8}{3}+\cos^{4}(bx+a)+\frac{4\left(\cos^{2}(bx+a)\right)}{3}\right)\sin(bx+a)}{105}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/9*sin(b*x+a)^3*cos(b*x+a)^6-1/21*sin(b*x+a)*cos(b*x+a)^6+1/105*(8/3
+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.37, size = 36, normalized size = 0.78

$$\frac{35\,\sin\left(bx+a\right)^{9}-90\,\sin\left(bx+a\right)^{7}+63\,\sin\left(bx+a\right)^{5}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/315*(35*sin(b*x + a)^9 - 90*sin(b*x + a)^7 + 63*sin(b*x + a)^5)/b
```

mupad [B] time = 0.38, size = 36, normalized size = 0.78

$$\frac{35\sin\left(a+b\,x\right)^{9}-90\sin\left(a+b\,x\right)^{7}+63\sin\left(a+b\,x\right)^{5}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5*sin(a + b*x)^4,x)
```

```
[Out] (63*sin(a + b*x)^5 - 90*sin(a + b*x)^7 + 35*sin(a + b*x)^9)/(315*b)
```

sympy [A] time = 20.52, size = 66, normalized size = 1.43

$$\begin{cases} \frac{8\sin^{9}(a+bx)}{315b}+\frac{4\sin^{7}(a+bx)\cos^{2}(a+bx)}{35b}+\frac{\sin^{5}(a+bx)\cos^{4}(a+bx)}{5b} & \text{for } b\neq 0\\ x\sin^{4}(a)\cos^{5}(a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((8*sin(a + b*x)**9/(315*b) + 4*sin(a + b*x)**7*cos(a + b*x)**2/(3
5*b) + sin(a + b*x)**5*cos(a + b*x)**4/(5*b), Ne(b, 0)), (x*sin(a)**4*cos(a
)**5, True))
```

3.82 $\int \cos^3(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sin^5(a+bx)}{5b}-\frac{\sin^7(a+bx)}{7b}$$

[Out] 1/5*sin(b*x+a)^5/b-1/7*sin(b*x+a)^7/b

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sin^5(a+bx)}{5b}-\frac{\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]^3*Sin[a + b*x]^4,x]

[Out] Sin[a + b*x]^5/(5*b) - Sin[a + b*x]^7/(7*b)

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sin^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\left(1-x^2\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^4-x^6\right)\,dx,x,\sin(a+bx)\right)}{b}\\ &= \frac{\sin^5(a+bx)}{5b}-\frac{\sin^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.07, size = 27, normalized size = 0.87

$$\frac{\sin^5(a+bx)(5\cos(2(a+bx))+9)}{70b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x]^4,x]
```

```
[Out] ((9 + 5*Cos[2*(a + b*x)])*Sin[a + b*x]^5)/(70*b)
```

fricas [A] time = 0.42, size = 41, normalized size = 1.32

$$\frac{\left(5 \cos\left(bx+a\right)^6 - 8 \cos\left(bx+a\right)^4 + \cos\left(bx+a\right)^2 + 2\right)\sin\left(bx+a\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(b*x + a)^6 - 8*cos(b*x + a)^4 + cos(b*x + a)^2 + 2)*sin(b*x + a
)/b
```

giac [A] time = 0.24, size = 26, normalized size = 0.84

$$-\frac{5 \sin\left(bx+a\right)^7 - 7 \sin\left(bx+a\right)^5}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/35*(5*sin(b*x + a)^7 - 7*sin(b*x + a)^5)/b
```

maple [B] time = 0.02, size = 58, normalized size = 1.87

$$\frac{-\frac{\left(\cos^4(bx+a)\right)\left(\sin^3(bx+a)\right)}{7} - \frac{3\sin(bx+a)\left(\cos^4(bx+a)\right)}{35} + \frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{35}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/7*cos(b*x+a)^4*sin(b*x+a)^3-3/35*sin(b*x+a)*cos(b*x+a)^4+1/35*(2+co
s(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.32, size = 26, normalized size = 0.84

$$-\frac{5\,\sin\left(bx+a\right)^{7}-7\,\sin\left(bx+a\right)^{5}}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/35*(5*sin(b*x + a)^7 - 7*sin(b*x + a)^5)/b
```

mupad [B] time = 0.03, size = 26, normalized size = 0.84

$$\frac{7\sin\left(a+b\,x\right)^5-5\sin\left(a+b\,x\right)^7}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^4,x)
```

```
[Out] (7*sin(a + b*x)^5 - 5*sin(a + b*x)^7)/(35*b)
```

sympy [A] time = 7.36, size = 44, normalized size = 1.42

$$\begin{cases} \frac{2\sin^{7}{\left(a+bx \right)}}{35b}+\frac{\sin^{5}{\left(a+bx \right)}\cos^{2}{\left(a+bx \right)}}{5b} & \text{for}\: b \neq 0 \\ x \sin^{4}{\left(a \right)} \cos^{3}{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((2*sin(a + b*x)**7/(35*b) + sin(a + b*x)**5*cos(a + b*x)**2/(5*b)
, Ne(b, 0)), (x*sin(a)**4*cos(a)**3, True))
```

3.83 $\int \cos(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^5(a+bx)}{5b}$$

```
[Out] 1/5*sin(b*x+a)^5/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[a + b*x]^4,x]
```

```
[Out] Sin[a + b*x]^5/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x]^4,x]
```

```
[Out] Sin[a + b*x]^5/(5*b)
```

fricas [B] time = 0.43, size = 31, normalized size = 2.07

$$\frac{\left(\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/b
```

giac [A] time = 0.22, size = 13, normalized size = 0.87

$$\frac{\sin\left(bx+a\right)^{5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/5*sin(b*x + a)^5/b
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$\frac{\sin^{5}\left(bx+a\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^4,x)
```

```
[Out] 1/5*sin(b*x+a)^5/b
```

maxima [A] time = 0.36, size = 13, normalized size = 0.87

$$\frac{\sin\left(bx+a\right)^{5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/5*sin(b*x + a)^5/b
```

mupad [B] time = 0.36, size = 13, normalized size = 0.87

$$\frac{\sin\left(a+b\,x\right)^5}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^4,x)
```

```
[Out] sin(a + b*x)^5/(5*b)
```

sympy [A] time = 2.25, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sin^{5}\left(a+bx\right)}{5b} & \text{for } b \neq 0 \\ x\sin^{4}\left(a\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((sin(a + b*x)**5/(5*b), Ne(b, 0)), (x*sin(a)**4*cos(a), True))
```

3.84 $\int \sin^2(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{3\tan(a+bx)}{2b}-\frac{\sin^2(a+bx)\tan(a+bx)}{2b}-\frac{3x}{2}$$

```
[Out] -3/2*x+3/2*tan(b*x+a)/b-1/2*sin(b*x+a)^2*tan(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 321, 203}

$$\frac{3\tan(a+bx)}{2b}-\frac{\sin^2(a+bx)\tan(a+bx)}{2b}-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Tan[a + b*x]^2,x]
```

```
[Out] (-3*x)/2 + (3*Tan[a + b*x])/(2*b) - (Sin[a + b*x]^2*Tan[a + b*x])/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sin^2(a+bx)\tan^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \tan(a+bx)\right)}{b}$$

$$= -\frac{\sin^2(a+bx)\tan(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(a+bx)\right)}{2b}$$

$$= \frac{3\tan(a+bx)}{2b} - \frac{\sin^2(a+bx)\tan(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(a+bx)\right)}{2b}$$

$$= -\frac{3x}{2} + \frac{3\tan(a+bx)}{2b} - \frac{\sin^2(a+bx)\tan(a+bx)}{2b}$$

Mathematica [A] time = 0.10, size = 31, normalized size = 0.78

$$\frac{-6(a+bx)+\sin(2(a+bx))+4\tan(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Tan[a + b*x]^2,x]
[Out] (-6*(a + b*x) + Sin[2*(a + b*x)] + 4*Tan[a + b*x])/(4*b)
```

fricas [A] time = 0.43, size = 42, normalized size = 1.05

$$-\frac{3\,bx\cos(bx+a)-\left(\cos(bx+a)^2+2\right)\sin(bx+a)}{2\,b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^4,x, algorithm="fricas")
[Out] -1/2*(3*b*x*cos(b*x + a) - (cos(b*x + a)^2 + 2)*sin(b*x + a))/(b*cos(b*x +
a))
```

giac [A] time = 0.36, size = 41, normalized size = 1.02

$$-\frac{3\,bx + 3\,a - \frac{\tan(bx+a)}{\tan(bx+a)^2+1} - 2\,\tan\left(bx + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/2*(3*b*x + 3*a - tan(b*x + a)/(tan(b*x + a)^2 + 1) - 2*tan(b*x + a))/b
```

maple [A] time = 0.03, size = 54, normalized size = 1.35

$$\frac{\frac{\sin^5(bx+a)}{\cos(bx+a)} + \left(\sin^3\left(bx + a\right) + \frac{3\sin(bx+a)}{2}\right)\cos\left(bx + a\right) - \frac{3bx}{2} - \frac{3a}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(sin(b*x+a)^5/cos(b*x+a)+(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/2*b
*x-3/2*a)
```

maxima [A] time = 0.43, size = 41, normalized size = 1.02

$$-\frac{3\,bx + 3\,a - \frac{\tan(bx+a)}{\tan(bx+a)^2+1} - 2\,\tan\left(bx + a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x + 3*a - tan(b*x + a)/(tan(b*x + a)^2 + 1) - 2*tan(b*x + a))/b
```

mupad [B] time = 0.48, size = 38, normalized size = 0.95

$$\frac{\frac{\cos(a+b\,x)\,\sin(a+b\,x)}{2} + \frac{\sin(a+b\,x)}{\cos(a+b\,x)}}{b} - \frac{3\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^2,x)
```

```
[Out] ((cos(a + b*x)*sin(a + b*x))/2 + sin(a + b*x)/cos(a + b*x))/b - (3*x)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^4(a + bx) \sec^2(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a)**4,x)
```

```
[Out] Integral(sin(a + b*x)**4*sec(a + b*x)**2, x)
```

3.85 $\int \tan^4(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{\tan^3(a+bx)}{3b}-\frac{\tan(a+bx)}{b}+x$$

```
[Out] x-tan(b*x+a)/b+1/3*tan(b*x+a)^3/b
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 8}

$$\frac{\tan^3(a+bx)}{3b}-\frac{\tan(a+bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x]^4,x]
```

```
[Out] x - Tan[a + b*x]/b + Tan[a + b*x]^3/(3*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \tan^4(a+bx)\,dx &= \frac{\tan^3(a+bx)}{3b}-\int \tan^2(a+bx)\,dx\\ &= -\frac{\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}+\int 1\,dx\\ &= x-\frac{\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.36

$$\frac{\tan^{-1}(\tan(a+bx))}{b}+\frac{\tan^3(a+bx)}{3b}-\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x]^4,x]
```

```
[Out] ArcTan[Tan[a + b*x]]/b - Tan[a + b*x]/b + Tan[a + b*x]^3/(3*b)
```

fricas [A] time = 0.45, size = 46, normalized size = 1.64

$$\frac{3\,bx\cos\left(bx+a\right)^{3}-\left(4\,\cos\left(bx+a\right)^{2}-1\right)\sin\left(bx+a\right)}{3\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x*cos(b*x + a)^3 - (4*cos(b*x + a)^2 - 1)*sin(b*x + a))/(b*cos(b*x
 + a)^3)
```

giac [A] time = 0.25, size = 29, normalized size = 1.04

$$\frac{\tan\left(bx+a\right)^{3}+3\,bx+3\,a-3\,\tan\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*(tan(b*x + a)^3 + 3*b*x + 3*a - 3*tan(b*x + a))/b
```

maple [A] time = 0.03, size = 28, normalized size = 1.00

$$\frac{\frac{\left(\tan^{3}(bx+a)\right)}{3}-\tan\left(bx+a\right)+bx+a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(1/3*tan(b*x+a)^3-tan(b*x+a)+b*x+a)
```

maxima [A] time = 0.42, size = 29, normalized size = 1.04

$$\frac{\tan\left(bx+a\right)^{3}+3\,bx+3\,a-3\,\tan\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^4,x, algorithm="maxima")
[Out] 1/3*(tan(b*x + a)^3 + 3*b*x + 3*a - 3*tan(b*x + a))/b
```

mupad [B] time = 0.40, size = 24, normalized size = 0.86

$$x-\frac{\tan\left(a+b\,x\right)-\frac{{\tan\left(a+b\,x\right)}^3}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^4,x)
[Out] x - (tan(a + b*x) - tan(a + b*x)^3/3)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a)**4,x)
[Out] Timed out
```

3.86 $\int \sec^2(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\tan^5(a+bx)}{5b}$$

```
[Out] 1/5*tan(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$\frac{\tan^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*Tan[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]^5/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\tan^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\tan^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tan^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]^5/(5*b)
```

fricas [B] time = 0.40, size = 39, normalized size = 2.60

$$\frac{\left(\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)}{5\,b\cos\left(bx+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/(b*cos(b*x + a)^5)
```

giac [A] time = 0.20, size = 13, normalized size = 0.87

$$\frac{\tan\left(bx+a\right)^{5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/5*tan(b*x + a)^5/b
```

maple [A] time = 0.04, size = 22, normalized size = 1.47

$$\frac{\sin^{5}\left(bx+a\right)}{5b\cos\left(bx+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^4,x)
```

```
[Out] 1/5/b*sin(b*x+a)^5/cos(b*x+a)^5
```

maxima [A] time = 0.32, size = 13, normalized size = 0.87

$$\frac{\tan\left(bx+a\right)^{5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/5*tan(b*x + a)^5/b
```

mupad [B] time = 0.41, size = 13, normalized size = 0.87

$$\frac{\tan\left(a+b\,x\right)^5}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^6,x)
[Out] tan(a + b*x)^5/(5*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**4,x)
[Out] Timed out
```

3.87 $\int \sec^4(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\tan^7(a+bx)}{7b}+\frac{\tan^5(a+bx)}{5b}$$

```
[Out] 1/5*tan(b*x+a)^5/b+1/7*tan(b*x+a)^7/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 14}

$$\frac{\tan^7(a+bx)}{7b}+\frac{\tan^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*Tan[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]^5/(5*b) + Tan[a + b*x]^7/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^4(a+bx)\tan^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\left(1+x^2\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^4+x^6\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\tan^5(a+bx)}{5b}+\frac{\tan^7(a+bx)}{7b}\end{aligned}$$

Mathematica [B] time = 0.03, size = 77, normalized size = 2.48

$$\frac{2\tan(a+bx)}{35b}+\frac{\tan(a+bx)\sec^6(a+bx)}{7b}-\frac{8\tan(a+bx)\sec^4(a+bx)}{35b}+\frac{\tan(a+bx)\sec^2(a+bx)}{35b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*Tan[a + b*x]^4,x]
```

```
[Out] (2*Tan[a + b*x])/(35*b) + (Sec[a + b*x]^2*Tan[a + b*x])/(35*b) - (8*Sec[a +
 b*x]^4*Tan[a + b*x])/(35*b) + (Sec[a + b*x]^6*Tan[a + b*x])/(7*b)
```

fricas [A] time = 0.42, size = 49, normalized size = 1.58

$$\frac{\left(2\,\cos\left(bx+a\right)^6+\cos\left(bx+a\right)^4-8\,\cos\left(bx+a\right)^2+5\right)\sin\left(bx+a\right)}{35\,b\cos\left(bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(2*cos(b*x + a)^6 + cos(b*x + a)^4 - 8*cos(b*x + a)^2 + 5)*sin(b*x + a
)/(b*cos(b*x + a)^7)
```

giac [A] time = 0.25, size = 26, normalized size = 0.84

$$\frac{5\,\tan\left(bx+a\right)^7+7\,\tan\left(bx+a\right)^5}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/35*(5*tan(b*x + a)^7 + 7*tan(b*x + a)^5)/b
```

maple [A] time = 0.04, size = 42, normalized size = 1.35

$$\frac{\frac{\sin^5(bx+a)}{7\cos(bx+a)^7}+\frac{2\left(\sin^5(bx+a)\right)}{35\cos(bx+a)^5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^8*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(1/7*sin(b*x+a)^5/cos(b*x+a)^7+2/35*sin(b*x+a)^5/cos(b*x+a)^5)
```

maxima [A] time = 0.32, size = 26, normalized size = 0.84

$$\frac{5\ \tan\left(bx+a\right)^{7}+7\ \tan\left(bx+a\right)^{5}}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/35*(5*tan(b*x + a)^7 + 7*tan(b*x + a)^5)/b
```

mupad [B] time = 0.40, size = 25, normalized size = 0.81

$$\frac{\tan\left(a+b\,x\right)^5\,\left(5\tan\left(a+b\,x\right)^2+7\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^8,x)
```

```
[Out] (tan(a + b*x)^5*(5*tan(a + b*x)^2 + 7))/(35*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**8*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

3.88 $\int \sec^6(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\tan^9(a+bx)}{9b}+\frac{2\tan^7(a+bx)}{7b}+\frac{\tan^5(a+bx)}{5b}$$

```
[Out] 1/5*tan(b*x+a)^5/b+2/7*tan(b*x+a)^7/b+1/9*tan(b*x+a)^9/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 270}

$$\frac{\tan^9(a+bx)}{9b}+\frac{2\tan^7(a+bx)}{7b}+\frac{\tan^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6*Tan[a + b*x]^4,x]
```

```
[Out] Tan[a + b*x]^5/(5*b) + (2*Tan[a + b*x]^7)/(7*b) + Tan[a + b*x]^9/(9*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^6(a+bx)\tan^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^4\left(1+x^2\right)^2\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^4+2x^6+x^8\right)dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\tan^5(a+bx)}{5b}+\frac{2\tan^7(a+bx)}{7b}+\frac{\tan^9(a+bx)}{9b}\end{aligned}$$

Mathematica [B] time = 0.03, size = 98, normalized size = 2.13

$$\frac{8\tan(a+bx)}{315b}+\frac{\tan(a+bx)\sec^8(a+bx)}{9b}-\frac{10\tan(a+bx)\sec^6(a+bx)}{63b}+\frac{\tan(a+bx)\sec^4(a+bx)}{105b}+\frac{4\tan(a+bx)}{315}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6*Tan[a + b*x]^4,x]
```

```
[Out] (8*Tan[a + b*x])/(315*b) + (4*Sec[a + b*x]^2*Tan[a + b*x])/(315*b) + (Sec[a
 + b*x]^4*Tan[a + b*x])/(105*b) - (10*Sec[a + b*x]^6*Tan[a + b*x])/(63*b) +
 (Sec[a + b*x]^8*Tan[a + b*x])/(9*b)
```

fricas [A] time = 0.44, size = 61, normalized size = 1.33

$$\frac{\left(8\cos\left(bx+a\right)^{8}+4\cos\left(bx+a\right)^{6}+3\cos\left(bx+a\right)^{4}-50\cos\left(bx+a\right)^{2}+35\right)\sin\left(bx+a\right)}{315\,b\cos\left(bx+a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/315*(8*cos(b*x + a)^8 + 4*cos(b*x + a)^6 + 3*cos(b*x + a)^4 - 50*cos(b*x
+ a)^2 + 35)*sin(b*x + a)/(b*cos(b*x + a)^9)
```

giac [A] time = 0.25, size = 36, normalized size = 0.78

$$\frac{35\tan\left(bx+a\right)^{9}+90\tan\left(bx+a\right)^{7}+63\tan\left(bx+a\right)^{5}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/315*(35*tan(b*x + a)^9 + 90*tan(b*x + a)^7 + 63*tan(b*x + a)^5)/b
```

maple [A] time = 0.04, size = 60, normalized size = 1.30

$$\frac{\frac{\sin^5(bx+a)}{9\cos(bx+a)^9}+\frac{4\left(\sin^5(bx+a)\right)}{63\cos(bx+a)^7}+\frac{8\left(\sin^5(bx+a)\right)}{315\cos(bx+a)^5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^10*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(1/9*sin(b*x+a)^5/cos(b*x+a)^9+4/63*sin(b*x+a)^5/cos(b*x+a)^7+8/315*sin
(b*x+a)^5/cos(b*x+a)^5)
```

maxima [A] time = 0.45, size = 36, normalized size = 0.78

$$\frac{35\tan\left(bx+a\right)^{9}+90\tan\left(bx+a\right)^{7}+63\tan\left(bx+a\right)^{5}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^4,x, algorithm="maxima")
[Out] 1/315*(35*tan(b*x + a)^9 + 90*tan(b*x + a)^7 + 63*tan(b*x + a)^5)/b
```

mupad [B] time = 0.43, size = 35, normalized size = 0.76

$$\frac{\tan\left(a+b\,x\right)^5\,\left(35\tan\left(a+b\,x\right)^4+90\tan\left(a+b\,x\right)^2+63\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^10,x)
[Out] (tan(a + b*x)^5*(90*tan(a + b*x)^2 + 35*tan(a + b*x)^4 + 63))/(315*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**10*sin(b*x+a)**4,x)
[Out] Timed out
```

3.89 $\int \cos^6(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=111

$$-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}-\frac{3\sin(a+bx)\cos^7(a+bx)}{80b}+\frac{\sin(a+bx)\cos^5(a+bx)}{160b}+\frac{\sin(a+bx)\cos^3(a+bx)}{128b}+\frac{3}{}$$

```
[Out] 3/256*x+3/256*cos(b*x+a)*sin(b*x+a)/b+1/128*cos(b*x+a)^3*sin(b*x+a)/b+1/160
*cos(b*x+a)^5*sin(b*x+a)/b-3/80*cos(b*x+a)^7*sin(b*x+a)/b-1/10*cos(b*x+a)^7
*sin(b*x+a)^3/b
```

Rubi [A] time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$-\frac{\sin^3(a+bx)\cos^7(a+bx)}{10b}-\frac{3\sin(a+bx)\cos^7(a+bx)}{80b}+\frac{\sin(a+bx)\cos^5(a+bx)}{160b}+\frac{\sin(a+bx)\cos^3(a+bx)}{128b}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^6*Sin[a + b*x]^4,x]
```

```
[Out] (3*x)/256 + (3*Cos[a + b*x]*Sin[a + b*x])/(256*b) + (Cos[a + b*x]^3*Sin[a +
 b*x])/(128*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(160*b) - (3*Cos[a + b*x]^7*
Sin[a + b*x])/(80*b) - (Cos[a + b*x]^7*Sin[a + b*x]^3)/(10*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cos^6(a+bx)\sin^4(a+bx)\,dx = -\frac{\cos^7(a+bx)\sin^3(a+bx)}{10b} + \frac{3}{10}\int \cos^6(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{3\cos^7(a+bx)\sin(a+bx)}{80b} - \frac{\cos^7(a+bx)\sin^3(a+bx)}{10b} + \frac{3}{80}\int \cos^6(a+bx)$$
$$= \frac{\cos^5(a+bx)\sin(a+bx)}{160b} - \frac{3\cos^7(a+bx)\sin(a+bx)}{80b} - \frac{\cos^7(a+bx)\sin^3(a+}{10b}$$
$$= \frac{\cos^3(a+bx)\sin(a+bx)}{128b} + \frac{\cos^5(a+bx)\sin(a+bx)}{160b} - \frac{3\cos^7(a+bx)\sin(a+b}{80b}$$
$$= \frac{3\cos(a+bx)\sin(a+bx)}{256b} + \frac{\cos^3(a+bx)\sin(a+bx)}{128b} + \frac{\cos^5(a+bx)\sin(a+b}{160b}$$
$$= \frac{3x}{256} + \frac{3\cos(a+bx)\sin(a+bx)}{256b} + \frac{\cos^3(a+bx)\sin(a+bx)}{128b} + \frac{\cos^5(a+bx)\sin}{160b}$$

Mathematica [A] time = 0.18, size = 62, normalized size = 0.56

$$\frac{20\sin(2(a+bx)) - 40\sin(4(a+bx)) - 10\sin(6(a+bx)) + 5\sin(8(a+bx)) + 2\sin(10(a+bx)) + 120bx}{10240b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^6*Sin[a + b*x]^4,x]
```

```
[Out] (120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
+ 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(10240*b)
```

fricas [A] time = 0.44, size = 66, normalized size = 0.59

$$\frac{15\,bx + \left(128\cos(bx+a)^9 - 176\cos(bx+a)^7 + 8\cos(bx+a)^5 + 10\cos(bx+a)^3 + 15\cos(bx+a)\right)\sin(bx+}{1280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^
5 + 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

giac [A] time = 1.87, size = 74, normalized size = 0.67

$$\frac{3}{256}x + \frac{\sin(10\,bx + 10\,a)}{5120\,b} + \frac{\sin(8\,bx + 8\,a)}{2048\,b} - \frac{\sin(6\,bx + 6\,a)}{1024\,b} - \frac{\sin(4\,bx + 4\,a)}{256\,b} + \frac{\sin(2\,bx + 2\,a)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/256*x + 1/5120*sin(10*b*x + 10*a)/b + 1/2048*sin(8*b*x + 8*a)/b - 1/1024*
sin(6*b*x + 6*a)/b - 1/256*sin(4*b*x + 4*a)/b + 1/512*sin(2*b*x + 2*a)/b
```

maple [A] time = 0.02, size = 82, normalized size = 0.74

$$\frac{-\frac{\left(\sin^3(bx+a)\right)\left(\cos^7(bx+a)\right)}{10}-\frac{3\sin(bx+a)\left(\cos^7(bx+a)\right)}{80}+\frac{\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{160}+\frac{3bx}{256}+\frac{3a}{256}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/10*sin(b*x+a)^3*cos(b*x+a)^7-3/80*sin(b*x+a)*cos(b*x+a)^7+1/160*(co
s(b*x+a)^5+5/4*cos(b*x+a)^3+15/8*cos(b*x+a))*sin(b*x+a)+3/256*b*x+3/256*a)
```

maxima [A] time = 0.69, size = 48, normalized size = 0.43

$$\frac{32\sin(2bx+2a)^5+120bx+120a+5\sin(8bx+8a)-40\sin(4bx+4a)}{10240b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/10240*(32*sin(2*b*x + 2*a)^5 + 120*b*x + 120*a + 5*sin(8*b*x + 8*a) - 40*
sin(4*b*x + 4*a))/b
```

mupad [B] time = 1.94, size = 109, normalized size = 0.98

$$\frac{3x}{256}+\frac{\frac{3\tan(a+bx)^9}{256}+\frac{7\tan(a+bx)^7}{128}+\frac{\tan(a+bx)^5}{10}-\frac{7\tan(a+bx)^3}{128}-\frac{3\tan(a+bx)}{256}}{b\left(\tan(a+bx)^{10}+5\tan(a+bx)^8+10\tan(a+bx)^6+10\tan(a+bx)^4+5\tan(a+bx)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^6*sin(a + b*x)^4,x)
```

```
[Out] (3*x)/256 + (tan(a + b*x)^5/10 - (7*tan(a + b*x)^3)/128 - (3*tan(a + b*x))/
256 + (7*tan(a + b*x)^7)/128 + (3*tan(a + b*x)^9)/256)/(b*(5*tan(a + b*x)^2
 + 10*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^
10 + 1))
```

sympy [A] time = 32.10, size = 231, normalized size = 2.08

$$\begin{cases}\frac{3x\sin^{10}(a+bx)}{256}+\frac{15x\sin^8(a+bx)\cos^2(a+bx)}{256}+\frac{15x\sin^6(a+bx)\cos^4(a+bx)}{128}+\frac{15x\sin^4(a+bx)\cos^6(a+bx)}{128}+\frac{15x\sin^2(a+bx)\cos^8(a+bx)}{256}+ \\ x\sin^4(a)\cos^6(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**10/256 + 15*x*sin(a + b*x)**8*cos(a + b*x)**2/
256 + 15*x*sin(a + b*x)**6*cos(a + b*x)**4/128 + 15*x*sin(a + b*x)**4*cos(a
 + b*x)**6/128 + 15*x*sin(a + b*x)**2*cos(a + b*x)**8/256 + 3*x*cos(a + b*x
)**10/256 + 3*sin(a + b*x)**9*cos(a + b*x)/(256*b) + 7*sin(a + b*x)**7*cos(
a + b*x)**3/(128*b) + sin(a + b*x)**5*cos(a + b*x)**5/(10*b) - 7*sin(a + b*
x)**3*cos(a + b*x)**7/(128*b) - 3*sin(a + b*x)*cos(a + b*x)**9/(256*b), Ne(
b, 0)), (x*sin(a)**4*cos(a)**6, True))
```

3.90 $\int \cos^4(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=90

$$-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}-\frac{\sin(a+bx)\cos^5(a+bx)}{16b}+\frac{\sin(a+bx)\cos^3(a+bx)}{64b}+\frac{3\sin(a+bx)\cos(a+bx)}{128b}+\frac{3x}{12}$$

```
[Out] 3/128*x+3/128*cos(b*x+a)*sin(b*x+a)/b+1/64*cos(b*x+a)^3*sin(b*x+a)/b-1/16*c
os(b*x+a)^5*sin(b*x+a)/b-1/8*cos(b*x+a)^5*sin(b*x+a)^3/b
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$-\frac{\sin^3(a+bx)\cos^5(a+bx)}{8b}-\frac{\sin(a+bx)\cos^5(a+bx)}{16b}+\frac{\sin(a+bx)\cos^3(a+bx)}{64b}+\frac{3\sin(a+bx)\cos(a+bx)}{128b}+\frac{3x}{12}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Sin[a + b*x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cos[a + b*x]*Sin[a + b*x])/(128*b) + (Cos[a + b*x]^3*Sin[a +
 b*x])/(64*b) - (Cos[a + b*x]^5*Sin[a + b*x])/(16*b) - (Cos[a + b*x]^5*Sin[
a + b*x]^3)/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule Rubi steps

$$\int \cos^4(a+bx)\sin^4(a+bx)\,dx = -\frac{\cos^5(a+bx)\sin^3(a+bx)}{8b} + \frac{3}{8}\int \cos^4(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{\cos^5(a+bx)\sin(a+bx)}{16b} - \frac{\cos^5(a+bx)\sin^3(a+bx)}{8b} + \frac{1}{16}\int \cos^4(a+bx)\,dx$$
$$= \frac{\cos^3(a+bx)\sin(a+bx)}{64b} - \frac{\cos^5(a+bx)\sin(a+bx)}{16b} - \frac{\cos^5(a+bx)\sin^3(a+b}{8b}$$
$$= \frac{3\cos(a+bx)\sin(a+bx)}{128b} + \frac{\cos^3(a+bx)\sin(a+bx)}{64b} - \frac{\cos^5(a+bx)\sin(a+b}{16b}$$
$$= \frac{3x}{128} + \frac{3\cos(a+bx)\sin(a+bx)}{128b} + \frac{\cos^3(a+bx)\sin(a+bx)}{64b} - \frac{\cos^5(a+bx)\sin}{16b}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 0.37

$$\frac{24(a+bx) - 8\sin(4(a+bx)) + \sin(8(a+bx))}{1024b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Sin[a + b*x]^4,x]
```

```
[Out] (24*(a + b*x) - 8*Sin[4*(a + b*x)] + Sin[8*(a + b*x)])/(1024*b)
```

fricas [A] time = 0.55, size = 56, normalized size = 0.62

$$\frac{3\,bx + \left(16\,\cos\left(bx+a\right)^7 - 24\,\cos\left(bx+a\right)^5 + 2\,\cos\left(bx+a\right)^3 + 3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(3*b*x + (16*cos(b*x + a)^7 - 24*cos(b*x + a)^5 + 2*cos(b*x + a)^3 +
3*cos(b*x + a))*sin(b*x + a))/b
```

giac [A] time = 1.13, size = 32, normalized size = 0.36

$$\frac{3}{128}\,x + \frac{\sin\left(8\,bx + 8\,a\right)}{1024\,b} - \frac{\sin\left(4\,bx + 4\,a\right)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/128*x + 1/1024*sin(8*b*x + 8*a)/b - 1/128*sin(4*b*x + 4*a)/b
```

maple [A] time = 0.02, size = 72, normalized size = 0.80

$$\frac{-\frac{\left(\cos^{5}(bx+a)\right)\left(\sin^{3}(bx+a)\right)}{8}-\frac{\sin(bx+a)\left(\cos^{5}(bx+a)\right)}{16}+\frac{\left(\cos^{3}(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{64}+\frac{3bx}{128}+\frac{3a}{128}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/8*cos(b*x+a)^5*sin(b*x+a)^3-1/16*sin(b*x+a)*cos(b*x+a)^5+1/64*(cos(
b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/128*b*x+3/128*a)
```

maxima [A] time = 0.40, size = 33, normalized size = 0.37

$$\frac{24\,bx+24\,a+\sin\left(8\,bx+8\,a\right)-8\,\sin\left(4\,bx+4\,a\right)}{1024\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/1024*(24*b*x + 24*a + sin(8*b*x + 8*a) - 8*sin(4*b*x + 4*a))/b
```

mupad [B] time = 1.50, size = 90, normalized size = 1.00

$$\frac{3\,x}{128}-\frac{-\frac{3\,\tan(a+b\,x)^7}{128}-\frac{11\,\tan(a+b\,x)^5}{128}+\frac{11\,\tan(a+b\,x)^3}{128}+\frac{3\,\tan(a+b\,x)}{128}}{b\,\left(\tan\left(a+b\,x\right)^8+4\,\tan\left(a+b\,x\right)^6+6\,\tan\left(a+b\,x\right)^4+4\,\tan\left(a+b\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4*sin(a + b*x)^4,x)
```

```
[Out] (3*x)/128 - ((3*tan(a + b*x))/128 + (11*tan(a + b*x)^3)/128 - (11*tan(a + b
*x)^5)/128 - (3*tan(a + b*x)^7)/128)/(b*(4*tan(a + b*x)^2 + 6*tan(a + b*x)^
4 + 4*tan(a + b*x)^6 + tan(a + b*x)^8 + 1))
```

sympy [A] time = 13.09, size = 189, normalized size = 2.10

$$\begin{cases}\frac{3x\sin^{8}(a+bx)}{128}+\frac{3x\sin^{6}(a+bx)\cos^{2}(a+bx)}{32}+\frac{9x\sin^{4}(a+bx)\cos^{4}(a+bx)}{64}+\frac{3x\sin^{2}(a+bx)\cos^{6}(a+bx)}{32}+\frac{3x\cos^{8}(a+bx)}{128}+\frac{3\sin^{7}(a+bx)\cos}{128b} \\ x\sin^{4}(a)\cos^{4}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**8/128 + 3*x*sin(a + b*x)**6*cos(a + b*x)**2/32
 + 9*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 3*x*sin(a + b*x)**2*cos(a + b*x
)**6/32 + 3*x*cos(a + b*x)**8/128 + 3*sin(a + b*x)**7*cos(a + b*x)/(128*b)
+ 11*sin(a + b*x)**5*cos(a + b*x)**3/(128*b) - 11*sin(a + b*x)**3*cos(a + b
*x)**5/(128*b) - 3*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)), (x*sin(
a)**4*cos(a)**4, True))
```

3.91 $\int \cos^2(a+bx)\sin^4(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}-\frac{\sin(a+bx)\cos^3(a+bx)}{8b}+\frac{\sin(a+bx)\cos(a+bx)}{16b}+\frac{x}{16}$$

```
[Out] 1/16*x+1/16*cos(b*x+a)*sin(b*x+a)/b-1/8*cos(b*x+a)^3*sin(b*x+a)/b-1/6*cos(b
*x+a)^3*sin(b*x+a)^3/b
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2568, 2635, 8}

$$-\frac{\sin^3(a+bx)\cos^3(a+bx)}{6b}-\frac{\sin(a+bx)\cos^3(a+bx)}{8b}+\frac{\sin(a+bx)\cos(a+bx)}{16b}+\frac{x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^4,x]
```

```
[Out] x/16 + (Cos[a + b*x]*Sin[a + b*x])/(16*b) - (Cos[a + b*x]^3*Sin[a + b*x])/(
8*b) - (Cos[a + b*x]^3*Sin[a + b*x]^3)/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^4(a+bx)\,dx = -\frac{\cos^3(a+bx)\sin^3(a+bx)}{6b} + \frac{1}{2}\int \cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{\cos^3(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin^3(a+bx)}{6b} + \frac{1}{8}\int \cos^2(a+bx)\,dx$$
$$= \frac{\cos(a+bx)\sin(a+bx)}{16b} - \frac{\cos^3(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin^3(a+bx}{6b}$$
$$= \frac{x}{16} + \frac{\cos(a+bx)\sin(a+bx)}{16b} - \frac{\cos^3(a+bx)\sin(a+bx)}{8b} - \frac{\cos^3(a+bx)\sin^3(a}{6b}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.58

$$\frac{-3\sin(2(a+bx)) - 3\sin(4(a+bx)) + \sin(6(a+bx)) + 12bx}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^4,x]
```

```
[Out] (12*b*x - 3*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)])/(192*
b)
```

fricas [A] time = 0.41, size = 46, normalized size = 0.67

$$\frac{3\,bx + \left(8\,\cos(bx+a)^5 - 14\,\cos(bx+a)^3 + 3\,\cos(bx+a)\right)\sin(bx+a)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(3*b*x + (8*cos(b*x + a)^5 - 14*cos(b*x + a)^3 + 3*cos(b*x + a))*sin(b
*x + a))/b
```

giac [A] time = 0.32, size = 46, normalized size = 0.67

$$\frac{1}{16}\,x + \frac{\sin(6\,bx + 6\,a)}{192\,b} - \frac{\sin(4\,bx + 4\,a)}{64\,b} - \frac{\sin(2\,bx + 2\,a)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*x + 1/192*sin(6*b*x + 6*a)/b - 1/64*sin(4*b*x + 4*a)/b - 1/64*sin(2*b*
x + 2*a)/b
```

maple [A] time = 0.02, size = 61, normalized size = 0.88

$$\frac{-\frac{\left(\cos^{3}(bx+a)\right)\left(\sin^{3}(bx+a)\right)}{6}-\frac{\left(\cos^{3}(bx+a)\right)\sin(bx+a)}{8}+\frac{\cos(bx+a)\sin(bx+a)}{16}+\frac{bx}{16}+\frac{a}{16}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/6*cos(b*x+a)^3*sin(b*x+a)^3-1/8*cos(b*x+a)^3*sin(b*x+a)+1/16*cos(b*
x+a)*sin(b*x+a)+1/16*b*x+1/16*a)
```

maxima [A] time = 0.46, size = 37, normalized size = 0.54

$$-\frac{4\sin\left(2\,bx+2\,a\right)^{3}-12\,bx-12\,a+3\,\sin\left(4\,bx+4\,a\right)}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/192*(4*sin(2*b*x + 2*a)^3 - 12*b*x - 12*a + 3*sin(4*b*x + 4*a))/b
```

mupad [B] time = 0.52, size = 43, normalized size = 0.62

$$\frac{x}{16}-\frac{\frac{\sin(2\,a+2\,b\,x)}{64}+\frac{\sin(4\,a+4\,b\,x)}{64}-\frac{\sin(6\,a+6\,b\,x)}{192}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^4,x)
```

```
[Out] x/16 - (sin(2*a + 2*b*x)/64 + sin(4*a + 4*b*x)/64 - sin(6*a + 6*b*x)/192)/b
```

sympy [A] time = 4.56, size = 136, normalized size = 1.97

$$\begin{cases}\frac{x\sin^{6}(a+bx)}{16}+\frac{3x\sin^{4}(a+bx)\cos^{2}(a+bx)}{16}+\frac{3x\sin^{2}(a+bx)\cos^{4}(a+bx)}{16}+\frac{x\cos^{6}(a+bx)}{16}+\frac{\sin^{5}(a+bx)\cos(a+bx)}{16b}-\frac{\sin^{3}(a+bx)\cos^{3}(a+bx)}{6b} \\ x\sin^{4}(a)\cos^{2}(a)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**4,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**6/16 + 3*x*sin(a + b*x)**4*cos(a + b*x)**2/16 +
3*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + x*cos(a + b*x)**6/16 + sin(a + b*x
)**5*cos(a + b*x)/(16*b) - sin(a + b*x)**3*cos(a + b*x)**3/(6*b) - sin(a +
b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*sin(a)**4*cos(a)**2, True))
```

3.92 $\int \sin^4(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}-\frac{3\sin(a+bx)\cos(a+bx)}{8b}+\frac{3x}{8}$$

```
[Out] 3/8*x-3/8*cos(b*x+a)*sin(b*x+a)/b-1/4*cos(b*x+a)*sin(b*x+a)^3/b
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}-\frac{3\sin(a+bx)\cos(a+bx)}{8b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[a + b*x]*Sin[a + b*x])/(8*b) - (Cos[a + b*x]*Sin[a + b*x]^
3)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \sin^4(a+bx)\,dx &= -\frac{\cos(a+bx)\sin^3(a+bx)}{4b}+\frac{3}{4}\int \sin^2(a+bx)\,dx\\ &= -\frac{3\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos(a+bx)\sin^3(a+bx)}{4b}+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3\cos(a+bx)\sin(a+bx)}{8b}-\frac{\cos(a+bx)\sin^3(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.72

$$\frac{12(a+bx)-8\sin(2(a+bx))+\sin(4(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4,x]
[Out] (12*(a + b*x) - 8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])/(32*b)
```

fricas [A] time = 0.44, size = 36, normalized size = 0.78

$$\frac{3\,bx+\left(2\,\cos\left(bx+a\right)^{3}-5\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4,x, algorithm="fricas")
[Out] 1/8*(3*b*x + (2*cos(b*x + a)^3 - 5*cos(b*x + a))*sin(b*x + a))/b
```

giac [A] time = 0.18, size = 32, normalized size = 0.70

$$\frac{3}{8}\,x+\frac{\sin\left(4\,bx+4\,a\right)}{32\,b}-\frac{\sin\left(2\,bx+2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4,x, algorithm="giac")
[Out] 3/8*x + 1/32*sin(4*b*x + 4*a)/b - 1/4*sin(2*b*x + 2*a)/b
```

maple [A] time = 0.00, size = 38, normalized size = 0.83

$$\frac{-\frac{\left(\sin^{3}(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4,x)
[Out] 1/b*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)
```

maxima [A] time = 0.33, size = 33, normalized size = 0.72

$$\frac{12\,bx+12\,a+\sin\left(4\,bx+4\,a\right)-8\,\sin\left(2\,bx+2\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/32*(12*b*x + 12*a + sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))/b
```

mupad [B] time = 0.42, size = 50, normalized size = 1.09

$$\frac{3x}{8}-\frac{\frac{5\tan(a+bx)^3}{8}+\frac{3\tan(a+bx)}{8}}{b\left(\tan\left(a+bx\right)^4+2\tan\left(a+bx\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4,x)
```

```
[Out] (3*x)/8 - ((3*tan(a + b*x))/8 + (5*tan(a + b*x)^3)/8)/(b*(2*tan(a + b*x)^2
+ tan(a + b*x)^4 + 1))
```

sympy [A] time = 1.24, size = 95, normalized size = 2.07

$$\begin{cases}\frac{3x\sin^{4}\left(a+bx\right)}{8}+\frac{3x\sin^{2}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{4}+\frac{3x\cos^{4}\left(a+bx\right)}{8}-\frac{5\sin^{3}\left(a+bx\right)\cos\left(a+bx\right)}{8b}-\frac{3\sin\left(a+bx\right)\cos^{3}\left(a+bx\right)}{8b} & \text{for } b\neq 0\\ x\sin^{4}\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**4/8 + 3*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
3*x*cos(a + b*x)**4/8 - 5*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*sin(a + b*
x)*cos(a + b*x)**3/(8*b), Ne(b, 0)), (x*sin(a)**4, True))
```

3.93 $\int \sin^3(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=38

$$-\frac{\sin^3(a+bx)}{3b}-\frac{\sin(a+bx)}{b}+\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

```
[Out] arctanh(sin(b*x+a))/b-sin(b*x+a)/b-1/3*sin(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2592, 302, 206}

$$-\frac{\sin^3(a+bx)}{3b}-\frac{\sin(a+bx)}{b}+\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Tan[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \sin^3(a+bx)\tan(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \sin(a+bx)\right)}{b}$$
$$= -\frac{\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{3b} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{b} - \frac{\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{3b}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.00

$$-\frac{\sin^3(a+bx)}{3b} - \frac{\sin(a+bx)}{b} + \frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Tan[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

fricas [A] time = 0.46, size = 48, normalized size = 1.26

$$\frac{2\left(\cos\left(bx+a\right)^2-4\right)\sin\left(bx+a\right)+3\,\log\left(\sin\left(bx+a\right)+1\right)-3\,\log\left(-\sin\left(bx+a\right)+1\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(cos(b*x + a)^2 - 4)*sin(b*x + a) + 3*log(sin(b*x + a) + 1) - 3*log(
-sin(b*x + a) + 1))/b
```

giac [A] time = 0.56, size = 48, normalized size = 1.26

$$-\frac{2\,\sin\left(bx+a\right)^3-3\,\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+3\,\log\left(\left|\sin\left(bx+a\right)-1\right|\right)+6\,\sin\left(bx+a\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^4,x, algorithm="giac")
```

[Out] -1/6*(2*sin(b*x + a)^3 - 3*log(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)) + 6*sin(b*x + a))/b

maple [A] time = 0.03, size = 44, normalized size = 1.16

$$-\frac{\sin^3(bx+a)}{3b}-\frac{\sin(bx+a)}{b}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)*sin(b*x+a)^4,x)

[Out] -1/3*sin(b*x+a)^3/b-sin(b*x+a)/b+1/b*ln(sec(b*x+a)+tan(b*x+a))

maxima [A] time = 0.58, size = 46, normalized size = 1.21

$$-\frac{2\sin(bx+a)^3-3\log(\sin(bx+a)+1)+3\log(\sin(bx+a)-1)+6\sin(bx+a)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)^4,x, algorithm="maxima")

[Out] -1/6*(2*sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1) + 6*sin(b*x + a))/b

mupad [B] time = 0.58, size = 53, normalized size = 1.39

$$\frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\right)}{b}-\frac{5\,\sin(a+b\,x)}{4\,b}+\frac{\sin(3\,a+3\,b\,x)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a + b*x)^4/cos(a + b*x),x)

[Out] (2*atanh(sin(a/2 + (b*x)/2)/cos(a/2 + (b*x)/2)))/b - (5*sin(a + b*x))/(4*b) + sin(3*a + 3*b*x)/(12*b)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)*sin(b*x+a)**4,x)

[Out] Timed out

3.94 $\int \sin(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\sin(a+bx)}{2b}+\frac{\sin(a+bx)\tan^2(a+bx)}{2b}-\frac{3\tanh^{-1}(\sin(a+bx))}{2b}$$

```
[Out] -3/2*arctanh(sin(b*x+a))/b+3/2*sin(b*x+a)/b+1/2*sin(b*x+a)*tan(b*x+a)^2/b
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2592, 288, 321, 206}

$$\frac{3\sin(a+bx)}{2b}+\frac{\sin(a+bx)\tan^2(a+bx)}{2b}-\frac{3\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[a + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Sin[a + b*x]])/(2*b) + (3*Sin[a + b*x])/(2*b) + (Sin[a + b*x]*T
an[a + b*x]^2)/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\tan^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin(a+bx)\tan^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}\\ &= \frac{3\sin(a+bx)}{2b} + \frac{\sin(a+bx)\tan^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}\\ &= -\frac{3\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3\sin(a+bx)}{2b} + \frac{\sin(a+bx)\tan^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.09, size = 40, normalized size = 0.82

$$\frac{(\cos(2(a+bx))+2)\tan(a+bx)\sec(a+bx) - 3\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Sin[a + b*x]] + (2 + Cos[2*(a + b*x)])*Sec[a + b*x]*Tan[a + b*x
])/(2*b)
```

fricas [A] time = 0.45, size = 74, normalized size = 1.51

$$-\frac{3\cos(bx+a)^2\log(\sin(bx+a)+1) - 3\cos(bx+a)^2\log(-\sin(bx+a)+1) - 2\left(2\cos(bx+a)^2+1\right)\sin(bx+a)}{4\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/4*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^2*log(-sin(b*
x + a) + 1) - 2*(2*cos(b*x + a)^2 + 1)*sin(b*x + a))/(b*cos(b*x + a)^2)
```

giac [A] time = 0.86, size = 58, normalized size = 1.18

$$-\frac{\frac{2\,\sin\left(bx+a\right)}{\sin\left(bx+a\right)^{2}-1}+3\,\log\left(\left|\sin\left(bx+a\right)+1\right|\right)-3\,\log\left(\left|\sin\left(bx+a\right)-1\right|\right)-4\,\sin\left(bx+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 3*log(abs(sin(b*x + a) + 1)) -
3*log(abs(sin(b*x + a) - 1)) - 4*sin(b*x + a))/b
```

maple [A] time = 0.04, size = 66, normalized size = 1.35

$$\frac{\sin^{5}\left(bx+a\right)}{2b\cos\left(bx+a\right)^{2}}+\frac{\sin^{3}\left(bx+a\right)}{2b}+\frac{3\sin\left(bx+a\right)}{2b}-\frac{3\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^4,x)
```

```
[Out] 1/2/b*sin(b*x+a)^5/cos(b*x+a)^2+1/2*sin(b*x+a)^3/b+3/2*sin(b*x+a)/b-3/2/b*l
n(sec(b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.40, size = 56, normalized size = 1.14

$$-\frac{\frac{2\,\sin\left(bx+a\right)}{\sin\left(bx+a\right)^{2}-1}+3\,\log\left(\sin\left(bx+a\right)+1\right)-3\,\log\left(\sin\left(bx+a\right)-1\right)-4\,\sin\left(bx+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 3*log(sin(b*x + a) + 1) - 3*log
(sin(b*x + a) - 1) - 4*sin(b*x + a))/b
```

mupad [B] time = 3.97, size = 98, normalized size = 2.00

$$-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}-\frac{3\,\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{5}-2\,\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{3}+3\,\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{b\,\left(-\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{6}+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{4}+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^3,x)
```

```
[Out] - (3*atanh(tan(a/2 + (b*x)/2)))/b - (3*tan(a/2 + (b*x)/2) - 2*tan(a/2 + (b*
x)/2)^3 + 3*tan(a/2 + (b*x)/2)^5)/(b*(tan(a/2 + (b*x)/2)^2 + tan(a/2 + (b*x
)/2)^4 - tan(a/2 + (b*x)/2)^6 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^4(a+bx)\sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**4,x)
```

```
[Out] Integral(sin(a + b*x)**4*sec(a + b*x)**3, x)
```

3.95 $\int \sec(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{3\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\tan^3(a+bx)\sec(a+bx)}{4b}-\frac{3\tan(a+bx)\sec(a+bx)}{8b}$$

```
[Out] 3/8*arctanh(sin(b*x+a))/b-3/8*sec(b*x+a)*tan(b*x+a)/b+1/4*sec(b*x+a)*tan(b*
x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$\frac{3\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\tan^3(a+bx)\sec(a+bx)}{4b}-\frac{3\tan(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Tan[a + b*x]^4,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(8*b) - (3*Sec[a + b*x]*Tan[a + b*x])/(8*b) + (Se
c[a + b*x]*Tan[a + b*x]^3)/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(a+bx)\tan^4(a+bx)\,dx = \frac{\sec(a+bx)\tan^3(a+bx)}{4b} - \frac{3}{4}\int \sec(a+bx)\tan^2(a+bx)\,dx$$
$$= -\frac{3\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec(a+bx)\tan^3(a+bx)}{4b} + \frac{3}{8}\int \sec(a+bx)\,dx$$
$$= \frac{3\tanh^{-1}(\sin(a+bx))}{8b} - \frac{3\sec(a+bx)\tan(a+bx)}{8b} + \frac{\sec(a+bx)\tan^3(a+bx)}{4b}$$

Mathematica [A] time = 0.11, size = 45, normalized size = 0.82

$$\frac{6\tanh^{-1}(\sin(a+bx)) - (5\cos(2(a+bx))+1)\tan(a+bx)\sec^3(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x]^4,x]
```

```
[Out] (6*ArcTanh[Sin[a + b*x]] - (1 + 5*Cos[2*(a + b*x)])*Sec[a + b*x]^3*Tan[a +
b*x])/(16*b)
```

fricas [A] time = 0.47, size = 74, normalized size = 1.35

$$\frac{3\cos(bx+a)^4\log(\sin(bx+a)+1) - 3\cos(bx+a)^4\log(-\sin(bx+a)+1) - 2\left(5\cos(bx+a)^2-2\right)\sin(bx}{16\,b\cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(b*x + a)^4*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^4*log(-sin(b*
x + a) + 1) - 2*(5*cos(b*x + a)^2 - 2)*sin(b*x + a))/(b*cos(b*x + a)^4)
```

giac [A] time = 0.39, size = 63, normalized size = 1.15

$$\frac{\frac{2\left(5\sin(bx+a)^3-3\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2} + 3\log(|\sin(bx+a)+1|) - 3\log(|\sin(bx+a)-1|)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/16*(2*(5*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 3*log(
abs(sin(b*x + a) + 1)) - 3*log(abs(sin(b*x + a) - 1)))/b
```

maple [A] time = 0.04, size = 87, normalized size = 1.58

$$\frac{\sin^5(bx+a)}{4b\cos(bx+a)^4}-\frac{\sin^5(bx+a)}{8b\cos(bx+a)^2}-\frac{\sin^3(bx+a)}{8b}-\frac{3\sin(bx+a)}{8b}+\frac{3\ln(\sec(bx+a)+\tan(bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5*sin(b*x+a)^4,x)
```

```
[Out] 1/4/b*sin(b*x+a)^5/cos(b*x+a)^4-1/8/b*sin(b*x+a)^5/cos(b*x+a)^2-1/8*sin(b*x
+a)^3/b-3/8*sin(b*x+a)/b+3/8/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.51, size = 71, normalized size = 1.29

$$\frac{\frac{2\left(5\sin(bx+a)^3-3\sin(bx+a)\right)}{\sin(bx+a)^4-2\sin(bx+a)^2+1}+3\log(\sin(bx+a)+1)-3\log(\sin(bx+a)-1)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(5*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x + a)^4 - 2*sin(b*x + a
)^2 + 1) + 3*log(sin(b*x + a) + 1) - 3*log(sin(b*x + a) - 1))/b
```

mupad [B] time = 6.59, size = 126, normalized size = 2.29

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{4\,b}-\frac{\frac{3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^7}{4}-\frac{11\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^5}{4}-\frac{11\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3}{4}+\frac{3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{4}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8-4\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6+6\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4-4\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^5,x)
```

```
[Out] (3*atanh(tan(a/2 + (b*x)/2)))/(4*b) - ((3*tan(a/2 + (b*x)/2))/4 - (11*tan(a
/2 + (b*x)/2)^3)/4 - (11*tan(a/2 + (b*x)/2)^5)/4 + (3*tan(a/2 + (b*x)/2)^7)
/4)/(b*(6*tan(a/2 + (b*x)/2)^4 - 4*tan(a/2 + (b*x)/2)^2 - 4*tan(a/2 + (b*x)
/2)^6 + tan(a/2 + (b*x)/2)^8 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

3.96 $\int \sec^3(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan^3(a+bx)\sec^3(a+bx)}{6b}-\frac{\tan(a+bx)\sec^3(a+bx)}{8b}+\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

```
[Out] 1/16*arctanh(sin(b*x+a))/b+1/16*sec(b*x+a)*tan(b*x+a)/b-1/8*sec(b*x+a)^3*ta
n(b*x+a)/b+1/6*sec(b*x+a)^3*tan(b*x+a)^3/b
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan^3(a+bx)\sec^3(a+bx)}{6b}-\frac{\tan(a+bx)\sec^3(a+bx)}{8b}+\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x]^4,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) + (Sec[a + b*x]*Tan[a + b*x])/(16*b) - (Sec[a
+ b*x]^3*Tan[a + b*x])/(8*b) + (Sec[a + b*x]^3*Tan[a + b*x]^3)/(6*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(a+bx)\tan^4(a+bx)\,dx = \frac{\sec^3(a+bx)\tan^3(a+bx)}{6b} - \frac{1}{2}\int \sec^3(a+bx)\tan^2(a+bx)\,dx$$
$$= -\frac{\sec^3(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan^3(a+bx)}{6b} + \frac{1}{8}\int \sec^3(a+bx)\,dx$$
$$= \frac{\sec(a+bx)\tan(a+bx)}{16b} - \frac{\sec^3(a+bx)\tan(a+bx)}{8b} + \frac{\sec^3(a+bx)\tan^3(a+bx)}{6b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{16b} + \frac{\sec(a+bx)\tan(a+bx)}{16b} - \frac{\sec^3(a+bx)\tan(a+bx)}{8b} + \frac{s}{}$$

Mathematica [A] time = 0.03, size = 99, normalized size = 1.27

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b} - \frac{\tan(a+bx)\sec^5(a+bx)}{6b} + \frac{\tan^3(a+bx)\sec^3(a+bx)}{3b} + \frac{\tan(a+bx)\sec^3(a+bx)}{24b} + \frac{\tan(a+bx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x]^4,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) + (Sec[a + b*x]*Tan[a + b*x])/(16*b) + (Sec[a
+ b*x]^3*Tan[a + b*x])/(24*b) - (Sec[a + b*x]^5*Tan[a + b*x])/(6*b) + (Sec[
a + b*x]^3*Tan[a + b*x]^3)/(3*b)
```

fricas [A] time = 0.44, size = 84, normalized size = 1.08

$$\frac{3\cos(bx+a)^6\log(\sin(bx+a)+1) - 3\cos(bx+a)^6\log(-\sin(bx+a)+1) + 2\left(3\cos(bx+a)^4 - 14\cos(bx+\right.}{96\,b\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(3*cos(b*x + a)^6*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^6*log(-sin(b*
x + a) + 1) + 2*(3*cos(b*x + a)^4 - 14*cos(b*x + a)^2 + 8)*sin(b*x + a))/(b
*cos(b*x + a)^6)
```

giac [A] time = 0.53, size = 73, normalized size = 0.94

$$-\frac{\frac{2\left(3\sin(bx+a)^5+8\sin(bx+a)^3-3\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^3} - 3\log(|\sin(bx+a)+1|) + 3\log(|\sin(bx+a)-1|)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/96*(2*(3*sin(b*x + a)^5 + 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x +
a)^2 - 1)^3 - 3*log(abs(sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/
b
```

maple [A] time = 0.04, size = 108, normalized size = 1.38

$$\frac{\sin^5(bx+a)}{6b\cos(bx+a)^6}+\frac{\sin^5(bx+a)}{24b\cos(bx+a)^4}-\frac{\sin^5(bx+a)}{48b\cos(bx+a)^2}-\frac{\sin^3(bx+a)}{48b}-\frac{\sin(bx+a)}{16b}+\frac{\ln(\sec(bx+a)+\tan(bx+a))}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^7*sin(b*x+a)^4,x)
```

```
[Out] 1/6/b*sin(b*x+a)^5/cos(b*x+a)^6+1/24/b*sin(b*x+a)^5/cos(b*x+a)^4-1/48/b*sin
(b*x+a)^5/cos(b*x+a)^2-1/48*sin(b*x+a)^3/b-1/16*sin(b*x+a)/b+1/16/b*ln(sec(
b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.53, size = 91, normalized size = 1.17

$$-\frac{\frac{2\left(3\sin(bx+a)^5+8\sin(bx+a)^3-3\sin(bx+a)\right)}{\sin(bx+a)^6-3\sin(bx+a)^4+3\sin(bx+a)^2-1}-3\log(\sin(bx+a)+1)+3\log(\sin(bx+a)-1)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(3*sin(b*x + a)^5 + 8*sin(b*x + a)^3 - 3*sin(b*x + a))/(sin(b*x +
a)^6 - 3*sin(b*x + a)^4 + 3*sin(b*x + a)^2 - 1) - 3*log(sin(b*x + a) + 1) +
 3*log(sin(b*x + a) - 1))/b
```

mupad [B] time = 7.38, size = 177, normalized size = 2.27

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b}+\frac{-\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{11}}{8}+\frac{17\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{9}}{24}+\frac{19\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{7}}{4}+\frac{19\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{5}}{4}+\frac{17\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{24}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{12}-6\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+15\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{8}-20\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{6}+15\tan\left(\frac{a}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^7,x)
```

```
[Out] atanh(tan(a/2 + (b*x)/2))/(8*b) + ((17*tan(a/2 + (b*x)/2)^3)/24 - tan(a/2 +
 (b*x)/2)/8 + (19*tan(a/2 + (b*x)/2)^5)/4 + (19*tan(a/2 + (b*x)/2)^7)/4 + (
17*tan(a/2 + (b*x)/2)^9)/24 - tan(a/2 + (b*x)/2)^11/8)/(b*(15*tan(a/2 + (b*
```

```
x)/2)^4 - 6*tan(a/2 + (b*x)/2)^2 - 20*tan(a/2 + (b*x)/2)^6 + 15*tan(a/2 + (
b*x)/2)^8 - 6*tan(a/2 + (b*x)/2)^10 + tan(a/2 + (b*x)/2)^12 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**7*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

3.97 $\int \sec^5(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=99

$$\frac{3\tanh^{-1}(\sin(a+bx))}{128b}+\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b}-\frac{\tan(a+bx)\sec^5(a+bx)}{16b}+\frac{\tan(a+bx)\sec^3(a+bx)}{64b}+\frac{3\tan(a$$

```
[Out] 3/128*arctanh(sin(b*x+a))/b+3/128*sec(b*x+a)*tan(b*x+a)/b+1/64*sec(b*x+a)^3
*tan(b*x+a)/b-1/16*sec(b*x+a)^5*tan(b*x+a)/b+1/8*sec(b*x+a)^5*tan(b*x+a)^3/
b
```

Rubi [A] time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$\frac{3\tanh^{-1}(\sin(a+bx))}{128b}+\frac{\tan^3(a+bx)\sec^5(a+bx)}{8b}-\frac{\tan(a+bx)\sec^5(a+bx)}{16b}+\frac{\tan(a+bx)\sec^3(a+bx)}{64b}+\frac{3\tan(a$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5*Tan[a + b*x]^4,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(128*b) + (3*Sec[a + b*x]*Tan[a + b*x])/(128*b) +
 (Sec[a + b*x]^3*Tan[a + b*x])/(64*b) - (Sec[a + b*x]^5*Tan[a + b*x])/(16*b
) + (Sec[a + b*x]^5*Tan[a + b*x]^3)/(8*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^5(a+bx)\tan^4(a+bx)\,dx = \frac{\sec^5(a+bx)\tan^3(a+bx)}{8b} - \frac{3}{8}\int \sec^5(a+bx)\tan^2(a+bx)\,dx$$
$$= -\frac{\sec^5(a+bx)\tan(a+bx)}{16b} + \frac{\sec^5(a+bx)\tan^3(a+bx)}{8b} + \frac{1}{16}\int \sec^5(a+bx)\,dx$$
$$= \frac{\sec^3(a+bx)\tan(a+bx)}{64b} - \frac{\sec^5(a+bx)\tan(a+bx)}{16b} + \frac{\sec^5(a+bx)\tan^3(a+b}{8b}$$
$$= \frac{3\sec(a+bx)\tan(a+bx)}{128b} + \frac{\sec^3(a+bx)\tan(a+bx)}{64b} - \frac{\sec^5(a+bx)\tan(a+b}{16b}$$
$$= \frac{3\tanh^{-1}(\sin(a+bx))}{128b} + \frac{3\sec(a+bx)\tan(a+bx)}{128b} + \frac{\sec^3(a+bx)\tan(a+bx)}{64b}$$

Mathematica [A] time = 0.29, size = 64, normalized size = 0.65

$$\frac{96\tanh^{-1}(\sin(a+bx)) + (-307\cos(2(a+bx)) + 26\cos(4(a+bx)) + 3\cos(6(a+bx)) + 182)\tan(a+bx)\sec^7(a}{4096b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5*Tan[a + b*x]^4,x]
```

```
[Out] (96*ArcTanh[Sin[a + b*x]] + (182 - 307*Cos[2*(a + b*x)] + 26*Cos[4*(a + b*x
)] + 3*Cos[6*(a + b*x)])*Sec[a + b*x]^7*Tan[a + b*x])/(4096*b)
```

fricas [A] time = 0.47, size = 94, normalized size = 0.95

$$\frac{3\cos(bx+a)^8\log(\sin(bx+a)+1) - 3\cos(bx+a)^8\log(-\sin(bx+a)+1) + 2\left(3\cos(bx+a)^6 + 2\cos(bx+\right.}{256\,b\cos(bx+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/256*(3*cos(b*x + a)^8*log(sin(b*x + a) + 1) - 3*cos(b*x + a)^8*log(-sin(b
*x + a) + 1) + 2*(3*cos(b*x + a)^6 + 2*cos(b*x + a)^4 - 24*cos(b*x + a)^2 +
 16)*sin(b*x + a))/(b*cos(b*x + a)^8)
```

giac [A] time = 1.02, size = 107, normalized size = 1.08

$$-\frac{\frac{4\left(3\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^3 - \frac{20}{\sin(bx+a)} - 20\sin(bx+a)\right)}{\left(\left(\frac{1}{\sin(bx+a)}+\sin(bx+a)\right)^2 - 4\right)^2} - 3\log\left(\left|\frac{1}{\sin(bx+a)} + \sin(bx+a) + 2\right|\right) + 3\log\left(\left|\frac{1}{\sin(bx+a)} + \sin(bx+a\right.\right.}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/512*(4*(3*(1/sin(b*x + a) + sin(b*x + a))^3 - 20/sin(b*x + a) - 20*sin(b
*x + a))/((1/sin(b*x + a) + sin(b*x + a))^2 - 4)^2 - 3*log(abs(1/sin(b*x +
a) + sin(b*x + a) + 2)) + 3*log(abs(1/sin(b*x + a) + sin(b*x + a) - 2)))/b
```

maple [A] time = 0.04, size = 129, normalized size = 1.30

$$\frac{\sin^5(bx+a)}{8b\cos(bx+a)^8}+\frac{\sin^5(bx+a)}{16b\cos(bx+a)^6}+\frac{\sin^5(bx+a)}{64b\cos(bx+a)^4}-\frac{\sin^5(bx+a)}{128b\cos(bx+a)^2}-\frac{\sin^3(bx+a)}{128b}-\frac{3\sin(bx+a)}{128b}+\frac{3\ln(\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^9*sin(b*x+a)^4,x)
```

```
[Out] 1/8/b*sin(b*x+a)^5/cos(b*x+a)^8+1/16/b*sin(b*x+a)^5/cos(b*x+a)^6+1/64/b*sin
(b*x+a)^5/cos(b*x+a)^4-1/128/b*sin(b*x+a)^5/cos(b*x+a)^2-1/128*sin(b*x+a)^3
/b-3/128*sin(b*x+a)/b+3/128/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.40, size = 111, normalized size = 1.12

$$-\frac{\frac{2\left(3\sin(bx+a)^7-11\sin(bx+a)^5-11\sin(bx+a)^3+3\sin(bx+a)\right)}{\sin(bx+a)^8-4\sin(bx+a)^6+6\sin(bx+a)^4-4\sin(bx+a)^2+1}-3\log(\sin(bx+a)+1)+3\log(\sin(bx+a)-1)}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/256*(2*(3*sin(b*x + a)^7 - 11*sin(b*x + a)^5 - 11*sin(b*x + a)^3 + 3*sin
(b*x + a))/(sin(b*x + a)^8 - 4*sin(b*x + a)^6 + 6*sin(b*x + a)^4 - 4*sin(b*
x + a)^2 + 1) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b
```

mupad [B] time = 7.44, size = 229, normalized size = 2.31

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{64\,b}+\frac{-\frac{3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{15}}{64}+\frac{23\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{13}}{64}+\frac{333\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{11}}{64}+\frac{671\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{9}}{64}+\frac{671\cdots}{}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{16}-8\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{14}+28\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{12}-56\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+70\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/cos(a + b*x)^9,x)
```

```
[Out] (3*atanh(tan(a/2 + (b*x)/2)))/(64*b) + ((23*tan(a/2 + (b*x)/2)^3)/64 - (3*t
an(a/2 + (b*x)/2))/64 + (333*tan(a/2 + (b*x)/2)^5)/64 + (671*tan(a/2 + (b*x
```

```
)/2)^7)/64 + (671*tan(a/2 + (b*x)/2)^9)/64 + (333*tan(a/2 + (b*x)/2)^11)/64
 + (23*tan(a/2 + (b*x)/2)^13)/64 - (3*tan(a/2 + (b*x)/2)^15)/64)/(b*(28*tan
(a/2 + (b*x)/2)^4 - 8*tan(a/2 + (b*x)/2)^2 - 56*tan(a/2 + (b*x)/2)^6 + 70*t
an(a/2 + (b*x)/2)^8 - 56*tan(a/2 + (b*x)/2)^10 + 28*tan(a/2 + (b*x)/2)^12 -
 8*tan(a/2 + (b*x)/2)^14 + tan(a/2 + (b*x)/2)^16 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**9*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

3.98 $\int \cos^7(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos^{12}(a+bx)}{12b}+\frac{\cos^{10}(a+bx)}{5b}-\frac{\cos^8(a+bx)}{8b}$$

```
[Out] -1/8*cos(b*x+a)^8/b+1/5*cos(b*x+a)^10/b-1/12*cos(b*x+a)^12/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2565, 266, 43}

$$-\frac{\cos^{12}(a+bx)}{12b}+\frac{\cos^{10}(a+bx)}{5b}-\frac{\cos^8(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^7*Sin[a + b*x]^5,x]
```

```
[Out] -Cos[a + b*x]^8/(8*b) + Cos[a + b*x]^10/(5*b) - Cos[a + b*x]^12/(12*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \cos^7(a+bx)\sin^5(a+bx)\,dx = -\frac{\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int (1-x)^2x^3\,dx,x,\cos^2(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(x^3-2x^4+x^5\right)\,dx,x,\cos^2(a+bx)\right)}{2b}$$
$$= -\frac{\cos^8(a+bx)}{8b}+\frac{\cos^{10}(a+bx)}{5b}-\frac{\cos^{12}(a+bx)}{12b}$$

Mathematica [A] time = 0.33, size = 68, normalized size = 1.48

$$-\frac{600\cos(2(a+bx))+75\cos(4(a+bx))-100\cos(6(a+bx))-30\cos(8(a+bx))+12\cos(10(a+bx))+5\cos(12(a+bx))}{122880b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^7*Sin[a + b*x]^5,x]
```

```
[Out] -1/122880*(600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] - 100*Cos[6*(a + b*x)
] - 30*Cos[8*(a + b*x)] + 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/b
```

fricas [A] time = 0.45, size = 36, normalized size = 0.78

$$-\frac{10\cos\left(bx+a\right)^{12}-24\cos\left(bx+a\right)^{10}+15\cos\left(bx+a\right)^{8}}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/120*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

giac [B] time = 0.26, size = 85, normalized size = 1.85

$$-\frac{\cos\left(12\,bx+12\,a\right)}{24576\,b}-\frac{\cos\left(10\,bx+10\,a\right)}{10240\,b}+\frac{\cos\left(8\,bx+8\,a\right)}{4096\,b}+\frac{5\cos\left(6\,bx+6\,a\right)}{6144\,b}-\frac{5\cos\left(4\,bx+4\,a\right)}{8192\,b}-\frac{5\cos\left(2\,bx+2\,a\right)}{1024\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/24576*cos(12*b*x + 12*a)/b - 1/10240*cos(10*b*x + 10*a)/b + 1/4096*cos(8
*b*x + 8*a)/b + 5/6144*cos(6*b*x + 6*a)/b - 5/8192*cos(4*b*x + 4*a)/b - 5/1
024*cos(2*b*x + 2*a)/b
```

maple [A] time = 0.03, size = 52, normalized size = 1.13

$$\frac{-\frac{\left(\sin^4(bx+a)\right)\left(\cos^8(bx+a)\right)}{12}-\frac{\left(\sin^2(bx+a)\right)\left(\cos^8(bx+a)\right)}{30}-\frac{\left(\cos^8(bx+a)\right)}{120}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/12*sin(b*x+a)^4*cos(b*x+a)^8-1/30*sin(b*x+a)^2*cos(b*x+a)^8-1/120*c
os(b*x+a)^8)
```

maxima [A] time = 0.39, size = 46, normalized size = 1.00

$$-\frac{10\,\sin\left(bx+a\right)^{12}-36\,\sin\left(bx+a\right)^{10}+45\,\sin\left(bx+a\right)^{8}-20\,\sin\left(bx+a\right)^{6}}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/120*(10*sin(b*x + a)^12 - 36*sin(b*x + a)^10 + 45*sin(b*x + a)^8 - 20*si
n(b*x + a)^6)/b
```

mupad [B] time = 0.41, size = 35, normalized size = 0.76

$$-\frac{\cos\left(a+b\,x\right)^8\,\left(10\cos\left(a+b\,x\right)^4-24\cos\left(a+b\,x\right)^2+15\right)}{120\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^7*sin(a + b*x)^5,x)
```

```
[Out] -(cos(a + b*x)^8*(10*cos(a + b*x)^4 - 24*cos(a + b*x)^2 + 15))/(120*b)
```

sympy [A] time = 71.14, size = 65, normalized size = 1.41

$$\begin{cases} -\frac{\sin^4(a+bx)\cos^8(a+bx)}{8b}-\frac{\sin^2(a+bx)\cos^{10}(a+bx)}{20b}-\frac{\cos^{12}(a+bx)}{120b} & \text{for } b\neq 0 \\ x\sin^5(a)\cos^7(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**8/(8*b) - sin(a + b*x)**2*cos(a +
 b*x)**10/(20*b) - cos(a + b*x)**12/(120*b), Ne(b, 0)), (x*sin(a)**5*cos(a)
**7, True))
```

3.99 $\int \cos^6(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos^{11}(a+bx)}{11b}+\frac{2\cos^9(a+bx)}{9b}-\frac{\cos^7(a+bx)}{7b}$$

```
[Out] -1/7*cos(b*x+a)^7/b+2/9*cos(b*x+a)^9/b-1/11*cos(b*x+a)^11/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 270}

$$-\frac{\cos^{11}(a+bx)}{11b}+\frac{2\cos^9(a+bx)}{9b}-\frac{\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^6*Sin[a + b*x]^5,x]
```

```
[Out] -Cos[a + b*x]^7/(7*b) + (2*Cos[a + b*x]^9)/(9*b) - Cos[a + b*x]^11/(11*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\int \cos^6(a+bx)\sin^5(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^6\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\cos^7(a+bx)}{7b}+\frac{2\cos^9(a+bx)}{9b}-\frac{\cos^{11}(a+bx)}{11b}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 37, normalized size = 0.80

$$\frac{\cos^7(a+bx)(364\cos(2(a+bx))-63\cos(4(a+bx))-365)}{5544b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^6*Sin[a + b*x]^5,x]
```

```
[Out] (Cos[a + b*x]^7*(-365 + 364*Cos[2*(a + b*x)] - 63*Cos[4*(a + b*x)]))/(5544*
b)
```

fricas [A] time = 0.48, size = 36, normalized size = 0.78

$$-\frac{63\cos(bx+a)^{11}-154\cos(bx+a)^{9}+99\cos(bx+a)^{7}}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

giac [B] time = 0.25, size = 82, normalized size = 1.78

$$-\frac{\cos(11\,bx+11\,a)}{11264\,b}-\frac{\cos(9\,bx+9\,a)}{9216\,b}+\frac{5\cos(7\,bx+7\,a)}{7168\,b}+\frac{\cos(5\,bx+5\,a)}{1024\,b}-\frac{5\cos(3\,bx+3\,a)}{1536\,b}-\frac{5\cos(bx+a)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/11264*cos(11*b*x + 11*a)/b - 1/9216*cos(9*b*x + 9*a)/b + 5/7168*cos(7*b*
x + 7*a)/b + 1/1024*cos(5*b*x + 5*a)/b - 5/1536*cos(3*b*x + 3*a)/b - 5/512*
cos(b*x + a)/b
```

maple [A] time = 0.03, size = 52, normalized size = 1.13

$$\frac{-\frac{\left(\cos^7(bx+a)\right)\left(\sin^4(bx+a)\right)}{11}-\frac{4\left(\cos^7(bx+a)\right)\left(\sin^2(bx+a)\right)}{99}-\frac{8\left(\cos^7(bx+a)\right)}{693}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/11*cos(b*x+a)^7*sin(b*x+a)^4-4/99*cos(b*x+a)^7*sin(b*x+a)^2-8/693*c
os(b*x+a)^7)
```

maxima [A] time = 0.36, size = 36, normalized size = 0.78

$$-\frac{63\,\cos\left(bx+a\right)^{11}-154\,\cos\left(bx+a\right)^{9}+99\,\cos\left(bx+a\right)^{7}}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

mupad [B] time = 0.41, size = 36, normalized size = 0.78

$$-\frac{63\cos\left(a+b\,x\right)^{11}-154\cos\left(a+b\,x\right)^{9}+99\cos\left(a+b\,x\right)^{7}}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^6*sin(a + b*x)^5,x)
```

```
[Out] -(99*cos(a + b*x)^7 - 154*cos(a + b*x)^9 + 63*cos(a + b*x)^11)/(693*b)
```

sympy [A] time = 47.18, size = 68, normalized size = 1.48

$$\begin{cases} -\frac{\sin^{4}\left(a+bx\right)\cos^{7}\left(a+bx\right)}{7b}-\frac{4\sin^{2}\left(a+bx\right)\cos^{9}\left(a+bx\right)}{63b}-\frac{8\cos^{11}\left(a+bx\right)}{693b} & \text{for } b\neq 0 \\ x\sin^{5}\left(a\right)\cos^{6}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**7/(7*b) - 4*sin(a + b*x)**2*cos(a
 + b*x)**9/(63*b) - 8*cos(a + b*x)**11/(693*b), Ne(b, 0)), (x*sin(a)**5*cos
(a)**6, True))
```

3.100 $\int \cos^5(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sin^{10}(a+bx)}{10b}-\frac{\sin^8(a+bx)}{4b}+\frac{\sin^6(a+bx)}{6b}$$

```
[Out] 1/6*sin(b*x+a)^6/b-1/4*sin(b*x+a)^8/b+1/10*sin(b*x+a)^10/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2564, 266, 43}

$$\frac{\sin^{10}(a+bx)}{10b}-\frac{\sin^8(a+bx)}{4b}+\frac{\sin^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Sin[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]^6/(6*b) - Sin[a + b*x]^8/(4*b) + Sin[a + b*x]^10/(10*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \cos^5(a+bx)\sin^5(a+bx)\,dx = \frac{\text{Subst}\left(\int x^5\left(1-x^2\right)^2\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int (1-x)^2x^2\,dx, x, \sin^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(x^2-2x^3+x^4\right)\,dx, x, \sin^2(a+bx)\right)}{2b}$$
$$= \frac{\sin^6(a+bx)}{6b} - \frac{\sin^8(a+bx)}{4b} + \frac{\sin^{10}(a+bx)}{10b}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 1.09

$$\frac{1}{32}\left(-\frac{5\cos(2(a+bx))}{16b} + \frac{5\cos(6(a+bx))}{96b} - \frac{\cos(10(a+bx))}{160b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Sin[a + b*x]^5,x]
```

```
[Out] ((-5*Cos[2*(a + b*x)])/(16*b) + (5*Cos[6*(a + b*x)])/(96*b) - Cos[10*(a + b
*x)]/(160*b))/32
```

fricas [A] time = 0.46, size = 36, normalized size = 0.78

$$-\frac{6\cos(bx+a)^{10} - 15\cos(bx+a)^8 + 10\cos(bx+a)^6}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(6*cos(b*x + a)^10 - 15*cos(b*x + a)^8 + 10*cos(b*x + a)^6)/b
```

giac [A] time = 0.22, size = 43, normalized size = 0.93

$$-\frac{\cos(10\,bx+10\,a)}{5120\,b} + \frac{5\cos(6\,bx+6\,a)}{3072\,b} - \frac{5\cos(2\,bx+2\,a)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/5120*cos(10*b*x + 10*a)/b + 5/3072*cos(6*b*x + 6*a)/b - 5/512*cos(2*b*x
+ 2*a)/b
```

maple [A] time = 0.02, size = 52, normalized size = 1.13

$$\frac{-\frac{\left(\cos^{6}(bx+a)\right)\left(\sin^{4}(bx+a)\right)}{10}-\frac{\left(\cos^{6}(bx+a)\right)\left(\sin^{2}(bx+a)\right)}{20}-\frac{\left(\cos^{6}(bx+a)\right)}{60}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/10*cos(b*x+a)^6*sin(b*x+a)^4-1/20*cos(b*x+a)^6*sin(b*x+a)^2-1/60*co
s(b*x+a)^6)
```

maxima [A] time = 0.31, size = 36, normalized size = 0.78

$$\frac{6\,\sin\left(bx+a\right)^{10}-15\,\sin\left(bx+a\right)^{8}+10\,\sin\left(bx+a\right)^{6}}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/60*(6*sin(b*x + a)^10 - 15*sin(b*x + a)^8 + 10*sin(b*x + a)^6)/b
```

mupad [B] time = 0.48, size = 36, normalized size = 0.78

$$-\frac{\frac{\cos(a+b\,x)^{10}}{10}-\frac{\cos(a+b\,x)^{8}}{4}+\frac{\cos(a+b\,x)^{6}}{6}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5*sin(a + b*x)^5,x)
```

```
[Out] -(cos(a + b*x)^6/6 - cos(a + b*x)^8/4 + cos(a + b*x)^10/10)/b
```

sympy [A] time = 30.08, size = 65, normalized size = 1.41

$$\begin{cases} -\frac{\sin^{4}(a+bx)\cos^{6}(a+bx)}{6b}-\frac{\sin^{2}(a+bx)\cos^{8}(a+bx)}{12b}-\frac{\cos^{10}(a+bx)}{60b} & \text{for } b\neq 0 \\ x\sin^{5}(a)\cos^{5}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**6/(6*b) - sin(a + b*x)**2*cos(a +
 b*x)**8/(12*b) - cos(a + b*x)**10/(60*b), Ne(b, 0)), (x*sin(a)**5*cos(a)**
5, True))
```

3.101 $\int \cos^4(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos^9(a+bx)}{9b}+\frac{2\cos^7(a+bx)}{7b}-\frac{\cos^5(a+bx)}{5b}$$

```
[Out] -1/5*cos(b*x+a)^5/b+2/7*cos(b*x+a)^7/b-1/9*cos(b*x+a)^9/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 270}

$$-\frac{\cos^9(a+bx)}{9b}+\frac{2\cos^7(a+bx)}{7b}-\frac{\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Sin[a + b*x]^5,x]
```

```
[Out] -Cos[a + b*x]^5/(5*b) + (2*Cos[a + b*x]^7)/(7*b) - Cos[a + b*x]^9/(9*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\int \cos^4(a+bx)\sin^5(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^4\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\cos^5(a+bx)}{5b}+\frac{2\cos^7(a+bx)}{7b}-\frac{\cos^9(a+bx)}{9b}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 37, normalized size = 0.80

$$\frac{\cos^5(a+bx)(220\cos(2(a+bx))-35\cos(4(a+bx))-249)}{2520b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Sin[a + b*x]^5,x]
```

```
[Out] (Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(2520*
b)
```

fricas [A] time = 0.43, size = 36, normalized size = 0.78

$$-\frac{35\cos(bx+a)^9-90\cos(bx+a)^7+63\cos(bx+a)^5}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

giac [A] time = 0.25, size = 68, normalized size = 1.48

$$-\frac{\cos(9\,bx+9\,a)}{2304\,b}+\frac{\cos(7\,bx+7\,a)}{1792\,b}+\frac{\cos(5\,bx+5\,a)}{320\,b}-\frac{\cos(3\,bx+3\,a)}{192\,b}-\frac{3\cos(bx+a)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/2304*cos(9*b*x + 9*a)/b + 1/1792*cos(7*b*x + 7*a)/b + 1/320*cos(5*b*x +
5*a)/b - 1/192*cos(3*b*x + 3*a)/b - 3/128*cos(b*x + a)/b
```

maple [A] time = 0.02, size = 52, normalized size = 1.13

$$\frac{-\frac{\left(\cos^5(bx+a)\right)\left(\sin^4(bx+a)\right)}{9}-\frac{4\left(\cos^5(bx+a)\right)\left(\sin^2(bx+a)\right)}{63}-\frac{8\left(\cos^5(bx+a)\right)}{315}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/9*cos(b*x+a)^5*sin(b*x+a)^4-4/63*cos(b*x+a)^5*sin(b*x+a)^2-8/315*co
s(b*x+a)^5)
```

maxima [A] time = 0.36, size = 36, normalized size = 0.78

$$-\frac{35\,\cos\left(bx+a\right)^9-90\,\cos\left(bx+a\right)^7+63\,\cos\left(bx+a\right)^5}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

mupad [B] time = 0.38, size = 36, normalized size = 0.78

$$-\frac{35\cos\left(a+b\,x\right)^9-90\cos\left(a+b\,x\right)^7+63\cos\left(a+b\,x\right)^5}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4*sin(a + b*x)^5,x)
```

```
[Out] -(63*cos(a + b*x)^5 - 90*cos(a + b*x)^7 + 35*cos(a + b*x)^9)/(315*b)
```

sympy [A] time = 20.09, size = 68, normalized size = 1.48

$$\begin{cases} -\frac{\sin^4\left(a+bx\right)\cos^5\left(a+bx\right)}{5b}-\frac{4\sin^2\left(a+bx\right)\cos^7\left(a+bx\right)}{35b}-\frac{8\cos^9\left(a+bx\right)}{315b} & \text{for } b\neq 0\\ x\sin^5\left(a\right)\cos^4\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**5/(5*b) - 4*sin(a + b*x)**2*cos(a
 + b*x)**7/(35*b) - 8*cos(a + b*x)**9/(315*b), Ne(b, 0)), (x*sin(a)**5*cos(
a)**4, True))
```

3.102 $\int \cos^3(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\sin^6(a+bx)}{6b}-\frac{\sin^8(a+bx)}{8b}$$

```
[Out] 1/6*sin(b*x+a)^6/b-1/8*sin(b*x+a)^8/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 14}

$$\frac{\sin^6(a+bx)}{6b}-\frac{\sin^8(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]^6/(6*b) - Sin[a + b*x]^8/(8*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(a+bx)\sin^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^5\left(1-x^2\right)\,dx, x, \sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(x^5-x^7\right)\,dx, x, \sin(a+bx)\right)}{b}\\
&= \frac{\sin^6(a+bx)}{6b}-\frac{\sin^8(a+bx)}{8b}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 48, normalized size = 1.55

$$\frac{-72\cos(2(a+bx))+12\cos(4(a+bx))+8\cos(6(a+bx))-3\cos(8(a+bx))}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[a + b*x]^5,x]
```

```
[Out] (-72*Cos[2*(a + b*x)] + 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] - 3*Cos[8*
(a + b*x)])/(3072*b)
```

fricas [A] time = 0.43, size = 36, normalized size = 1.16

$$-\frac{3\cos\left(bx+a\right)^{8}-8\cos\left(bx+a\right)^{6}+6\cos\left(bx+a\right)^{4}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(3*cos(b*x + a)^8 - 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4)/b
```

giac [A] time = 0.52, size = 26, normalized size = 0.84

$$-\frac{3\sin\left(bx+a\right)^{8}-4\sin\left(bx+a\right)^{6}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/24*(3*sin(b*x + a)^8 - 4*sin(b*x + a)^6)/b
```

maple [A] time = 0.02, size = 52, normalized size = 1.68

$$\frac{-\frac{\left(\cos^{4}(bx+a)\right)\left(\sin^{4}(bx+a)\right)}{8}-\frac{\left(\cos^{4}(bx+a)\right)\left(\sin^{2}(bx+a)\right)}{12}-\frac{\left(\cos^{4}(bx+a)\right)}{24}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/8*cos(b*x+a)^4*sin(b*x+a)^4-1/12*cos(b*x+a)^4*sin(b*x+a)^2-1/24*cos
(b*x+a)^4)
```

maxima [A] time = 0.38, size = 26, normalized size = 0.84

$$-\frac{3\sin\left(bx+a\right)^{8}-4\sin\left(bx+a\right)^{6}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(3*sin(b*x + a)^8 - 4*sin(b*x + a)^6)/b
```

mupad [**B**] time = 0.38, size = 26, normalized size = 0.84

$$\frac{4\sin{(a+b\,x)}^6-3\sin{(a+b\,x)}^8}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^5,x)
```

```
[Out] (4*sin(a + b*x)^6 - 3*sin(a + b*x)^8)/(24*b)
```

sympy [**A**] time = 12.76, size = 65, normalized size = 2.10

$$\begin{cases} -\frac{\sin^4{(a+bx)}\cos^4{(a+bx)}}{4b}-\frac{\sin^2{(a+bx)}\cos^6{(a+bx)}}{6b}-\frac{\cos^8{(a+bx)}}{24b} & \text{for } b\neq 0 \\ x\sin^5{(a)}\cos^3{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**4/(4*b) - sin(a + b*x)**2*cos(a +
 b*x)**6/(6*b) - cos(a + b*x)**8/(24*b), Ne(b, 0)), (x*sin(a)**5*cos(a)**3,
 True))
```

3.103 $\int \cos^2(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos^7(a+bx)}{7b}+\frac{2\cos^5(a+bx)}{5b}-\frac{\cos^3(a+bx)}{3b}$$

```
[Out] -1/3*cos(b*x+a)^3/b+2/5*cos(b*x+a)^5/b-1/7*cos(b*x+a)^7/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2565, 270}

$$-\frac{\cos^7(a+bx)}{7b}+\frac{2\cos^5(a+bx)}{5b}-\frac{\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^5,x]
```

```
[Out] -Cos[a + b*x]^3/(3*b) + (2*Cos[a + b*x]^5)/(5*b) - Cos[a + b*x]^7/(7*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^5(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{\cos^3(a+bx)}{3b}+\frac{2\cos^5(a+bx)}{5b}-\frac{\cos^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 37, normalized size = 0.80

$$\frac{\cos^3(a+bx)(108\cos(2(a+bx))-15\cos(4(a+bx))-157)}{840b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^5,x]
```

```
[Out] (Cos[a + b*x]^3*(-157 + 108*Cos[2*(a + b*x)] - 15*Cos[4*(a + b*x)]))/(840*b
)
```

fricas [A] time = 0.44, size = 36, normalized size = 0.78

$$-\frac{15\cos(bx+a)^7-42\cos(bx+a)^5+35\cos(bx+a)^3}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

giac [A] time = 0.21, size = 54, normalized size = 1.17

$$-\frac{\cos(7\,bx+7\,a)}{448\,b}+\frac{3\cos(5\,bx+5\,a)}{320\,b}-\frac{\cos(3\,bx+3\,a)}{192\,b}-\frac{5\cos(bx+a)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/448*cos(7*b*x + 7*a)/b + 3/320*cos(5*b*x + 5*a)/b - 1/192*cos(3*b*x + 3*
a)/b - 5/64*cos(b*x + a)/b
```

maple [A] time = 0.02, size = 52, normalized size = 1.13

$$\frac{-\frac{\left(\cos^3(bx+a)\right)\left(\sin^4(bx+a)\right)}{7}-\frac{4\left(\cos^3(bx+a)\right)\left(\sin^2(bx+a)\right)}{35}-\frac{8\left(\cos^3(bx+a)\right)}{105}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(-1/7*cos(b*x+a)^3*sin(b*x+a)^4-4/35*cos(b*x+a)^3*sin(b*x+a)^2-8/105*co
s(b*x+a)^3)
```

maxima [A] time = 0.44, size = 36, normalized size = 0.78

$$-\frac{15\ \cos\left(bx+a\right)^{7}-42\ \cos\left(bx+a\right)^{5}+35\ \cos\left(bx+a\right)^{3}}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

mupad [B] time = 0.37, size = 36, normalized size = 0.78

$$-\frac{15\cos\left(a+b\,x\right)^{7}-42\cos\left(a+b\,x\right)^{5}+35\cos\left(a+b\,x\right)^{3}}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^5,x)
```

```
[Out] -(35*cos(a + b*x)^3 - 42*cos(a + b*x)^5 + 15*cos(a + b*x)^7)/(105*b)
```

sympy [A] time = 7.50, size = 68, normalized size = 1.48

$$\begin{cases} -\frac{\sin^{4}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{3b}-\frac{4\sin^{2}\left(a+bx\right)\cos^{5}\left(a+bx\right)}{15b}-\frac{8\cos^{7}\left(a+bx\right)}{105b} & \text{for}\ b\neq 0 \\ x\sin^{5}\left(a\right)\cos^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-sin(a + b*x)**4*cos(a + b*x)**3/(3*b) - 4*sin(a + b*x)**2*cos(a
 + b*x)**5/(15*b) - 8*cos(a + b*x)**7/(105*b), Ne(b, 0)), (x*sin(a)**5*cos(
a)**2, True))
```

3.104 $\int \cos(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^6(a+bx)}{6b}$$

[Out] 1/6*sin(b*x+a)^6/b

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2564, 30}

$$\frac{\sin^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]*Sin[a + b*x]^5,x]

[Out] Sin[a + b*x]^6/(6*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^5\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{\sin^6(a+bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sin^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]^6/(6*b)
```

fricas [B] time = 0.43, size = 34, normalized size = 2.27

$$-\frac{\cos(bx+a)^6 - 3\cos(bx+a)^4 + 3\cos(bx+a)^2}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/6*(cos(b*x + a)^6 - 3*cos(b*x + a)^4 + 3*cos(b*x + a)^2)/b
```

giac [A] time = 0.17, size = 13, normalized size = 0.87

$$\frac{\sin(bx+a)^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/6*sin(b*x + a)^6/b
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\frac{\sin^6(bx+a)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^5,x)
```

```
[Out] 1/6*sin(b*x+a)^6/b
```

maxima [A] time = 0.33, size = 13, normalized size = 0.87

$$\frac{\sin(bx+a)^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/6*sin(b*x + a)^6/b
```

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$\frac{\sin\left(a+b\,x\right)^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^5,x)
```

```
[Out] sin(a + b*x)^6/(6*b)
```

sympy [A] time = 4.04, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sin^{6}{\left(a+bx \right)}}{6b} & \text{for}\: b \neq 0 \\ x \sin^{5}{\left(a \right)} \cos{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((sin(a + b*x)**6/(6*b), Ne(b, 0)), (x*sin(a)**5*cos(a), True))
```

3.105 $\int \sin^4(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=40

$$-\frac{\cos^4(a+bx)}{4b}+\frac{\cos^2(a+bx)}{b}-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] cos(b*x+a)^2/b-1/4*cos(b*x+a)^4/b-ln(cos(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2590, 266, 43}

$$-\frac{\cos^4(a+bx)}{4b}+\frac{\cos^2(a+bx)}{b}-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4*Tan[a + b*x],x]
```

```
[Out] Cos[a + b*x]^2/b - Cos[a + b*x]^4/(4*b) - Log[Cos[a + b*x]]/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \sin^4(a+bx)\tan(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, \cos^2(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)\,dx, x, \cos^2(a+bx)\right)}{2b}$$
$$= \frac{\cos^2(a+bx)}{b} - \frac{\cos^4(a+bx)}{4b} - \frac{\log(\cos(a+bx))}{b}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.88

$$-\frac{\frac{1}{4}\cos^4(a+bx) - \cos^2(a+bx) + \log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4*Tan[a + b*x],x]
[Out] -((-Cos[a + b*x]^2 + Cos[a + b*x]^4/4 + Log[Cos[a + b*x]])/b)
```

fricas [A] time = 0.53, size = 35, normalized size = 0.88

$$-\frac{\cos\left(bx+a\right)^4 - 4\cos\left(bx+a\right)^2 + 4\log\left(-\cos\left(bx+a\right)\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/4*(cos(b*x + a)^4 - 4*cos(b*x + a)^2 + 4*log(-cos(b*x + a)))/b
```

giac [B] time = 0.33, size = 226, normalized size = 5.65

$$-\frac{\frac{3\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)^2-\frac{20\,(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{20\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+44}{\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-2\right)^2} - 2\log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+2\right|\right)+2\log\left(\left|-\frac{\cos(bx+a}{\cos(bx+a}\right.\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*((3*((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b
*x + a) + 1))^2 - 20*(cos(b*x + a) + 1)/(cos(b*x + a) - 1) - 20*(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 44)/((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (
cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2)^2 - 2*log(abs(-(cos(b*x + a) + 1)
/(cos(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 2)) + 2*log(a
bs(-(cos(b*x + a) + 1)/(cos(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a
) + 1) - 2)))/b
```

maple [A] time = 0.03, size = 40, normalized size = 1.00

$$-\frac{\sin^4(bx+a)}{4b}-\frac{\sin^2(bx+a)}{2b}-\frac{\ln(\cos(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^5,x)
```

```
[Out] -1/4*sin(b*x+a)^4/b-1/2*sin(b*x+a)^2/b-ln(cos(b*x+a))/b
```

maxima [A] time = 0.37, size = 37, normalized size = 0.92

$$-\frac{\sin(bx+a)^4+2\sin(bx+a)^2+2\log\left(\sin(bx+a)^2-1\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*(sin(b*x + a)^4 + 2*sin(b*x + a)^2 + 2*log(sin(b*x + a)^2 - 1))/b
```

mupad [B] time = 0.54, size = 53, normalized size = 1.32

$$\frac{\ln\left(\tan(a+bx)^2+1\right)}{2b}+\frac{\tan(a+bx)^2+\frac{3}{4}}{b\left(\tan(a+bx)^4+2\tan(a+bx)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x),x)
```

```
[Out] log(tan(a + b*x)^2 + 1)/(2*b) + (tan(a + b*x)^2 + 3/4)/(b*(2*tan(a + b*x)^2
 + tan(a + b*x)^4 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.106 $\int \sin^3(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=37

$$-\frac{\cos^3(a+bx)}{3b}+\frac{2\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

```
[Out] 2*cos(b*x+a)/b-1/3*cos(b*x+a)^3/b+sec(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$-\frac{\cos^3(a+bx)}{3b}+\frac{2\cos(a+bx)}{b}+\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Tan[a + b*x]^2,x]
```

```
[Out] (2*Cos[a + b*x])/b - Cos[a + b*x]^3/(3*b) + Sec[a + b*x]/b
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sin^3(a+bx)\tan^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)\,dx,x,\cos(a+bx)\right)}{b}\\
&= \frac{2\cos(a+bx)}{b}-\frac{\cos^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.05

$$\frac{7\cos(a+bx)}{4b}-\frac{\cos(3(a+bx))}{12b}+\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Tan[a + b*x]^2,x]
[Out] (7*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(12*b) + Sec[a + b*x]/b
```

fricas [A] time = 0.49, size = 33, normalized size = 0.89

$$-\frac{\cos\left(bx+a\right)^{4}-6\,\cos\left(bx+a\right)^{2}-3}{3\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^5,x, algorithm="fricas")
[Out] -1/3*(cos(b*x + a)^4 - 6*cos(b*x + a)^2 - 3)/(b*cos(b*x + a))
```

giac [B] time = 0.31, size = 99, normalized size = 2.68

$$\frac{2\left(\frac{3}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1}+\frac{\frac{12\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{3\,(\cos(bx+a)-1)^{2}}{(\cos(bx+a)+1)^{2}}-5}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{3}}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^5,x, algorithm="giac")
[Out] 2/3*(3/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + (12*(cos(b*x + a) - 1)
/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 - 5)/((co
s(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^3)/b
```

maple [A] time = 0.03, size = 50, normalized size = 1.35

$$\frac{\frac{\sin^{6}(bx+a)}{\cos(bx+a)}+\left(\frac{8}{3}+\sin^{4}\left(bx+a\right)+\frac{4\left(\sin^{2}(bx+a)\right)}{3}\right)\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(sin(b*x+a)^6/cos(b*x+a)+(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)
)
```

maxima [A] time = 0.32, size = 32, normalized size = 0.86

$$-\frac{\cos\left(bx+a\right)^{3}-\frac{3}{\cos(bx+a)}-6\,\cos\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/3*(cos(b*x + a)^3 - 3/cos(b*x + a) - 6*cos(b*x + a))/b
```

mupad [B] time = 0.50, size = 31, normalized size = 0.84

$$-\frac{\left(\cos\left(a+b\,x\right)+1\right)^{3}\,\left(\cos\left(a+b\,x\right)-3\right)}{3\,b\,\cos\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^2,x)
```

```
[Out] -((cos(a + b*x) + 1)^3*(cos(a + b*x) - 3))/(3*b*cos(a + b*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.107 $\int \sin^2(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\cos^2(a+bx)}{2b}+\frac{\sec^2(a+bx)}{2b}+\frac{2\log(\cos(a+bx))}{b}$$

```
[Out] -1/2*cos(b*x+a)^2/b+2*ln(cos(b*x+a))/b+1/2*sec(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$-\frac{\cos^2(a+bx)}{2b}+\frac{\sec^2(a+bx)}{2b}+\frac{2\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Tan[a + b*x]^3,x]
```

```
[Out] -Cos[a + b*x]^2/(2*b) + (2*Log[Cos[a + b*x]])/b + Sec[a + b*x]^2/(2*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \sin^2(a+bx)\tan^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \cos^2(a+bx)\right)}{2b}$$
$$= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx, x, \cos^2(a+bx)\right)}{2b}$$
$$= -\frac{\cos^2(a+bx)}{2b} + \frac{2\log(\cos(a+bx))}{b} + \frac{\sec^2(a+bx)}{2b}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.77

$$\frac{\sin^2(a+bx)+\sec^2(a+bx)+4\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Tan[a + b*x]^3,x]
```

```
[Out] (4*Log[Cos[a + b*x]] + Sec[a + b*x]^2 + Sin[a + b*x]^2)/(2*b)
```

fricas [A] time = 0.50, size = 54, normalized size = 1.26

$$-\frac{2\cos(bx+a)^4 - 8\cos(bx+a)^2\log(-\cos(bx+a)) - \cos(bx+a)^2 - 2}{4\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(b*x + a)^4 - 8*cos(b*x + a)^2*log(-cos(b*x + a)) - cos(b*x + a)
^2 - 2)/(b*cos(b*x + a)^2)
```

giac [B] time = 0.30, size = 182, normalized size = 4.23

$$-\frac{\frac{4\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)^2-4} + \log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+2\right|\right) - \log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-2\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -(4*((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*x +
a) + 1))/(((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(
b*x + a) + 1))^2 - 4) + log(abs(-(cos(b*x + a) + 1)/(cos(b*x + a) - 1) - (c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + 2)) - log(abs(-(cos(b*x + a) + 1)/(co
s(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2)))/b
```

maple [A] time = 0.03, size = 60, normalized size = 1.40

$$\frac{\sin^6(bx+a)}{2b\cos(bx+a)^2}+\frac{\sin^4(bx+a)}{2b}+\frac{\sin^2(bx+a)}{b}+\frac{2\ln(\cos(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^5,x)
```

```
[Out] 1/2/b*sin(b*x+a)^6/cos(b*x+a)^2+1/2*sin(b*x+a)^4/b+sin(b*x+a)^2/b+2*ln(cos(
b*x+a))/b
```

maxima [A] time = 0.32, size = 41, normalized size = 0.95

$$\frac{\sin(bx+a)^2-\frac{1}{\sin(bx+a)^2-1}+2\log\left(\sin(bx+a)^2-1\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/2*(sin(b*x + a)^2 - 1/(sin(b*x + a)^2 - 1) + 2*log(sin(b*x + a)^2 - 1))/b
```

mupad [B] time = 0.40, size = 37, normalized size = 0.86

$$-\frac{\ln\left(\tan(a+b\,x)^2+1\right)+\frac{\cos(a+b\,x)^2}{2}-\frac{\tan(a+b\,x)^2}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^3,x)
```

```
[Out] -(log(tan(a + b*x)^2 + 1) + cos(a + b*x)^2/2 - tan(a + b*x)^2/2)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.108 $\int \sin(a+bx)\tan^4(a+bx)\,dx$

Optimal. Leaf size=38

$$-\frac{\cos(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}-\frac{2\sec(a+bx)}{b}$$

```
[Out] -cos(b*x+a)/b-2*sec(b*x+a)/b+1/3*sec(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 270}

$$-\frac{\cos(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}-\frac{2\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[a + b*x]^4,x]
```

```
[Out] -(Cos[a + b*x]/b) - (2*Sec[a + b*x])/b + Sec[a + b*x]^3/(3*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \sin(a+bx)\tan^4(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^4}\,dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx, x, \cos(a+bx)\right)}{b}\\
&= -\frac{\cos(a+bx)}{b}-\frac{2\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$-\frac{\cos(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}-\frac{2\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x]^4,x]
```

```
[Out] -(Cos[a + b*x]/b) - (2*Sec[a + b*x])/b + Sec[a + b*x]^3/(3*b)
```

fricas [A] time = 0.42, size = 35, normalized size = 0.92

$$-\frac{3\cos(bx+a)^4+6\cos(bx+a)^2-1}{3\,b\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(b*x + a)^4 + 6*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3)
```

giac [B] time = 0.24, size = 100, normalized size = 2.63

$$\frac{2\left(\frac{3}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1}-\frac{\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+5}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 2/3*(3/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - (12*(cos(b*x + a) - 1)
/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 5)/((co
s(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3)/b
```

maple [A] time = 0.03, size = 70, normalized size = 1.84

$$\frac{\frac{\sin^6(bx+a)}{3\cos(bx+a)^3}-\frac{\sin^6(bx+a)}{\cos(bx+a)}-\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/3*sin(b*x+a)^6/cos(b*x+a)^3-sin(b*x+a)^6/cos(b*x+a)-(8/3+sin(b*x+a)^
4+4/3*sin(b*x+a)^2)*cos(b*x+a))
```

maxima [A] time = 0.32, size = 35, normalized size = 0.92

$$-\frac{\frac{6 \cos(bx+a)^2-1}{\cos(bx+a)^3} + 3 \cos(bx+a)}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*sin(b*x+a)^5,x, algorithm="maxima")
[Out] -1/3*((6*cos(b*x + a)^2 - 1)/cos(b*x + a)^3 + 3*cos(b*x + a))/b
```

mupad [B] time = 0.53, size = 35, normalized size = 0.92

$$-\frac{3 \cos(a+b x)^4 + 6 \cos(a+b x)^2 - 1}{3 b \cos(a+b x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^4,x)
[Out] -(6*cos(a + b*x)^2 + 3*cos(a + b*x)^4 - 1)/(3*b*cos(a + b*x)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*sin(b*x+a)**5,x)
[Out] Timed out
```

3.109 $\int \tan^5(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\tan^4(a+bx)}{4b}-\frac{\tan^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] -ln(cos(b*x+a))/b-1/2*tan(b*x+a)^2/b+1/4*tan(b*x+a)^4/b
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 3475}

$$\frac{\tan^4(a+bx)}{4b}-\frac{\tan^2(a+bx)}{2b}-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x]^5,x]
```

```
[Out] -(Log[Cos[a + b*x]]/b) - Tan[a + b*x]^2/(2*b) + Tan[a + b*x]^4/(4*b)
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \tan^5(a+bx)\,dx &= \frac{\tan^4(a+bx)}{4b}-\int \tan^3(a+bx)\,dx\\
&= -\frac{\tan^2(a+bx)}{2b}+\frac{\tan^4(a+bx)}{4b}+\int \tan(a+bx)\,dx\\
&= -\frac{\log(\cos(a+bx))}{b}-\frac{\tan^2(a+bx)}{2b}+\frac{\tan^4(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 37, normalized size = 0.86

$$-\frac{-\tan^4(a+bx)+2\tan^2(a+bx)+4\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x]^5,x]
```

```
[Out] -1/4*(4*Log[Cos[a + b*x]] + 2*Tan[a + b*x]^2 - Tan[a + b*x]^4)/b
```

fricas [A] time = 0.44, size = 44, normalized size = 1.02

$$-\frac{4\cos\left(bx+a\right)^4\log\left(-\cos\left(bx+a\right)\right)+4\cos\left(bx+a\right)^2-1}{4\,b\cos\left(bx+a\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*cos(b*x + a)^4*log(-cos(b*x + a)) + 4*cos(b*x + a)^2 - 1)/(b*cos(b*
x + a)^4)
```

giac [B] time = 0.25, size = 226, normalized size = 5.26

$$\frac{\frac{3\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)^2+\frac{20\left(\cos(bx+a)+1\right)}{\cos(bx+a)-1}+\frac{20\left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}+44}{\left(\frac{\cos(bx+a)+1}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+2\right)^2}+2\log\left(\left|-\frac{\cos(bx+a)+1}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+2\right|\right)-2\log\left(\left|-\frac{\cos(bx+a)}{\cos(bx+a)}\right.\right.}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/4*((3*((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*
x + a) + 1))^2 + 20*(cos(b*x + a) + 1)/(cos(b*x + a) - 1) + 20*(cos(b*x + a
) - 1)/(cos(b*x + a) + 1) + 44)/((cos(b*x + a) + 1)/(cos(b*x + a) - 1) + (c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + 2)^2 + 2*log(abs(-(cos(b*x + a) + 1)/
(cos(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 2)) - 2*log(ab
s(-(cos(b*x + a) + 1)/(cos(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a)
 + 1) - 2)))/b
```

maple [A] time = 0.04, size = 40, normalized size = 0.93

$$-\frac{\ln\left(\cos\left(bx+a\right)\right)}{b}-\frac{\tan^2\left(bx+a\right)}{2b}+\frac{\tan^4\left(bx+a\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5*sin(b*x+a)^5,x)
```

```
[Out] -ln(cos(b*x+a))/b-1/2*tan(b*x+a)^2/b+1/4*tan(b*x+a)^4/b
```

maxima [A] time = 0.67, size = 54, normalized size = 1.26

$$\frac{\frac{4 \sin(bx+a)^2-3}{\sin(bx+a)^4-2 \sin(bx+a)^2+1} - 2 \log\left(\sin(bx+a)^2 - 1\right)}{4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/4*((4*sin(b*x + a)^2 - 3)/(sin(b*x + a)^4 - 2*sin(b*x + a)^2 + 1) - 2*log
(sin(b*x + a)^2 - 1))/b
```

mupad [B] time = 0.40, size = 38, normalized size = 0.88

$$\frac{\frac{\ln\left(\tan(a+b x)^2+1\right)}{2} - \frac{\tan(a+b x)^2}{2} + \frac{\tan(a+b x)^4}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^5,x)
```

```
[Out] (log(tan(a + b*x)^2 + 1)/2 - tan(a + b*x)^2/2 + tan(a + b*x)^4/4)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.110 $\int \sec(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\sec^5(a+bx)}{5b}-\frac{2\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}$$

```
[Out] sec(b*x+a)/b-2/3*sec(b*x+a)^3/b+1/5*sec(b*x+a)^5/b
```

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 194}

$$\frac{\sec^5(a+bx)}{5b}-\frac{2\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]/b - (2*Sec[a + b*x]^3)/(3*b) + Sec[a + b*x]^5/(5*b)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \sec(a+bx)\right)}{b}\\ &= \frac{\sec(a+bx)}{b}-\frac{2\sec^3(a+bx)}{3b}+\frac{\sec^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.00

$$\frac{\sec^5(a+bx)}{5b}-\frac{2\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*Tan[a + b*x]^5,x]
[Out] Sec[a + b*x]/b - (2*Sec[a + b*x]^3)/(3*b) + Sec[a + b*x]^5/(5*b)
```

fricas [A] time = 0.42, size = 35, normalized size = 0.85

$$\frac{15\,\cos(bx+a)^4-10\,\cos(bx+a)^2+3}{15\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/15*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^5)
```

giac [A] time = 0.23, size = 72, normalized size = 1.76

$$\frac{16\left(\frac{5(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{10(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)}{15\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^5,x, algorithm="giac")
[Out] 16/15*(5*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 10*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^5)
```

maple [B] time = 0.03, size = 88, normalized size = 2.15

$$\frac{\frac{\sin^6(bx+a)}{5\cos(bx+a)^5}-\frac{\sin^6(bx+a)}{15\cos(bx+a)^3}+\frac{\sin^6(bx+a)}{5\cos(bx+a)}+\frac{\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)\cos(bx+a)}{5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/5*sin(b*x+a)^6/cos(b*x+a)^5-1/15*sin(b*x+a)^6/cos(b*x+a)^3+1/5*sin(b
*x+a)^6/cos(b*x+a)+1/5*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))
```

maxima [A] time = 0.30, size = 35, normalized size = 0.85

$$\frac{15 \cos\left(bx+a\right)^{4}-10 \cos\left(bx+a\right)^{2}+3}{15 \, b \cos\left(bx+a\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/15*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^5)
```

mupad [B] time = 0.54, size = 35, normalized size = 0.85

$$\frac{15\cos\left(a+b\,x\right)^4-10\cos\left(a+b\,x\right)^2+3}{15\,b\cos\left(a+b\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^6,x)
```

```
[Out] (15*cos(a + b*x)^4 - 10*cos(a + b*x)^2 + 3)/(15*b*cos(a + b*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.111 $\int \sec^2(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{\tan^6(a+bx)}{6b}$$

[Out] 1/6*tan(b*x+a)^6/b

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$\frac{\tan^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b*x]^2*Tan[a + b*x]^5,x]

[Out] Tan[a + b*x]^6/(6*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^5\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\tan^6(a+bx)}{6b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{\tan^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*Tan[a + b*x]^5,x]
```

```
[Out] Tan[a + b*x]^6/(6*b)
```

fricas [B] time = 0.42, size = 35, normalized size = 2.33

$$\frac{3 \cos\left(bx+a\right)^{4} - 3 \cos\left(bx+a\right)^{2} + 1}{6\, b \cos\left(bx+a\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(3*cos(b*x + a)^4 - 3*cos(b*x + a)^2 + 1)/(b*cos(b*x + a)^6)
```

giac [B] time = 0.37, size = 48, normalized size = 3.20

$$-\frac{32 \left(\cos\left(bx+a\right) - 1\right)^{3}}{3\, b \left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + 1\right)^{6} \left(\cos\left(bx+a\right) + 1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -32/3*(cos(b*x + a) - 1)^3/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^6
*(cos(b*x + a) + 1)^3)
```

maple [A] time = 0.04, size = 22, normalized size = 1.47

$$\frac{\sin^{6}\left(bx+a\right)}{6b \cos\left(bx+a\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^7*sin(b*x+a)^5,x)
```

```
[Out] 1/6/b*sin(b*x+a)^6/cos(b*x+a)^6
```

maxima [B] time = 0.37, size = 59, normalized size = 3.93

$$-\frac{3 \sin\left(bx+a\right)^{4} - 3 \sin\left(bx+a\right)^{2} + 1}{6 \left(\sin\left(bx+a\right)^{6} - 3 \sin\left(bx+a\right)^{4} + 3 \sin\left(bx+a\right)^{2} - 1\right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/6*(3*sin(b*x + a)^4 - 3*sin(b*x + a)^2 + 1)/((sin(b*x + a)^6 - 3*sin(b*x
 + a)^4 + 3*sin(b*x + a)^2 - 1)*b)
```

mupad [B] time = 0.42, size = 13, normalized size = 0.87

$$\frac{\tan\left(a+b\,x\right)^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^7,x)
```

```
[Out] tan(a + b*x)^6/(6*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**7*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.112 $\int \sec^3(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sec^7(a+bx)}{7b}-\frac{2\sec^5(a+bx)}{5b}+\frac{\sec^3(a+bx)}{3b}$$

```
[Out] 1/3*sec(b*x+a)^3/b-2/5*sec(b*x+a)^5/b+1/7*sec(b*x+a)^7/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 270}

$$\frac{\sec^7(a+bx)}{7b}-\frac{2\sec^5(a+bx)}{5b}+\frac{\sec^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^3/(3*b) - (2*Sec[a + b*x]^5)/(5*b) + Sec[a + b*x]^7/(7*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\int \sec^3(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,\sec(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\sec(a+bx)\right)}{b}\\
&= \frac{\sec^3(a+bx)}{3b}-\frac{2\sec^5(a+bx)}{5b}+\frac{\sec^7(a+bx)}{7b}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{\sec^7(a+bx)}{7b}-\frac{2\sec^5(a+bx)}{5b}+\frac{\sec^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*Tan[a + b*x]^5,x]
[Out] Sec[a + b*x]^3/(3*b) - (2*Sec[a + b*x]^5)/(5*b) + Sec[a + b*x]^7/(7*b)
```

fricas [A] time = 0.43, size = 35, normalized size = 0.76

$$\frac{35\cos\left(bx+a\right)^4-42\cos\left(bx+a\right)^2+15}{105\,b\cos\left(bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/105*(35*cos(b*x + a)^4 - 42*cos(b*x + a)^2 + 15)/(b*cos(b*x + a)^7)
```

giac [B] time = 0.26, size = 116, normalized size = 2.52

$$\frac{16\left(\frac{7(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{21(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-\frac{35(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{70(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+1\right)}{105\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^5,x, algorithm="giac")
[Out] 16/105*(7*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 21*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 - 35*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 70*(co
s(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x
 + a) + 1) + 1)^7)
```

maple [B] time = 0.04, size = 106, normalized size = 2.30

$$\frac{\frac{\sin^6(bx+a)}{7\cos(bx+a)^7}+\frac{\sin^6(bx+a)}{35\cos(bx+a)^5}-\frac{\sin^6(bx+a)}{105\cos(bx+a)^3}+\frac{\sin^6(bx+a)}{35\cos(bx+a)}+\frac{\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)\cos(bx+a)}{35}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^8*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/7*sin(b*x+a)^6/cos(b*x+a)^7+1/35*sin(b*x+a)^6/cos(b*x+a)^5-1/105*sin
(b*x+a)^6/cos(b*x+a)^3+1/35*sin(b*x+a)^6/cos(b*x+a)+1/35*(8/3+sin(b*x+a)^4+
4/3*sin(b*x+a)^2)*cos(b*x+a))
```

maxima [**A**] time = 0.75, size = 35, normalized size = 0.76

$$\frac{35\cos\left(bx+a\right)^{4}-42\cos\left(bx+a\right)^{2}+15}{105\,b\cos\left(bx+a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^8*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/105*(35*cos(b*x + a)^4 - 42*cos(b*x + a)^2 + 15)/(b*cos(b*x + a)^7)
```

mupad [**B**] time = 0.59, size = 35, normalized size = 0.76

$$\frac{35\cos\left(a+b\,x\right)^{4}-42\cos\left(a+b\,x\right)^{2}+15}{105\,b\cos\left(a+b\,x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^8,x)
```

```
[Out] (35*cos(a + b*x)^4 - 42*cos(a + b*x)^2 + 15)/(105*b*cos(a + b*x)^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**8*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.113 $\int \sec^4(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=31

$$\frac{\tan^8(a+bx)}{8b}+\frac{\tan^6(a+bx)}{6b}$$

```
[Out] 1/6*tan(b*x+a)^6/b+1/8*tan(b*x+a)^8/b
```

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 14}

$$\frac{\tan^8(a+bx)}{8b}+\frac{\tan^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*Tan[a + b*x]^5,x]
```

```
[Out] Tan[a + b*x]^6/(6*b) + Tan[a + b*x]^8/(8*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \sec^4(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^5\left(1+x^2\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^5+x^7\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\tan^6(a+bx)}{6b}+\frac{\tan^8(a+bx)}{8b}\end{aligned}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 1.23

$$\frac{3\sec^8(a+bx)-8\sec^6(a+bx)+6\sec^4(a+bx)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*Tan[a + b*x]^5,x]
```

```
[Out] (6*Sec[a + b*x]^4 - 8*Sec[a + b*x]^6 + 3*Sec[a + b*x]^8)/(24*b)
```

fricas [A] time = 0.42, size = 35, normalized size = 1.13

$$\frac{6\cos(bx+a)^4-8\cos(bx+a)^2+3}{24\,b\cos(bx+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*(6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 + 3)/(b*cos(b*x + a)^8)
```

giac [B] time = 0.31, size = 93, normalized size = 3.00

$$-\frac{32\left(\frac{(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}\right)}{3\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -32/3*((cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - (cos(b*x + a) - 1)^4/(co
s(b*x + a) + 1)^4 + (cos(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5)/(b*((cos(b*x
 + a) - 1)/(cos(b*x + a) + 1) + 1)^8)
```

maple [A] time = 0.05, size = 42, normalized size = 1.35

$$\frac{\frac{\sin^6(bx+a)}{8\cos(bx+a)^8}+\frac{\sin^6(bx+a)}{24\cos(bx+a)^6}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^9*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/8*sin(b*x+a)^6/cos(b*x+a)^8+1/24*sin(b*x+a)^6/cos(b*x+a)^6)
```

maxima [B] time = 0.36, size = 69, normalized size = 2.23

$$\frac{6 \sin(bx+a)^4 - 4 \sin(bx+a)^2 + 1}{24 \left(\sin(bx+a)^8 - 4 \sin(bx+a)^6 + 6 \sin(bx+a)^4 - 4 \sin(bx+a)^2 + 1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^9*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(6*sin(b*x + a)^4 - 4*sin(b*x + a)^2 + 1)/((sin(b*x + a)^8 - 4*sin(b*x
 + a)^6 + 6*sin(b*x + a)^4 - 4*sin(b*x + a)^2 + 1)*b)
```

mupad [B] time = 0.42, size = 25, normalized size = 0.81

$$\frac{\tan(a+b\,x)^6\,\left(3\tan(a+b\,x)^2+4\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^9,x)
```

```
[Out] (tan(a + b*x)^6*(3*tan(a + b*x)^2 + 4))/(24*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**9*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.114 $\int \sec^5(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sec^9(a+bx)}{9b}-\frac{2\sec^7(a+bx)}{7b}+\frac{\sec^5(a+bx)}{5b}$$

```
[Out] 1/5*sec(b*x+a)^5/b-2/7*sec(b*x+a)^7/b+1/9*sec(b*x+a)^9/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 270}

$$\frac{\sec^9(a+bx)}{9b}-\frac{2\sec^7(a+bx)}{7b}+\frac{\sec^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^5*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^5/(5*b) - (2*Sec[a + b*x]^7)/(7*b) + Sec[a + b*x]^9/(9*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^5(a+bx)\tan^5(a+bx)\,dx&=\frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)^2\,dx,x,\sec(a+bx)\right)}{b}\\&=\frac{\text{Subst}\left(\int \left(x^4-2x^6+x^8\right)dx,x,\sec(a+bx)\right)}{b}\\&=\frac{\sec^5(a+bx)}{5b}-\frac{2\sec^7(a+bx)}{7b}+\frac{\sec^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{\sec^9(a+bx)}{9b}-\frac{2\sec^7(a+bx)}{7b}+\frac{\sec^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^5*Tan[a + b*x]^5,x]
[Out] Sec[a + b*x]^5/(5*b) - (2*Sec[a + b*x]^7)/(7*b) + Sec[a + b*x]^9/(9*b)
```

fricas [A] time = 0.41, size = 35, normalized size = 0.76

$$\frac{63\cos(bx+a)^4-90\cos(bx+a)^2+35}{315\,b\cos(bx+a)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/315*(63*cos(b*x + a)^4 - 90*cos(b*x + a)^2 + 35)/(b*cos(b*x + a)^9)
```

giac [B] time = 0.22, size = 160, normalized size = 3.48

$$\frac{16\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{36(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-\frac{126(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{441(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-\frac{315(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{210(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}+1\right)}{315\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^5,x, algorithm="giac")
[Out] 16/315*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 36*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 - 126*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 441*(
cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 315*(cos(b*x + a) - 1)^5/(cos(b*
x + a) + 1)^5 + 210*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 1)/(b*((cos
(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^9)
```

maple [B] time = 0.03, size = 124, normalized size = 2.70

$$\frac{\frac{\sin^6(bx+a)}{9\cos(bx+a)^9}+\frac{\sin^6(bx+a)}{21\cos(bx+a)^7}+\frac{\sin^6(bx+a)}{105\cos(bx+a)^5}-\frac{\sin^6(bx+a)}{315\cos(bx+a)^3}+\frac{\sin^6(bx+a)}{105\cos(bx+a)}+\frac{\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)\cos(bx+a)}{105}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^10*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/9*sin(b*x+a)^6/cos(b*x+a)^9+1/21*sin(b*x+a)^6/cos(b*x+a)^7+1/105*sin
(b*x+a)^6/cos(b*x+a)^5-1/315*sin(b*x+a)^6/cos(b*x+a)^3+1/105*sin(b*x+a)^6/c
os(b*x+a)+1/105*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))
```

maxima [A] time = 0.30, size = 35, normalized size = 0.76

$$\frac{63 \cos\left(bx+a\right)^{4} - 90 \cos\left(bx+a\right)^{2} + 35}{315 \, b \cos\left(bx+a\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^10*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/315*(63*cos(b*x + a)^4 - 90*cos(b*x + a)^2 + 35)/(b*cos(b*x + a)^9)
```

mupad [B] time = 0.77, size = 35, normalized size = 0.76

$$\frac{63 \cos\left(a+b\,x\right)^4-90 \cos\left(a+b\,x\right)^2+35}{315\,b\cos\left(a+b\,x\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^10,x)
```

```
[Out] (63*cos(a + b*x)^4 - 90*cos(a + b*x)^2 + 35)/(315*b*cos(a + b*x)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**10*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.115 $\int \sec^6(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sec^{10}(a+bx)}{10b}-\frac{\sec^8(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}$$

```
[Out] 1/6*sec(b*x+a)^6/b-1/4*sec(b*x+a)^8/b+1/10*sec(b*x+a)^10/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2606, 266, 43}

$$\frac{\sec^{10}(a+bx)}{10b}-\frac{\sec^8(a+bx)}{4b}+\frac{\sec^6(a+bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^6*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^6/(6*b) - Sec[a + b*x]^8/(4*b) + Sec[a + b*x]^10/(10*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\int \sec^6(a+bx)\tan^5(a+bx)\,dx = \frac{\text{Subst}\left(\int x^5\left(-1+x^2\right)^2\,dx, x, \sec(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int (-1+x)^2x^2\,dx, x, \sec^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(x^2-2x^3+x^4\right)\,dx, x, \sec^2(a+bx)\right)}{2b}$$
$$= \frac{\sec^6(a+bx)}{6b} - \frac{\sec^8(a+bx)}{4b} + \frac{\sec^{10}(a+bx)}{10b}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 0.83

$$\frac{6\sec^{10}(a+bx) - 15\sec^8(a+bx) + 10\sec^6(a+bx)}{60b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^6*Tan[a + b*x]^5,x]
[Out] (10*Sec[a + b*x]^6 - 15*Sec[a + b*x]^8 + 6*Sec[a + b*x]^10)/(60*b)
```

fricas [A] time = 0.42, size = 35, normalized size = 0.76

$$\frac{10\,\cos(bx+a)^4 - 15\,\cos(bx+a)^2 + 6}{60\,b\cos(bx+a)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^11*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/60*(10*cos(b*x + a)^4 - 15*cos(b*x + a)^2 + 6)/(b*cos(b*x + a)^10)
```

giac [B] time = 0.32, size = 139, normalized size = 3.02

$$-\frac{32\left(\frac{5\,(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{10\,(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} + \frac{18\,(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5} - \frac{10\,(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6} + \frac{5\,(\cos(bx+a)-1)^7}{(\cos(bx+a)+1)^7}\right)}{15\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^11*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -32/15*(5*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 10*(cos(b*x + a) - 1)
^4/(cos(b*x + a) + 1)^4 + 18*(cos(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5 - 10
*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 5*(cos(b*x + a) - 1)^7/(cos(b*
x + a) + 1)^7)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^10)
```

maple [A] time = 0.04, size = 60, normalized size = 1.30

$$\frac{\frac{\sin^6(bx+a)}{10\cos(bx+a)^{10}}+\frac{\sin^6(bx+a)}{20\cos(bx+a)^8}+\frac{\sin^6(bx+a)}{60\cos(bx+a)^6}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^11*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/10*sin(b*x+a)^6/cos(b*x+a)^10+1/20*sin(b*x+a)^6/cos(b*x+a)^8+1/60*si
n(b*x+a)^6/cos(b*x+a)^6)
```

maxima [A] time = 0.33, size = 79, normalized size = 1.72

$$-\frac{10\,\sin\left(bx+a\right)^4-5\,\sin\left(bx+a\right)^2+1}{60\left(\sin\left(bx+a\right)^{10}-5\,\sin\left(bx+a\right)^8+10\,\sin\left(bx+a\right)^6-10\,\sin\left(bx+a\right)^4+5\,\sin\left(bx+a\right)^2-1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^11*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(10*sin(b*x + a)^4 - 5*sin(b*x + a)^2 + 1)/((sin(b*x + a)^10 - 5*sin(
b*x + a)^8 + 10*sin(b*x + a)^6 - 10*sin(b*x + a)^4 + 5*sin(b*x + a)^2 - 1)*
b)
```

mupad [B] time = 0.44, size = 35, normalized size = 0.76

$$\frac{\tan\left(a+b\,x\right)^6\,\left(6\tan\left(a+b\,x\right)^4+15\tan\left(a+b\,x\right)^2+10\right)}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^11,x)
```

```
[Out] (tan(a + b*x)^6*(15*tan(a + b*x)^2 + 6*tan(a + b*x)^4 + 10))/(60*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**11*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.116 $\int \sec^7(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sec^{11}(a+bx)}{11b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^7(a+bx)}{7b}$$

```
[Out] 1/7*sec(b*x+a)^7/b-2/9*sec(b*x+a)^9/b+1/11*sec(b*x+a)^11/b
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2606, 270}

$$\frac{\sec^{11}(a+bx)}{11b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^7*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^7/(7*b) - (2*Sec[a + b*x]^9)/(9*b) + Sec[a + b*x]^11/(11*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \sec^7(a+bx)\tan^5(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^6\left(-1+x^2\right)^2\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(x^6-2x^8+x^{10}\right)\,dx,x,\sec(a+bx)\right)}{b}\\ &= \frac{\sec^7(a+bx)}{7b}-\frac{2\sec^9(a+bx)}{9b}+\frac{\sec^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{\sec^{11}(a+bx)}{11b}-\frac{2\sec^{9}(a+bx)}{9b}+\frac{\sec^{7}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^7*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^7/(7*b) - (2*Sec[a + b*x]^9)/(9*b) + Sec[a + b*x]^11/(11*b)
```

fricas [A] time = 0.45, size = 35, normalized size = 0.76

$$\frac{99\,\cos\left(bx+a\right)^{4}-154\,\cos\left(bx+a\right)^{2}+63}{693\,b\cos\left(bx+a\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^12*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/693*(99*cos(b*x + a)^4 - 154*cos(b*x + a)^2 + 63)/(b*cos(b*x + a)^11)
```

giac [B] time = 0.30, size = 204, normalized size = 4.43

$$\frac{16\left(\frac{11\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{55\,(\cos(bx+a)-1)^{2}}{(\cos(bx+a)+1)^{2}}-\frac{297\,(\cos(bx+a)-1)^{3}}{(\cos(bx+a)+1)^{3}}+\frac{1485\,(\cos(bx+a)-1)^{4}}{(\cos(bx+a)+1)^{4}}-\frac{2079\,(\cos(bx+a)-1)^{5}}{(\cos(bx+a)+1)^{5}}+\frac{2541\,(\cos(bx+a)-1)^{6}}{(\cos(bx+a)+1)^{6}}-1\cdots}{693\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^12*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 16/693*(11*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 55*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 - 297*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 1485
*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 2079*(cos(b*x + a) - 1)^5/(cos
(b*x + a) + 1)^5 + 2541*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 - 1155*(c
os(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 462*(cos(b*x + a) - 1)^8/(cos(b*x
 + a) + 1)^8 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^11)
```

maple [B] time = 0.04, size = 142, normalized size = 3.09

$$\frac{\frac{\sin^{6}(bx+a)}{11\cos(bx+a)^{11}}+\frac{5\left(\sin^{6}(bx+a)\right)}{99\cos(bx+a)^{9}}+\frac{5\left(\sin^{6}(bx+a)\right)}{231\cos(bx+a)^{7}}+\frac{\sin^{6}(bx+a)}{231\cos(bx+a)^{5}}-\frac{\sin^{6}(bx+a)}{693\cos(bx+a)^{3}}+\frac{\sin^{6}(bx+a)}{231\cos(bx+a)}+\frac{\left(\frac{8}{3}+\sin^{4}(bx+a)+\frac{4\left(\sin^{2}(bx+a)\right)}{3}\right)\cos\cdots}{231}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^12*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/11*sin(b*x+a)^6/cos(b*x+a)^11+5/99*sin(b*x+a)^6/cos(b*x+a)^9+5/231*s
in(b*x+a)^6/cos(b*x+a)^7+1/231*sin(b*x+a)^6/cos(b*x+a)^5-1/693*sin(b*x+a)^6
/cos(b*x+a)^3+1/231*sin(b*x+a)^6/cos(b*x+a)+1/231*(8/3+sin(b*x+a)^4+4/3*sin
(b*x+a)^2)*cos(b*x+a))
```

maxima [A] time = 0.45, size = 35, normalized size = 0.76

$$\frac{99\,\cos\left(bx+a\right)^{4}-154\,\cos\left(bx+a\right)^{2}+63}{693\,b\cos\left(bx+a\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^12*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/693*(99*cos(b*x + a)^4 - 154*cos(b*x + a)^2 + 63)/(b*cos(b*x + a)^11)
```

mupad [B] time = 1.02, size = 35, normalized size = 0.76

$$\frac{99\cos\left(a+b\,x\right)^{4}-154\cos\left(a+b\,x\right)^{2}+63}{693\,b\cos\left(a+b\,x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^12,x)
```

```
[Out] (99*cos(a + b*x)^4 - 154*cos(a + b*x)^2 + 63)/(693*b*cos(a + b*x)^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**12*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.117 $\int \sec^8(a+bx)\tan^5(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\sec^{12}(a+bx)}{12b}-\frac{\sec^{10}(a+bx)}{5b}+\frac{\sec^8(a+bx)}{8b}$$

```
[Out] 1/8*sec(b*x+a)^8/b-1/5*sec(b*x+a)^10/b+1/12*sec(b*x+a)^12/b
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2606, 266, 43}

$$\frac{\sec^{12}(a+bx)}{12b}-\frac{\sec^{10}(a+bx)}{5b}+\frac{\sec^8(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^8*Tan[a + b*x]^5,x]
```

```
[Out] Sec[a + b*x]^8/(8*b) - Sec[a + b*x]^10/(5*b) + Sec[a + b*x]^12/(12*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\int \sec^8(a+bx)\tan^5(a+bx)\,dx = \frac{\text{Subst}\left(\int x^7\left(-1+x^2\right)^2\,dx, x, \sec(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int(-1+x)^2x^3\,dx, x, \sec^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int\left(x^3-2x^4+x^5\right)\,dx, x, \sec^2(a+bx)\right)}{2b}$$
$$= \frac{\sec^8(a+bx)}{8b} - \frac{\sec^{10}(a+bx)}{5b} + \frac{\sec^{12}(a+bx)}{12b}$$

Mathematica [A] time = 0.13, size = 38, normalized size = 0.83

$$\frac{10\sec^{12}(a+bx)-24\sec^{10}(a+bx)+15\sec^8(a+bx)}{120b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^8*Tan[a + b*x]^5,x]
[Out] (15*Sec[a + b*x]^8 - 24*Sec[a + b*x]^10 + 10*Sec[a + b*x]^12)/(120*b)
```

fricas [A] time = 0.43, size = 35, normalized size = 0.76

$$\frac{15\cos\left(bx+a\right)^4-24\cos\left(bx+a\right)^2+10}{120\,b\cos\left(bx+a\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^13*sin(b*x+a)^5,x, algorithm="fricas")
[Out] 1/120*(15*cos(b*x + a)^4 - 24*cos(b*x + a)^2 + 10)/(b*cos(b*x + a)^12)
```

giac [B] time = 0.38, size = 183, normalized size = 3.98

$$-\frac{32\left(\frac{5(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{15(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{39(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}-\frac{42(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}+\frac{39(\cos(bx+a)-1)^7}{(\cos(bx+a)+1)^7}-\frac{15(\cos(bx+a)-1)^8}{(\cos(bx+a)+1)^8}+\frac{5(\cos}{(\cos(}\right.}{15\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^13*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -32/15*(5*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 15*(cos(b*x + a) - 1)
^4/(cos(b*x + a) + 1)^4 + 39*(cos(b*x + a) - 1)^5/(cos(b*x + a) + 1)^5 - 42
*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 39*(cos(b*x + a) - 1)^7/(cos(b
*x + a) + 1)^7 - 15*(cos(b*x + a) - 1)^8/(cos(b*x + a) + 1)^8 + 5*(cos(b*x
+ a) - 1)^9/(cos(b*x + a) + 1)^9)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1)
 + 1)^12)
```

maple [A] time = 0.04, size = 78, normalized size = 1.70

$$\frac{\frac{\sin^6(bx+a)}{12\cos(bx+a)^{12}}+\frac{\sin^6(bx+a)}{20\cos(bx+a)^{10}}+\frac{\sin^6(bx+a)}{40\cos(bx+a)^{8}}+\frac{\sin^6(bx+a)}{120\cos(bx+a)^{6}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^13*sin(b*x+a)^5,x)
```

```
[Out] 1/b*(1/12*sin(b*x+a)^6/cos(b*x+a)^12+1/20*sin(b*x+a)^6/cos(b*x+a)^10+1/40*s
in(b*x+a)^6/cos(b*x+a)^8+1/120*sin(b*x+a)^6/cos(b*x+a)^6)
```

maxima [B] time = 0.39, size = 89, normalized size = 1.93

$$\frac{15\sin(bx+a)^4-6\sin(bx+a)^2+1}{120\left(\sin(bx+a)^{12}-6\sin(bx+a)^{10}+15\sin(bx+a)^8-20\sin(bx+a)^6+15\sin(bx+a)^4-6\sin(bx+a)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^13*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/120*(15*sin(b*x + a)^4 - 6*sin(b*x + a)^2 + 1)/((sin(b*x + a)^12 - 6*sin(
b*x + a)^10 + 15*sin(b*x + a)^8 - 20*sin(b*x + a)^6 + 15*sin(b*x + a)^4 - 6
*sin(b*x + a)^2 + 1)*b)
```

mupad [B] time = 0.42, size = 45, normalized size = 0.98

$$\frac{\frac{\tan(a+b\,x)^{12}}{12}+\frac{3\tan(a+b\,x)^{10}}{10}+\frac{3\tan(a+b\,x)^{8}}{8}+\frac{\tan(a+b\,x)^{6}}{6}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5/cos(a + b*x)^13,x)
```

```
[Out] (tan(a + b*x)^6/6 + (3*tan(a + b*x)^8)/8 + (3*tan(a + b*x)^10)/10 + tan(a +
 b*x)^12/12)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**13*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.118 $\int \sin^3(a+bx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{5\sin^3(a+bx)}{6b}+\frac{5\sin(a+bx)}{2b}+\frac{\sin^3(a+bx)\tan^2(a+bx)}{2b}-\frac{5\tanh^{-1}(\sin(a+bx))}{2b}$$

```
[Out] -5/2*arctanh(sin(b*x+a))/b+5/2*sin(b*x+a)/b+5/6*sin(b*x+a)^3/b+1/2*sin(b*x+
a)^3*tan(b*x+a)^2/b
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2592, 288, 302, 206}

$$\frac{5\sin^3(a+bx)}{6b}+\frac{5\sin(a+bx)}{2b}+\frac{\sin^3(a+bx)\tan^2(a+bx)}{2b}-\frac{5\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Tan[a + b*x]^3,x]
```

```
[Out] (-5*ArcTanh[Sin[a + b*x]])/(2*b) + (5*Sin[a + b*x])/(2*b) + (5*Sin[a + b*x]
^3)/(6*b) + (Sin[a + b*x]^3*Tan[a + b*x]^2)/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \sin^3(a+bx)\tan^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \sin(a+bx)\right)}{b}$$

$$= \frac{\sin^3(a+bx)\tan^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}$$

$$= \frac{\sin^3(a+bx)\tan^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \sin(a+bx)\right)}{2b}$$

$$= \frac{5\sin(a+bx)}{2b} + \frac{5\sin^3(a+bx)}{6b} + \frac{\sin^3(a+bx)\tan^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{1}{1-x^2}$$

$$= -\frac{5\tanh^{-1}(\sin(a+bx))}{2b} + \frac{5\sin(a+bx)}{2b} + \frac{5\sin^3(a+bx)}{6b} + \frac{\sin^3(a+bx)\tan^2}{2b}$$

Mathematica [A] time = 0.17, size = 52, normalized size = 0.79

$$\frac{(24\cos(2(a+bx))-\cos(4(a+bx))+37)\tan(a+bx)\sec(a+bx)-60\tanh^{-1}(\sin(a+bx))}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Tan[a + b*x]^3,x]
```

```
[Out] (-60*ArcTanh[Sin[a + b*x]] + (37 + 24*Cos[2*(a + b*x)] - Cos[4*(a + b*x)])*
Sec[a + b*x]*Tan[a + b*x])/(24*b)
```

fricas [A] time = 0.45, size = 84, normalized size = 1.27

$$-\frac{15\cos(bx+a)^2\log(\sin(bx+a)+1)-15\cos(bx+a)^2\log(-\sin(bx+a)+1)+2\left(2\cos(bx+a)^4-14\cos\right.}{12\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/12*(15*cos(b*x + a)^2*log(sin(b*x + a) + 1) - 15*cos(b*x + a)^2*log(-sin
(b*x + a) + 1) + 2*(2*cos(b*x + a)^4 - 14*cos(b*x + a)^2 - 3)*sin(b*x + a))
/(b*cos(b*x + a)^2)
```

giac [A] time = 0.34, size = 68, normalized size = 1.03

$$\frac{4\,\sin\left(bx+a\right)^{3}-\frac{6\,\sin(bx+a)}{\sin(bx+a)^{2}-1}-15\,\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+15\,\log\left(\left|\sin\left(bx+a\right)-1\right|\right)+24\,\sin\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 1/12*(4*sin(b*x + a)^3 - 6*sin(b*x + a)/(sin(b*x + a)^2 - 1) - 15*log(abs(s
in(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) - 1)) + 24*sin(b*x + a))/b
```

maple [A] time = 0.04, size = 79, normalized size = 1.20

$$\frac{\sin^{7}\left(bx+a\right)}{2b\cos\left(bx+a\right)^{2}}+\frac{\sin^{5}\left(bx+a\right)}{2b}+\frac{5\left(\sin^{3}\left(bx+a\right)\right)}{6b}+\frac{5\sin\left(bx+a\right)}{2b}-\frac{5\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*sin(b*x+a)^6,x)
```

```
[Out] 1/2/b*sin(b*x+a)^7/cos(b*x+a)^2+1/2*sin(b*x+a)^5/b+5/6*sin(b*x+a)^3/b+5/2*s
in(b*x+a)/b-5/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.48, size = 66, normalized size = 1.00

$$\frac{4\,\sin\left(bx+a\right)^{3}-\frac{6\,\sin(bx+a)}{\sin(bx+a)^{2}-1}-15\,\log\left(\sin\left(bx+a\right)+1\right)+15\,\log\left(\sin\left(bx+a\right)-1\right)+24\,\sin\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*sin(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] 1/12*(4*sin(b*x + a)^3 - 6*sin(b*x + a)/(sin(b*x + a)^2 - 1) - 15*log(sin(b
*x + a) + 1) + 15*log(sin(b*x + a) - 1) + 24*sin(b*x + a))/b
```

mupad [B] time = 7.24, size = 147, normalized size = 2.23

$$\frac{5\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{9}+\frac{20\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{7}}{3}-\frac{22\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{5}}{3}+\frac{20\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{3}}{3}+5\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{8}-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{6}-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{4}+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{2}+1\right)}-\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{a}{2}+\right.\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^6/cos(a + b*x)^3,x)
```

```
[Out] (5*tan(a/2 + (b*x)/2) + (20*tan(a/2 + (b*x)/2)^3)/3 - (22*tan(a/2 + (b*x)/2
)^5)/3 + (20*tan(a/2 + (b*x)/2)^7)/3 + 5*tan(a/2 + (b*x)/2)^9)/(b*(tan(a/2
+ (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2)^4 - 2*tan(a/2 + (b*x)/2)^6 + tan(a/2 +
(b*x)/2)^8 + tan(a/2 + (b*x)/2)^10 + 1)) - (5*atanh(tan(a/2 + (b*x)/2)))/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*sin(b*x+a)**6,x)
```

```
[Out] Timed out
```

3.119 $\int \sin(a+bx)\tan^6(a+bx)\,dx$

Optimal. Leaf size=50

$$\frac{\cos(a+bx)}{b}+\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{b}+\frac{3\sec(a+bx)}{b}$$

```
[Out] cos(b*x+a)/b+3*sec(b*x+a)/b-sec(b*x+a)^3/b+1/5*sec(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 270}

$$\frac{\cos(a+bx)}{b}+\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{b}+\frac{3\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[a + b*x]^6,x]
```

```
[Out] Cos[a + b*x]/b + (3*Sec[a + b*x])/b - Sec[a + b*x]^3/b + Sec[a + b*x]^5/(5*
b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \sin(a+bx)\tan^6(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^6}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)dx, x, \cos(a+bx)\right)}{b}$$
$$= \frac{\cos(a+bx)}{b}+\frac{3\sec(a+bx)}{b}-\frac{\sec^3(a+bx)}{b}+\frac{\sec^5(a+bx)}{5b}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 1.00

$$\frac{\cos(a+bx)}{b}+\frac{\sec^5(a+bx)}{5b}-\frac{\sec^3(a+bx)}{b}+\frac{3\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[a + b*x]^6,x]
```

```
[Out] Cos[a + b*x]/b + (3*Sec[a + b*x])/b - Sec[a + b*x]^3/b + Sec[a + b*x]^5/(5*
b)
```

fricas [A] time = 0.44, size = 45, normalized size = 0.90

$$\frac{5\,\cos(bx+a)^6+15\,\cos(bx+a)^4-5\,\cos(bx+a)^2+1}{5\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/5*(5*cos(b*x + a)^6 + 15*cos(b*x + a)^4 - 5*cos(b*x + a)^2 + 1)/(b*cos(b*
x + a)^5)
```

giac [B] time = 0.28, size = 144, normalized size = 2.88

$$-\frac{2\left(\frac{5}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1}-\frac{\frac{50(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{80(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{30(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{5(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+11}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^5}\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -2/5*(5/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - (50*(cos(b*x + a) - 1
)/(cos(b*x + a) + 1) + 80*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 30*(c
os(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 5*(cos(b*x + a) - 1)^4/(cos(b*x +
 a) + 1)^4 + 11)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^5)/b
```

maple [A] time = 0.04, size = 96, normalized size = 1.92

$$\frac{\frac{\sin^8(bx+a)}{5\cos(bx+a)^5} - \frac{\sin^8(bx+a)}{5\cos(bx+a)^3} + \frac{\sin^8(bx+a)}{\cos(bx+a)} + \left(\frac{16}{5} + \sin^6(bx+a) + \frac{6\left(\sin^4(bx+a)\right)}{5} + \frac{8\left(\sin^2(bx+a)\right)}{5}\right)\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6*sin(b*x+a)^7,x)
```

```
[Out] 1/b*(1/5*sin(b*x+a)^8/cos(b*x+a)^5-1/5*sin(b*x+a)^8/cos(b*x+a)^3+sin(b*x+a)
^8/cos(b*x+a)+(16/5+sin(b*x+a)^6+6/5*sin(b*x+a)^4+8/5*sin(b*x+a)^2)*cos(b*x
+a))
```

maxima [A] time = 0.40, size = 45, normalized size = 0.90

$$\frac{\frac{15\cos(bx+a)^4 - 5\cos(bx+a)^2 + 1}{\cos(bx+a)^5} + 5\cos(bx+a)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6*sin(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/5*((15*cos(b*x + a)^4 - 5*cos(b*x + a)^2 + 1)/cos(b*x + a)^5 + 5*cos(b*x
+ a))/b
```

mupad [B] time = 0.54, size = 50, normalized size = 1.00

$$\frac{\cos(a+b\,x)}{b} + \frac{3}{b\,\cos(a+b\,x)} - \frac{1}{b\cos(a+b\,x)^3} + \frac{1}{5\,b\cos(a+b\,x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^7/cos(a + b*x)^6,x)
```

```
[Out] cos(a + b*x)/b + 3/(b*cos(a + b*x)) - 1/(b*cos(a + b*x)^3) + 1/(5*b*cos(a +
 b*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6*sin(b*x+a)**7,x)
```

```
[Out] Timed out
```

3.120 $\int \cos^5(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{\cos^5(a+bx)}{5b}+\frac{\cos^3(a+bx)}{3b}+\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -arctanh(cos(b*x+a))/b+cos(b*x+a)/b+1/3*cos(b*x+a)^3/b+1/5*cos(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2592, 302, 206}

$$\frac{\cos^5(a+bx)}{5b}+\frac{\cos^3(a+bx)}{3b}+\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Cot[a + b*x],x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Cos[a + b*x]/b + Cos[a + b*x]^3/(3*b) + Cos[a
+ b*x]^5/(5*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \cos^5(a+bx)\cot(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \left(-1-x^2-x^4+\frac{1}{1-x^2}\right)dx, x, \cos(a+bx)\right)}{b}$$

$$= \frac{\cos(a+bx)}{b} + \frac{\cos^3(a+bx)}{3b} + \frac{\cos^5(a+bx)}{5b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right.}{b}$$

$$= -\frac{\tanh^{-1}(\cos(a+bx))}{b} + \frac{\cos(a+bx)}{b} + \frac{\cos^3(a+bx)}{3b} + \frac{\cos^5(a+bx)}{5b}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 1.42

$$\frac{11\cos(a+bx)}{8b} + \frac{7\cos(3(a+bx))}{48b} + \frac{\cos(5(a+bx))}{80b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Cot[a + b*x],x]
```

```
[Out] (11*Cos[a + b*x])/(8*b) + (7*Cos[3*(a + b*x)])/(48*b) + Cos[5*(a + b*x)]/(8
0*b) - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b
```

fricas [A] time = 0.43, size = 60, normalized size = 1.13

$$\frac{6\cos(bx+a)^5 + 10\cos(bx+a)^3 + 30\cos(bx+a) - 15\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) + 15\log\left(-\frac{1}{2}\cos(bx+a)+\right.}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/30*(6*cos(b*x + a)^5 + 10*cos(b*x + a)^3 + 30*cos(b*x + a) - 15*log(1/2*c
os(b*x + a) + 1/2) + 15*log(-1/2*cos(b*x + a) + 1/2))/b
```

giac [B] time = 0.27, size = 145, normalized size = 2.74

$$\frac{\frac{4\left(\frac{70(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{140(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{90(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{45(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} - 23\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^5} + 15\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/30*(4*(70*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 140*(cos(b*x + a) - 1)^
2/(cos(b*x + a) + 1)^2 + 90*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 45*
(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 23)/((cos(b*x + a) - 1)/(cos(b*
x + a) + 1) - 1)^5 + 15*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/
b
```

maple [A] time = 0.03, size = 58, normalized size = 1.09

$$\frac{\cos^5(bx+a)}{5b}+\frac{\cos^3(bx+a)}{3b}+\frac{\cos(bx+a)}{b}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6/sin(b*x+a),x)
```

```
[Out] 1/5*cos(b*x+a)^5/b+1/3*cos(b*x+a)^3/b+cos(b*x+a)/b+1/b*ln(csc(b*x+a)-cot(b*
x+a))
```

maxima [A] time = 0.32, size = 56, normalized size = 1.06

$$\frac{6\cos(bx+a)^5+10\cos(bx+a)^3+30\cos(bx+a)-15\log(\cos(bx+a)+1)+15\log(\cos(bx+a)-1)}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/30*(6*cos(b*x + a)^5 + 10*cos(b*x + a)^3 + 30*cos(b*x + a) - 15*log(cos(b
*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

mupad [B] time = 5.37, size = 88, normalized size = 1.66

$$\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}+\frac{6\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8+12\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6+\frac{56\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{3}+\frac{28\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{3}+\frac{46}{15}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^6/sin(a + b*x),x)
```

```
[Out] log(tan(a/2 + (b*x)/2))/b + ((28*tan(a/2 + (b*x)/2)^2)/3 + (56*tan(a/2 + (b
*x)/2)^4)/3 + 12*tan(a/2 + (b*x)/2)^6 + 6*tan(a/2 + (b*x)/2)^8 + 46/15)/(b*
(tan(a/2 + (b*x)/2)^2 + 1)^5)
```

sympy [A] time = 8.16, size = 1085, normalized size = 20.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a),x)
```

```
[Out] Piecewise((15*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**10/(15*b*tan(a/2 + b*
x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan
(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 75*log(tan(a/2 + b*x/
2))*tan(a/2 + b*x/2)**8/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)*
*8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 +
 b*x/2)**2 + 15*b) + 150*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(15*b*ta
n(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 +
 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 150*log(tan
(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/
2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*
b*tan(a/2 + b*x/2)**2 + 15*b) + 75*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**
2/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b
*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 1
5*log(tan(a/2 + b*x/2))/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)*
*8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 +
 b*x/2)**2 + 15*b) + 90*tan(a/2 + b*x/2)**8/(15*b*tan(a/2 + b*x/2)**10 + 75
*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)
**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 180*tan(a/2 + b*x/2)**6/(15*b*tan(
a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 1
50*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 + 15*b) + 280*tan(a/2 +
 b*x/2)**4/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 + b*x/2)**8 + 150*b*ta
n(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*tan(a/2 + b*x/2)**2 +
15*b) + 140*tan(a/2 + b*x/2)**2/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 +
 b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*t
an(a/2 + b*x/2)**2 + 15*b) + 46/(15*b*tan(a/2 + b*x/2)**10 + 75*b*tan(a/2 +
 b*x/2)**8 + 150*b*tan(a/2 + b*x/2)**6 + 150*b*tan(a/2 + b*x/2)**4 + 75*b*t
an(a/2 + b*x/2)**2 + 15*b), Ne(b, 0)), (x*cos(a)**6/sin(a), True))
```

3.121 $\int \cos^4(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{\sin^4(a+bx)}{4b}-\frac{\sin^2(a+bx)}{b}+\frac{\log(\sin(a+bx))}{b}$$

```
[Out] ln(sin(b*x+a))/b-sin(b*x+a)^2/b+1/4*sin(b*x+a)^4/b
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2590, 266, 43}

$$\frac{\sin^4(a+bx)}{4b}-\frac{\sin^2(a+bx)}{b}+\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/b + Sin[a + b*x]^4/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cos^4(a+bx)\cot(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x}\,dx, x, -\sin(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, \sin^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)dx, x, \sin^2(a+bx)\right)}{2b}$$
$$= \frac{\log(\sin(a+bx))}{b} - \frac{\sin^2(a+bx)}{b} + \frac{\sin^4(a+bx)}{4b}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{\sin^4(a+bx)}{4b} - \frac{\sin^2(a+bx)}{b} + \frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x],x]
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/b + Sin[a + b*x]^4/(4*b)
```

fricas [A] time = 0.45, size = 35, normalized size = 0.88

$$\frac{\cos(bx+a)^4 + 2\cos(bx+a)^2 + 4\log\left(\frac{1}{2}\sin(bx+a)\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a),x, algorithm="fricas")
[Out] 1/4*(cos(b*x + a)^4 + 2*cos(b*x + a)^2 + 4*log(1/2*sin(b*x + a)))/b
```

giac [B] time = 0.27, size = 170, normalized size = 4.25

$$-\frac{\frac{\frac{52(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{102(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{52(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} - \frac{25(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} - 25}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^4} - 6\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) + 12\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right|\right)}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/12*((52*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 102*(cos(b*x + a) - 1)^2
/(cos(b*x + a) + 1)^2 + 52*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 25*(
cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 25)/((cos(b*x + a) - 1)/(cos(b*x
 + a) + 1) - 1)^4 - 6*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 1
2*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

maple [A] time = 0.03, size = 39, normalized size = 0.98

$$\frac{\cos^4(bx+a)}{4b}+\frac{\cos^2(bx+a)}{2b}+\frac{\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5/sin(b*x+a),x)
```

```
[Out] 1/4*cos(b*x+a)^4/b+1/2*cos(b*x+a)^2/b+ln(sin(b*x+a))/b
```

maxima [A] time = 0.40, size = 35, normalized size = 0.88

$$\frac{\sin(bx+a)^4-4\sin(bx+a)^2+2\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(sin(b*x + a)^4 - 4*sin(b*x + a)^2 + 2*log(sin(b*x + a)^2))/b
```

mupad [B] time = 0.50, size = 66, normalized size = 1.65

$$\frac{\ln(\tan(a+b\,x))}{b}-\frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b}+\frac{\frac{\tan(a+b\,x)^2}{2}+\frac{3}{4}}{b\,\left(\tan(a+b\,x)^4+2\tan(a+b\,x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5/sin(a + b*x),x)
```

```
[Out] log(tan(a + b*x))/b - log(tan(a + b*x)^2 + 1)/(2*b) + (tan(a + b*x)^2/2 + 3
/4)/(b*(2*tan(a + b*x)^2 + tan(a + b*x)^4 + 1))
```

sympy [A] time = 6.54, size = 1086, normalized size = 27.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b
*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2
+ b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(b*ta
n(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b
*tan(a/2 + b*x/2)**2 + b) - 6*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)
**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)
**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2
 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2
 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2)**2 + 1)/
(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4
+ 4*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/
(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4
+ 4*b*tan(a/2 + b*x/2)**2 + b) + 4*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**
6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**
4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 6*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)
**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)
**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 4*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/
2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/
2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b
*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2
+ b*x/2)**2 + b) - 4*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a
/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4
*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b
*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*tan(a/2 + b*x/2)**2
/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4
 + 4*b*tan(a/2 + b*x/2)**2 + b), Ne(b, 0)), (x*cos(a)**5/sin(a), True))
```

3.122 $\int \cos^3(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\cos^3(a+bx)}{3b}+\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -arctanh(cos(b*x+a))/b+cos(b*x+a)/b+1/3*cos(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2592, 302, 206}

$$\frac{\cos^3(a+bx)}{3b}+\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x],x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Cos[a + b*x]/b + Cos[a + b*x]^3/(3*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \cos^3(a+bx)\cot(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cos(a+bx)\right)}{b}$$
$$= \frac{\cos(a+bx)}{b} + \frac{\cos^3(a+bx)}{3b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cos(a+bx))}{b} + \frac{\cos(a+bx)}{b} + \frac{\cos^3(a+bx)}{3b}$$

Mathematica [A] time = 0.02, size = 60, normalized size = 1.58

$$\frac{5\cos(a+bx)}{4b} + \frac{\cos(3(a+bx))}{12b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x],x]
```

```
[Out] (5*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b) - Log[Cos[(a + b*x)/2]]/b
+ Log[Sin[(a + b*x)/2]]/b
```

fricas [A] time = 0.46, size = 50, normalized size = 1.32

$$\frac{2\cos(bx+a)^3 + 6\cos(bx+a) - 3\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) + 3\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(b*x + a)^3 + 6*cos(b*x + a) - 3*log(1/2*cos(b*x + a) + 1/2) + 3*
log(-1/2*cos(b*x + a) + 1/2))/b
```

giac [B] time = 0.19, size = 101, normalized size = 2.66

$$\frac{\frac{8\left(\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - 2\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3} + 3\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(8*(3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 - 2)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^3 + 3*l
og(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.02, size = 45, normalized size = 1.18

$$\frac{\cos^3(bx+a)}{3b}+\frac{\cos(bx+a)}{b}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/sin(b*x+a),x)
```

```
[Out] 1/3*cos(b*x+a)^3/b+cos(b*x+a)/b+1/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [A] time = 0.44, size = 46, normalized size = 1.21

$$\frac{2\cos(bx+a)^3+6\cos(bx+a)-3\log(\cos(bx+a)+1)+3\log(\cos(bx+a)-1)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*cos(b*x + a)^3 + 6*cos(b*x + a) - 3*log(cos(b*x + a) + 1) + 3*log(co
s(b*x + a) - 1))/b
```

mupad [B] time = 1.68, size = 62, normalized size = 1.63

$$\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}+\frac{4\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+4\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+\frac{8}{3}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4/sin(a + b*x),x)
```

```
[Out] log(tan(a/2 + (b*x)/2))/b + (4*tan(a/2 + (b*x)/2)^2 + 4*tan(a/2 + (b*x)/2)^
4 + 8/3)/(b*(tan(a/2 + (b*x)/2)^2 + 1)^3)
```

sympy [A] time = 3.00, size = 473, normalized size = 12.45

$$\begin{cases} \frac{3\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+3b} + \frac{9\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{3b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+3b} + \frac{9\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right.}{3b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+9b\tan^4\left(\frac{a}{2}+\right.} \\ \frac{x\cos^4(a)}{\sin(a)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a),x)
```

```
[Out] Piecewise((3*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(3*b*tan(a/2 + b*x/2
)**6 + 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b) + 9*log(tan
(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(3*b*tan(a/2 + b*x/2)**6 + 9*b*tan(a/2 +
 b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b) + 9*log(tan(a/2 + b*x/2))*tan(a
/2 + b*x/2)**2/(3*b*tan(a/2 + b*x/2)**6 + 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan
(a/2 + b*x/2)**2 + 3*b) + 3*log(tan(a/2 + b*x/2))/(3*b*tan(a/2 + b*x/2)**6
+ 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b) + 12*tan(a/2 + b
*x/2)**4/(3*b*tan(a/2 + b*x/2)**6 + 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan(a/2 +
 b*x/2)**2 + 3*b) + 12*tan(a/2 + b*x/2)**2/(3*b*tan(a/2 + b*x/2)**6 + 9*b*t
an(a/2 + b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b) + 8/(3*b*tan(a/2 + b*x/
2)**6 + 9*b*tan(a/2 + b*x/2)**4 + 9*b*tan(a/2 + b*x/2)**2 + 3*b), Ne(b, 0))
, (x*cos(a)**4/sin(a), True))
```

3.123 $\int \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

```
[Out] ln(sin(b*x+a))/b-1/2*sin(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(a+bx)\cot(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, -\sin(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x}-x\right)\,dx, x, -\sin(a+bx)\right)}{b}\\
&= \frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\log(\sin(a+bx))}{b}-\frac{\sin^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x],x]
[Out] Log[Sin[a + b*x]]/b - Sin[a + b*x]^2/(2*b)
```

fricas [A] time = 0.44, size = 25, normalized size = 0.93

$$\frac{\cos(bx+a)^2+2\log\left(\frac{1}{2}\sin(bx+a)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a),x, algorithm="fricas")
[Out] 1/2*(cos(b*x + a)^2 + 2*log(1/2*sin(b*x + a)))/b
```

giac [A] time = 0.57, size = 25, normalized size = 0.93

$$-\frac{\sin(bx+a)^2-\log\left(\sin(bx+a)^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a),x, algorithm="giac")
[Out] -1/2*(sin(b*x + a)^2 - log(sin(b*x + a)^2))/b
```

maple [A] time = 0.03, size = 26, normalized size = 0.96

$$\frac{\cos^2(bx+a)}{2b}+\frac{\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(b*x+a),x)
[Out] 1/2*cos(b*x+a)^2/b+ln(sin(b*x+a))/b
```

maxima [A] time = 0.54, size = 25, normalized size = 0.93

$$-\frac{\sin(bx+a)^2-\log\left(\sin(bx+a)^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(sin(b*x + a)^2 - log(sin(b*x + a)^2))/b
```

mupad [B] time = 0.41, size = 35, normalized size = 1.30

$$\frac{\frac{\cos(a+b\,x)^2}{2}-\frac{\ln\left(\tan(a+b\,x)^2+1\right)}{2}+\ln\left(\tan\left(a+b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(a + b*x),x)
```

```
[Out] (log(tan(a + b*x)) - log(tan(a + b*x)^2 + 1)/2 + cos(a + b*x)^2/2)/b
```

sympy [A] time = 2.35, size = 369, normalized size = 13.67

$$\begin{cases} -\frac{\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}-\frac{2\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}-\frac{\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^4\left(\frac{a}{2}\right.}{b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^2\left(\frac{a}{2}+\right.} \\ \frac{x\cos^3(a)}{\sin(a)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b
*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - 2*log(tan(a/2 + b*x/2)**2 + 1)*ta
n(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - l
og(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**
2 + b) + log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 +
 2*b*tan(a/2 + b*x/2)**2 + b) + 2*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2
/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/
2))/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x
/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b), Ne(b, 0)), (x
*cos(a)**3/sin(a), True))
```

3.124 $\int \cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -arctanh(cos(b*x+a))/b+cos(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2592, 321, 206}

$$\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Cos[a + b*x]/b
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \cos(a+bx)\cot(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= \frac{\cos(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cos(a+bx))}{b} + \frac{\cos(a+bx)}{b}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.83

$$\frac{\cos(a+bx)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b
```

fricas [A] time = 0.44, size = 38, normalized size = 1.65

$$\frac{2\cos(bx+a) - \log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + \log\left(-\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a) +
 1/2))/b
```

giac [B] time = 0.24, size = 57, normalized size = 2.48

$$-\frac{\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1} - \log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - log(abs(-cos(b*x + a)
 + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.02, size = 32, normalized size = 1.39

$$\frac{\cos(bx+a)}{b}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(b*x+a),x)
```

```
[Out] cos(b*x+a)/b+1/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [A] time = 0.36, size = 34, normalized size = 1.48

$$\frac{2\cos(bx+a)-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
```

mupad [B] time = 0.46, size = 35, normalized size = 1.52

$$\frac{2}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)}+\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(a + b*x),x)
```

```
[Out] 2/(b*(tan(a/2 + (b*x)/2)^2 + 1)) + log(tan(a/2 + (b*x)/2))/b
```

sympy [A] time = 1.31, size = 92, normalized size = 4.00

$$\begin{cases} \frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{2}{b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+b} & \text{for}\ b\neq 0 \\ \frac{x\cos^2(a)}{\sin(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**2
 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b*x/2)**2 + b) + 2/(b*tan(a/2 +
b*x/2)**2 + b), Ne(b, 0)), (x*cos(a)**2/sin(a), True))
```

3.125 $\int \cot(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\sin(a+bx))}{b}$$

[Out] ln(sin(b*x+a))/b

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Int[Cot[a + b*x],x]

[Out] Log[Sin[a + b*x]]/b

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(a+bx)\,dx = \frac{\log(\sin(a+bx))}{b}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.73

$$\frac{\log(\tan(a+bx)) + \log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Cot[a + b*x],x]

[Out] (Log[Cos[a + b*x]] + Log[Tan[a + b*x]])/b

fricas [A] time = 0.42, size = 13, normalized size = 1.18

$$\frac{\log\left(\frac{1}{2}\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] log(1/2*sin(b*x + a))/b
```

giac [A] time = 0.21, size = 12, normalized size = 1.09

$$\frac{\log\left(\left|\sin\left(bx+a\right)\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(sin(b*x + a)))/b
```

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$\frac{\ln\left(\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+a),x)
```

```
[Out] ln(sin(b*x+a))/b
```

maxima [A] time = 0.30, size = 11, normalized size = 1.00

$$\frac{\log\left(\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] log(sin(b*x + a))/b
```

mupad [B] time = 0.41, size = 26, normalized size = 2.36

$$-\frac{\ln\left(\tan\left(a+b\,x\right)^2+1\right)-2\,\ln\left(\tan\left(a+b\,x\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(a + b*x),x)
```

```
[Out] -(log(tan(a + b*x)^2 + 1) - 2*log(tan(a + b*x)))/(2*b)
```

sympy [A] time = 0.61, size = 17, normalized size = 1.55

$$\begin{cases} \frac{\log(\sin(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x\cos(a)}{\sin(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a),x)
[Out] Piecewise((log(sin(a + b*x))/b, Ne(b, 0)), (x*cos(a)/sin(a), True))
```

3.126 $\int \csc(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\tan(a+bx))}{b}$$

```
[Out] ln(tan(b*x+a))/b
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2620, 29}

$$\frac{\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] Log[Tan[a + b*x]]/b
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sec(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\log(\tan(a+bx))}{b}\end{aligned}$$

Mathematica [B] time = 0.02, size = 31, normalized size = 2.82

$$2\left(\frac{\log(\sin(a+bx))}{2b}-\frac{\log(\cos(a+bx))}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x],x]
[Out] 2*(-1/2*Log[Cos[a + b*x]]/b + Log[Sin[a + b*x]]/(2*b))
```

fricas [B] time = 0.45, size = 30, normalized size = 2.73

$$-\frac{\log\left(\cos\left(bx+a\right)^{2}\right)-\log\left(-\frac{1}{4}\cos\left(bx+a\right)^{2}+\frac{1}{4}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a),x, algorithm="fricas")
[Out] -1/2*(log(cos(b*x + a)^2) - log(-1/4*cos(b*x + a)^2 + 1/4))/b
```

giac [B] time = 0.24, size = 56, normalized size = 5.09

$$\frac{\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-2\,\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a),x, algorithm="giac")
[Out] 1/2*(log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*
x + a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

maple [A] time = 0.04, size = 12, normalized size = 1.09

$$\frac{\ln\left(\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/sin(b*x+a),x)
[Out] ln(tan(b*x+a))/b
```

maxima [B] time = 0.38, size = 28, normalized size = 2.55

$$-\frac{\log\left(\sin\left(bx+a\right)^{2}-1\right)-\log\left(\sin\left(bx+a\right)^{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a),x, algorithm="maxima")
[Out] -1/2*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b
```

mupad [B] time = 0.39, size = 11, normalized size = 1.00

$$\frac{\ln\left(\tan\left(a+b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)),x)
```

```
[Out] log(tan(a + b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(a+bx\right)}{\sin\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)/sin(a + b*x), x)
```

3.127 $\int \csc(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\sec(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -arctanh(cos(b*x+a))/b+sec(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2622, 321, 207}

$$\frac{\sec(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Sec[a + b*x]/b
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc(a+bx)\sec^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{b}$$
$$= \frac{\sec(a+bx)}{b} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cos(a+bx))}{b} + \frac{\sec(a+bx)}{b}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.83

$$\frac{\sec(a+bx)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] -(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b
```

fricas [B] time = 0.50, size = 52, normalized size = 2.26

$$-\frac{\cos(bx+a)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) - \cos(bx+a)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) - 2}{2\,b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(b*x + a)*log(1/2*cos(b*x + a) + 1/2) - cos(b*x + a)*log(-1/2*cos(
b*x + a) + 1/2) - 2)/(b*cos(b*x + a))
```

giac [B] time = 0.27, size = 55, normalized size = 2.39

$$\frac{\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1} + \log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + log(abs(-cos(b*x + a)
+ 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.04, size = 34, normalized size = 1.48

$$\frac{1}{b\cos(bx+a)}+\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/sin(b*x+a),x)
[Out] 1/b/cos(b*x+a)+1/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [A] time = 0.31, size = 36, normalized size = 1.57

$$\frac{\frac{2}{\cos(bx+a)}-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a),x, algorithm="maxima")
[Out] 1/2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
```

mupad [B] time = 0.08, size = 23, normalized size = 1.00

$$-\frac{\operatorname{atanh}(\cos(a+b\,x))-\frac{1}{\cos(a+b\,x)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)),x)
[Out] -(atanh(cos(a + b*x)) - 1/cos(a + b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2(a+bx)}{\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a),x)
[Out] Integral(sec(a + b*x)**2/sin(a + b*x), x)
```

3.128 $\int \csc(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\tan^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}$$

```
[Out] ln(tan(b*x+a))/b+1/2*tan(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2620, 14}

$$\frac{\tan^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] Log[Tan[a + b*x]]/b + Tan[a + b*x]^2/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sec^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 1.33

$$-\frac{-\sec^2(a+bx)-2\log(\sin(a+bx))+2\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*(2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]] - Sec[a + b*x]^2)/b
```

fricas [B] time = 0.52, size = 56, normalized size = 2.07

$$-\frac{\cos(bx+a)^2\log\left(\cos(bx+a)^2\right)-\cos(bx+a)^2\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-1}{2\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(b*x + a)^2*log(cos(b*x + a)^2) - cos(b*x + a)^2*log(-1/4*cos(b*x
+ a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2)
```

giac [B] time = 0.23, size = 124, normalized size = 4.59

$$\frac{\frac{\frac{2(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+3}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^2}+\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-2\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*((2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos
(b*x + a) + 1)^2 + 3)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^2 + log(a
bs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*x + a) - 1
)/(cos(b*x + a) + 1) - 1)))/b
```

maple [A] time = 0.04, size = 26, normalized size = 0.96

$$\frac{1}{2b\cos(bx+a)^2}+\frac{\ln(\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/sin(b*x+a),x)
```

```
[Out] 1/2/b/cos(b*x+a)^2+ln(tan(b*x+a))/b
```

maxima [A] time = 0.48, size = 40, normalized size = 1.48

$$-\frac{\frac{1}{\sin(bx+a)^2-1} + \log\left(\sin\left(bx+a\right)^2 - 1\right) - \log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2
))/b
```

mupad [B] time = 0.10, size = 35, normalized size = 1.30

$$\frac{\frac{\ln\left(\sin(a+b\,x)^2\right)}{2} - \ln\left(\cos\left(a+b\,x\right)\right) + \frac{1}{2\cos(a+b\,x)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)),x)
```

```
[Out] (log(sin(a + b*x)^2)/2 - log(cos(a + b*x)) + 1/(2*cos(a + b*x)^2))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3\left(a+bx\right)}{\sin\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)**3/sin(a + b*x), x)
```

3.129 $\int \csc(a+bx)\sec^4(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -arctanh(cos(b*x+a))/b+sec(b*x+a)/b+1/3*sec(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2622, 302, 207}

$$\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^4,x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Sec[a + b*x]/b + Sec[a + b*x]^3/(3*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc(a+bx)\sec^4(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \sec(a+bx)\right)}{b}$$
$$= \frac{\sec(a+bx)}{b} + \frac{\sec^3(a+bx)}{3b} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cos(a+bx))}{b} + \frac{\sec(a+bx)}{b} + \frac{\sec^3(a+bx)}{3b}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 1.50

$$\frac{\sec^3(a+bx)}{3b} + \frac{\sec(a+bx)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^4,x]
```

```
[Out] -(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + Sec
[a + b*x]^3/(3*b)
```

fricas [A] time = 0.45, size = 67, normalized size = 1.76

$$-\frac{3\cos(bx+a)^3\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) - 3\cos(bx+a)^3\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) - 6\cos(bx+a)^2 - 2}{6\,b\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*cos(b*x + a)^3*log(1/2*cos(b*x + a) + 1/2) - 3*cos(b*x + a)^3*log(-
1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^3)
```

giac [B] time = 0.21, size = 101, normalized size = 2.66

$$\frac{\frac{8\left(\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+2\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3} + 3\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(8*(3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 + 2)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3 + 3*l
og(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.04, size = 47, normalized size = 1.24

$$\frac{1}{3b\cos\left(bx+a\right)^{3}}+\frac{1}{b\cos\left(bx+a\right)}+\frac{\ln\left(\csc\left(bx+a\right)-\cot\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/sin(b*x+a),x)
```

```
[Out] 1/3/b/cos(b*x+a)^3+1/b/cos(b*x+a)+1/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [A] time = 0.31, size = 50, normalized size = 1.32

$$\frac{\frac{2\left(3\cos\left(bx+a\right)^{2}+1\right)}{\cos\left(bx+a\right)^{3}}-3\log\left(\cos\left(bx+a\right)+1\right)+3\log\left(\cos\left(bx+a\right)-1\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(3*cos(b*x + a)^2 + 1)/cos(b*x + a)^3 - 3*log(cos(b*x + a) + 1) + 3*
log(cos(b*x + a) - 1))/b
```

mupad [B] time = 0.39, size = 33, normalized size = 0.87

$$-\frac{\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)-\frac{\cos\left(a+b\,x\right)^{2}+\frac{1}{3}}{\cos\left(a+b\,x\right)^{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^4*sin(a + b*x)),x)
```

```
[Out] -(atanh(cos(a + b*x)) - (cos(a + b*x)^2 + 1/3)/cos(a + b*x)^3)/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{4}\left(a+bx\right)}{\sin\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)**4/sin(a + b*x), x)
```

3.130 $\int \csc(a+bx)\sec^5(a+bx)\,dx$

Optimal. Leaf size=39

$$\frac{\tan^4(a+bx)}{4b}+\frac{\tan^2(a+bx)}{b}+\frac{\log(\tan(a+bx))}{b}$$

```
[Out] ln(tan(b*x+a))/b+tan(b*x+a)^2/b+1/4*tan(b*x+a)^4/b
```

Rubi [A] time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2620, 266, 43}

$$\frac{\tan^4(a+bx)}{4b}+\frac{\tan^2(a+bx)}{b}+\frac{\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^5,x]
```

```
[Out] Log[Tan[a + b*x]]/b + Tan[a + b*x]^2/b + Tan[a + b*x]^4/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \csc(a+bx)\sec^5(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x}\,dx, x, \tan^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x}+x\right)dx, x, \tan^2(a+bx)\right)}{2b}$$
$$= \frac{\log(\tan(a+bx))}{b} + \frac{\tan^2(a+bx)}{b} + \frac{\tan^4(a+bx)}{4b}$$

Mathematica [A] time = 0.08, size = 46, normalized size = 1.18

$$-\frac{-\sec^4(a+bx) - 2\sec^2(a+bx) - 4\log(\sin(a+bx)) + 4\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^5,x]
```

```
[Out] -1/4*(4*Log[Cos[a + b*x]] - 4*Log[Sin[a + b*x]] - 2*Sec[a + b*x]^2 - Sec[a
+ b*x]^4)/b
```

fricas [A] time = 0.46, size = 67, normalized size = 1.72

$$-\frac{2\cos(bx+a)^4\log\left(\cos(bx+a)^2\right) - 2\cos(bx+a)^4\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right) - 2\cos(bx+a)^2 - 1}{4\,b\cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(b*x + a)^4*log(cos(b*x + a)^2) - 2*cos(b*x + a)^4*log(-1/4*cos(
b*x + a)^2 + 1/4) - 2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4)
```

giac [B] time = 0.27, size = 170, normalized size = 4.36

$$\frac{\frac{\frac{52(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{102(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{52(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{25(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+25}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^4} + 6\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) - 12\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right|\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*((52*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 102*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 52*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 25*(c
os(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 25)/((cos(b*x + a) - 1)/(cos(b*x
+ a) + 1) + 1)^4 + 6*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 12
*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

maple [A] time = 0.04, size = 39, normalized size = 1.00

$$\frac{1}{4b\cos(bx+a)^4} + \frac{1}{2b\cos(bx+a)^2} + \frac{\ln(\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5/sin(b*x+a),x)
```

```
[Out] 1/4/b/cos(b*x+a)^4+1/2/b/cos(b*x+a)^2+ln(tan(b*x+a))/b
```

maxima [A] time = 0.55, size = 65, normalized size = 1.67

$$-\frac{\frac{2\sin(bx+a)^2-3}{\sin(bx+a)^4-2\sin(bx+a)^2+1} + 2\log\left(\sin(bx+a)^2-1\right) - 2\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*((2*sin(b*x + a)^2 - 3)/(sin(b*x + a)^4 - 2*sin(b*x + a)^2 + 1) + 2*lo
g(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b
```

mupad [B] time = 0.41, size = 46, normalized size = 1.18

$$\frac{\frac{\ln\left(\sin(a+b\,x)^2\right)}{2} - \ln(\cos(a+b\,x)) + \frac{\frac{\cos(a+b\,x)^2}{2}+\frac{1}{4}}{\cos(a+b\,x)^4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^5*sin(a + b*x)),x)
```

```
[Out] (log(sin(a + b*x)^2)/2 - log(cos(a + b*x)) + (cos(a + b*x)^2/2 + 1/4)/cos(a
 + b*x)^4)/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(a+bx)}{\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)**5/sin(a + b*x), x)
```

3.131 $\int \csc(a+bx)\sec^6(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{\sec^5(a+bx)}{5b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -arctanh(cos(b*x+a))/b+sec(b*x+a)/b+1/3*sec(b*x+a)^3/b+1/5*sec(b*x+a)^5/b
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2622, 302, 207}

$$\frac{\sec^5(a+bx)}{5b}+\frac{\sec^3(a+bx)}{3b}+\frac{\sec(a+bx)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^6,x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b) + Sec[a + b*x]/b + Sec[a + b*x]^3/(3*b) + Sec[a
+ b*x]^5/(5*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc(a+bx)\sec^6(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)dx, x, \sec(a+bx)\right)}{b}$$
$$= \frac{\sec(a+bx)}{b} + \frac{\sec^3(a+bx)}{3b} + \frac{\sec^5(a+bx)}{5b} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{b}$$
$$= -\frac{\tanh^{-1}(\cos(a+bx))}{b} + \frac{\sec(a+bx)}{b} + \frac{\sec^3(a+bx)}{3b} + \frac{\sec^5(a+bx)}{5b}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.36

$$\frac{\sec^5(a+bx)}{5b} + \frac{\sec^3(a+bx)}{3b} + \frac{\sec(a+bx)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^6,x]
```

```
[Out] -(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + Sec
[a + b*x]^3/(3*b) + Sec[a + b*x]^5/(5*b)
```

fricas [A] time = 0.50, size = 77, normalized size = 1.45

$$-\frac{15\cos(bx+a)^5\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) - 15\cos(bx+a)^5\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) - 30\cos(bx+a)^4 - 10\,\cdots}{30\,b\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/30*(15*cos(b*x + a)^5*log(1/2*cos(b*x + a) + 1/2) - 15*cos(b*x + a)^5*lo
g(-1/2*cos(b*x + a) + 1/2) - 30*cos(b*x + a)^4 - 10*cos(b*x + a)^2 - 6)/(b*
cos(b*x + a)^5)
```

giac [B] time = 0.66, size = 145, normalized size = 2.74

$$\frac{\frac{4\left(\frac{70(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{140(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{90(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3} + \frac{45(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4} + 23\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^5} + 15\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/30*(4*(70*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 140*(cos(b*x + a) - 1)^
2/(cos(b*x + a) + 1)^2 + 90*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 45*
(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 23)/((cos(b*x + a) - 1)/(cos(b*
x + a) + 1) + 1)^5 + 15*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/
b
```

maple [A] time = 0.04, size = 60, normalized size = 1.13

$$\frac{1}{5b\cos\left(bx+a\right)^{5}}+\frac{1}{3b\cos\left(bx+a\right)^{3}}+\frac{1}{b\cos\left(bx+a\right)}+\frac{\ln\left(\csc\left(bx+a\right)-\cot\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^6/sin(b*x+a),x)
```

```
[Out] 1/5/b/cos(b*x+a)^5+1/3/b/cos(b*x+a)^3+1/b/cos(b*x+a)+1/b*ln(csc(b*x+a)-cot(
b*x+a))
```

maxima [A] time = 0.46, size = 60, normalized size = 1.13

$$\frac{\frac{2\left(15\cos\left(bx+a\right)^{4}+5\cos\left(bx+a\right)^{2}+3\right)}{\cos\left(bx+a\right)^{5}}-15\log\left(\cos\left(bx+a\right)+1\right)+15\log\left(\cos\left(bx+a\right)-1\right)}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^6/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/30*(2*(15*cos(b*x + a)^4 + 5*cos(b*x + a)^2 + 3)/cos(b*x + a)^5 - 15*log(
cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

mupad [B] time = 0.40, size = 45, normalized size = 0.85

$$\frac{\cos\left(a+b\,x\right)^4+\frac{\cos\left(a+b\,x\right)^2}{3}+\frac{1}{5}}{b\cos\left(a+b\,x\right)^5}-\frac{\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^6*sin(a + b*x)),x)
```

```
[Out] (cos(a + b*x)^2/3 + cos(a + b*x)^4 + 1/5)/(b*cos(a + b*x)^5) - atanh(cos(a
+ b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^6(a+bx)}{\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**6/sin(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)**6/sin(a + b*x), x)
```

3.132 $\int \csc(a+bx)\sec^7(a+bx)\,dx$

Optimal. Leaf size=57

$$\frac{\tan^6(a+bx)}{6b}+\frac{3\tan^4(a+bx)}{4b}+\frac{3\tan^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}$$

```
[Out] ln(tan(b*x+a))/b+3/2*tan(b*x+a)^2/b+3/4*tan(b*x+a)^4/b+1/6*tan(b*x+a)^6/b
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2620, 266, 43}

$$\frac{\tan^6(a+bx)}{6b}+\frac{3\tan^4(a+bx)}{4b}+\frac{3\tan^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sec[a + b*x]^7,x]
```

```
[Out] Log[Tan[a + b*x]]/b + (3*Tan[a + b*x]^2)/(2*b) + (3*Tan[a + b*x]^4)/(4*b) +
 Tan[a + b*x]^6/(6*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \csc(a+bx)\sec^7(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x}\,dx, x, \tan^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x}+3x+x^2\right)dx, x, \tan^2(a+bx)\right)}{2b}$$
$$= \frac{\log(\tan(a+bx))}{b} + \frac{3\tan^2(a+bx)}{2b} + \frac{3\tan^4(a+bx)}{4b} + \frac{\tan^6(a+bx)}{6b}$$

Mathematica [A] time = 0.14, size = 56, normalized size = 0.98

$$-\frac{-2\sec^6(a+bx)-3\sec^4(a+bx)-6\sec^2(a+bx)-12\log(\sin(a+bx))+12\log(\cos(a+bx))}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sec[a + b*x]^7,x]
```

```
[Out] -1/12*(12*Log[Cos[a + b*x]] - 12*Log[Sin[a + b*x]] - 6*Sec[a + b*x]^2 - 3*S
ec[a + b*x]^4 - 2*Sec[a + b*x]^6)/b
```

fricas [A] time = 0.49, size = 77, normalized size = 1.35

$$-\frac{6\cos(bx+a)^6\log\left(\cos(bx+a)^2\right)-6\cos(bx+a)^6\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-6\cos(bx+a)^4-3\cos(bx+}{12\,b\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(b*x + a)^6*log(cos(b*x + a)^2) - 6*cos(b*x + a)^6*log(-1/4*cos
(b*x + a)^2 + 1/4) - 6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 2)/(b*cos(b*x +
a)^6)
```

giac [B] time = 0.34, size = 214, normalized size = 3.75

$$\frac{\frac{\frac{522(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{1485(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{1580(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{1485(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+\frac{522(\cos(bx+a)-1)^5}{(\cos(bx+a)+1)^5}+\frac{147(\cos(bx+a)-1)^6}{(\cos(bx+a)+1)^6}+147}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^6}+30\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/60*((522*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1485*(cos(b*x + a) - 1)^
2/(cos(b*x + a) + 1)^2 + 1580*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 1
485*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 522*(cos(b*x + a) - 1)^5/(c
os(b*x + a) + 1)^5 + 147*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 147)/(
(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^6 + 30*log(abs(-cos(b*x + a) + 1
)/abs(cos(b*x + a) + 1)) - 60*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1
) - 1)))/b
```

maple [A] time = 0.05, size = 52, normalized size = 0.91

$$\frac{1}{6b\cos\left(bx+a\right)^{6}}+\frac{1}{4b\cos\left(bx+a\right)^{4}}+\frac{1}{2b\cos\left(bx+a\right)^{2}}+\frac{\ln\left(\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^7/sin(b*x+a),x)
```

```
[Out] 1/6/b/cos(b*x+a)^6+1/4/b/cos(b*x+a)^4+1/2/b/cos(b*x+a)^2+ln(tan(b*x+a))/b
```

maxima [A] time = 0.40, size = 85, normalized size = 1.49

$$-\frac{\frac{6\,\sin\left(bx+a\right)^{4}-15\,\sin\left(bx+a\right)^{2}+11}{\sin\left(bx+a\right)^{6}-3\,\sin\left(bx+a\right)^{4}+3\,\sin\left(bx+a\right)^{2}-1}+6\,\log\left(\sin\left(bx+a\right)^{2}-1\right)-6\,\log\left(\sin\left(bx+a\right)^{2}\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^7/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/12*((6*sin(b*x + a)^4 - 15*sin(b*x + a)^2 + 11)/(sin(b*x + a)^6 - 3*sin(
b*x + a)^4 + 3*sin(b*x + a)^2 - 1) + 6*log(sin(b*x + a)^2 - 1) - 6*log(sin(
b*x + a)^2))/b
```

mupad [B] time = 0.40, size = 56, normalized size = 0.98

$$\frac{\frac{\ln\left(\sin\left(a+b\,x\right)^{2}\right)}{2}-\ln\left(\cos\left(a+b\,x\right)\right)+\frac{\frac{\cos\left(a+b\,x\right)^{4}}{2}+\frac{\cos\left(a+b\,x\right)^{2}}{4}+\frac{1}{6}}{\cos\left(a+b\,x\right)^{6}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^7*sin(a + b*x)),x)
```

```
[Out] (log(sin(a + b*x)^2)/2 - log(cos(a + b*x)) + (cos(a + b*x)^2/4 + cos(a + b*
x)^4/2 + 1/6)/cos(a + b*x)^6)/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^7(a + bx)}{\sin(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**7/sin(b*x+a),x)
```

```
[Out] Integral(sec(a + b*x)**7/sin(a + b*x), x)
```

3.133 $\int \cos^5(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=50

$$-\frac{\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{b}-\frac{3\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

```
[Out] -csc(b*x+a)/b-3*sin(b*x+a)/b+sin(b*x+a)^3/b-1/5*sin(b*x+a)^5/b
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$-\frac{\sin^5(a+bx)}{5b}+\frac{\sin^3(a+bx)}{b}-\frac{3\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Cot[a + b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]/b) - (3*Sin[a + b*x])/b + Sin[a + b*x]^3/b - Sin[a + b*x]^5/
(5*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cos^5(a+bx)\cot^2(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^2}\,dx, x, -\sin(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(-3+\frac{1}{x^2}+3x^2-x^4\right)dx, x, -\sin(a+bx)\right)}{b}$$
$$= -\frac{\csc(a+bx)}{b} - \frac{3\sin(a+bx)}{b} + \frac{\sin^3(a+bx)}{b} - \frac{\sin^5(a+bx)}{5b}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.00

$$-\frac{\sin^5(a+bx)}{5b} + \frac{\sin^3(a+bx)}{b} - \frac{3\sin(a+bx)}{b} - \frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Cot[a + b*x]^2,x]
[Out] -(Csc[a + b*x]/b) - (3*Sin[a + b*x])/b + Sin[a + b*x]^3/b - Sin[a + b*x]^5/
(5*b)
```

fricas [A] time = 0.42, size = 43, normalized size = 0.86

$$\frac{\cos(bx+a)^6 + 2\cos(bx+a)^4 + 8\cos(bx+a)^2 - 16}{5\,b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/5*(cos(b*x + a)^6 + 2*cos(b*x + a)^4 + 8*cos(b*x + a)^2 - 16)/(b*sin(b*x
+ a))
```

giac [A] time = 0.26, size = 42, normalized size = 0.84

$$-\frac{\sin(bx+a)^5 - 5\sin(bx+a)^3 + \frac{5}{\sin(bx+a)} + 15\sin(bx+a)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^2,x, algorithm="giac")
[Out] -1/5*(sin(b*x + a)^5 - 5*sin(b*x + a)^3 + 5/sin(b*x + a) + 15*sin(b*x + a))
/b
```

maple [A] time = 0.02, size = 62, normalized size = 1.24

$$\frac{-\frac{\cos^8(bx+a)}{\sin(bx+a)} - \left(\frac{16}{5} + \cos^6(bx+a) + \frac{6\left(\cos^4(bx+a)\right)}{5} + \frac{8\left(\cos^2(bx+a)\right)}{5}\right)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/sin(b*x+a)*cos(b*x+a)^8-(16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos
(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.38, size = 42, normalized size = 0.84

$$-\frac{\sin(bx+a)^5 - 5\sin(bx+a)^3 + \frac{5}{\sin(bx+a)} + 15\sin(bx+a)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/5*(sin(b*x + a)^5 - 5*sin(b*x + a)^3 + 5/sin(b*x + a) + 15*sin(b*x + a))
/b
```

mupad [B] time = 0.48, size = 43, normalized size = 0.86

$$-\frac{\sin(a+bx)^6 - 5\sin(a+bx)^4 + 15\sin(a+bx)^2 + 5}{5b\sin(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^7/sin(a + b*x)^2,x)
```

```
[Out] -(15*sin(a + b*x)^2 - 5*sin(a + b*x)^4 + sin(a + b*x)^6 + 5)/(5*b*sin(a + b
*x))
```

sympy [A] time = 8.67, size = 82, normalized size = 1.64

$$\begin{cases} -\frac{16\sin^5(a+bx)}{5b} - \frac{8\sin^3(a+bx)\cos^2(a+bx)}{b} - \frac{6\sin(a+bx)\cos^4(a+bx)}{b} - \frac{\cos^6(a+bx)}{b\sin(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^7(a)}{\sin^2(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-16*sin(a + b*x)**5/(5*b) - 8*sin(a + b*x)**3*cos(a + b*x)**2/b
- 6*sin(a + b*x)*cos(a + b*x)**4/b - cos(a + b*x)**6/(b*sin(a + b*x)), Ne(b
, 0)), (x*cos(a)**7/sin(a)**2, True))
```

3.134 $\int \cos^4(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=61

$$-\frac{15\cot(a+bx)}{8b}+\frac{\cos^4(a+bx)\cot(a+bx)}{4b}+\frac{5\cos^2(a+bx)\cot(a+bx)}{8b}-\frac{15x}{8}$$

```
[Out] -15/8*x-15/8*cot(b*x+a)/b+5/8*cos(b*x+a)^2*cot(b*x+a)/b+1/4*cos(b*x+a)^4*co
t(b*x+a)/b
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 321, 203}

$$-\frac{15\cot(a+bx)}{8b}+\frac{\cos^4(a+bx)\cot(a+bx)}{4b}+\frac{5\cos^2(a+bx)\cot(a+bx)}{8b}-\frac{15x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x]^2,x]
```

```
[Out] (-15*x)/8 - (15*Cot[a + b*x])/(8*b) + (5*Cos[a + b*x]^2*Cot[a + b*x])/(8*b)
 + (Cos[a + b*x]^4*Cot[a + b*x])/(4*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^4(a+bx)\cot^2(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^3}\,dx, x, \cot(a+bx)\right)}{b}$$
$$= \frac{\cos^4(a+bx)\cot(a+bx)}{4b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^2}\,dx, x, \cot(a+bx)\right)}{4b}$$
$$= \frac{5\cos^2(a+bx)\cot(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot(a+bx)}{4b} - \frac{15\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x$$
$$= -\frac{15\cot(a+bx)}{8b} + \frac{5\cos^2(a+bx)\cot(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot(a+bx)}{4b} + \frac{15}{}$$
$$= -\frac{15x}{8} - \frac{15\cot(a+bx)}{8b} + \frac{5\cos^2(a+bx)\cot(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot(a+bx)}{4b}$$

Mathematica [A] time = 0.13, size = 41, normalized size = 0.67

$$-\frac{16\sin(2(a+bx)) + \sin(4(a+bx)) + 32\cot(a+bx) + 60a + 60bx}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x]^2,x]
```

```
[Out] -1/32*(60*a + 60*b*x + 32*Cot[a + b*x] + 16*Sin[2*(a + b*x)] + Sin[4*(a + b
*x)])/b
```

fricas [A] time = 0.45, size = 52, normalized size = 0.85

$$\frac{2\cos\left(bx+a\right)^5 + 5\cos\left(bx+a\right)^3 - 15\,bx\sin\left(bx+a\right) - 15\cos\left(bx+a\right)}{8\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(b*x + a)^5 + 5*cos(b*x + a)^3 - 15*b*x*sin(b*x + a) - 15*cos(b*x
 + a))/(b*sin(b*x + a))
```

giac [A] time = 0.20, size = 55, normalized size = 0.90

$$-\frac{15\,bx + 15\,a + \frac{7\,\tan(bx+a)^3 + 9\,\tan(bx+a)}{\left(\tan(bx+a)^2 + 1\right)^2} + \frac{8}{\tan(bx+a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(15*b*x + 15*a + (7*tan(b*x + a)^3 + 9*tan(b*x + a))/(tan(b*x + a)^2 +
 1)^2 + 8/tan(b*x + a))/b
```

maple [A] time = 0.02, size = 66, normalized size = 1.08

$$\frac{-\frac{\cos^7(bx+a)}{\sin(bx+a)} - \left(\cos^5(bx+a) + \frac{5\left(\cos^3(bx+a)\right)}{4} + \frac{15\cos(bx+a)}{8}\right)\sin(bx+a) - \frac{15bx}{8} - \frac{15a}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/sin(b*x+a)*cos(b*x+a)^7-(cos(b*x+a)^5+5/4*cos(b*x+a)^3+15/8*cos(b*x
+a))*sin(b*x+a)-15/8*b*x-15/8*a)
```

maxima [A] time = 0.59, size = 63, normalized size = 1.03

$$-\frac{15\,bx + 15\,a + \frac{15\,\tan(bx+a)^4 + 25\,\tan(bx+a)^2 + 8}{\tan(bx+a)^5 + 2\,\tan(bx+a)^3 + \tan(bx+a)}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(15*b*x + 15*a + (15*tan(b*x + a)^4 + 25*tan(b*x + a)^2 + 8)/(tan(b*x
+ a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a)))/b
```

mupad [B] time = 0.59, size = 47, normalized size = 0.77

$$-\frac{15\,x}{8} - \frac{\cos\left(a + b\,x\right)^4\,\left(\frac{15\tan(a+b\,x)^4}{8} + \frac{25\tan(a+b\,x)^2}{8} + 1\right)}{b\tan\left(a + b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^6/sin(a + b*x)^2,x)
```

```
[Out] - (15*x)/8 - (cos(a + b*x)^4*((25*tan(a + b*x)^2)/8 + (15*tan(a + b*x)^4)/8
 + 1))/(b*tan(a + b*x))
```

sympy [A] time = 5.34, size = 119, normalized size = 1.95

$$\begin{cases} -\frac{15x\sin^4(a+bx)}{8} - \frac{15x\sin^2(a+bx)\cos^2(a+bx)}{4} - \frac{15x\cos^4(a+bx)}{8} - \frac{15\sin^3(a+bx)\cos(a+bx)}{8b} - \frac{25\sin(a+bx)\cos^3(a+bx)}{8b} - \frac{\cos^5(a+bx)}{b\sin(a+bx)} & \text{f} \\ \frac{x\cos^6(a)}{\sin^2(a)} & \text{c} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-15*x*sin(a + b*x)**4/8 - 15*x*sin(a + b*x)**2*cos(a + b*x)**2/4
 - 15*x*cos(a + b*x)**4/8 - 15*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 25*sin(
a + b*x)*cos(a + b*x)**3/(8*b) - cos(a + b*x)**5/(b*sin(a + b*x)), Ne(b, 0)
), (x*cos(a)**6/sin(a)**2, True))
```

3.135 $\int \cos^3(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\sin^3(a+bx)}{3b}-\frac{2\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

```
[Out] -csc(b*x+a)/b-2*sin(b*x+a)/b+1/3*sin(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$\frac{\sin^3(a+bx)}{3b}-\frac{2\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]/b) - (2*Sin[a + b*x])/b + Sin[a + b*x]^3/(3*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(a+bx)\cot^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,dx,x,-\sin(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)dx,x,-\sin(a+bx)\right)}{b}\\
&= -\frac{\csc(a+bx)}{b}-\frac{2\sin(a+bx)}{b}+\frac{\sin^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{\sin^3(a+bx)}{3b}-\frac{2\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x]^2,x]
[Out] -(Csc[a + b*x]/b) - (2*Sin[a + b*x])/b + Sin[a + b*x]^3/(3*b)
```

fricas [A] time = 0.41, size = 33, normalized size = 0.87

$$\frac{\cos\left(bx+a\right)^4+4\,\cos\left(bx+a\right)^2-8}{3\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/3*(cos(b*x + a)^4 + 4*cos(b*x + a)^2 - 8)/(b*sin(b*x + a))
```

giac [A] time = 0.25, size = 32, normalized size = 0.84

$$\frac{\sin\left(bx+a\right)^3-\frac{3}{\sin(bx+a)}-6\,\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^2,x, algorithm="giac")
[Out] 1/3*(sin(b*x + a)^3 - 3/sin(b*x + a) - 6*sin(b*x + a))/b
```

maple [A] time = 0.02, size = 52, normalized size = 1.37

$$\frac{-\frac{\cos^6(bx+a)}{\sin(bx+a)}-\left(\frac{8}{3}+\cos^4\left(bx+a\right)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5/sin(b*x+a)^2,x)
[Out] 1/b*(-cos(b*x+a)^6/sin(b*x+a)-(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a
))
```

maxima [A] time = 0.37, size = 32, normalized size = 0.84

$$\frac{\sin(bx+a)^3 - \frac{3}{\sin(bx+a)} - 6\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^2,x, algorithm="maxima")
[Out] 1/3*(sin(b*x + a)^3 - 3/sin(b*x + a) - 6*sin(b*x + a))/b
```

mupad [B] time = 0.45, size = 35, normalized size = 0.92

$$-\frac{-\sin(a+bx)^4 + 6\sin(a+bx)^2 + 3}{3b\sin(a+bx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5/sin(a + b*x)^2,x)
[Out] -(6*sin(a + b*x)^2 - sin(a + b*x)^4 + 3)/(3*b*sin(a + b*x))
```

sympy [A] time = 3.25, size = 61, normalized size = 1.61

$$\begin{cases} -\frac{8\sin^3(a+bx)}{3b} - \frac{4\sin(a+bx)\cos^2(a+bx)}{b} - \frac{\cos^4(a+bx)}{b\sin(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^5(a)}{\sin^2(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a)**2,x)
[Out] Piecewise((-8*sin(a + b*x)**3/(3*b) - 4*sin(a + b*x)*cos(a + b*x)**2/b - co
s(a + b*x)**4/(b*sin(a + b*x)), Ne(b, 0)), (x*cos(a)**5/sin(a)**2, True))
```

3.136 $\int \cos^2(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=40

$$-\frac{3\cot(a+bx)}{2b}+\frac{\cos^2(a+bx)\cot(a+bx)}{2b}-\frac{3x}{2}$$

```
[Out] -3/2*x-3/2*cot(b*x+a)/b+1/2*cos(b*x+a)^2*cot(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 321, 203}

$$-\frac{3\cot(a+bx)}{2b}+\frac{\cos^2(a+bx)\cot(a+bx)}{2b}-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x]^2,x]
```

```
[Out] (-3*x)/2 - (3*Cot[a + b*x])/(2*b) + (Cos[a + b*x]^2*Cot[a + b*x])/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^2(a+bx)\cot^2(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \cot(a+bx)\right)}{b}$$

$$= \frac{\cos^2(a+bx)\cot(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \cot(a+bx)\right)}{2b}$$

$$= -\frac{3\cot(a+bx)}{2b} + \frac{\cos^2(a+bx)\cot(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \cot(a+bx)\right)}{2b}$$

$$= -\frac{3x}{2} - \frac{3\cot(a+bx)}{2b} + \frac{\cos^2(a+bx)\cot(a+bx)}{2b}$$

Mathematica [A] time = 0.12, size = 31, normalized size = 0.78

$$-\frac{6(a+bx)+\sin(2(a+bx))+4\cot(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x]^2,x]
[Out] -1/4*(6*(a + b*x) + 4*Cot[a + b*x] + Sin[2*(a + b*x)])/b
```

fricas [A] time = 0.43, size = 40, normalized size = 1.00

$$\frac{\cos(bx+a)^3 - 3\,bx\sin(bx+a) - 3\cos(bx+a)}{2\,b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(cos(b*x + a)^3 - 3*b*x*sin(b*x + a) - 3*cos(b*x + a))/(b*sin(b*x + a))
```

giac [A] time = 0.17, size = 43, normalized size = 1.08

$$-\frac{3\,bx + 3\,a + \frac{3\tan(bx+a)^2+2}{\tan(bx+a)^3+\tan(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b*x + 3*a + (3*tan(b*x + a)^2 + 2)/(tan(b*x + a)^3 + tan(b*x + a)))
/b
```

maple [A] time = 0.02, size = 56, normalized size = 1.40

$$\frac{-\frac{\cos^5(bx+a)}{\sin(bx+a)}-\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)-\frac{3bx}{2}-\frac{3a}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-cos(b*x+a)^5/sin(b*x+a)-(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-3/2*
b*x-3/2*a)
```

maxima [A] time = 0.85, size = 43, normalized size = 1.08

$$-\frac{3\,bx+3\,a+\frac{3\,\tan(bx+a)^2+2}{\tan(bx+a)^3+\tan(bx+a)}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x + 3*a + (3*tan(b*x + a)^2 + 2)/(tan(b*x + a)^3 + tan(b*x + a)))
/b
```

mupad [B] time = 0.57, size = 43, normalized size = 1.08

$$-\frac{9\,\cos(a+b\,x)-\cos(3\,a+3\,b\,x)+12\,b\,x\,\sin(a+b\,x)}{8\,b\,\sin(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4/sin(a + b*x)^2,x)
```

```
[Out] -(9*cos(a + b*x) - cos(3*a + 3*b*x) + 12*b*x*sin(a + b*x))/(8*b*sin(a + b*x
))
```

sympy [A] time = 1.77, size = 75, normalized size = 1.88

$$\begin{cases} -\frac{3x\sin^2(a+bx)}{2}-\frac{3x\cos^2(a+bx)}{2}-\frac{3\sin(a+bx)\cos(a+bx)}{2b}-\frac{\cos^3(a+bx)}{b\sin(a+bx)} & \text{for } b\neq 0 \\ \frac{x\cos^4(a)}{\sin^2(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-3*x*sin(a + b*x)**2/2 - 3*x*cos(a + b*x)**2/2 - 3*sin(a + b*x)*
cos(a + b*x)/(2*b) - cos(a + b*x)**3/(b*sin(a + b*x)), Ne(b, 0)), (x*cos(a)
**4/sin(a)**2, True))
```

3.137 $\int \cos(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=23

$$-\frac{\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

```
[Out] -csc(b*x+a)/b-sin(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 14}

$$-\frac{\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -(Csc[a + b*x]/b) - Sin[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cot^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -\sin(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, -\sin(a+bx)\right)}{b}\\ &= -\frac{\csc(a+bx)}{b}-\frac{\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{\sin(a+bx)}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x]^2,x]
[Out] -(Csc[a + b*x]/b) - Sin[a + b*x]/b
```

fricas [A] time = 0.42, size = 22, normalized size = 0.96

$$\frac{\cos\left(bx+a\right)^2-2}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^2,x, algorithm="fricas")
[Out] (cos(b*x + a)^2 - 2)/(b*sin(b*x + a))
```

giac [A] time = 0.23, size = 20, normalized size = 0.87

$$-\frac{\frac{1}{\sin(bx+a)}+\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^2,x, algorithm="giac")
[Out] -(1/sin(b*x + a) + sin(b*x + a))/b
```

maple [A] time = 0.02, size = 42, normalized size = 1.83

$$\frac{-\frac{\cos^4(bx+a)}{\sin(bx+a)}-\left(2+\cos^2\left(bx+a\right)\right)\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(b*x+a)^2,x)
[Out] 1/b*(-cos(b*x+a)^4/sin(b*x+a)-(2+cos(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.39, size = 20, normalized size = 0.87

$$-\frac{\frac{1}{\sin(bx+a)}+\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(1/sin(b*x + a) + sin(b*x + a))/b
```

mupad [B] time = 0.45, size = 23, normalized size = 1.00

$$-\frac{\sin(a+b\,x)^2+1}{b\,\sin(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(a + b*x)^2,x)
```

```
[Out] -(sin(a + b*x)^2 + 1)/(b*sin(a + b*x))
```

sympy [A] time = 1.40, size = 39, normalized size = 1.70

$$\begin{cases} -\frac{2\sin(a+bx)}{b} - \frac{\cos^2(a+bx)}{b\sin(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^3(a)}{\sin^2(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-2*sin(a + b*x)/b - cos(a + b*x)**2/(b*sin(a + b*x)), Ne(b, 0)),
 (x*cos(a)**3/sin(a)**2, True))
```

3.138 $\int \cot^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cot(a+bx)}{b}-x$$

```
[Out] -x-cot(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 8}

$$-\frac{\cot(a+bx)}{b}-x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^2,x]
```

```
[Out] -x - Cot[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^2(a+bx)\,dx &= -\frac{\cot(a+bx)}{b}-\int 1\,dx\\ &= -x-\frac{\cot(a+bx)}{b}\end{aligned}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 1.93

$$-\frac{\cot(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^2,x]
[Out] -((Cot[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + b*x]^2])/b)
```

fricas [A] time = 0.44, size = 29, normalized size = 1.93

$$-\frac{bx\sin(bx+a)+\cos(bx+a)}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^2,x, algorithm="fricas")
[Out] -(b*x*sin(b*x + a) + cos(b*x + a))/(b*sin(b*x + a))
```

giac [B] time = 0.18, size = 35, normalized size = 2.33

$$-\frac{2\,bx+2\,a+\frac{1}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}-\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^2,x, algorithm="giac")
[Out] -1/2*(2*b*x + 2*a + 1/tan(1/2*b*x + 1/2*a) - tan(1/2*b*x + 1/2*a))/b
```

maple [A] time = 0.02, size = 21, normalized size = 1.40

$$\frac{-\cot(bx+a)-bx-a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(b*x+a)^2,x)
[Out] 1/b*(-cot(b*x+a)-b*x-a)
```

maxima [A] time = 0.63, size = 18, normalized size = 1.20

$$-\frac{bx+a+\frac{1}{\tan(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^2,x, algorithm="maxima")
[Out] -(b*x + a + 1/tan(b*x + a))/b
```

mupad [B] time = 0.41, size = 15, normalized size = 1.00

$$-x-\frac{\cot\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(a + b*x)^2,x)
```

```
[Out] - x - cot(a + b*x)/b
```

sympy [A] time = 1.08, size = 29, normalized size = 1.93

$$\begin{cases} -x-\frac{\cos\left(a+bx\right)}{b\sin\left(a+bx\right)} & \text{for}\ b\neq 0 \\ \frac{x\cos^{2}\left(a\right)}{\sin^{2}\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-x - cos(a + b*x)/(b*sin(a + b*x)), Ne(b, 0)), (x*cos(a)**2/sin(
a)**2, True))
```

3.139 $\int \cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\csc(a+bx)}{b}$$

```
[Out] -csc(b*x+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 8}

$$-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -(Csc[a + b*x]/b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot(a+bx)\csc(a+bx)\,dx &= -\frac{\text{Subst}\left(\int 1\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{\csc(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]*Csc[a + b*x],x]
[Out] -(Csc[a + b*x]/b)
```

fricas [A] time = 0.45, size = 13, normalized size = 1.18

$$-\frac{1}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^2,x, algorithm="fricas")
[Out] -1/(b*sin(b*x + a))
```

giac [A] time = 0.59, size = 13, normalized size = 1.18

$$-\frac{1}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^2,x, algorithm="giac")
[Out] -1/(b*sin(b*x + a))
```

maple [A] time = 0.00, size = 14, normalized size = 1.27

$$-\frac{1}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+a)^2,x)
[Out] -1/b/sin(b*x+a)
```

maxima [A] time = 0.44, size = 13, normalized size = 1.18

$$-\frac{1}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^2,x, algorithm="maxima")
[Out] -1/(b*sin(b*x + a))
```

mupad [B] time = 0.42, size = 13, normalized size = 1.18

$$-\frac{1}{b\,\sin\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(a + b*x)^2,x)
```

```
[Out] -1/(b*sin(a + b*x))
```

sympy [A] time = 1.02, size = 20, normalized size = 1.82

$$\begin{cases} -\frac{1}{b\sin\left(a+bx\right)} & \text{for}\ b \neq 0 \\ \frac{x\cos\left(a\right)}{\sin^{2}\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)**2,x)
```

```
[Out] Piecewise((-1/(b*sin(a + b*x)), Ne(b, 0)), (x*cos(a)/sin(a)**2, True))
```

3.140 $\int \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}$$

[Out] arctanh(sin(b*x+a))/b-csc(b*x+a)/b

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2621, 321, 207}

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b*x]^2*Sec[a + b*x],x]

[Out] ArcTanh[Sin[a + b*x]]/b - Csc[a + b*x]/b

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^2(a+bx)\sec(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{b}$$
$$= -\frac{\csc(a+bx)}{b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{b} - \frac{\csc(a+bx)}{b}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 1.17

$$-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x],x]
[Out] -((Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b)
```

fricas [B] time = 0.46, size = 50, normalized size = 2.17

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right) - \log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right) - 2}{2\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^2,x, algorithm="fricas")
[Out] 1/2*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
a) - 2)/(b*sin(b*x + a))
```

giac [A] time = 0.34, size = 38, normalized size = 1.65

$$-\frac{\frac{2}{\sin(bx+a)} - \log\left(\left|\sin\left(bx+a\right)+1\right|\right) + \log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^2,x, algorithm="giac")
[Out] -1/2*(2/sin(b*x + a) - log(abs(sin(b*x + a) + 1)) + log(abs(sin(b*x + a) -
1)))/b
```

maple [A] time = 0.03, size = 33, normalized size = 1.43

$$-\frac{1}{b\sin\left(bx+a\right)}+\frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/sin(b*x+a)^2,x)
[Out] -1/b/sin(b*x+a)+1/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.41, size = 36, normalized size = 1.57

$$-\frac{\frac{2}{\sin(bx+a)}-\log\left(\sin\left(bx+a\right)+1\right)+\log\left(\sin\left(bx+a\right)-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^2,x, algorithm="maxima")
[Out] -1/2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b
```

mupad [B] time = 0.02, size = 22, normalized size = 0.96

$$\frac{\operatorname{atanh}\left(\sin\left(a+b\,x\right)\right)-\frac{1}{\sin(a+b\,x)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^2),x)
[Out] (atanh(sin(a + b*x)) - 1/sin(a + b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(a+bx\right)}{\sin^{2}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)**2,x)
[Out] Integral(sec(a + b*x)/sin(a + b*x)**2, x)
```

3.141 $\int \csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=22

$$\frac{\tan(a+bx)}{b}-\frac{\cot(a+bx)}{b}$$

```
[Out] -cot(b*x+a)/b+tan(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 14}

$$\frac{\tan(a+bx)}{b}-\frac{\cot(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] -(Cot[a + b*x]/b) + Tan[a + b*x]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sec^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx, x, \tan(a+bx)\right)}{b}\\ &= -\frac{\cot(a+bx)}{b}+\frac{\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.59

$$-\frac{2\cot(2(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (-2*Cot[2*(a + b*x)])/b
```

fricas [A] time = 0.41, size = 33, normalized size = 1.50

$$-\frac{2\cos\left(bx+a\right)^{2}-1}{b\cos\left(bx+a\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)*sin(b*x + a))
```

giac [A] time = 0.70, size = 16, normalized size = 0.73

$$-\frac{2}{b\tan\left(2\,bx+2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/(b*tan(2*b*x + 2*a))
```

maple [A] time = 0.06, size = 31, normalized size = 1.41

$$\frac{\frac{1}{\sin(bx+a)\cos(bx+a)}-2\cot\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/sin(b*x+a)/cos(b*x+a)-2*cot(b*x+a))
```

maxima [A] time = 0.49, size = 22, normalized size = 1.00

$$-\frac{\frac{1}{\tan(bx+a)}-\tan\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(1/tan(b*x + a) - tan(b*x + a))/b
```

mupad [B] time = 0.39, size = 14, normalized size = 0.64

$$-\frac{2\cot(2a+2bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

```
[Out] -(2*cot(2*a + 2*b*x))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)}{\sin^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a)**2,x)
```

```
[Out] Integral(sec(a + b*x)**2/sin(a + b*x)**2, x)
```

3.142 $\int \csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\csc(a+bx)}{2b}+\frac{3\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\csc(a+bx)\sec^2(a+bx)}{2b}$$

```
[Out] 3/2*arctanh(sin(b*x+a))/b-3/2*csc(b*x+a)/b+1/2*csc(b*x+a)*sec(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 321, 207}

$$-\frac{3\csc(a+bx)}{2b}+\frac{3\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\csc(a+bx)\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(2*b) - (3*Csc[a + b*x])/(2*b) + (Csc[a + b*x]*Se
c[a + b*x]^2)/(2*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^2(a+bx)\sec^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \csc(a+bx)\right)}{b}$$
$$= \frac{\csc(a+bx)\sec^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}$$
$$= -\frac{3\csc(a+bx)}{2b} + \frac{\csc(a+bx)\sec^2(a+bx)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx\right.}{2b}$$
$$= \frac{3\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3\csc(a+bx)}{2b} + \frac{\csc(a+bx)\sec^2(a+bx)}{2b}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.55

$$-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\sin^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] -((Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b)
```

fricas [A] time = 0.44, size = 85, normalized size = 1.73

$$\frac{3\cos(bx+a)^2\log(\sin(bx+a)+1)\sin(bx+a) - 3\cos(bx+a)^2\log(-\sin(bx+a)+1)\sin(bx+a) - 6\cos(b}{4\,b\cos(bx+a)^2\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*cos(b*x + a)^2
*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 2)/(b*cos(b*x + a
)^2*sin(b*x + a))
```

giac [A] time = 0.35, size = 63, normalized size = 1.29

$$-\frac{\frac{2\,\left(3\,\sin\left(bx+a\right)^{2}-2\right)}{\sin\left(bx+a\right)^{3}-\sin\left(bx+a\right)}-3\,\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+3\,\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(abs(
sin(b*x + a) + 1)) + 3*log(abs(sin(b*x + a) - 1)))/b
```

maple [A] time = 0.04, size = 55, normalized size = 1.12

$$\frac{1}{2b\sin\left(bx+a\right)\cos\left(bx+a\right)^{2}}-\frac{3}{2b\sin\left(bx+a\right)}+\frac{3\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/sin(b*x+a)^2,x)
```

```
[Out] 1/2/b/sin(b*x+a)/cos(b*x+a)^2-3/2/b/sin(b*x+a)+3/2/b*ln(sec(b*x+a)+tan(b*x+
a))
```

maxima [A] time = 0.42, size = 61, normalized size = 1.24

$$-\frac{\frac{2\,\left(3\,\sin\left(bx+a\right)^{2}-2\right)}{\sin\left(bx+a\right)^{3}-\sin\left(bx+a\right)}-3\,\log\left(\sin\left(bx+a\right)+1\right)+3\,\log\left(\sin\left(bx+a\right)-1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(
b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b
```

mupad [B] time = 0.43, size = 48, normalized size = 0.98

$$\frac{3\,\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{2\,b}+\frac{\frac{3\,\sin\left(a+b\,x\right)^2}{2}-1}{b\,\left(\sin\left(a+b\,x\right)-\sin\left(a+b\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

```
[Out] (3*atanh(sin(a + b*x)))/(2*b) + ((3*sin(a + b*x)^2)/2 - 1)/(b*(sin(a + b*x)
 - sin(a + b*x)^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(a+bx)}{\sin^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a)**2,x)
```

```
[Out] Integral(sec(a + b*x)**3/sin(a + b*x)**2, x)
```

3.143 $\int \csc^2(a+bx)\sec^4(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\tan^3(a+bx)}{3b}+\frac{2\tan(a+bx)}{b}-\frac{\cot(a+bx)}{b}$$

```
[Out] -cot(b*x+a)/b+2*tan(b*x+a)/b+1/3*tan(b*x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 270}

$$\frac{\tan^3(a+bx)}{3b}+\frac{2\tan(a+bx)}{b}-\frac{\cot(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x]^4,x]
```

```
[Out] -(Cot[a + b*x]/b) + (2*Tan[a + b*x])/b + Tan[a + b*x]^3/(3*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(a+bx)\sec^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2}\,dx, x, \tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx, x, \tan(a+bx)\right)}{b}\\
&= -\frac{\cot(a+bx)}{b}+\frac{2\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.21

$$\frac{5\tan(a+bx)}{3b}-\frac{\cot(a+bx)}{b}+\frac{\tan(a+bx)\sec^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x]^4,x]
```

```
[Out] -(Cot[a + b*x]/b) + (5*Tan[a + b*x])/(3*b) + (Sec[a + b*x]^2*Tan[a + b*x])/
(3*b)
```

fricas [A] time = 0.41, size = 43, normalized size = 1.13

$$-\frac{8\cos(bx+a)^4-4\cos(bx+a)^2-1}{3\,b\cos(bx+a)^3\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(8*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3*sin(b*x +
a))
```

giac [A] time = 0.19, size = 32, normalized size = 0.84

$$\frac{\tan(bx+a)^3-\frac{3}{\tan(bx+a)}+6\tan(bx+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(tan(b*x + a)^3 - 3/tan(b*x + a) + 6*tan(b*x + a))/b
```

maple [A] time = 0.04, size = 50, normalized size = 1.32

$$\frac{\frac{1}{3\sin(bx+a)\cos(bx+a)^3}+\frac{4}{3\sin(bx+a)\cos(bx+a)}-\frac{8\cot(bx+a)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/3/sin(b*x+a)/cos(b*x+a)^3+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+a))
```

maxima [A] time = 0.47, size = 32, normalized size = 0.84

$$\frac{\tan\left(bx+a\right)^{3}-\frac{3}{\tan\left(bx+a\right)}+6\,\tan\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(b*x + a)^3 - 3/tan(b*x + a) + 6*tan(b*x + a))/b
```

mupad [B] time = 0.41, size = 33, normalized size = 0.87

$$\frac{\tan\left(a+b\,x\right)^{4}+6\tan\left(a+b\,x\right)^{2}-3}{3\,b\tan\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^4*sin(a + b*x)^2),x)
```

```
[Out] (6*tan(a + b*x)^2 + tan(a + b*x)^4 - 3)/(3*b*tan(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(a+bx\right)}{\sin^{2}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a)**2,x)
```

```
[Out] Integral(sec(a + b*x)**4/sin(a + b*x)**2, x)
```

3.144 $\int \csc^2(a+bx)\sec^5(a+bx)\,dx$

Optimal. Leaf size=70

$$-\frac{15\csc(a+bx)}{8b}+\frac{15\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\csc(a+bx)\sec^4(a+bx)}{4b}+\frac{5\csc(a+bx)\sec^2(a+bx)}{8b}$$

```
[Out] 15/8*arctanh(sin(b*x+a))/b-15/8*csc(b*x+a)/b+5/8*csc(b*x+a)*sec(b*x+a)^2/b+
1/4*csc(b*x+a)*sec(b*x+a)^4/b
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 321, 207}

$$-\frac{15\csc(a+bx)}{8b}+\frac{15\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\csc(a+bx)\sec^4(a+bx)}{4b}+\frac{5\csc(a+bx)\sec^2(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sec[a + b*x]^5,x]
```

```
[Out] (15*ArcTanh[Sin[a + b*x]])/(8*b) - (15*Csc[a + b*x])/(8*b) + (5*Csc[a + b*x
]*Sec[a + b*x]^2)/(8*b) + (Csc[a + b*x]*Sec[a + b*x]^4)/(4*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^2(a+bx)\sec^5(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \csc(a+bx)\right)}{b}$$

$$= \frac{\csc(a+bx)\sec^4(a+bx)}{4b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{4b}$$

$$= \frac{5\csc(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc(a+bx)\sec^4(a+bx)}{4b} - \frac{15\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx\right.}{8b}$$

$$= -\frac{15\csc(a+bx)}{8b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc(a+bx)\sec^4(a+bx)}{4b} - \frac{15}{}$$

$$= \frac{15\tanh^{-1}(\sin(a+bx))}{8b} - \frac{15\csc(a+bx)}{8b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{8b} + \csc($$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.39

$$-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sec[a + b*x]^5,x]
[Out] -((Csc[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[a + b*x]^2])/b)
```

fricas [A] time = 0.45, size = 95, normalized size = 1.36

$$\frac{15\cos(bx+a)^4\log(\sin(bx+a)+1)\sin(bx+a) - 15\cos(bx+a)^4\log(-\sin(bx+a)+1)\sin(bx+a) - 30\,c}{16\,b\cos(bx+a)^4\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(15*cos(b*x + a)^4*log(sin(b*x + a) + 1)*sin(b*x + a) - 15*cos(b*x + a
)^4*log(-sin(b*x + a) + 1)*sin(b*x + a) - 30*cos(b*x + a)^4 + 10*cos(b*x +
a)^2 + 4)/(b*cos(b*x + a)^4*sin(b*x + a))
```

giac [A] time = 0.69, size = 73, normalized size = 1.04

$$-\frac{\frac{2\left(7\,\sin(bx+a)^{3}-9\,\sin(bx+a)\right)}{\left(\sin(bx+a)^{2}-1\right)^{2}}+\frac{16}{\sin(bx+a)}-15\,\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+15\,\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*(7*sin(b*x + a)^3 - 9*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 16/si
n(b*x + a) - 15*log(abs(sin(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) - 1)))
/b
```

maple [A] time = 0.04, size = 76, normalized size = 1.09

$$\frac{1}{4b\sin\left(bx+a\right)\cos\left(bx+a\right)^{4}}+\frac{5}{8b\sin\left(bx+a\right)\cos\left(bx+a\right)^{2}}-\frac{15}{8b\sin\left(bx+a\right)}+\frac{15\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5/sin(b*x+a)^2,x)
```

```
[Out] 1/4/b/sin(b*x+a)/cos(b*x+a)^4+5/8/b/sin(b*x+a)/cos(b*x+a)^2-15/8/b/sin(b*x+
a)+15/8/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.44, size = 79, normalized size = 1.13

$$-\frac{\frac{2\left(15\,\sin(bx+a)^{4}-25\,\sin(bx+a)^{2}+8\right)}{\sin(bx+a)^{5}-2\,\sin(bx+a)^{3}+\sin(bx+a)}-15\,\log\left(\sin\left(bx+a\right)+1\right)+15\,\log\left(\sin\left(bx+a\right)-1\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(15*sin(b*x + a)^4 - 25*sin(b*x + a)^2 + 8)/(sin(b*x + a)^5 - 2*si
n(b*x + a)^3 + sin(b*x + a)) - 15*log(sin(b*x + a) + 1) + 15*log(sin(b*x +
a) - 1))/b
```

mupad [B] time = 0.09, size = 67, normalized size = 0.96

$$\frac{15\,\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{8\,b}-\frac{\frac{15\sin(a+b\,x)^4}{8}-\frac{25\sin(a+b\,x)^2}{8}+1}{b\,\left(\sin\left(a+b\,x\right)^5-2\sin\left(a+b\,x\right)^3+\sin\left(a+b\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^5*sin(a + b*x)^2),x)
```

```
[Out] (15*atanh(sin(a + b*x)))/(8*b) - ((15*sin(a + b*x)^4)/8 - (25*sin(a + b*x)^
2)/8 + 1)/(b*(sin(a + b*x) - 2*sin(a + b*x)^3 + sin(a + b*x)^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(a+bx)}{\sin^2(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a)**2,x)
```

```
[Out] Integral(sec(a + b*x)**5/sin(a + b*x)**2, x)
```

3.145 $\int \cos^4(a+bx)\cot^3(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{\sin^4(a+bx)}{4b}+\frac{3\sin^2(a+bx)}{2b}-\frac{\csc^2(a+bx)}{2b}-\frac{3\log(\sin(a+bx))}{b}$$

```
[Out] -1/2*csc(b*x+a)^2/b-3*ln(sin(b*x+a))/b+3/2*sin(b*x+a)^2/b-1/4*sin(b*x+a)^4/
b
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$-\frac{\sin^4(a+bx)}{4b}+\frac{3\sin^2(a+bx)}{2b}-\frac{\csc^2(a+bx)}{2b}-\frac{3\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x]^3,x]
```

```
[Out] -Csc[a + b*x]^2/(2*b) - (3*Log[Sin[a + b*x]])/b + (3*Sin[a + b*x]^2)/(2*b)
- Sin[a + b*x]^4/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cos^4(a+bx)\cot^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^3}\,dx, x, -\sin(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x^2}\,dx, x, \sin^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}-\frac{3}{x}-x\right)dx, x, \sin^2(a+bx)\right)}{2b}$$
$$= -\frac{\csc^2(a+bx)}{2b} - \frac{3\log(\sin(a+bx))}{b} + \frac{3\sin^2(a+bx)}{2b} - \frac{\sin^4(a+bx)}{4b}$$

Mathematica [A] time = 0.10, size = 45, normalized size = 0.78

$$-\frac{\sin^4(a+bx)-6\sin^2(a+bx)+2\csc^2(a+bx)+12\log(\sin(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x]^3,x]
```

```
[Out] -1/4*(2*Csc[a + b*x]^2 + 12*Log[Sin[a + b*x]] - 6*Sin[a + b*x]^2 + Sin[a + 
b*x]^4)/b
```

fricas [A] time = 0.47, size = 71, normalized size = 1.22

$$-\frac{8\cos(bx+a)^6+24\cos(bx+a)^4-51\cos(bx+a)^2+96\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\sin(bx+a)\right)+3}{32\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*(8*cos(b*x + a)^6 + 24*cos(b*x + a)^4 - 51*cos(b*x + a)^2 + 96*(cos(b
*x + a)^2 - 1)*log(1/2*sin(b*x + a)) + 3)/(b*cos(b*x + a)^2 - b)
```

giac [B] time = 0.26, size = 231, normalized size = 3.98

$$\frac{\frac{\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}+1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1} + \frac{\cos(bx+a)-1}{\cos(bx+a)+1} + \frac{2\left(\frac{76(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{118(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{76(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{25(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-25\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^4} - 12\log\left(\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(cos
(b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 2*(76*(cos(b*x + a
) - 1)/(cos(b*x + a) + 1) - 118*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 +
 76*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 25*(cos(b*x + a) - 1)^4/(co
s(b*x + a) + 1)^4 - 25)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^4 - 12*
log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 24*log(abs(-(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

maple [A] time = 0.02, size = 74, normalized size = 1.28

$$-\frac{\cos^{8}(bx+a)}{2b\sin(bx+a)^{2}}-\frac{\cos^{6}(bx+a)}{2b}-\frac{3\left(\cos^{4}(bx+a)\right)}{4b}-\frac{3\left(\cos^{2}(bx+a)\right)}{2b}-\frac{3\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7/sin(b*x+a)^3,x)
```

```
[Out] -1/2/b/sin(b*x+a)^2*cos(b*x+a)^8-1/2*cos(b*x+a)^6/b-3/4*cos(b*x+a)^4/b-3/2*
cos(b*x+a)^2/b-3*ln(sin(b*x+a))/b
```

maxima [A] time = 0.53, size = 45, normalized size = 0.78

$$-\frac{\sin(bx+a)^{4}-6\,\sin(bx+a)^{2}+\frac{2}{\sin(bx+a)^{2}}+6\,\log\left(\sin(bx+a)^{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(sin(b*x + a)^4 - 6*sin(b*x + a)^2 + 2/sin(b*x + a)^2 + 6*log(sin(b*x
+ a)^2))/b
```

mupad [B] time = 0.69, size = 85, normalized size = 1.47

$$\frac{3\,\ln\left(\tan(a+b\,x)^{2}+1\right)}{2\,b}-\frac{3\,\ln(\tan(a+b\,x))}{b}-\frac{\frac{3\tan(a+b\,x)^{4}}{2}+\frac{9\tan(a+b\,x)^{2}}{4}+\frac{1}{2}}{b\,\left(\tan(a+b\,x)^{6}+2\tan(a+b\,x)^{4}+\tan(a+b\,x)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^7/sin(a + b*x)^3,x)
```

```
[Out] (3*log(tan(a + b*x)^2 + 1))/(2*b) - (3*log(tan(a + b*x)))/b - ((9*tan(a + b
*x)^2)/4 + (3*tan(a + b*x)^4)/2 + 1/2)/(b*(tan(a + b*x)^2 + 2*tan(a + b*x)^
4 + tan(a + b*x)^6))
```

sympy [A] time = 12.97, size = 1484, normalized size = 25.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((24*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**10/(8*b*tan(a/
2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b
*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 96*log(tan(a/2 + b*x/2)**
2 + 1)*tan(a/2 + b*x/2)**8/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2
)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 +
b*x/2)**2) + 144*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(8*b*tan(
a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32
*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 96*log(tan(a/2 + b*x/2)
**2 + 1)*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x
/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2
+ b*x/2)**2) + 24*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(8*b*tan
(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 3
2*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 24*log(tan(a/2 + b*x/2
))*tan(a/2 + b*x/2)**10/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**
8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x
/2)**2) - 96*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(8*b*tan(a/2 + b*x/2
)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2
+ b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 144*log(tan(a/2 + b*x/2))*tan(a/2
+ b*x/2)**6/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan
(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 96
*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**10 + 32*b
*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4
+ 8*b*tan(a/2 + b*x/2)**2) - 24*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(
8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 + b*x/2)
**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - tan(a/2 + b*x/2
)**12/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*tan(a/2 +
 b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 57*tan(a
/2 + b*x/2)**8/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 + 48*b*
tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) +
 80*tan(a/2 + b*x/2)**6/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**
8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x
/2)**2) + 57*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 +
 b*x/2)**8 + 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(
a/2 + b*x/2)**2) - 1/(8*b*tan(a/2 + b*x/2)**10 + 32*b*tan(a/2 + b*x/2)**8 +
```

```
 48*b*tan(a/2 + b*x/2)**6 + 32*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)
**2), Ne(b, 0)), (x*cos(a)**7/sin(a)**3, True))
```

3.146 $\int \cos^3(a+bx)\cot^3(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\cos^3(a+bx)}{6b}-\frac{5\cos(a+bx)}{2b}-\frac{\cos^3(a+bx)\cot^2(a+bx)}{2b}+\frac{5\tanh^{-1}(\cos(a+bx))}{2b}$$

```
[Out] 5/2*arctanh(cos(b*x+a))/b-5/2*cos(b*x+a)/b-5/6*cos(b*x+a)^3/b-1/2*cos(b*x+a
)^3*cot(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2592, 288, 302, 206}

$$-\frac{5\cos^3(a+bx)}{6b}-\frac{5\cos(a+bx)}{2b}-\frac{\cos^3(a+bx)\cot^2(a+bx)}{2b}+\frac{5\tanh^{-1}(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x]^3,x]
```

```
[Out] (5*ArcTanh[Cos[a + b*x]])/(2*b) - (5*Cos[a + b*x])/(2*b) - (5*Cos[a + b*x]^
3)/(6*b) - (Cos[a + b*x]^3*Cot[a + b*x]^2)/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \cos^3(a+bx)\cot^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\cos^3(a+bx)\cot^2(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}$$
$$= -\frac{\cos^3(a+bx)\cot^2(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \left(-1-x^2+\frac{1}{1-x^2}\right)dx, x, \cos(a+bx)\right)}{2b}$$
$$= -\frac{5\cos(a+bx)}{2b} - \frac{5\cos^3(a+bx)}{6b} - \frac{\cos^3(a+bx)\cot^2(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \frac{1}{1-x^{}}\right.}{}$$
$$= \frac{5\tanh^{-1}(\cos(a+bx))}{2b} - \frac{5\cos(a+bx)}{2b} - \frac{5\cos^3(a+bx)}{6b} - \frac{\cos^3(a+bx)\cot^2(a}{2b}$$

Mathematica [A] time = 0.03, size = 103, normalized size = 1.56

$$-\frac{9\cos(a+bx)}{4b} - \frac{\cos(3(a+bx))}{12b} - \frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{5\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{5\log\left(\cos\left(\frac{1}{2}(a+\right.\right.}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x]^3,x]
```

```
[Out] (-9*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(12*b) - Csc[(a + b*x)/2]^2/(8*b
) + (5*Log[Cos[(a + b*x)/2]])/(2*b) - (5*Log[Sin[(a + b*x)/2]])/(2*b) + Sec
[(a + b*x)/2]^2/(8*b)
```

fricas [A] time = 0.45, size = 93, normalized size = 1.41

$$-\frac{4\cos(bx+a)^5 + 20\cos(bx+a)^3 - 15\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) + 15\left(\cos(bx+a)^2-1\right)\log}{12\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(4*cos(b*x + a)^5 + 20*cos(b*x + a)^3 - 15*(cos(b*x + a)^2 - 1)*log(1
/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^2 - 1)*log(-1/2*cos(b*x + a) + 1/
2) - 30*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

giac [B] time = 0.60, size = 163, normalized size = 2.47

$$\frac{\frac{3\left(\frac{10(\cos(bx+a)-1)}{\cos(bx+a)+1}+1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{16\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{9(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-7\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3}-30\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*(3*(10*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(
cos(b*x + a) - 1) - 3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 16*(12*(cos(b
*x + a) - 1)/(cos(b*x + a) + 1) - 9*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)
^2 - 7)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^3 - 30*log(abs(-cos(b*x
 + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.02, size = 81, normalized size = 1.23

$$-\frac{\cos^7(bx+a)}{2b\sin(bx+a)^2}-\frac{\cos^5(bx+a)}{2b}-\frac{5\left(\cos^3(bx+a)\right)}{6b}-\frac{5\cos(bx+a)}{2b}-\frac{5\ln(\csc(bx+a)-\cot(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6/sin(b*x+a)^3,x)
```

```
[Out] -1/2/b*cos(b*x+a)^7/sin(b*x+a)^2-1/2*cos(b*x+a)^5/b-5/6*cos(b*x+a)^3/b-5/2*
cos(b*x+a)/b-5/2/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [A] time = 0.47, size = 66, normalized size = 1.00

$$-\frac{4\cos(bx+a)^3-\frac{6\cos(bx+a)}{\cos(bx+a)^2-1}+24\cos(bx+a)-15\log(\cos(bx+a)+1)+15\log(\cos(bx+a)-1)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*cos(b*x + a)^3 - 6*cos(b*x + a)/(cos(b*x + a)^2 - 1) + 24*cos(b*x
+ a) - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

mupad [B] time = 1.35, size = 129, normalized size = 1.95

$$\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b}-\frac{5\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{2\,b}-\frac{\frac{49\,\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6}{8}+\frac{67\,\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{8}+\frac{121\,\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{24}+\frac{1}{8}}{b\,\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8+3\,\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6+3\,\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^6/sin(a + b*x)^3,x)
```

```
[Out] tan(a/2 + (b*x)/2)^2/(8*b) - (5*log(tan(a/2 + (b*x)/2)))/(2*b) - ((121*tan(
a/2 + (b*x)/2)^2)/24 + (67*tan(a/2 + (b*x)/2)^4)/8 + (49*tan(a/2 + (b*x)/2)
^6)/8 + 1/8)/(b*(tan(a/2 + (b*x)/2)^2 + 3*tan(a/2 + (b*x)/2)^4 + 3*tan(a/2
+ (b*x)/2)^6 + tan(a/2 + (b*x)/2)^8))
```

sympy [A] time = 11.08, size = 719, normalized size = 10.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-60*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(24*b*tan(a/2 + b*
x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/
2 + b*x/2)**2) - 180*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(24*b*tan(a/
2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*
tan(a/2 + b*x/2)**2) - 180*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(24*b*
tan(a/2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 +
 24*b*tan(a/2 + b*x/2)**2) - 60*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(
24*b*tan(a/2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)
**4 + 24*b*tan(a/2 + b*x/2)**2) + 3*tan(a/2 + b*x/2)**10/(24*b*tan(a/2 + b*
x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/
2 + b*x/2)**2) - 165*tan(a/2 + b*x/2)**6/(24*b*tan(a/2 + b*x/2)**8 + 72*b*t
an(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/2 + b*x/2)**2) -
 225*tan(a/2 + b*x/2)**4/(24*b*tan(a/2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)*
*6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/2 + b*x/2)**2) - 130*tan(a/2 + b
*x/2)**2/(24*b*tan(a/2 + b*x/2)**8 + 72*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/
2 + b*x/2)**4 + 24*b*tan(a/2 + b*x/2)**2) - 3/(24*b*tan(a/2 + b*x/2)**8 + 7
2*b*tan(a/2 + b*x/2)**6 + 72*b*tan(a/2 + b*x/2)**4 + 24*b*tan(a/2 + b*x/2)*
*2), Ne(b, 0)), (x*cos(a)**6/sin(a)**3, True))
```

3.147 $\int \cos^2(a+bx)\cot^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\sin^2(a+bx)}{2b}-\frac{\csc^2(a+bx)}{2b}-\frac{2\log(\sin(a+bx))}{b}$$

```
[Out] -1/2*csc(b*x+a)^2/b-2*ln(sin(b*x+a))/b+1/2*sin(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$\frac{\sin^2(a+bx)}{2b}-\frac{\csc^2(a+bx)}{2b}-\frac{2\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x]^3,x]
```

```
[Out] -Csc[a + b*x]^2/(2*b) - (2*Log[Sin[a + b*x]])/b + Sin[a + b*x]^2/(2*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cos^2(a+bx)\cot^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^3}\,dx, x, -\sin(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \sin^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx, x, \sin^2(a+bx)\right)}{2b}$$
$$= -\frac{\csc^2(a+bx)}{2b} - \frac{2\log(\sin(a+bx))}{b} + \frac{\sin^2(a+bx)}{2b}$$

Mathematica [A] time = 0.06, size = 35, normalized size = 0.81

$$-\frac{-\sin^2(a+bx)+\csc^2(a+bx)+4\log(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x]^3,x]
[Out] -1/2*(Csc[a + b*x]^2 + 4*Log[Sin[a + b*x]] - Sin[a + b*x]^2)/b
```

fricas [A] time = 0.44, size = 61, normalized size = 1.42

$$-\frac{2\cos(bx+a)^4-3\cos(bx+a)^2+8\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\sin(bx+a)\right)-1}{4\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^3,x, algorithm="fricas")
[Out] -1/4*(2*cos(b*x + a)^4 - 3*cos(b*x + a)^2 + 8*(cos(b*x + a)^2 - 1)*log(1/2*
sin(b*x + a)) - 1)/(b*cos(b*x + a)^2 - b)
```

giac [B] time = 0.27, size = 187, normalized size = 4.35

$$\frac{\frac{\left(\frac{8(\cos(bx+a)-1)}{\cos(bx+a)+1}+1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{8\left(\frac{4(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-3\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^2}-8\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+16\log\left(\left|-\frac{\cos(bx+a}{\cos(bx+a}\right.\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*((8*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(cos(
b*x + a) - 1) + (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 8*(4*(cos(b*x + a)
- 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 - 3)/
((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^2 - 8*log(abs(-cos(b*x + a) + 1
)/abs(cos(b*x + a) + 1)) + 16*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1
) + 1)))/b
```

maple [A] time = 0.02, size = 61, normalized size = 1.42

$$-\frac{\cos^6(bx+a)}{2b\sin(bx+a)^2}-\frac{\cos^4(bx+a)}{2b}-\frac{\cos^2(bx+a)}{b}-\frac{2\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5/sin(b*x+a)^3,x)
```

```
[Out] -1/2/b*cos(b*x+a)^6/sin(b*x+a)^2-1/2*cos(b*x+a)^4/b-cos(b*x+a)^2/b-2*ln(sin
(b*x+a))/b
```

maxima [A] time = 0.54, size = 35, normalized size = 0.81

$$\frac{\sin(bx+a)^2-\frac{1}{\sin(bx+a)^2}-2\log\left(\sin(bx+a)^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(sin(b*x + a)^2 - 1/sin(b*x + a)^2 - 2*log(sin(b*x + a)^2))/b
```

mupad [B] time = 0.46, size = 62, normalized size = 1.44

$$\frac{\ln\left(\tan(a+bx)^2+1\right)}{b}-\frac{2\ln(\tan(a+bx))}{b}-\frac{\tan(a+bx)^2+\frac{1}{2}}{b\left(\tan(a+bx)^4+\tan(a+bx)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5/sin(a + b*x)^3,x)
```

```
[Out] log(tan(a + b*x)^2 + 1)/b - (2*log(tan(a + b*x)))/b - (tan(a + b*x)^2 + 1/2
)/(b*(tan(a + b*x)^2 + tan(a + b*x)^4))
```

sympy [A] time = 4.83, size = 614, normalized size = 14.28

$$\begin{cases} \frac{16\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+16b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)} + \frac{32\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+16b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)} + \frac{16\log\left(\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^2(}{8b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+16b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8} \\ \frac{x\cos^5(a)}{\sin^3(a)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((16*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(8*b*tan(a/2
 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 32*log
(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**6 + 16
*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 16*log(tan(a/2 + b*x/2)
**2 + 1)*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/
2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2
)**6/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*
x/2)**2) - 32*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/
2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16*log(tan(a/
2 + b*x/2))*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b
*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - tan(a/2 + b*x/2)**8/(8*b*tan(a/2 + b*
x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 18*tan(a/2
+ b*x/2)**4/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a
/2 + b*x/2)**2) - 1/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8
*b*tan(a/2 + b*x/2)**2), Ne(b, 0)), (x*cos(a)**5/sin(a)**3, True))
```

3.148 $\int \cos(a+bx)\cot^3(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\cos(a+bx)}{2b}-\frac{\cos(a+bx)\cot^2(a+bx)}{2b}+\frac{3\tanh^{-1}(\cos(a+bx))}{2b}$$

```
[Out] 3/2*arctanh(cos(b*x+a))/b-3/2*cos(b*x+a)/b-1/2*cos(b*x+a)*cot(b*x+a)^2/b
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2592, 288, 321, 206}

$$-\frac{3\cos(a+bx)}{2b}-\frac{\cos(a+bx)\cot^2(a+bx)}{2b}+\frac{3\tanh^{-1}(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[a + b*x]^3,x]
```

```
[Out] (3*ArcTanh[Cos[a + b*x]])/(2*b) - (3*Cos[a + b*x])/(2*b) - (Cos[a + b*x]*Co
t[a + b*x]^2)/(2*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \cos(a+bx)\cot^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\cos(a+bx)\cot^2(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}$$
$$= -\frac{3\cos(a+bx)}{2b} - \frac{\cos(a+bx)\cot^2(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}$$
$$= \frac{3\tanh^{-1}(\cos(a+bx))}{2b} - \frac{3\cos(a+bx)}{2b} - \frac{\cos(a+bx)\cot^2(a+bx)}{2b}$$

Mathematica [A] time = 0.03, size = 86, normalized size = 1.76

$$-\frac{\cos(a+bx)}{b} - \frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{3\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{3\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x]^3,x]
```

```
[Out] -(Cos[a + b*x]/b) - Csc[(a + b*x)/2]^2/(8*b) + (3*Log[Cos[(a + b*x)/2]])/(2
*b) - (3*Log[Sin[(a + b*x)/2]])/(2*b) + Sec[(a + b*x)/2]^2/(8*b)
```

fricas [A] time = 0.45, size = 83, normalized size = 1.69

$$-\frac{4\cos(bx+a)^3 - 3\left(\cos(bx+a)^2-1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right) + 3\left(\cos(bx+a)^2-1\right)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right.}{4\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*cos(b*x + a)^3 - 3*(cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2)
 + 3*(cos(b*x + a)^2 - 1)*log(-1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a))/(b
*cos(b*x + a)^2 - b)
```

giac [B] time = 0.31, size = 140, normalized size = 2.86

$$-\frac{\frac{\frac{14\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}}+\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+6\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*((14*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 - 1)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x +
 a) - 1)^2/(cos(b*x + a) + 1)^2) + (cos(b*x + a) - 1)/(cos(b*x + a) + 1) +
6*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.02, size = 68, normalized size = 1.39

$$-\frac{\cos^5(bx+a)}{2b\sin(bx+a)^2}-\frac{\cos^3(bx+a)}{2b}-\frac{3\cos(bx+a)}{2b}-\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/sin(b*x+a)^3,x)
```

```
[Out] -1/2/b*cos(b*x+a)^5/sin(b*x+a)^2-1/2*cos(b*x+a)^3/b-3/2*cos(b*x+a)/b-3/2/b*
ln(csc(b*x+a)-cot(b*x+a))
```

maxima [A] time = 0.33, size = 56, normalized size = 1.14

$$\frac{\frac{2\,\cos(bx+a)}{\cos(bx+a)^2-1}-4\,\cos(bx+a)+3\,\log(\cos(bx+a)+1)-3\,\log(\cos(bx+a)-1)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - 4*cos(b*x + a) + 3*log(cos(b*x +
 a) + 1) - 3*log(cos(b*x + a) - 1))/b
```

mupad [B] time = 0.54, size = 77, normalized size = 1.57

$$\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b}-\frac{3\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{2\,b}-\frac{\frac{17\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8}+\frac{1}{8}}{b\,\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4/sin(a + b*x)^3,x)
```

```
[Out] tan(a/2 + (b*x)/2)^2/(8*b) - (3*log(tan(a/2 + (b*x)/2)))/(2*b) - ((17*tan(a
/2 + (b*x)/2)^2)/8 + 1/8)/(b*(tan(a/2 + (b*x)/2)^2 + tan(a/2 + (b*x)/2)^4))
```

sympy [A] time = 3.92, size = 241, normalized size = 4.92

$$\begin{cases} -\frac{12\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}-\frac{12\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}+\frac{\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}-\frac{18\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)+8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)} \\ \frac{x\cos^4(a)}{\sin^3(a)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-12*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x
/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 12*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/
2)**2/(8*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + tan(a/2 + b*x/2
)**6/(8*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 18*tan(a/2 + b*x
/2)**2/(8*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 1/(8*b*tan(a/2
 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2), Ne(b, 0)), (x*cos(a)**4/sin(a)**3,
 True))
```

3.149 $\int \cot^3(a+bx)\,dx$

Optimal. Leaf size=28

$$-\frac{\cot^2(a+bx)}{2b}-\frac{\log(\sin(a+bx))}{b}$$

```
[Out] -1/2*cot(b*x+a)^2/b-ln(sin(b*x+a))/b
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 3475}

$$-\frac{\cot^2(a+bx)}{2b}-\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^3,x]
```

```
[Out] -Cot[a + b*x]^2/(2*b) - Log[Sin[a + b*x]]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^3(a+bx)\,dx &= -\frac{\cot^2(a+bx)}{2b}-\int \cot(a+bx)\,dx\\ &= -\frac{\cot^2(a+bx)}{2b}-\frac{\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 34, normalized size = 1.21

$$-\frac{\cot^2(a+bx)+2\log(\tan(a+bx))+2\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^3,x]
```

```
[Out] -1/2*(Cot[a + b*x]^2 + 2*Log[Cos[a + b*x]] + 2*Log[Tan[a + b*x]])/b
```

fricas [A] time = 0.46, size = 41, normalized size = 1.46

$$-\frac{2\left(\cos\left(bx+a\right)^2-1\right)\log\left(\frac{1}{2}\sin\left(bx+a\right)\right)-1}{2\left(b\cos\left(bx+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(cos(b*x + a)^2 - 1)*log(1/2*sin(b*x + a)) - 1)/(b*cos(b*x + a)^2 -
 b)
```

giac [A] time = 0.18, size = 36, normalized size = 1.29

$$\frac{\frac{\sin(bx+a)^2-1}{\sin(bx+a)^2}-\log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*((sin(b*x + a)^2 - 1)/sin(b*x + a)^2 - log(sin(b*x + a)^2))/b
```

maple [A] time = 0.02, size = 27, normalized size = 0.96

$$-\frac{\cot^2\left(bx+a\right)}{2b}-\frac{\ln\left(\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(b*x+a)^3,x)
```

```
[Out] -1/2*cot(b*x+a)^2/b-ln(sin(b*x+a))/b
```

maxima [A] time = 0.39, size = 23, normalized size = 0.82

$$-\frac{\frac{1}{\sin(bx+a)^2}+\log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b
```

mupad [B] time = 0.43, size = 36, normalized size = 1.29

$$-\frac{\cot(a+b\,x)^2-\ln\left(\tan(a+b\,x)^2+1\right)+2\,\ln(\tan(a+b\,x))}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(a + b*x)^3,x)
[Out] -(2*log(tan(a + b*x)) - log(tan(a + b*x)^2 + 1) + cot(a + b*x)^2)/(2*b)
```

sympy [A] time = 1.50, size = 42, normalized size = 1.50

$$\begin{cases} -\frac{\log(\sin(a+bx))}{b}-\frac{\cos^2(a+bx)}{2b\sin^2(a+bx)} & \text{for } b\neq 0 \\ \frac{x\cos^3(a)}{\sin^3(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a)**3,x)
[Out] Piecewise((-log(sin(a + b*x))/b - cos(a + b*x)**2/(2*b*sin(a + b*x)**2), Ne
(b, 0)), (x*cos(a)**3/sin(a)**3, True))
```

3.150 $\int \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}$$

```
[Out] 1/2*arctanh(cos(b*x+a))/b-1/2*cot(b*x+a)*csc(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$\frac{\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] ArcTanh[Cos[a + b*x]]/(2*b) - (Cot[a + b*x]*Csc[a + b*x])/(2*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(a+bx)\csc(a+bx)\,dx &= -\frac{\cot(a+bx)\csc(a+bx)}{2b}-\frac{1}{2}\int \csc(a+bx)\,dx\\ &= \frac{\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\cot(a+bx)\csc(a+bx)}{2b}\end{aligned}$$

Mathematica [B] time = 0.02, size = 75, normalized size = 2.21

$$-\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b}+\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -1/8*Csc[(a + b*x)/2]^2/b + Log[Cos[(a + b*x)/2]]/(2*b) - Log[Sin[(a + b*x)
/2]]/(2*b) + Sec[(a + b*x)/2]^2/(8*b)
```

fricas [B] time = 0.44, size = 72, normalized size = 2.12

$$\frac{\left(\cos\left(bx+a\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)+2\cos\left(bx+a\right)}{4\left(b\cos\left(bx+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2) - (cos(b*x + a)^2 - 1
)*log(-1/2*cos(b*x + a) + 1/2) + 2*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

giac [B] time = 0.22, size = 93, normalized size = 2.74

$$\frac{\frac{\left(\frac{2\left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}+1\right)\left(\cos(bx+a)+1\right)}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-2\log\left(\frac{\left|-\cos(bx+a)+1\right|}{\left|\cos(bx+a)+1\right|}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*((2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)*(cos(b*x + a) + 1)/(cos(
b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2*log(abs(-cos(b*x
+ a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.02, size = 55, normalized size = 1.62

$$-\frac{\cos^3\left(bx+a\right)}{2b\sin\left(bx+a\right)^2}-\frac{\cos\left(bx+a\right)}{2b}-\frac{\ln\left(\csc\left(bx+a\right)-\cot\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(b*x+a)^3,x)
```

```
[Out] -1/2/b*cos(b*x+a)^3/sin(b*x+a)^2-1/2*cos(b*x+a)/b-1/2/b*ln(csc(b*x+a)-cot(b
*x+a))
```

maxima [A] time = 0.58, size = 46, normalized size = 1.35

$$\frac{\frac{2\cos(bx+a)}{\cos(bx+a)^2-1} + \log\left(\cos\left(bx+a\right)+1\right) - \log\left(\cos\left(bx+a\right)-1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(
b*x + a) - 1))/b
```

mupad [B] time = 0.45, size = 48, normalized size = 1.41

$$\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b} - \frac{1}{8\,b\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2} - \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(a + b*x)^3,x)
```

```
[Out] tan(a/2 + (b*x)/2)^2/(8*b) - 1/(8*b*tan(a/2 + (b*x)/2)^2) - log(tan(a/2 + (
b*x)/2))/(2*b)
```

sympy [A] time = 1.61, size = 58, normalized size = 1.71

$$\begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{2b} + \frac{\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} - \frac{1}{8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for}\ b \neq 0 \\ \frac{x\cos^2(a)}{\sin^3(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2))/(2*b) + tan(a/2 + b*x/2)**2/(8*b) - 1/(8*
b*tan(a/2 + b*x/2)**2), Ne(b, 0)), (x*cos(a)**2/sin(a)**3, True))
```

3.151 $\int \cot(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\csc^2(a+bx)}{2b}$$

```
[Out] -1/2*csc(b*x+a)^2/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -Csc[a + b*x]^2/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot(a+bx)\csc^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{\csc^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -1/2*Csc[a + b*x]^2/b
```

fricas [A] time = 1.20, size = 18, normalized size = 1.20

$$\frac{1}{2\left(b\cos\left(bx+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2/(b*cos(b*x + a)^2 - b)
```

giac [A] time = 0.20, size = 13, normalized size = 0.87

$$-\frac{1}{2\,b\sin\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2/(b*sin(b*x + a)^2)
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$-\frac{1}{2\sin\left(bx+a\right)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+a)^3,x)
```

```
[Out] -1/2/sin(b*x+a)^2/b
```

maxima [A] time = 0.63, size = 13, normalized size = 0.87

$$-\frac{1}{2\,b\sin\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(b*sin(b*x + a)^2)
```

mupad [B] time = 0.39, size = 13, normalized size = 0.87

$$-\frac{1}{2\,b\sin\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(a + b*x)^3,x)
```

```
[Out] -1/(2*b*sin(a + b*x)^2)
```

sympy [A] time = 1.46, size = 24, normalized size = 1.60

$$\begin{cases} -\frac{1}{2b\sin^2\left(a+bx\right)} & \text{for}\ b \neq 0 \\ \frac{x\cos\left(a\right)}{\sin^3\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-1/(2*b*sin(a + b*x)**2), Ne(b, 0)), (x*cos(a)/sin(a)**3, True))
```

3.152 $\int \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\log(\tan(a+bx))}{b}-\frac{\cot^2(a+bx)}{2b}$$

```
[Out] -1/2*cot(b*x+a)^2/b+ln(tan(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2620, 14}

$$\frac{\log(\tan(a+bx))}{b}-\frac{\cot^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -Cot[a + b*x]^2/(2*b) + Log[Tan[a + b*x]]/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(a+bx)\sec(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx, x, \tan(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)\,dx, x, \tan(a+bx)\right)}{b}\\
&= -\frac{\cot^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 1.26

$$-\frac{\csc^2(a+bx)-2\log(\sin(a+bx))+2\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -1/2*(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]])/b
```

fricas [B] time = 0.44, size = 65, normalized size = 2.41

$$-\frac{\left(\cos(bx+a)^2-1\right)\log\left(\cos(bx+a)^2\right)-\left(\cos(bx+a)^2-1\right)\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-1}{2\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log(-
1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

giac [B] time = 0.33, size = 119, normalized size = 4.41

$$-\frac{\frac{\left(\frac{4(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-4\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+8\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right|\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*((4*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(cos
(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 4*log(abs(-cos(b*x
 + a) + 1)/abs(cos(b*x + a) + 1)) + 8*log(abs(-(cos(b*x + a) - 1)/(cos(b*x
+ a) + 1) - 1)))/b
```

maple [A] time = 0.03, size = 26, normalized size = 0.96

$$-\frac{1}{2\sin(bx+a)^2 b}+\frac{\ln(\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/sin(b*x+a)^3,x)
```

```
[Out] -1/2/sin(b*x+a)^2/b+ln(tan(b*x+a))/b
```

maxima [A] time = 0.35, size = 36, normalized size = 1.33

$$-\frac{\frac{1}{\sin(bx+a)^2} + \log\left(\sin\left(bx+a\right)^2 - 1\right) - \log\left(\sin\left(bx+a\right)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b
```

mupad [B] time = 0.05, size = 34, normalized size = 1.26

$$-\frac{\ln\left(\cos\left(a+b\,x\right)\right) - \frac{\ln\left(\sin(a+b\,x)^2\right)}{2} + \frac{1}{2\sin(a+b\,x)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^3),x)
[Out] -(log(cos(a + b*x)) - log(sin(a + b*x)^2)/2 + 1/(2*sin(a + b*x)^2))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(a+bx\right)}{\sin^3\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)**3,x)
[Out] Integral(sec(a + b*x)/sin(a + b*x)**3, x)
```

3.153 $\int \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\sec(a+bx)}{2b}-\frac{3\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\csc^2(a+bx)\sec(a+bx)}{2b}$$

```
[Out] -3/2*arctanh(cos(b*x+a))/b+3/2*sec(b*x+a)/b-1/2*csc(b*x+a)^2*sec(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 321, 207}

$$\frac{3\sec(a+bx)}{2b}-\frac{3\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\csc^2(a+bx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(2*b) + (3*Sec[a + b*x])/(2*b) - (Csc[a + b*x]^2
*Sec[a + b*x])/(2*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^3(a+bx)\sec^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{b}$$
$$= -\frac{\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{2b}$$
$$= \frac{3\sec(a+bx)}{2b} - \frac{\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{2b}$$
$$= -\frac{3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3\sec(a+bx)}{2b} - \frac{\csc^2(a+bx)\sec(a+bx)}{2b}$$

Mathematica [B] time = 0.23, size = 143, normalized size = 2.92

$$\frac{\csc^4(a+bx)\left(-6\cos(2(a+bx)) + 2\cos(3(a+bx)) + 3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right) - 3\cos(3(a+bx))\log\ldots\right)}{2b\left(\csc^2\left(\frac{1}{2}(a+bx)\right) - \sec^2\left(\frac{1}{2}\ldots\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a +
 b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + C
os[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(2*b
*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

fricas [B] time = 0.45, size = 96, normalized size = 1.96

$$\frac{6\cos(bx+a)^2 - 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + 3\left(\cos(bx+a)^3 - \cos(bx+a)\right)\log\ldots}{4\left(b\cos(bx+a)^3 - b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x +
 a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2)
 - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

giac [B] time = 0.40, size = 140, normalized size = 2.86

$$\frac{\frac{\frac{14\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{3\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+6\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*((14*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(co
s(b*x + a) + 1)^2 + 1)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x +
a) - 1)^2/(cos(b*x + a) + 1)^2) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 6
*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.03, size = 57, normalized size = 1.16

$$-\frac{1}{2b\sin(bx+a)^2\cos(bx+a)}+\frac{3}{2b\cos(bx+a)}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/sin(b*x+a)^3,x)
```

```
[Out] -1/2/b/sin(b*x+a)^2/cos(b*x+a)+3/2/b/cos(b*x+a)+3/2/b*ln(csc(b*x+a)-cot(b*x
+a))
```

maxima [A] time = 0.29, size = 61, normalized size = 1.24

$$\frac{\frac{2\left(3\,\cos(bx+a)^2-2\right)}{\cos(bx+a)^3-\cos(bx+a)}-3\,\log(\cos(bx+a)+1)+3\,\log(\cos(bx+a)-1)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b
*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b
```

mupad [B] time = 0.03, size = 49, normalized size = 1.00

$$-\frac{3\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{2\,b}-\frac{\frac{3\cos\left(a+b\,x\right)^2}{2}-1}{b\,\left(\cos\left(a+b\,x\right)-\cos\left(a+b\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] - (3*atanh(cos(a + b*x)))/(2*b) - ((3*cos(a + b*x)^2)/2 - 1)/(b*(cos(a + b*
x) - cos(a + b*x)^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)}{\sin^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a)**3,x)
```

```
[Out] Integral(sec(a + b*x)**2/sin(a + b*x)**3, x)
```

3.154 $\int \csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=43

$$\frac{\tan^2(a+bx)}{2b}-\frac{\cot^2(a+bx)}{2b}+\frac{2\log(\tan(a+bx))}{b}$$

```
[Out] -1/2*cot(b*x+a)^2/b+2*ln(tan(b*x+a))/b+1/2*tan(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tan^2(a+bx)}{2b}-\frac{\cot^2(a+bx)}{2b}+\frac{2\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] -Cot[a + b*x]^2/(2*b) + (2*Log[Tan[a + b*x]])/b + Tan[a + b*x]^2/(2*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sec^3(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^3}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^2}\,dx, x, \tan^2(a+bx)\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}+\frac{2}{x}\right)dx, x, \tan^2(a+bx)\right)}{2b}\\ &= -\frac{\cot^2(a+bx)}{2b}+\frac{2\log(\tan(a+bx))}{b}+\frac{\tan^2(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 1.42

$$8\left(-\frac{\csc^2(a+bx)}{16b}+\frac{\sec^2(a+bx)}{16b}+\frac{\log(\sin(a+bx))}{4b}-\frac{\log(\cos(a+bx))}{4b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] 8*(-1/16*Csc[a + b*x]^2/b - Log[Cos[a + b*x]]/(4*b) + Log[Sin[a + b*x]]/(4*
b) + Sec[a + b*x]^2/(16*b))
```

fricas [B] time = 0.43, size = 102, normalized size = 2.37

$$\frac{2\cos(bx+a)^2-2\left(\cos(bx+a)^4-\cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right)+2\left(\cos(bx+a)^4-\cos(bx+a)^2\right)\log\left(-\frac{1}{4}\right.}{2\left(b\cos(bx+a)^4-b\cos(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + a)^2 - 2*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(cos(b*x + a
)^2) + 2*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) -
 1)/(b*cos(b*x + a)^4 - b*cos(b*x + a)^2)
```

giac [B] time = 0.25, size = 188, normalized size = 4.37

$$-\frac{\frac{\left(\frac{8(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-\frac{8\left(\frac{4(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+3\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^2}-8\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+16\log\left(\left|-\frac{\cos(bx}{\cos(bx}\right.\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*((8*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(cos
(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 8*(4*(cos(b*x + a)
 - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 3)
/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^2 - 8*log(abs(-cos(b*x + a) +
1)/abs(cos(b*x + a) + 1)) + 16*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) +
1) - 1)))/b
```

maple [A] time = 0.04, size = 48, normalized size = 1.12

$$\frac{1}{2b\sin(bx+a)^{2}\cos(bx+a)^{2}}-\frac{1}{\sin(bx+a)^{2}b}+\frac{2\ln(\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/sin(b*x+a)^3,x)
```

```
[Out] 1/2/b/sin(b*x+a)^2/cos(b*x+a)^2-1/sin(b*x+a)^2/b+2*ln(tan(b*x+a))/b
```

maxima [A] time = 0.30, size = 64, normalized size = 1.49

$$-\frac{\frac{2\sin(bx+a)^{2}-1}{\sin(bx+a)^{4}-\sin(bx+a)^{2}}+2\log\left(\sin(bx+a)^{2}-1\right)-2\log\left(\sin(bx+a)^{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(
b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b
```

mupad [B] time = 0.38, size = 39, normalized size = 0.91

$$\frac{\tan(a+b\,x)^{2}}{2\,b}-\frac{1}{2\,b\tan(a+b\,x)^{2}}+\frac{2\,\ln(\tan(a+b\,x))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

```
[Out] tan(a + b*x)^2/(2*b) - 1/(2*b*tan(a + b*x)^2) + (2*log(tan(a + b*x)))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^{3}(a+bx)}{\sin^{3}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a)**3,x)
```

```
[Out] Integral(sec(a + b*x)**3/sin(a + b*x)**3, x)
```

3.155 $\int \csc^3(a+bx)\sec^4(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{5\sec^3(a+bx)}{6b}+\frac{5\sec(a+bx)}{2b}-\frac{5\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{2b}$$

```
[Out] -5/2*arctanh(cos(b*x+a))/b+5/2*sec(b*x+a)/b+5/6*sec(b*x+a)^3/b-1/2*csc(b*x+
a)^2*sec(b*x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 302, 207}

$$\frac{5\sec^3(a+bx)}{6b}+\frac{5\sec(a+bx)}{2b}-\frac{5\tanh^{-1}(\cos(a+bx))}{2b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x]^4,x]
```

```
[Out] (-5*ArcTanh[Cos[a + b*x]])/(2*b) + (5*Sec[a + b*x])/(2*b) + (5*Sec[a + b*x]
^3)/(6*b) - (Csc[a + b*x]^2*Sec[a + b*x]^3)/(2*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^3(a+bx)\sec^4(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{b}$$

$$= -\frac{\csc^2(a+bx)\sec^3(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{2b}$$

$$= -\frac{\csc^2(a+bx)\sec^3(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \sec(a+bx)\right)}{2b}$$

$$= \frac{5\sec(a+bx)}{2b} + \frac{5\sec^3(a+bx)}{6b} - \frac{\csc^2(a+bx)\sec^3(a+bx)}{2b} + \frac{5\,\text{Subst}\left(\int \frac{1}{-1+x^2}\right.}{}$$

$$= -\frac{5\tanh^{-1}(\cos(a+bx))}{2b} + \frac{5\sec(a+bx)}{2b} + \frac{5\sec^3(a+bx)}{6b} - \frac{\csc^2(a+bx)\sec^3(}{2b}$$

Mathematica [B] time = 0.39, size = 205, normalized size = 3.11

$$\frac{2\csc^8(a+bx)\left(-40\cos(2(a+bx))+13\cos(3(a+bx))-30\cos(4(a+bx))+13\cos(5(a+bx))+15\cos(3(a+bx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x]^4,x]
```

```
[Out] (2*Csc[a + b*x]^8*(22 - 40*Cos[2*(a + b*x)] + 13*Cos[3*(a + b*x)] - 30*Cos[
4*(a + b*x)] + 13*Cos[5*(a + b*x)] + 15*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/
2]] + 15*Cos[5*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 15*Cos[3*(a + b*x)]*Log[S
in[(a + b*x)/2]] - 15*Cos[5*(a + b*x)]*Log[Sin[(a + b*x)/2]] + Cos[a + b*x]
*(-26 - 30*Log[Cos[(a + b*x)/2]] + 30*Log[Sin[(a + b*x)/2]])))/(3*b*(Csc[(a
 + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

fricas [A] time = 0.44, size = 112, normalized size = 1.70

$$\frac{30 \cos(bx+a)^4 - 20 \cos(bx+a)^2 - 15\left(\cos(bx+a)^5 - \cos(bx+a)^3\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + 15\left(\cos(bx\right.}{12\left(b\cos(bx+a)^5 - b\cos(bx+a)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(30*cos(b*x + a)^4 - 20*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - cos(b*x
+ a)^3)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 - cos(b*x + a)^3)*
log(-1/2*cos(b*x + a) + 1/2) - 4)/(b*cos(b*x + a)^5 - b*cos(b*x + a)^3)
```

giac [B] time = 0.22, size = 163, normalized size = 2.47

$$-\frac{\frac{3\left(\frac{10(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1} + \frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{16\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{9(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + 7\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3} - 30\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/24*(3*(10*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/
(cos(b*x + a) - 1) + 3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 16*(12*(cos(
b*x + a) - 1)/(cos(b*x + a) + 1) + 9*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1
)^2 + 7)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3 - 30*log(abs(-cos(b*
x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.04, size = 78, normalized size = 1.18

$$\frac{1}{3b\sin(bx+a)^2\cos(bx+a)^3} - \frac{5}{6b\sin(bx+a)^2\cos(bx+a)} + \frac{5}{2b\cos(bx+a)} + \frac{5\ln(\csc(bx+a) - \cot(bx+a))}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/sin(b*x+a)^3,x)
```

```
[Out] 1/3/b/sin(b*x+a)^2/cos(b*x+a)^3-5/6/b/sin(b*x+a)^2/cos(b*x+a)+5/2/b/cos(b*x
+a)+5/2/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [A] time = 0.29, size = 73, normalized size = 1.11

$$\frac{\frac{2\left(15\cos(bx+a)^4 - 10\cos(bx+a)^2 - 2\right)}{\cos(bx+a)^5 - \cos(bx+a)^3} - 15\log(\cos(bx+a)+1) + 15\log(\cos(bx+a)-1)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(2*(15*cos(b*x + a)^4 - 10*cos(b*x + a)^2 - 2)/(cos(b*x + a)^5 - cos(b
*x + a)^3) - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x + a) - 1))/b
```

mupad [B] time = 0.46, size = 60, normalized size = 0.91

$$\frac{-\frac{5\cos(a+b\,x)^4}{2}+\frac{5\cos(a+b\,x)^2}{3}+\frac{1}{3}}{b\,\left(\cos\left(a+b\,x\right)^3-\cos\left(a+b\,x\right)^5\right)}-\frac{5\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^4*sin(a + b*x)^3),x)
```

```
[Out] ((5*cos(a + b*x)^2)/3 - (5*cos(a + b*x)^4)/2 + 1/3)/(b*(cos(a + b*x)^3 - co
s(a + b*x)^5)) - (5*atanh(cos(a + b*x)))/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(a+bx)}{\sin^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a)**3,x)
```

```
[Out] Integral(sec(a + b*x)**4/sin(a + b*x)**3, x)
```

3.156 $\int \csc^3(a+bx)\sec^5(a+bx)\,dx$

Optimal. Leaf size=58

$$\frac{\tan^4(a+bx)}{4b}+\frac{3\tan^2(a+bx)}{2b}-\frac{\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}$$

```
[Out] -1/2*cot(b*x+a)^2/b+3*ln(tan(b*x+a))/b+3/2*tan(b*x+a)^2/b+1/4*tan(b*x+a)^4/
b
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tan^4(a+bx)}{4b}+\frac{3\tan^2(a+bx)}{2b}-\frac{\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sec[a + b*x]^5,x]
```

```
[Out] -Cot[a + b*x]^2/(2*b) + (3*Log[Tan[a + b*x]])/b + (3*Tan[a + b*x]^2)/(2*b)
+ Tan[a + b*x]^4/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \csc^3(a+bx)\sec^5(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, \tan^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)dx, x, \tan^2(a+bx)\right)}{2b}$$
$$= -\frac{\cot^2(a+bx)}{2b} + \frac{3\log(\tan(a+bx))}{b} + \frac{3\tan^2(a+bx)}{2b} + \frac{\tan^4(a+bx)}{4b}$$

Mathematica [A] time = 0.21, size = 56, normalized size = 0.97

$$-\frac{2\csc^2(a+bx)-\sec^4(a+bx)-4\sec^2(a+bx)-12\log(\sin(a+bx))+12\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sec[a + b*x]^5,x]
```

```
[Out] -1/4*(2*Csc[a + b*x]^2 + 12*Log[Cos[a + b*x]] - 12*Log[Sin[a + b*x]] - 4*Se
c[a + b*x]^2 - Sec[a + b*x]^4)/b
```

fricas [B] time = 0.49, size = 112, normalized size = 1.93

$$\frac{6\cos(bx+a)^4-3\cos(bx+a)^2-6\left(\cos(bx+a)^6-\cos(bx+a)^4\right)\log\left(\cos(bx+a)^2\right)+6\left(\cos(bx+a)^6-\cos(\right.}{4\left(b\cos(bx+a)^6-b\cos(bx+a)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - cos(b*x + a)
^4)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*cos(
b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^6 - b*cos(b*x + a)^4)
```

giac [B] time = 0.25, size = 232, normalized size = 4.00

$$-\frac{\frac{\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-\frac{2\left(\frac{76(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{118(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{76(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{25(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+25\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^4}-12\log\left(\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)/(co
s(b*x + a) - 1) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 2*(76*(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 118*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2
+ 76*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 25*(cos(b*x + a) - 1)^4/(c
os(b*x + a) + 1)^4 + 25)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^4 - 12
*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 24*log(abs(-(cos(b*x +
 a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

maple [A] time = 0.04, size = 69, normalized size = 1.19

$$\frac{1}{4b\sin\left(bx+a\right)^{2}\cos\left(bx+a\right)^{4}}+\frac{3}{4b\sin\left(bx+a\right)^{2}\cos\left(bx+a\right)^{2}}-\frac{3}{2\sin\left(bx+a\right)^{2}b}+\frac{3\ln\left(\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5/sin(b*x+a)^3,x)
```

```
[Out] 1/4/b/sin(b*x+a)^2/cos(b*x+a)^4+3/4/b/sin(b*x+a)^2/cos(b*x+a)^2-3/2/sin(b*x
+a)^2/b+3*ln(tan(b*x+a))/b
```

maxima [A] time = 0.30, size = 82, normalized size = 1.41

$$-\frac{\frac{6\,\sin\left(bx+a\right)^{4}-9\,\sin\left(bx+a\right)^{2}+2}{\sin\left(bx+a\right)^{6}-2\,\sin\left(bx+a\right)^{4}+\sin\left(bx+a\right)^{2}}+6\,\log\left(\sin\left(bx+a\right)^{2}-1\right)-6\,\log\left(\sin\left(bx+a\right)^{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((6*sin(b*x + a)^4 - 9*sin(b*x + a)^2 + 2)/(sin(b*x + a)^6 - 2*sin(b*x
 + a)^4 + sin(b*x + a)^2) + 6*log(sin(b*x + a)^2 - 1) - 6*log(sin(b*x + a)^
2))/b
```

mupad [B] time = 0.44, size = 74, normalized size = 1.28

$$\frac{3\,\ln\left(\sin\left(a+b\,x\right)^{2}\right)}{2\,b}-\frac{3\,\ln\left(\cos\left(a+b\,x\right)\right)}{b}+\frac{-\frac{3\cos\left(a+b\,x\right)^{4}}{2}+\frac{3\cos\left(a+b\,x\right)^{2}}{4}+\frac{1}{4}}{b\,\left(\cos\left(a+b\,x\right)^{4}-\cos\left(a+b\,x\right)^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^5*sin(a + b*x)^3),x)
```

```
[Out] (3*log(sin(a + b*x)^2))/(2*b) - (3*log(cos(a + b*x)))/b + ((3*cos(a + b*x)^
2)/4 - (3*cos(a + b*x)^4)/2 + 1/4)/(b*(cos(a + b*x)^4 - cos(a + b*x)^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(a+bx)}{\sin^3(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a)**3,x)
```

```
[Out] Integral(sec(a + b*x)**5/sin(a + b*x)**3, x)
```

3.157 $\int \cos^5(a+bx)\cot^4(a+bx)\,dx$

Optimal. Leaf size=68

$$\frac{\sin^5(a+bx)}{5b}-\frac{4\sin^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{4\csc(a+bx)}{b}$$

```
[Out] 4*csc(b*x+a)/b-1/3*csc(b*x+a)^3/b+6*sin(b*x+a)/b-4/3*sin(b*x+a)^3/b+1/5*sin
(b*x+a)^5/b
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$\frac{\sin^5(a+bx)}{5b}-\frac{4\sin^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{4\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*Cot[a + b*x]^4,x]
```

```
[Out] (4*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (6*Sin[a + b*x])/b - (4*Sin[a +
 b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cos^5(a+bx)\cot^4(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{x^4}\,dx, x, -\sin(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(6+\frac{1}{x^4}-\frac{4}{x^2}-4x^2+x^4\right)dx, x, -\sin(a+bx)\right)}{b}$$
$$= \frac{4\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{4\sin^3(a+bx)}{3b}+\frac{\sin^5(a+bx)}{5b}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.00

$$\frac{\sin^5(a+bx)}{5b}-\frac{4\sin^3(a+bx)}{3b}+\frac{6\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{4\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*Cot[a + b*x]^4,x]
```

```
[Out] (4*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (6*Sin[a + b*x])/b - (4*Sin[a +
 b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

fricas [A] time = 0.43, size = 68, normalized size = 1.00

$$-\frac{3\cos(bx+a)^8+8\cos(bx+a)^6+48\cos(bx+a)^4-192\cos(bx+a)^2+128}{15\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(b*x + a)^8 + 8*cos(b*x + a)^6 + 48*cos(b*x + a)^4 - 192*cos(b*
x + a)^2 + 128)/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

giac [A] time = 0.22, size = 56, normalized size = 0.82

$$\frac{3\sin(bx+a)^5-20\sin(bx+a)^3+\frac{5\left(12\sin(bx+a)^2-1\right)}{\sin(bx+a)^3}+90\sin(bx+a)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(b*x + a)^5 - 20*sin(b*x + a)^3 + 5*(12*sin(b*x + a)^2 - 1)/sin(
b*x + a)^3 + 90*sin(b*x + a))/b
```

maple [A] time = 0.06, size = 90, normalized size = 1.32

$$\frac{-\frac{\cos^{10}(bx+a)}{3\sin(bx+a)^3}+\frac{7\left(\cos^{10}(bx+a)\right)}{3\sin(bx+a)}+\frac{7\left(\frac{128}{35}+\cos^{8}(bx+a)+\frac{8\left(\cos^{6}(bx+a)\right)}{7}+\frac{48\left(\cos^{4}(bx+a)\right)}{35}+\frac{64\left(\cos^{2}(bx+a)\right)}{35}\right)\sin(bx+a)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^9/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3/sin(b*x+a)^3*cos(b*x+a)^10+7/3/sin(b*x+a)*cos(b*x+a)^10+7/3*(128/
35+cos(b*x+a)^8+8/7*cos(b*x+a)^6+48/35*cos(b*x+a)^4+64/35*cos(b*x+a)^2)*sin
(b*x+a))
```

maxima [A] time = 0.36, size = 56, normalized size = 0.82

$$\frac{3\,\sin\left(bx+a\right)^{5}-20\,\sin\left(bx+a\right)^{3}+\frac{5\left(12\,\sin(bx+a)^{2}-1\right)}{\sin(bx+a)^{3}}+90\,\sin\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/15*(3*sin(b*x + a)^5 - 20*sin(b*x + a)^3 + 5*(12*sin(b*x + a)^2 - 1)/sin(
b*x + a)^3 + 90*sin(b*x + a))/b
```

mupad [B] time = 0.52, size = 55, normalized size = 0.81

$$\frac{3\sin\left(a+b\,x\right)^{8}-20\sin\left(a+b\,x\right)^{6}+90\sin\left(a+b\,x\right)^{4}+60\sin\left(a+b\,x\right)^{2}-5}{15\,b\sin\left(a+b\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^9/sin(a + b*x)^4,x)
```

```
[Out] (60*sin(a + b*x)^2 + 90*sin(a + b*x)^4 - 20*sin(a + b*x)^6 + 3*sin(a + b*x)
^8 - 5)/(15*b*sin(a + b*x)^3)
```

sympy [A] time = 22.23, size = 105, normalized size = 1.54

$$\begin{cases} \frac{128\sin^{5}{\left(a+bx\right)}}{15b}+\frac{64\sin^{3}{\left(a+bx\right)}\cos^{2}{\left(a+bx\right)}}{3b}+\frac{16\sin{\left(a+bx\right)}\cos^{4}{\left(a+bx\right)}}{b}+\frac{8\cos^{6}{\left(a+bx\right)}}{3b\sin{\left(a+bx\right)}}-\frac{\cos^{8}{\left(a+bx\right)}}{3b\sin^{3}{\left(a+bx\right)}} & \text{for}\: b\neq 0 \\ \frac{x\cos^{9}{\left(a\right)}}{\sin^{4}{\left(a\right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**9/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((128*sin(a + b*x)**5/(15*b) + 64*sin(a + b*x)**3*cos(a + b*x)**2/
(3*b) + 16*sin(a + b*x)*cos(a + b*x)**4/b + 8*cos(a + b*x)**6/(3*b*sin(a +
b*x)) - cos(a + b*x)**8/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**9/sin(
a)**4, True))
```

3.158 $\int \cos^4(a+bx)\cot^4(a+bx)\,dx$

Optimal. Leaf size=80

$$-\frac{35\cot^3(a+bx)}{24b}+\frac{35\cot(a+bx)}{8b}+\frac{\cos^4(a+bx)\cot^3(a+bx)}{4b}+\frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b}+\frac{35x}{8}$$

```
[Out] 35/8*x+35/8*cot(b*x+a)/b-35/24*cot(b*x+a)^3/b+7/8*cos(b*x+a)^2*cot(b*x+a)^3
/b+1/4*cos(b*x+a)^4*cot(b*x+a)^3/b
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 302, 203}

$$-\frac{35\cot^3(a+bx)}{24b}+\frac{35\cot(a+bx)}{8b}+\frac{\cos^4(a+bx)\cot^3(a+bx)}{4b}+\frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b}+\frac{35x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x]^4,x]
```

```
[Out] (35*x)/8 + (35*Cot[a + b*x])/(8*b) - (35*Cot[a + b*x]^3)/(24*b) + (7*Cos[a
+ b*x]^2*Cot[a + b*x]^3)/(8*b) + (Cos[a + b*x]^4*Cot[a + b*x]^3)/(4*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^4(a+bx)\cot^4(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^8}{\left(1+x^2\right)^3}\,dx, x, \cot(a+bx)\right)}{b}$$

$$= \frac{\cos^4(a+bx)\cot^3(a+bx)}{4b} - \frac{7\,\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2}\,dx, x, \cot(a+bx)\right)}{4b}$$

$$= \frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot^3(a+bx)}{4b} - \frac{35\,\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx\right.}{8b}$$

$$= \frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b} + \frac{\cos^4(a+bx)\cot^3(a+bx)}{4b} - \frac{35\,\text{Subst}\left(\int \left(-1+x\right.\right.}{}$$

$$= \frac{35\cot(a+bx)}{8b} - \frac{35\cot^3(a+bx)}{24b} + \frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b} + \frac{\cos^4(a+bx)}{4}$$

$$= \frac{35x}{8} + \frac{35\cot(a+bx)}{8b} - \frac{35\cot^3(a+bx)}{24b} + \frac{7\cos^2(a+bx)\cot^3(a+bx)}{8b} + \cos^4($$

Mathematica [A] time = 0.26, size = 53, normalized size = 0.66

$$\frac{420(a+bx)+72\sin(2(a+bx))+3\sin(4(a+bx))-32\cot(a+bx)\left(\csc^2(a+bx)-10\right)}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x]^4,x]
```

```
[Out] (420*(a + b*x) - 32*Cot[a + b*x]*(-10 + Csc[a + b*x]^2) + 72*Sin[2*(a + b*x
)] + 3*Sin[4*(a + b*x)])/(96*b)
```

fricas [A] time = 0.44, size = 89, normalized size = 1.11

$$-\frac{6\cos(bx+a)^7+21\cos(bx+a)^5-140\cos(bx+a)^3-105\left(bx\cos(bx+a)^2-bx\right)\sin(bx+a)+105\cos(bx}{24\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(6*cos(b*x + a)^7 + 21*cos(b*x + a)^5 - 140*cos(b*x + a)^3 - 105*(b*x
*cos(b*x + a)^2 - b*x)*sin(b*x + a) + 105*cos(b*x + a))/((b*cos(b*x + a)^2
- b)*sin(b*x + a))
```

giac [A] time = 0.17, size = 68, normalized size = 0.85

$$\frac{105\,bx+105\,a+\frac{3\left(11\,\tan\left(bx+a\right)^{3}+13\,\tan\left(bx+a\right)\right)}{\left(\tan\left(bx+a\right)^{2}+1\right)^{2}}+\frac{8\left(9\,\tan\left(bx+a\right)^{2}-1\right)}{\tan\left(bx+a\right)^{3}}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/24*(105*b*x + 105*a + 3*(11*tan(b*x + a)^3 + 13*tan(b*x + a))/(tan(b*x +
a)^2 + 1)^2 + 8*(9*tan(b*x + a)^2 - 1)/tan(b*x + a)^3)/b
```

maple [A] time = 0.06, size = 94, normalized size = 1.18

$$\frac{-\frac{\cos^{9}\left(bx+a\right)}{3\sin\left(bx+a\right)^{3}}+\frac{2\left(\cos^{9}\left(bx+a\right)\right)}{\sin\left(bx+a\right)}+2\left(\cos^{7}\left(bx+a\right)+\frac{7\left(\cos^{5}\left(bx+a\right)\right)}{6}+\frac{35\left(\cos^{3}\left(bx+a\right)\right)}{24}+\frac{35\cos\left(bx+a\right)}{16}\right)\sin\left(bx+a\right)+\frac{35bx}{8}+\frac{35}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^8/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3/sin(b*x+a)^3*cos(b*x+a)^9+2/sin(b*x+a)*cos(b*x+a)^9+2*(cos(b*x+a)
^7+7/6*cos(b*x+a)^5+35/24*cos(b*x+a)^3+35/16*cos(b*x+a))*sin(b*x+a)+35/8*b*
x+35/8*a)
```

maxima [A] time = 0.46, size = 75, normalized size = 0.94

$$\frac{105\,bx+105\,a+\frac{105\,\tan\left(bx+a\right)^{6}+175\,\tan\left(bx+a\right)^{4}+56\,\tan\left(bx+a\right)^{2}-8}{\tan\left(bx+a\right)^{7}+2\,\tan\left(bx+a\right)^{5}+\tan\left(bx+a\right)^{3}}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/24*(105*b*x + 105*a + (105*tan(b*x + a)^6 + 175*tan(b*x + a)^4 + 56*tan(b
*x + a)^2 - 8)/(tan(b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3))/b
```

mupad [B] time = 1.58, size = 56, normalized size = 0.70

$$\frac{35\,x}{8}+\frac{\cos\left(a+b\,x\right)^4\,\left(\frac{35\,\tan\left(a+b\,x\right)^6}{8}+\frac{175\,\tan\left(a+b\,x\right)^4}{24}+\frac{7\,\tan\left(a+b\,x\right)^2}{3}-\frac{1}{3}\right)}{b\,\tan\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^8/sin(a + b*x)^4,x)
```

```
[Out] (35*x)/8 + (cos(a + b*x)^4*((7*tan(a + b*x)^2)/3 + (175*tan(a + b*x)^4)/24
+ (35*tan(a + b*x)^6)/8 - 1/3))/(b*tan(a + b*x)^3)
```

sympy [A] time = 14.83, size = 141, normalized size = 1.76

$$\begin{cases}\frac{35x\sin^4(a+bx)}{8}+\frac{35x\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{35x\cos^4(a+bx)}{8}+\frac{35\sin^3(a+bx)\cos(a+bx)}{8b}+\frac{175\sin(a+bx)\cos^3(a+bx)}{24b}+\frac{7\cos^5(a+bx)}{3b\sin(a+bx)}- \\ \frac{x\cos^8(a)}{\sin^4(a)}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**8/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((35*x*sin(a + b*x)**4/8 + 35*x*sin(a + b*x)**2*cos(a + b*x)**2/4
+ 35*x*cos(a + b*x)**4/8 + 35*sin(a + b*x)**3*cos(a + b*x)/(8*b) + 175*sin(
a + b*x)*cos(a + b*x)**3/(24*b) + 7*cos(a + b*x)**5/(3*b*sin(a + b*x)) - co
s(a + b*x)**7/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**8/sin(a)**4, Tru
e))
```

3.159 $\int \cos^3(a+bx)\cot^4(a+bx)\,dx$

Optimal. Leaf size=53

$$-\frac{\sin^3(a+bx)}{3b}+\frac{3\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{3\csc(a+bx)}{b}$$

```
[Out] 3*csc(b*x+a)/b-1/3*csc(b*x+a)^3/b+3*sin(b*x+a)/b-1/3*sin(b*x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2590, 270}

$$-\frac{\sin^3(a+bx)}{3b}+\frac{3\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{3\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x]^4,x]
```

```
[Out] (3*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (3*Sin[a + b*x])/b - Sin[a + b*
x]^3/(3*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cos^3(a+bx)\cot^4(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^4}\,dx, x, -\sin(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}-x^2\right)dx, x, -\sin(a+bx)\right)}{b}$$
$$= \frac{3\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b} + \frac{3\sin(a+bx)}{b} - \frac{\sin^3(a+bx)}{3b}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.00

$$-\frac{\sin^3(a+bx)}{3b} + \frac{3\sin(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b} + \frac{3\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x]^4,x]
```

```
[Out] (3*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + (3*Sin[a + b*x])/b - Sin[a + b*
x]^3/(3*b)
```

fricas [A] time = 0.42, size = 56, normalized size = 1.06

$$-\frac{\cos(bx+a)^6 + 6\cos(bx+a)^4 - 24\cos(bx+a)^2 + 16}{3\left(b\cos(bx+a)^2 - b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(b*x + a)^6 + 6*cos(b*x + a)^4 - 24*cos(b*x + a)^2 + 16)/((b*cos(b
*x + a)^2 - b)*sin(b*x + a))
```

giac [A] time = 0.21, size = 41, normalized size = 0.77

$$-\frac{\left(\frac{1}{\sin(bx+a)} + \sin(bx+a)\right)^3 - \frac{12}{\sin(bx+a)} - 12\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*((1/sin(b*x + a) + sin(b*x + a))^3 - 12/sin(b*x + a) - 12*sin(b*x + a)
)/b
```

maple [A] time = 0.02, size = 80, normalized size = 1.51

$$\frac{-\frac{\cos^8(bx+a)}{3\sin(bx+a)^3}+\frac{5\left(\cos^8(bx+a)\right)}{3\sin(bx+a)}+\frac{5\left(\frac{16}{5}+\cos^6(bx+a)+\frac{6\left(\cos^4(bx+a)\right)}{5}+\frac{8\left(\cos^2(bx+a)\right)}{5}\right)\sin(bx+a)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3/sin(b*x+a)^3*cos(b*x+a)^8+5/3/sin(b*x+a)*cos(b*x+a)^8+5/3*(16/5+c
os(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.32, size = 44, normalized size = 0.83

$$-\frac{\sin(bx+a)^3-\frac{9\sin(bx+a)^2-1}{\sin(bx+a)^3}-9\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(sin(b*x + a)^3 - (9*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 - 9*sin(b*x +
a))/b
```

mupad [B] time = 0.48, size = 45, normalized size = 0.85

$$\frac{-\sin(a+b\,x)^6+9\sin(a+b\,x)^4+9\sin(a+b\,x)^2-1}{3\,b\sin(a+b\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^7/sin(a + b*x)^4,x)
```

```
[Out] (9*sin(a + b*x)^2 + 9*sin(a + b*x)^4 - sin(a + b*x)^6 - 1)/(3*b*sin(a + b*x
)^3)
```

sympy [A] time = 9.59, size = 82, normalized size = 1.55

$$\begin{cases} \frac{16\sin^3(a+bx)}{3b}+\frac{8\sin(a+bx)\cos^2(a+bx)}{b}+\frac{2\cos^4(a+bx)}{b\sin(a+bx)}-\frac{\cos^6(a+bx)}{3b\sin^3(a+bx)} & \text{for } b\neq 0 \\ \frac{x\cos^7(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((16*sin(a + b*x)**3/(3*b) + 8*sin(a + b*x)*cos(a + b*x)**2/b + 2*
cos(a + b*x)**4/(b*sin(a + b*x)) - cos(a + b*x)**6/(3*b*sin(a + b*x)**3), N
e(b, 0)), (x*cos(a)**7/sin(a)**4, True))
```

3.160 $\int \cos^2(a+bx)\cot^4(a+bx)\,dx$

Optimal. Leaf size=57

$$-\frac{5\cot^3(a+bx)}{6b}+\frac{5\cot(a+bx)}{2b}+\frac{\cos^2(a+bx)\cot^3(a+bx)}{2b}+\frac{5x}{2}$$

```
[Out] 5/2*x+5/2*cot(b*x+a)/b-5/6*cot(b*x+a)^3/b+1/2*cos(b*x+a)^2*cot(b*x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2591, 288, 302, 203}

$$-\frac{5\cot^3(a+bx)}{6b}+\frac{5\cot(a+bx)}{2b}+\frac{\cos^2(a+bx)\cot^3(a+bx)}{2b}+\frac{5x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x]^4,x]
```

```
[Out] (5*x)/2 + (5*Cot[a + b*x])/(2*b) - (5*Cot[a + b*x]^3)/(6*b) + (Cos[a + b*x]
^2*Cot[a + b*x]^3)/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
```

```
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \cos^2(a+bx)\cot^4(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \cot(a+bx)\right)}{b}$$

$$= \frac{\cos^2(a+bx)\cot^3(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \cot(a+bx)\right)}{2b}$$

$$= \frac{\cos^2(a+bx)\cot^3(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right) dx, x, \cot(a+bx)\right)}{2b}$$

$$= \frac{5\cot(a+bx)}{2b} - \frac{5\cot^3(a+bx)}{6b} + \frac{\cos^2(a+bx)\cot^3(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{1}{1+x^2}\,d\right.}{2}$$

$$= \frac{5x}{2} + \frac{5\cot(a+bx)}{2b} - \frac{5\cot^3(a+bx)}{6b} + \frac{\cos^2(a+bx)\cot^3(a+bx)}{2b}$$

Mathematica [A] time = 0.17, size = 43, normalized size = 0.75

$$\frac{30(a+bx)+3\sin(2(a+bx))-4\cot(a+bx)\left(\csc^2(a+bx)-7\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x]^4,x]
```

```
[Out] (30*(a + b*x) - 4*Cot[a + b*x]*(-7 + Csc[a + b*x]^2) + 3*Sin[2*(a + b*x)])/
(12*b)
```

fricas [A] time = 0.44, size = 79, normalized size = 1.39

$$-\frac{3\cos(bx+a)^5 - 20\cos(bx+a)^3 - 15\left(bx\cos(bx+a)^2 - bx\right)\sin(bx+a) + 15\cos(bx+a)}{6\left(b\cos(bx+a)^2 - b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*cos(b*x + a)^5 - 20*cos(b*x + a)^3 - 15*(b*x*cos(b*x + a)^2 - b*x)*
sin(b*x + a) + 15*cos(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

giac [A] time = 0.74, size = 55, normalized size = 0.96

$$\frac{15\,bx + 15\,a + \frac{3\,\tan(bx+a)}{\tan(bx+a)^2+1} + \frac{2\left(6\,\tan(bx+a)^2-1\right)}{\tan(bx+a)^3}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/6*(15*b*x + 15*a + 3*tan(b*x + a)/(tan(b*x + a)^2 + 1) + 2*(6*tan(b*x + a
)^2 - 1)/tan(b*x + a)^3)/b
```

maple [A] time = 0.02, size = 84, normalized size = 1.47

$$\frac{-\frac{\cos^7(bx+a)}{3\sin(bx+a)^3} + \frac{4\left(\cos^7(bx+a)\right)}{3\sin(bx+a)} + \frac{4\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{3} + \frac{5bx}{2} + \frac{5a}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3*cos(b*x+a)^7/sin(b*x+a)^3+4/3/sin(b*x+a)*cos(b*x+a)^7+4/3*(cos(b*
x+a)^5+5/4*cos(b*x+a)^3+15/8*cos(b*x+a))*sin(b*x+a)+5/2*b*x+5/2*a)
```

maxima [A] time = 0.43, size = 55, normalized size = 0.96

$$\frac{15\,bx + 15\,a + \frac{15\,\tan(bx+a)^4+10\,\tan(bx+a)^2-2}{\tan(bx+a)^5+\tan(bx+a)^3}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(15*b*x + 15*a + (15*tan(b*x + a)^4 + 10*tan(b*x + a)^2 - 2)/(tan(b*x +
 a)^5 + tan(b*x + a)^3))/b
```

mupad [B] time = 0.72, size = 46, normalized size = 0.81

$$\frac{5\,x}{2} + \frac{\cos\left(a+b\,x\right)^2\,\left(\frac{5\tan(a+b\,x)^4}{2}+\frac{5\tan(a+b\,x)^2}{3}-\frac{1}{3}\right)}{b\tan\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^6/sin(a + b*x)^4,x)
```

```
[Out] (5*x)/2 + (cos(a + b*x)^2*((5*tan(a + b*x)^2)/3 + (5*tan(a + b*x)^4)/2 - 1/
3))/(b*tan(a + b*x)^3)
```

sympy [A] time = 6.72, size = 97, normalized size = 1.70

$$\begin{cases} \frac{5x\sin^2(a+bx)}{2} + \frac{5x\cos^2(a+bx)}{2} + \frac{5\sin(a+bx)\cos(a+bx)}{2b} + \frac{5\cos^3(a+bx)}{3b\sin(a+bx)} - \frac{\cos^5(a+bx)}{3b\sin^3(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^6(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((5*x*sin(a + b*x)**2/2 + 5*x*cos(a + b*x)**2/2 + 5*sin(a + b*x)*c
os(a + b*x)/(2*b) + 5*cos(a + b*x)**3/(3*b*sin(a + b*x)) - cos(a + b*x)**5/
(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**6/sin(a)**4, True))
```

3.161 $\int \cos(a+bx)\cot^4(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{2\csc(a+bx)}{b}$$

```
[Out] 2*csc(b*x+a)/b-1/3*csc(b*x+a)^3/b+sin(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2590, 270}

$$\frac{\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[a + b*x]^4,x]
```

```
[Out] (2*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + Sin[a + b*x]/b
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \cos(a+bx)\cot^4(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^4}\,dx,x,-\sin(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx,x,-\sin(a+bx)\right)}{b}\\
&= \frac{2\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{\sin(a+bx)}{b}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{\sin(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}+\frac{2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x]^4,x]
```

```
[Out] (2*Csc[a + b*x])/b - Csc[a + b*x]^3/(3*b) + Sin[a + b*x]/b
```

fricas [A] time = 0.42, size = 48, normalized size = 1.30

$$-\frac{3\cos(bx+a)^4-12\cos(bx+a)^2+8}{3\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 8)/((b*cos(b*x + a)^2 - b)*sin
(b*x + a))
```

giac [A] time = 0.21, size = 35, normalized size = 0.95

$$\frac{\frac{6\sin(bx+a)^2-1}{\sin(bx+a)^3}+3\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*((6*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 + 3*sin(b*x + a))/b
```

maple [A] time = 0.02, size = 68, normalized size = 1.84

$$\frac{-\frac{\cos^6(bx+a)}{3\sin(bx+a)^3}+\frac{\cos^6(bx+a)}{\sin(bx+a)}+\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3*cos(b*x+a)^6/sin(b*x+a)^3+cos(b*x+a)^6/sin(b*x+a)+(8/3+cos(b*x+a)
^4+4/3*cos(b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.42, size = 35, normalized size = 0.95

$$\frac{\frac{6\sin(bx+a)^2-1}{\sin(bx+a)^3}+3\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((6*sin(b*x + a)^2 - 1)/sin(b*x + a)^3 + 3*sin(b*x + a))/b
```

mupad [B] time = 0.45, size = 32, normalized size = 0.86

$$\frac{\sin(a+b\,x)^4+2\sin(a+b\,x)^2-\frac{1}{3}}{b\sin(a+b\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5/sin(a + b*x)^4,x)
```

```
[Out] (2*sin(a + b*x)^2 + sin(a + b*x)^4 - 1/3)/(b*sin(a + b*x)^3)
```

sympy [A] time = 3.76, size = 63, normalized size = 1.70

$$\begin{cases} \frac{8\sin(a+bx)}{3b}+\frac{4\cos^2(a+bx)}{3b\sin(a+bx)}-\frac{\cos^4(a+bx)}{3b\sin^3(a+bx)} & \text{for } b\neq 0 \\ \frac{x\cos^5(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((8*sin(a + b*x)/(3*b) + 4*cos(a + b*x)**2/(3*b*sin(a + b*x)) - co
s(a + b*x)**4/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)**5/sin(a)**4, Tru
e))
```

3.162 $\int \cot^4(a+bx)\,dx$

Optimal. Leaf size=27

$$-\frac{\cot^3(a+bx)}{3b}+\frac{\cot(a+bx)}{b}+x$$

```
[Out] x+cot(b*x+a)/b-1/3*cot(b*x+a)^3/b
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 8}

$$-\frac{\cot^3(a+bx)}{3b}+\frac{\cot(a+bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^4,x]
```

```
[Out] x + Cot[a + b*x]/b - Cot[a + b*x]^3/(3*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int \cot^4(a+bx)\,dx &= -\frac{\cot^3(a+bx)}{3b}-\int \cot^2(a+bx)\,dx\\ &= \frac{\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\int 1\,dx\\ &= x+\frac{\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}\end{aligned}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 1.22

$$-\frac{\cot^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^4,x]
```

```
[Out] -1/3*(Cot[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[a + b*x]^2])/b
```

fricas [B] time = 0.42, size = 69, normalized size = 2.56

$$\frac{4\cos(bx+a)^3 + 3\left(bx\cos(bx+a)^2 - bx\right)\sin(bx+a) - 3\cos(bx+a)}{3\left(b\cos(bx+a)^2 - b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(b*x + a)^3 + 3*(b*x*cos(b*x + a)^2 - b*x)*sin(b*x + a) - 3*cos(b
*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

giac [B] time = 0.23, size = 62, normalized size = 2.30

$$\frac{\tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)^3 + 24\,bx + 24\,a + \frac{15\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2 - 1}{\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^3} - 15\,\tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/24*(tan(1/2*b*x + 1/2*a)^3 + 24*b*x + 24*a + (15*tan(1/2*b*x + 1/2*a)^2 -
 1)/tan(1/2*b*x + 1/2*a)^3 - 15*tan(1/2*b*x + 1/2*a))/b
```

maple [A] time = 0.02, size = 26, normalized size = 0.96

$$\frac{-\frac{\left(\cot^3(bx+a)\right)}{3} + \cot(bx+a) + bx + a}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3*cot(b*x+a)^3+cot(b*x+a)+b*x+a)
```

maxima [A] time = 0.43, size = 34, normalized size = 1.26

$$\frac{3\,bx + 3\,a + \frac{3\,\tan(bx+a)^2 - 1}{\tan(bx+a)^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*x + 3*a + (3*tan(b*x + a)^2 - 1)/tan(b*x + a)^3)/b
```

mupad [B] time = 0.45, size = 24, normalized size = 0.89

$$x + \frac{\tan(a + b\,x)^2 - \frac{1}{3}}{b\tan(a + b\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4/sin(a + b*x)^4,x)
```

```
[Out] x + (tan(a + b*x)^2 - 1/3)/(b*tan(a + b*x)^3)
```

sympy [A] time = 2.54, size = 48, normalized size = 1.78

$$\begin{cases} x + \frac{\cos(a+bx)}{b\sin(a+bx)} - \frac{\cos^3(a+bx)}{3b\sin^3(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos^4(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((x + cos(a + b*x)/(b*sin(a + b*x)) - cos(a + b*x)**3/(3*b*sin(a +
 b*x)**3), Ne(b, 0)), (x*cos(a)**4/sin(a)**4, True))
```

3.163 $\int \cot^3(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}$$

```
[Out] csc(b*x+a)/b-1/3*csc(b*x+a)^3/b
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2606}

$$\frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^3*Csc[a + b*x],x]
```

```
[Out] Csc[a + b*x]/b - Csc[a + b*x]^3/(3*b)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot^3(a+bx)\csc(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,\csc(a+bx)\right)}{b}\\ &= \frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{\csc(a+bx)}{b}-\frac{\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^3*Csc[a + b*x],x]
```

```
[Out] Csc[a + b*x]/b - Csc[a + b*x]^3/(3*b)
```

fricas [A] time = 0.42, size = 38, normalized size = 1.46

$$\frac{3 \cos(bx+a)^2 - 2}{3 \left(b \cos(bx+a)^2 - b\right) \sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*cos(b*x + a)^2 - 2)/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

giac [A] time = 0.18, size = 25, normalized size = 0.96

$$\frac{3 \sin(bx+a)^2 - 1}{3 b \sin(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^3)
```

maple [B] time = 0.02, size = 60, normalized size = 2.31

$$\frac{-\frac{\cos^4(bx+a)}{3\sin(bx+a)^3} + \frac{\cos^4(bx+a)}{3\sin(bx+a)} + \frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3*cos(b*x+a)^4/sin(b*x+a)^3+1/3*cos(b*x+a)^4/sin(b*x+a)+1/3*(2+cos(
b*x+a)^2)*sin(b*x+a))
```

maxima [A] time = 0.30, size = 25, normalized size = 0.96

$$\frac{3 \sin(bx+a)^2 - 1}{3 b \sin(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^3)
```

mupad [B] time = 0.43, size = 22, normalized size = 0.85

$$\frac{\sin\left(a+b\,x\right)^2-\frac{1}{3}}{b\sin\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(a + b*x)^4,x)
```

```
[Out] (sin(a + b*x)^2 - 1/3)/(b*sin(a + b*x)^3)
```

sympy [A] time = 2.26, size = 42, normalized size = 1.62

$$\begin{cases} \frac{2}{3b\sin\left(a+bx\right)}-\frac{\cos^2\left(a+bx\right)}{3b\sin^3\left(a+bx\right)} & \text{for } b\neq 0 \\ \frac{x\cos^3\left(a\right)}{\sin^4\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((2/(3*b*sin(a + b*x)) - cos(a + b*x)**2/(3*b*sin(a + b*x)**3), Ne
(b, 0)), (x*cos(a)**3/sin(a)**4, True))
```

3.164 $\int \cot^2(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cot^3(a+bx)}{3b}$$

```
[Out] -1/3*cot(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$-\frac{\cot^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^2*Csc[a + b*x]^2,x]
```

```
[Out] -Cot[a + b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \cot^2(a+bx)\csc^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int x^2\,dx, x, -\cot(a+bx)\right)}{b}\\ &= -\frac{\cot^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\cot^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^2*Csc[a + b*x]^2,x]
```

```
[Out] -1/3*Cot[a + b*x]^3/b
```

fricas [B] time = 0.45, size = 34, normalized size = 2.27

$$\frac{\cos(bx+a)^3}{3\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*cos(b*x + a)^3/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

giac [A] time = 0.23, size = 13, normalized size = 0.87

$$-\frac{1}{3\,b\tan(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3/(b*tan(b*x + a)^3)
```

maple [A] time = 0.02, size = 22, normalized size = 1.47

$$-\frac{\cos^3(bx+a)}{3\sin(bx+a)^3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(b*x+a)^4,x)
```

```
[Out] -1/3*cos(b*x+a)^3/sin(b*x+a)^3/b
```

maxima [A] time = 0.43, size = 13, normalized size = 0.87

$$-\frac{1}{3\,b\tan(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3/(b*tan(b*x + a)^3)
```

mupad [B] time = 0.38, size = 13, normalized size = 0.87

$$-\frac{\cot(a+b\,x)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(a + b*x)^4,x)
```

```
[Out] -cot(a + b*x)^3/(3*b)
```

sympy [A] time = 2.61, size = 71, normalized size = 4.73

$$\begin{cases} \frac{\tan^3\left(\frac{a}{2}+\frac{bx}{2}\right)}{24b} - \frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} + \frac{1}{8b\tan\left(\frac{a}{2}+\frac{bx}{2}\right)} - \frac{1}{24b\tan^3\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for } b \neq 0 \\ \frac{x\cos^2(a)}{\sin^4(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a)**4,x)
```

```
[Out] Piecewise((tan(a/2 + b*x/2)**3/(24*b) - tan(a/2 + b*x/2)/(8*b) + 1/(8*b*tan
(a/2 + b*x/2)) - 1/(24*b*tan(a/2 + b*x/2)**3), Ne(b, 0)), (x*cos(a)**2/sin(
a)**4, True))
```

3.165 $\int \cot(a+bx)\csc^3(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\csc^3(a+bx)}{3b}$$

```
[Out] -1/3*csc(b*x+a)^3/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]*Csc[a + b*x]^3,x]
```

```
[Out] -Csc[a + b*x]^3/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot(a+bx)\csc^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^2\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{\csc^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]*Csc[a + b*x]^3,x]
```

```
[Out] -1/3*Csc[a + b*x]^3/b
```

fricas [A] time = 0.42, size = 26, normalized size = 1.73

$$\frac{1}{3\left(b\cos\left(bx+a\right)^{2}-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

giac [A] time = 0.57, size = 13, normalized size = 0.87

$$-\frac{1}{3\,b\sin\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3/(b*sin(b*x + a)^3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$-\frac{1}{3\sin\left(bx+a\right)^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+a)^4,x)
```

```
[Out] -1/3/sin(b*x+a)^3/b
```

maxima [A] time = 0.32, size = 13, normalized size = 0.87

$$-\frac{1}{3\,b\sin\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3/(b*sin(b*x + a)^3)
```

mupad [**B**]   time = 0.43, size = 13, normalized size = 0.87

$$-\frac{1}{3\,b\sin\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(a + b*x)^4,x)
[Out] -1/(3*b*sin(a + b*x)^3)
```

sympy [**A**]   time = 1.57, size = 24, normalized size = 1.60

$$\begin{cases} -\frac{1}{3b\sin^3\left(a+bx\right)} & \text{for}\ b \neq 0 \\ \frac{x\cos\left(a\right)}{\sin^4\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)**4,x)
[Out] Piecewise((-1/(3*b*sin(a + b*x)**3), Ne(b, 0)), (x*cos(a)/sin(a)**4, True))
```

3.166 $\int \csc^4(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=38

$$-\frac{\csc^3(a+bx)}{3b}-\frac{\csc(a+bx)}{b}+\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

```
[Out] arctanh(sin(b*x+a))/b-csc(b*x+a)/b-1/3*csc(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2621, 302, 207}

$$-\frac{\csc^3(a+bx)}{3b}-\frac{\csc(a+bx)}{b}+\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b - Csc[a + b*x]/b - Csc[a + b*x]^3/(3*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^4(a+bx)\sec(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(a+bx)\right)}{b}$$
$$= -\frac{\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{b}$$
$$= \frac{\tanh^{-1}(\sin(a+bx))}{b} - \frac{\csc(a+bx)}{b} - \frac{\csc^3(a+bx)}{3b}$$

Mathematica [C] time = 0.01, size = 31, normalized size = 0.82

$$-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x],x]
```

```
[Out] -1/3*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/b
```

fricas [B] time = 0.42, size = 94, normalized size = 2.47

$$\frac{3\left(\cos\left(bx+a\right)^2-1\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-3\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right.}{6\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b*x
 + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)/((
b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

giac [A] time = 0.23, size = 52, normalized size = 1.37

$$-\frac{\frac{2\left(3\sin\left(bx+a\right)^2+1\right)}{\sin\left(bx+a\right)^3} - 3\log\left(\left|\sin\left(bx+a\right)+1\right|\right) + 3\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(abs(sin(b*x + a) + 1)
) + 3*log(abs(sin(b*x + a) - 1)))/b
```

maple [A] time = 0.03, size = 46, normalized size = 1.21

$$-\frac{1}{3\sin\left(bx+a\right)^{3}b}-\frac{1}{b\sin\left(bx+a\right)}+\frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/sin(b*x+a)^4,x)
```

```
[Out] -1/3/sin(b*x+a)^3/b-1/b/sin(b*x+a)+1/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.36, size = 50, normalized size = 1.32

$$-\frac{\frac{2\left(3\,\sin\left(bx+a\right)^{2}+1\right)}{\sin\left(bx+a\right)^{3}}-3\,\log\left(\sin\left(bx+a\right)+1\right)+3\,\log\left(\sin\left(bx+a\right)-1\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) + 3
*log(sin(b*x + a) - 1))/b
```

mupad [B] time = 0.02, size = 32, normalized size = 0.84

$$\frac{\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)-\frac{\sin\left(a+b\,x\right)^{2}+\frac{1}{3}}{\sin\left(a+b\,x\right)^{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^4),x)
```

```
[Out] (atanh(sin(a + b*x)) - (sin(a + b*x)^2 + 1/3)/sin(a + b*x)^3)/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(a+bx\right)}{\sin^{4}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)/sin(a + b*x)**4, x)
```

3.167 $\int \csc^4(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{\tan(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}-\frac{2\cot(a+bx)}{b}$$

```
[Out] -2*cot(b*x+a)/b-1/3*cot(b*x+a)^3/b+tan(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 270}

$$\frac{\tan(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}-\frac{2\cot(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x]^2,x]
```

```
[Out] (-2*Cot[a + b*x])/b - Cot[a + b*x]^3/(3*b) + Tan[a + b*x]/b
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \csc^4(a+bx)\sec^2(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4}\,dx, x, \tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx, x, \tan(a+bx)\right)}{b}\\ &= -\frac{2\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\frac{\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.22

$$\frac{\tan(a+bx)}{b}-\frac{5\cot(a+bx)}{3b}-\frac{\cot(a+bx)\csc^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x]^2,x]
```

```
[Out] (-5*Cot[a + b*x])/(3*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(3*b) + Tan[a + b*x
]/b
```

fricas [A] time = 0.44, size = 54, normalized size = 1.46

$$-\frac{8\cos\left(bx+a\right)^4-12\cos\left(bx+a\right)^2+3}{3\left(b\cos\left(bx+a\right)^3-b\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*cos(
b*x + a))*sin(b*x + a))
```

giac [A] time = 0.22, size = 35, normalized size = 0.95

$$-\frac{\frac{6\tan(bx+a)^2+1}{\tan(bx+a)^3}-3\tan\left(bx+a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

maple [A] time = 0.04, size = 50, normalized size = 1.35

$$\frac{-\frac{1}{3\sin(bx+a)^3\cos(bx+a)}+\frac{4}{3\sin(bx+a)\cos(bx+a)}-\frac{8\cot(bx+a)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+a))
```

maxima [A] time = 0.53, size = 35, normalized size = 0.95

$$-\frac{\frac{6 \tan(bx+a)^2+1}{\tan(bx+a)^3} - 3 \tan(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

mupad [B] time = 0.42, size = 36, normalized size = 0.97

$$\frac{\tan(a+b\,x)}{b} - \frac{2\tan(a+b\,x)^2+\frac{1}{3}}{b\tan(a+b\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^4),x)
```

```
[Out] tan(a + b*x)/b - (2*tan(a + b*x)^2 + 1/3)/(b*tan(a + b*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)}{\sin^4(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)**2/sin(a + b*x)**4, x)
```

3.168 $\int \csc^4(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\csc^3(a+bx)}{6b}-\frac{5\csc(a+bx)}{2b}+\frac{5\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{2b}$$

```
[Out] 5/2*arctanh(sin(b*x+a))/b-5/2*csc(b*x+a)/b-5/6*csc(b*x+a)^3/b+1/2*csc(b*x+a
)^3*sec(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 302, 207}

$$-\frac{5\csc^3(a+bx)}{6b}-\frac{5\csc(a+bx)}{2b}+\frac{5\tanh^{-1}(\sin(a+bx))}{2b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x]^3,x]
```

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(2*b) - (5*Csc[a + b*x])/(2*b) - (5*Csc[a + b*x]^
3)/(6*b) + (Csc[a + b*x]^3*Sec[a + b*x]^2)/(2*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^4(a+bx)\sec^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \csc(a+bx)\right)}{b}$$

$$= \frac{\csc^3(a+bx)\sec^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}$$

$$= \frac{\csc^3(a+bx)\sec^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(a+bx)\right)}{2b}$$

$$= -\frac{5\csc(a+bx)}{2b} - \frac{5\csc^3(a+bx)}{6b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{2b} - \frac{5\,\text{Subst}\left(\int \frac{}{-1}\right.}{}$$

$$= \frac{5\tanh^{-1}(\sin(a+bx))}{2b} - \frac{5\csc(a+bx)}{2b} - \frac{5\csc^3(a+bx)}{6b} + \frac{\csc^3(a+bx)\sec^2(a}{2b}$$

Mathematica [C] time = 0.01, size = 31, normalized size = 0.47

$$-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x]^3,x]
[Out] -1/3*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/b
```

fricas [B] time = 0.44, size = 130, normalized size = 1.97

$$-\frac{30\cos(bx+a)^4 - 15\left(\cos(bx+a)^4 - \cos(bx+a)^2\right)\log(\sin(bx+a)+1)\sin(bx+a) + 15\left(\cos(bx+a)^4 - \text{c}\right.}{12\left(b\cos(bx+a)^4 - b\cos(bx+a)^2\right)\sin(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/12*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b*x
 + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(b*x
 + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b*cos
(b*x + a)^2)*sin(b*x + a))
```

giac [A] time = 0.54, size = 72, normalized size = 1.09

$$-\frac{\frac{6\,\sin(bx+a)}{\sin(bx+a)^2-1}+\frac{4\left(6\,\sin(bx+a)^2+1\right)}{\sin(bx+a)^3}-15\,\log\left(|\sin\left(bx+a\right)+1|\right)+15\,\log\left(|\sin\left(bx+a\right)-1|\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/12*(6*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(6*sin(b*x + a)^2 + 1)/sin(b
*x + a)^3 - 15*log(abs(sin(b*x + a) + 1)) + 15*log(abs(sin(b*x + a) - 1)))/
b
```

maple [A] time = 0.04, size = 76, normalized size = 1.15

$$-\frac{1}{3b\sin\left(bx+a\right)^3\cos\left(bx+a\right)^2}+\frac{5}{6b\sin\left(bx+a\right)\cos\left(bx+a\right)^2}-\frac{5}{2b\sin\left(bx+a\right)}+\frac{5\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/sin(b*x+a)^4,x)
```

```
[Out] -1/3/b/sin(b*x+a)^3/cos(b*x+a)^2+5/6/b/sin(b*x+a)/cos(b*x+a)^2-5/2/b/sin(b*
x+a)+5/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.31, size = 73, normalized size = 1.11

$$-\frac{\frac{2\left(15\,\sin(bx+a)^4-10\,\sin(bx+a)^2-2\right)}{\sin(bx+a)^5-\sin(bx+a)^3}-15\,\log\left(\sin\left(bx+a\right)+1\right)+15\,\log\left(\sin\left(bx+a\right)-1\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/12*(2*(15*sin(b*x + a)^4 - 10*sin(b*x + a)^2 - 2)/(sin(b*x + a)^5 - sin(
b*x + a)^3) - 15*log(sin(b*x + a) + 1) + 15*log(sin(b*x + a) - 1))/b
```

mupad [B] time = 0.38, size = 61, normalized size = 0.92

$$\frac{5\,\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{2\,b}-\frac{-\frac{5\,\sin(a+b\,x)^4}{2}+\frac{5\,\sin(a+b\,x)^2}{3}+\frac{1}{3}}{b\,\left(\sin\left(a+b\,x\right)^3-\sin\left(a+b\,x\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^4),x)
```

```
[Out] (5*atanh(sin(a + b*x)))/(2*b) - ((5*sin(a + b*x)^2)/3 - (5*sin(a + b*x)^4)/
2 + 1/3)/(b*(sin(a + b*x)^3 - sin(a + b*x)^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(a+bx)}{\sin^4(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)**3/sin(a + b*x)**4, x)
```

3.169 $\int \csc^4(a+bx)\sec^4(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{\tan^3(a+bx)}{3b}+\frac{3\tan(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}-\frac{3\cot(a+bx)}{b}$$

```
[Out] -3*cot(b*x+a)/b-1/3*cot(b*x+a)^3/b+3*tan(b*x+a)/b+1/3*tan(b*x+a)^3/b
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2620, 270}

$$\frac{\tan^3(a+bx)}{3b}+\frac{3\tan(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}-\frac{3\cot(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x]^4,x]
```

```
[Out] (-3*Cot[a + b*x])/b - Cot[a + b*x]^3/(3*b) + (3*Tan[a + b*x])/b + Tan[a + b
*x]^3/(3*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\begin{aligned}\int \csc^4(a+bx)\sec^4(a+bx)\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^4}\,dx,x,\tan(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^4}+\frac{3}{x^2}+x^2\right)\,dx,x,\tan(a+bx)\right)}{b}\\ &= -\frac{3\cot(a+bx)}{b}-\frac{\cot^3(a+bx)}{3b}+\frac{3\tan(a+bx)}{b}+\frac{\tan^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.81

$$16\left(-\frac{\cot(2(a+bx))}{3b}-\frac{\cot(2(a+bx))\csc^2(2(a+bx))}{6b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x]^4,x]
```

```
[Out] 16*(-1/3*Cot[2*(a + b*x)]/b - (Cot[2*(a + b*x)]*Csc[2*(a + b*x)]^2)/(6*b))
```

fricas [A] time = 0.50, size = 66, normalized size = 1.25

$$-\frac{16\cos(bx+a)^6-24\cos(bx+a)^4+6\cos(bx+a)^2+1}{3\left(b\cos(bx+a)^5-b\cos(bx+a)^3\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(16*cos(b*x + a)^6 - 24*cos(b*x + a)^4 + 6*cos(b*x + a)^2 + 1)/((b*cos
(b*x + a)^5 - b*cos(b*x + a)^3)*sin(b*x + a))
```

giac [A] time = 0.26, size = 31, normalized size = 0.58

$$-\frac{8\left(3\tan(2bx+2a)^2+1\right)}{3b\tan(2bx+2a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -8/3*(3*tan(2*b*x + 2*a)^2 + 1)/(b*tan(2*b*x + 2*a)^3)
```

maple [A] time = 0.05, size = 68, normalized size = 1.28

$$\frac{\frac{1}{3\sin(bx+a)^3\cos(bx+a)^3}-\frac{2}{3\sin(bx+a)^3\cos(bx+a)}+\frac{8}{3\sin(bx+a)\cos(bx+a)}-\frac{16\cot(bx+a)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/sin(b*x+a)^4,x)
```

```
[Out] 1/b*(1/3/sin(b*x+a)^3/cos(b*x+a)^3-2/3/sin(b*x+a)^3/cos(b*x+a)+8/3/sin(b*x+
a)/cos(b*x+a)-16/3*cot(b*x+a))
```

maxima [A] time = 0.32, size = 44, normalized size = 0.83

$$\frac{\tan\left(bx+a\right)^{3} - \frac{9\,\tan\left(bx+a\right)^{2}+1}{\tan\left(bx+a\right)^{3}} + 9\,\tan\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(b*x + a)^3 - (9*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 + 9*tan(b*x + a
))/b
```

mupad [B] time = 0.46, size = 45, normalized size = 0.85

$$-\frac{-\tan\left(a+b\,x\right)^6-9\tan\left(a+b\,x\right)^4+9\tan\left(a+b\,x\right)^2+1}{3\,b\tan\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^4*sin(a + b*x)^4),x)
```

```
[Out] -(9*tan(a + b*x)^2 - 9*tan(a + b*x)^4 - tan(a + b*x)^6 + 1)/(3*b*tan(a + b*
x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(a+bx\right)}{\sin^{4}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)**4/sin(a + b*x)**4, x)
```

3.170 $\int \csc^4(a+bx)\sec^5(a+bx)\,dx$

Optimal. Leaf size=89

$$-\frac{35\csc^3(a+bx)}{24b}-\frac{35\csc(a+bx)}{8b}+\frac{35\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\csc^3(a+bx)\sec^4(a+bx)}{4b}+\frac{7\csc^3(a+bx)\sec^2(a+$$

```
[Out] 35/8*arctanh(sin(b*x+a))/b-35/8*csc(b*x+a)/b-35/24*csc(b*x+a)^3/b+7/8*csc(b
*x+a)^3*sec(b*x+a)^2/b+1/4*csc(b*x+a)^3*sec(b*x+a)^4/b
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2621, 288, 302, 207}

$$-\frac{35\csc^3(a+bx)}{24b}-\frac{35\csc(a+bx)}{8b}+\frac{35\tanh^{-1}(\sin(a+bx))}{8b}+\frac{\csc^3(a+bx)\sec^4(a+bx)}{4b}+\frac{7\csc^3(a+bx)\sec^2(a+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^4*Sec[a + b*x]^5,x]
```

```
[Out] (35*ArcTanh[Sin[a + b*x]])/(8*b) - (35*Csc[a + b*x])/(8*b) - (35*Csc[a + b*
x]^3)/(24*b) + (7*Csc[a + b*x]^3*Sec[a + b*x]^2)/(8*b) + (Csc[a + b*x]^3*Se
c[a + b*x]^4)/(4*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^4(a+bx)\sec^5(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^3}\,dx, x, \csc(a+bx)\right)}{b}$$

$$= \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b} - \frac{7\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \csc(a+bx)\right)}{4b}$$

$$= \frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b} - \frac{35\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx \right.}{8b}$$

$$= \frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{4b} - \frac{35\,\text{Subst}\left(\int \left(1+x^2\right.\right.}{}$$

$$= -\frac{35\csc(a+bx)}{8b} - \frac{35\csc^3(a+bx)}{24b} + \frac{7\csc^3(a+bx)\sec^2(a+bx)}{8b} + \frac{\csc^3(a+bx}{}$$

$$= \frac{35\tanh^{-1}(\sin(a+bx))}{8b} - \frac{35\csc(a+bx)}{8b} - \frac{35\csc^3(a+bx)}{24b} + \frac{7\csc^3(a+bx)\,\text{se}}{8b}$$

Mathematica [C] time = 0.01, size = 31, normalized size = 0.35

$$-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^4*Sec[a + b*x]^5,x]
[Out] -1/3*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[a + b*x]^2])/b
```

fricas [A] time = 0.48, size = 140, normalized size = 1.57

$$-\frac{210\,\cos(bx+a)^6 - 280\,\cos(bx+a)^4 - 105\left(\cos(bx+a)^6 - \cos(bx+a)^4\right)\log(\sin(bx+a)+1)\sin(bx+a) +}{48\left(b\cos(bx+a)^6 - b\cos(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(210*cos(b*x + a)^6 - 280*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - cos(
b*x + a)^4)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*(cos(b*x + a)^6 - cos(
b*x + a)^4)*log(-sin(b*x + a) + 1)*sin(b*x + a) + 42*cos(b*x + a)^2 + 12)/(
(b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

giac [A] time = 0.60, size = 85, normalized size = 0.96

$$-\frac{\frac{6\left(11\,\sin(bx+a)^{3}-13\,\sin(bx+a)\right)}{\left(\sin(bx+a)^{2}-1\right)^{2}}+\frac{16\left(9\,\sin(bx+a)^{2}+1\right)}{\sin(bx+a)^{3}}-105\,\log\left(\left|\sin\left(bx+a\right)+1\right|\right)+105\,\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/48*(6*(11*sin(b*x + a)^3 - 13*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 16*
(9*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 105*log(abs(sin(b*x + a) + 1)) + 10
5*log(abs(sin(b*x + a) - 1)))/b
```

maple [A] time = 0.05, size = 97, normalized size = 1.09

$$\frac{1}{4b\sin\left(bx+a\right)^{3}\cos\left(bx+a\right)^{4}}-\frac{7}{12b\sin\left(bx+a\right)^{3}\cos\left(bx+a\right)^{2}}+\frac{35}{24b\sin\left(bx+a\right)\cos\left(bx+a\right)^{2}}-\frac{35}{8b\sin\left(bx+a\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5/sin(b*x+a)^4,x)
```

```
[Out] 1/4/b/sin(b*x+a)^3/cos(b*x+a)^4-7/12/b/sin(b*x+a)^3/cos(b*x+a)^2+35/24/b/si
n(b*x+a)/cos(b*x+a)^2-35/8/b/sin(b*x+a)+35/8/b*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [A] time = 0.35, size = 91, normalized size = 1.02

$$-\frac{\frac{2\left(105\,\sin(bx+a)^{6}-175\,\sin(bx+a)^{4}+56\,\sin(bx+a)^{2}+8\right)}{\sin(bx+a)^{7}-2\,\sin(bx+a)^{5}+\sin(bx+a)^{3}}-105\,\log\left(\sin\left(bx+a\right)+1\right)+105\,\log\left(\sin\left(bx+a\right)-1\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(105*sin(b*x + a)^6 - 175*sin(b*x + a)^4 + 56*sin(b*x + a)^2 + 8)/
(sin(b*x + a)^7 - 2*sin(b*x + a)^5 + sin(b*x + a)^3) - 105*log(sin(b*x + a)
 + 1) + 105*log(sin(b*x + a) - 1))/b
```

mupad [B] time = 0.45, size = 79, normalized size = 0.89

$$\frac{35\,\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{8\,b}-\frac{\frac{35\,\sin(a+b\,x)^6}{8}-\frac{175\,\sin(a+b\,x)^4}{24}+\frac{7\,\sin(a+b\,x)^2}{3}+\frac{1}{3}}{b\,\left(\sin\left(a+b\,x\right)^7-2\,\sin\left(a+b\,x\right)^5+\sin\left(a+b\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^5*sin(a + b*x)^4),x)
```

```
[Out] (35*atanh(sin(a + b*x)))/(8*b) - ((7*sin(a + b*x)^2)/3 - (175*sin(a + b*x)^
4)/24 + (35*sin(a + b*x)^6)/8 + 1/3)/(b*(sin(a + b*x)^3 - 2*sin(a + b*x)^5
+ sin(a + b*x)^7))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^5\left(a+bx\right)}{\sin^4\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a)**4,x)
```

```
[Out] Integral(sec(a + b*x)**5/sin(a + b*x)**4, x)
```

3.171 $\int \cos^4(a+bx)\cot^5(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{\sin^4(a+bx)}{4b}-\frac{2\sin^2(a+bx)}{b}-\frac{\csc^4(a+bx)}{4b}+\frac{2\csc^2(a+bx)}{b}+\frac{6\log(\sin(a+bx))}{b}$$

```
[Out] 2*csc(b*x+a)^2/b-1/4*csc(b*x+a)^4/b+6*ln(sin(b*x+a))/b-2*sin(b*x+a)^2/b+1/4
*sin(b*x+a)^4/b
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$\frac{\sin^4(a+bx)}{4b}-\frac{2\sin^2(a+bx)}{b}-\frac{\csc^4(a+bx)}{4b}+\frac{2\csc^2(a+bx)}{b}+\frac{6\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Cot[a + b*x]^5,x]
```

```
[Out] (2*Csc[a + b*x]^2)/b - Csc[a + b*x]^4/(4*b) + (6*Log[Sin[a + b*x]])/b - (2*
Sin[a + b*x]^2)/b + Sin[a + b*x]^4/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cos^4(a+bx)\cot^5(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{x^5}\,dx, x, -\sin(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^4}{x^3}\,dx, x, \sin^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(-4+\frac{1}{x^3}-\frac{4}{x^2}+\frac{6}{x}+x\right)dx, x, \sin^2(a+bx)\right)}{2b}$$
$$= \frac{2\csc^2(a+bx)}{b} - \frac{\csc^4(a+bx)}{4b} + \frac{6\log(\sin(a+bx))}{b} - \frac{2\sin^2(a+bx)}{b} + \frac{\sin^4(a\cdots}{4b}$$

Mathematica [A] time = 0.11, size = 55, normalized size = 0.80

$$\frac{\sin^4(a+bx) - 8\sin^2(a+bx) - \csc^4(a+bx) + 8\csc^2(a+bx) + 24\log(\sin(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Cot[a + b*x]^5,x]
```

```
[Out] (8*Csc[a + b*x]^2 - Csc[a + b*x]^4 + 24*Log[Sin[a + b*x]] - 8*Sin[a + b*x]^
2 + Sin[a + b*x]^4)/(4*b)
```

fricas [A] time = 0.54, size = 100, normalized size = 1.45

$$\frac{8\cos(bx+a)^8 + 32\cos(bx+a)^6 - 115\cos(bx+a)^4 + 38\cos(bx+a)^2 + 192\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 \cdots\right.}{32\left(b\cos(bx+a)^4 - 2b\cos(bx+a)^2 + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/32*(8*cos(b*x + a)^8 + 32*cos(b*x + a)^6 - 115*cos(b*x + a)^4 + 38*cos(b*
x + a)^2 + 192*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*sin(b*x + a)
) + 29)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

giac [B] time = 0.44, size = 277, normalized size = 4.01

$$-\frac{\frac{\left(\frac{28(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{288(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} + \frac{28(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{32\left(\frac{84(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{126(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{84(\cos(bx\cdots}{(\cos(bx+\cdots}\cdots\right.}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^4}\cdots}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*((28*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 288*(cos(b*x + a) - 1)^2
/(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 28*(
cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) +
 1)^2 + 32*(84*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 126*(cos(b*x + a) -
1)^2/(cos(b*x + a) + 1)^2 + 84*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 -
25*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 25)/((cos(b*x + a) - 1)/(cos
(b*x + a) + 1) - 1)^4 - 192*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1
)) + 384*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

maple [A] time = 0.03, size = 107, normalized size = 1.55

$$-\frac{\cos^{10}(bx+a)}{4b\sin(bx+a)^4}+\frac{3\left(\cos^{10}(bx+a)\right)}{4b\sin(bx+a)^2}+\frac{3\left(\cos^{8}(bx+a)\right)}{4b}+\frac{\cos^{6}(bx+a)}{b}+\frac{3\left(\cos^{4}(bx+a)\right)}{2b}+\frac{3\left(\cos^{2}(bx+a)\right)}{b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^9/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b/sin(b*x+a)^4*cos(b*x+a)^10+3/4/b/sin(b*x+a)^2*cos(b*x+a)^10+3/4*cos(
b*x+a)^8/b+cos(b*x+a)^6/b+3/2*cos(b*x+a)^4/b+3*cos(b*x+a)^2/b+6*ln(sin(b*x+
a))/b
```

maxima [A] time = 0.45, size = 56, normalized size = 0.81

$$\frac{\sin(bx+a)^4-8\sin(bx+a)^2+\frac{8\sin(bx+a)^2-1}{\sin(bx+a)^4}+12\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^9/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(sin(b*x + a)^4 - 8*sin(b*x + a)^2 + (8*sin(b*x + a)^2 - 1)/sin(b*x + a
)^4 + 12*log(sin(b*x + a)^2))/b
```

mupad [B] time = 1.42, size = 92, normalized size = 1.33

$$\frac{6\ln(\tan(a+bx))}{b}-\frac{3\ln\left(\tan(a+bx)^2+1\right)}{b}+\frac{3\tan(a+bx)^6+\frac{9\tan(a+bx)^4}{2}+\tan(a+bx)^2-\frac{1}{4}}{b\left(\tan(a+bx)^8+2\tan(a+bx)^6+\tan(a+bx)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^9/sin(a + b*x)^5,x)
```

```
[Out] (6*log(tan(a + b*x)))/b - (3*log(tan(a + b*x)^2 + 1))/b + (tan(a + b*x)^2 +
 (9*tan(a + b*x)^4)/2 + 3*tan(a + b*x)^6 - 1/4)/(b*(tan(a + b*x)^4 + 2*tan(
a + b*x)^6 + tan(a + b*x)^8))
```

sympy [A] time = 27.69, size = 1664, normalized size = 24.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**9/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-384*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**12/(64*b*tan
(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8
+ 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 1536*log(tan(a/2
+ b*x/2)**2 + 1)*tan(a/2 + b*x/2)**10/(64*b*tan(a/2 + b*x/2)**12 + 256*b*ta
n(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6
+ 64*b*tan(a/2 + b*x/2)**4) - 2304*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b
*x/2)**8/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*ta
n(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) -
 1536*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(64*b*tan(a/2 + b*x/
2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan
(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 384*log(tan(a/2 + b*x/2)**2
+ 1)*tan(a/2 + b*x/2)**4/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2
)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/
2 + b*x/2)**4) + 384*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**12/(64*b*tan(a
/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 +
256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 1536*log(tan(a/2 +
b*x/2))*tan(a/2 + b*x/2)**10/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b
*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*ta
n(a/2 + b*x/2)**4) + 2304*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(64*b*t
an(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**
8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 1536*log(tan(a/
2 + b*x/2))*tan(a/2 + b*x/2)**6/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2
+ b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b
*tan(a/2 + b*x/2)**4) + 384*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(64*b
*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)
**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - tan(a/2 + b*x
/2)**16/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan
(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) +
24*tan(a/2 + b*x/2)**14/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)
**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2
 + b*x/2)**4) - 744*tan(a/2 + b*x/2)**10/(64*b*tan(a/2 + b*x/2)**12 + 256*b
*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)*
```

```
*6 + 64*b*tan(a/2 + b*x/2)**4) - 1182*tan(a/2 + b*x/2)**8/(64*b*tan(a/2 + b
*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*x/2)**8 + 256*b*
tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 744*tan(a/2 + b*x/2)**6/(
64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan(a/2 + b*
x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 24*tan(a/
2 + b*x/2)**2/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384
*b*tan(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)*
*4) - 1/(64*b*tan(a/2 + b*x/2)**12 + 256*b*tan(a/2 + b*x/2)**10 + 384*b*tan
(a/2 + b*x/2)**8 + 256*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4), N
e(b, 0)), (x*cos(a)**9/sin(a)**5, True))
```

3.172 $\int \cos^3(a+bx)\cot^5(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{35\cos^3(a+bx)}{24b}+\frac{35\cos(a+bx)}{8b}-\frac{\cos^3(a+bx)\cot^4(a+bx)}{4b}+\frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b}-\frac{35\tanh^{-1}(\cos(a+bx))}{8b}$$

```
[Out] -35/8*arctanh(cos(b*x+a))/b+35/8*cos(b*x+a)/b+35/24*cos(b*x+a)^3/b+7/8*cos(
b*x+a)^3*cot(b*x+a)^2/b-1/4*cos(b*x+a)^3*cot(b*x+a)^4/b
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2592, 288, 302, 206}

$$\frac{35\cos^3(a+bx)}{24b}+\frac{35\cos(a+bx)}{8b}-\frac{\cos^3(a+bx)\cot^4(a+bx)}{4b}+\frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b}-\frac{35\tanh^{-1}(\cos(a+bx))}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cot[a + b*x]^5,x]
```

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(8*b) + (35*Cos[a + b*x])/(8*b) + (35*Cos[a + b
*x]^3)/(24*b) + (7*Cos[a + b*x]^3*Cot[a + b*x]^2)/(8*b) - (Cos[a + b*x]^3*C
ot[a + b*x]^4)/(4*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \cos^3(a+bx)\cot^5(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^8}{\left(1-x^2\right)^3}\,dx, x, \cos(a+bx)\right)}{b}$$

$$= -\frac{\cos^3(a+bx)\cot^4(a+bx)}{4b} + \frac{7\,\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^2}\,dx, x, \cos(a+bx)\right)}{4b}$$

$$= \frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos^3(a+bx)\cot^4(a+bx)}{4b} - \frac{35\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,d\right.}{8b}$$

$$= \frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos^3(a+bx)\cot^4(a+bx)}{4b} - \frac{35\,\text{Subst}\left(\int\left(-1-\right.\right.}{}$$

$$= \frac{35\cos(a+bx)}{8b} + \frac{35\cos^3(a+bx)}{24b} + \frac{7\cos^3(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos^3(a+b}{}$$

$$= -\frac{35\tanh^{-1}(\cos(a+bx))}{8b} + \frac{35\cos(a+bx)}{8b} + \frac{35\cos^3(a+bx)}{24b} + \frac{7\cos^3(a+b}{}$$

Mathematica [A] time = 0.04, size = 141, normalized size = 1.58

$$\frac{13\cos(a+bx)}{4b} + \frac{\cos(3(a+bx))}{12b} - \frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{13\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{13\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b} -$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cot[a + b*x]^5,x]
```

```
[Out] (13*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b) + (13*Csc[(a + b*x)/2]^2)
/(32*b) - Csc[(a + b*x)/2]^4/(64*b) - (35*Log[Cos[(a + b*x)/2]])/(8*b) + (3
5*Log[Sin[(a + b*x)/2]])/(8*b) - (13*Sec[(a + b*x)/2]^2)/(32*b) + Sec[(a +
b*x)/2]^4/(64*b)
```

fricas [A] time = 0.47, size = 132, normalized size = 1.48

$$\frac{16 \cos(bx+a)^7 + 112 \cos(bx+a)^5 - 350 \cos(bx+a)^3 - 105 \left(\cos(bx+a)^4 - 2 \cos(bx+a)^2 + 1\right) \log\left(\frac{1}{2} \cos(b\right.}{48 \left(b \cos(bx+a)^4 - 2 b \cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(16*cos(b*x + a)^7 + 112*cos(b*x + a)^5 - 350*cos(b*x + a)^3 - 105*(co
s(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*cos(b*x + a) + 1/2) + 105*(cos
(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/2*cos(b*x + a) + 1/2) + 210*cos(
b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

giac [B] time = 0.24, size = 209, normalized size = 2.35

$$-\frac{\frac{3\left(\frac{24(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{210(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} - \frac{72(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - \frac{256\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{6(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-5\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3} - 42}{192\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/192*(3*(24*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 210*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 7
2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(cos(b*x +
 a) + 1)^2 - 256*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 6*(cos(b*x + a)
 - 1)^2/(cos(b*x + a) + 1)^2 - 5)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) -
1)^3 - 420*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.02, size = 115, normalized size = 1.29

$$-\frac{\cos^9(bx+a)}{4b\sin(bx+a)^4} + \frac{5\left(\cos^9(bx+a)\right)}{8b\sin(bx+a)^2} + \frac{5\left(\cos^7(bx+a)\right)}{8b} + \frac{7\left(\cos^5(bx+a)\right)}{8b} + \frac{35\left(\cos^3(bx+a)\right)}{24b} + \frac{35\cos(bx+a)}{8b} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^8/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b*cos(b*x+a)^9/sin(b*x+a)^4+5/8/b/sin(b*x+a)^2*cos(b*x+a)^9+5/8*cos(b*
x+a)^7/b+7/8*cos(b*x+a)^5/b+35/24*cos(b*x+a)^3/b+35/8*cos(b*x+a)/b+35/8/b*l
n(csc(b*x+a)-cot(b*x+a))
```

maxima [A] time = 0.35, size = 89, normalized size = 1.00

$$\frac{16 \cos(bx+a)^3 - \frac{6\left(13 \cos(bx+a)^3 - 11 \cos(bx+a)\right)}{\cos(bx+a)^4 - 2 \cos(bx+a)^2 + 1} + 144 \cos(bx+a) - 105 \log(\cos(bx+a)+1) + 105 \log(\cos(bx$$
$$48\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(16*cos(b*x + a)^3 - 6*(13*cos(b*x + a)^3 - 11*cos(b*x + a))/(cos(b*x
+ a)^4 - 2*cos(b*x + a)^2 + 1) + 144*cos(b*x + a) - 105*log(cos(b*x + a) +
1) + 105*log(cos(b*x + a) - 1))/b
```

mupad [B] time = 0.55, size = 157, normalized size = 1.76

$$\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{64\,b}-\frac{3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{8\,b}+\frac{35\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b}+\frac{\frac{67\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8}{8}+\frac{839\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6}{64}+\frac{1487\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{192}+\frac{21\,\mathrm{ta}}{}}{b\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^{10}+3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^8+3\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^8/sin(a + b*x)^5,x)
```

```
[Out] tan(a/2 + (b*x)/2)^4/(64*b) - (3*tan(a/2 + (b*x)/2)^2)/(8*b) + (35*log(tan(
a/2 + (b*x)/2)))/(8*b) + ((21*tan(a/2 + (b*x)/2)^2)/64 + (1487*tan(a/2 + (b
*x)/2)^4)/192 + (839*tan(a/2 + (b*x)/2)^6)/64 + (67*tan(a/2 + (b*x)/2)^8)/8
 - 1/64)/(b*(tan(a/2 + (b*x)/2)^4 + 3*tan(a/2 + (b*x)/2)^6 + 3*tan(a/2 + (b
*x)/2)^8 + tan(a/2 + (b*x)/2)^10))
```

sympy [A] time = 18.45, size = 869, normalized size = 9.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**8/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((840*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**10/(192*b*tan(a/2 +
b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*b*
tan(a/2 + b*x/2)**4) + 2520*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(192*
b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)
**6 + 192*b*tan(a/2 + b*x/2)**4) + 2520*log(tan(a/2 + b*x/2))*tan(a/2 + b*x
/2)**6/(192*b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(
a/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**4) + 840*log(tan(a/2 + b*x/2))*ta
```

```
n(a/2 + b*x/2)**4/(192*b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 +
 576*b*tan(a/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**4) + 3*tan(a/2 + b*x/2
)**14/(192*b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a
/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**4) - 63*tan(a/2 + b*x/2)**12/(192*
b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)
**6 + 192*b*tan(a/2 + b*x/2)**4) + 2016*tan(a/2 + b*x/2)**8/(192*b*tan(a/2
+ b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*
b*tan(a/2 + b*x/2)**4) + 3066*tan(a/2 + b*x/2)**6/(192*b*tan(a/2 + b*x/2)**
10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*b*tan(a/2
+ b*x/2)**4) + 1694*tan(a/2 + b*x/2)**4/(192*b*tan(a/2 + b*x/2)**10 + 576*b
*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**
4) + 63*tan(a/2 + b*x/2)**2/(192*b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b
*x/2)**8 + 576*b*tan(a/2 + b*x/2)**6 + 192*b*tan(a/2 + b*x/2)**4) - 3/(192*
b*tan(a/2 + b*x/2)**10 + 576*b*tan(a/2 + b*x/2)**8 + 576*b*tan(a/2 + b*x/2)
**6 + 192*b*tan(a/2 + b*x/2)**4), Ne(b, 0)), (x*cos(a)**8/sin(a)**5, True))
```

3.173 $\int \cos^2(a+bx)\cot^5(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{\sin^2(a+bx)}{2b}-\frac{\csc^4(a+bx)}{4b}+\frac{3\csc^2(a+bx)}{2b}+\frac{3\log(\sin(a+bx))}{b}$$

```
[Out] 3/2*csc(b*x+a)^2/b-1/4*csc(b*x+a)^4/b+3*ln(sin(b*x+a))/b-1/2*sin(b*x+a)^2/b
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2590, 266, 43}

$$-\frac{\sin^2(a+bx)}{2b}-\frac{\csc^4(a+bx)}{4b}+\frac{3\csc^2(a+bx)}{2b}+\frac{3\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cot[a + b*x]^5,x]
```

```
[Out] (3*Csc[a + b*x]^2)/(2*b) - Csc[a + b*x]^4/(4*b) + (3*Log[Sin[a + b*x]])/b -
 Sin[a + b*x]^2/(2*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rubi steps

$$\int \cos^2(a+bx)\cot^5(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^5}\,dx, x, -\sin(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x^3}\,dx, x, \sin^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^3}-\frac{3}{x^2}+\frac{3}{x}\right)dx, x, \sin^2(a+bx)\right)}{2b}$$
$$= \frac{3\csc^2(a+bx)}{2b} - \frac{\csc^4(a+bx)}{4b} + \frac{3\log(\sin(a+bx))}{b} - \frac{\sin^2(a+bx)}{2b}$$

Mathematica [A] time = 0.15, size = 47, normalized size = 0.81

$$\frac{-2\sin^2(a+bx)-\csc^4(a+bx)+6\csc^2(a+bx)+12\log(\sin(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cot[a + b*x]^5,x]
```

```
[Out] (6*Csc[a + b*x]^2 - Csc[a + b*x]^4 + 12*Log[Sin[a + b*x]] - 2*Sin[a + b*x]^
2)/(4*b)
```

fricas [A] time = 0.48, size = 90, normalized size = 1.55

$$\frac{2\cos(bx+a)^6-5\cos(bx+a)^4-2\cos(bx+a)^2+12\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\sin(bx+a)\right)}{4\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*cos(b*x + a)^6 - 5*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 12*(cos(b*x +
 a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*sin(b*x + a)) + 4)/(b*cos(b*x + a)^4
- 2*b*cos(b*x + a)^2 + b)
```

giac [B] time = 0.29, size = 232, normalized size = 4.00

$$-\frac{\frac{20(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{\frac{18(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{111(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{36(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}-\frac{72(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}+1}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}\right)^2}-96\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*(20*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos
(b*x + a) + 1)^2 + (18*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 111*(cos(b*x
 + a) - 1)^2/(cos(b*x + a) + 1)^2 + 36*(cos(b*x + a) - 1)^3/(cos(b*x + a) +
 1)^3 - 72*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 1)/((cos(b*x + a) -
1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2)^2 - 96*l
og(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 192*log(abs(-(cos(b*x +
a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

maple [A] time = 0.02, size = 95, normalized size = 1.64

$$-\frac{\cos^8(bx+a)}{4b\sin(bx+a)^4}+\frac{\cos^8(bx+a)}{2b\sin(bx+a)^2}+\frac{\cos^6(bx+a)}{2b}+\frac{3\left(\cos^4(bx+a)\right)}{4b}+\frac{3\left(\cos^2(bx+a)\right)}{2b}+\frac{3\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b*cos(b*x+a)^8/sin(b*x+a)^4+1/2/b/sin(b*x+a)^2*cos(b*x+a)^8+1/2*cos(b*
x+a)^6/b+3/4*cos(b*x+a)^4/b+3/2*cos(b*x+a)^2/b+3*ln(sin(b*x+a))/b
```

maxima [A] time = 0.48, size = 49, normalized size = 0.84

$$-\frac{2\sin(bx+a)^2-\frac{6\sin(bx+a)^2-1}{\sin(bx+a)^4}-6\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(b*x + a)^2 - (6*sin(b*x + a)^2 - 1)/sin(b*x + a)^4 - 6*log(sin(
b*x + a)^2))/b
```

mupad [B] time = 0.64, size = 74, normalized size = 1.28

$$\frac{3\ln(\tan(a+bx))}{b}-\frac{3\ln\left(\tan(a+bx)^2+1\right)}{2b}+\frac{\frac{3\tan(a+bx)^4}{2}+\frac{3\tan(a+bx)^2}{4}-\frac{1}{4}}{b\left(\tan(a+bx)^6+\tan(a+bx)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^7/sin(a + b*x)^5,x)
```

```
[Out] (3*log(tan(a + b*x)))/b - (3*log(tan(a + b*x)^2 + 1))/(2*b) + ((3*tan(a + b
*x)^2)/4 + (3*tan(a + b*x)^4)/2 - 1/4)/(b*(tan(a + b*x)^4 + tan(a + b*x)^6)
)
```

sympy [A] time = 11.57, size = 733, normalized size = 12.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-192*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**8/(64*b*tan(
a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 3
84*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(64*b*tan(a/2 + b*x/2)*
*8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 192*log(tan(a/
2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(64*b*tan(a/2 + b*x/2)**8 + 128*b*ta
n(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 192*log(tan(a/2 + b*x/2))*t
an(a/2 + b*x/2)**8/(64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 +
64*b*tan(a/2 + b*x/2)**4) + 384*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(
64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2
)**4) + 192*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(64*b*tan(a/2 + b*x/2
)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - tan(a/2 + b*
x/2)**12/(64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a
/2 + b*x/2)**4) + 18*tan(a/2 + b*x/2)**10/(64*b*tan(a/2 + b*x/2)**8 + 128*b
*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 166*tan(a/2 + b*x/2)**6/
(64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/
2)**4) + 18*tan(a/2 + b*x/2)**2/(64*b*tan(a/2 + b*x/2)**8 + 128*b*tan(a/2 +
 b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 1/(64*b*tan(a/2 + b*x/2)**8 + 128*
b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4), Ne(b, 0)), (x*cos(a)**7/
sin(a)**5, True))
```

3.174 $\int \cos(a+bx)\cot^5(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{15\cos(a+bx)}{8b}-\frac{\cos(a+bx)\cot^4(a+bx)}{4b}+\frac{5\cos(a+bx)\cot^2(a+bx)}{8b}-\frac{15\tanh^{-1}(\cos(a+bx))}{8b}$$

```
[Out] -15/8*arctanh(cos(b*x+a))/b+15/8*cos(b*x+a)/b+5/8*cos(b*x+a)*cot(b*x+a)^2/b
-1/4*cos(b*x+a)*cot(b*x+a)^4/b
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2592, 288, 321, 206}

$$\frac{15\cos(a+bx)}{8b}-\frac{\cos(a+bx)\cot^4(a+bx)}{4b}+\frac{5\cos(a+bx)\cot^2(a+bx)}{8b}-\frac{15\tanh^{-1}(\cos(a+bx))}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[a + b*x]^5,x]
```

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(8*b) + (15*Cos[a + b*x])/(8*b) + (5*Cos[a + b*
x]*Cot[a + b*x]^2)/(8*b) - (Cos[a + b*x]*Cot[a + b*x]^4)/(4*b)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rubi steps

$$\int \cos(a+bx)\cot^5(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(a+bx)\right)}{b}$$

$$= -\frac{\cos(a+bx)\cot^4(a+bx)}{4b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(a+bx)\right)}{4b}$$

$$= \frac{5\cos(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos(a+bx)\cot^4(a+bx)}{4b} - \frac{15\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x,\right.}{8b}$$

$$= \frac{15\cos(a+bx)}{8b} + \frac{5\cos(a+bx)\cot^2(a+bx)}{8b} - \frac{\cos(a+bx)\cot^4(a+bx)}{4b} - \frac{15\,\text{Su}}{}$$

$$= -\frac{15\tanh^{-1}(\cos(a+bx))}{8b} + \frac{15\cos(a+bx)}{8b} + \frac{5\cos(a+bx)\cot^2(a+bx)}{8b} - \cos($$

Mathematica [A] time = 0.03, size = 123, normalized size = 1.76

$$\frac{\cos(a+bx)}{b} - \frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b} + \frac{9\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b} - \frac{9\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b} + \frac{15\log\left(\sin\left(\frac{1}{2}(a+bx)\right.\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[a + b*x]^5,x]
```

```
[Out] Cos[a + b*x]/b + (9*Csc[(a + b*x)/2]^2)/(32*b) - Csc[(a + b*x)/2]^4/(64*b)
- (15*Log[Cos[(a + b*x)/2]])/(8*b) + (15*Log[Sin[(a + b*x)/2]])/(8*b) - (9*
Sec[(a + b*x)/2]^2)/(32*b) + Sec[(a + b*x)/2]^4/(64*b)
```

fricas [A] time = 0.47, size = 122, normalized size = 1.74

$$\frac{16\cos(bx+a)^5 - 50\cos(bx+a)^3 - 15\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right) + 15\left(\cos\right.}{16\left(b\cos(bx+a)^4 - 2b\cos(bx+a)^2 +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*cos(b*x + a)^5 - 50*cos(b*x + a)^3 - 15*(cos(b*x + a)^4 - 2*cos(b*
x + a)^2 + 1)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^4 - 2*cos(b*x
+ a)^2 + 1)*log(-1/2*cos(b*x + a) + 1/2) + 30*cos(b*x + a))/(b*cos(b*x + a)
^4 - 2*b*cos(b*x + a)^2 + b)
```

giac [B] time = 0.27, size = 164, normalized size = 2.34

$$-\frac{\frac{\left(\frac{16\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{90\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{16\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{128}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1}-60\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*((16*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 90*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 16*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) +
1)^2 + 128/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - 60*log(abs(-cos(b*
x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.02, size = 102, normalized size = 1.46

$$-\frac{\cos^7(bx+a)}{4b\sin(bx+a)^4}+\frac{3\left(\cos^7(bx+a)\right)}{8b\sin(bx+a)^2}+\frac{3\left(\cos^5(bx+a)\right)}{8b}+\frac{5\left(\cos^3(bx+a)\right)}{8b}+\frac{15\cos(bx+a)}{8b}+\frac{15\ln(\csc(bx+a)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b*cos(b*x+a)^7/sin(b*x+a)^4+3/8/b*cos(b*x+a)^7/sin(b*x+a)^2+3/8*cos(b*
x+a)^5/b+5/8*cos(b*x+a)^3/b+15/8*cos(b*x+a)/b+15/8/b*ln(csc(b*x+a)-cot(b*x+
a))
```

maxima [A] time = 0.44, size = 79, normalized size = 1.13

$$-\frac{\frac{2\left(9\,\cos(bx+a)^3-7\,\cos(bx+a)\right)}{\cos(bx+a)^4-2\,\cos(bx+a)^2+1}-16\,\cos(bx+a)+15\,\log(\cos(bx+a)+1)-15\,\log(\cos(bx+a)-1)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(9*cos(b*x + a)^3 - 7*cos(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x +
a)^2 + 1) - 16*cos(b*x + a) + 15*log(cos(b*x + a) + 1) - 15*log(cos(b*x + a
) - 1))/b
```

mupad [B] time = 0.58, size = 105, normalized size = 1.50

$$\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{64\,b}-\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{4\,b}+\frac{15\,\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b}+\frac{\frac{9\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{4}+\frac{15\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2}{64}-\frac{1}{64}}{b\,\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^6+\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^6/sin(a + b*x)^5,x)
```

```
[Out] tan(a/2 + (b*x)/2)^4/(64*b) - tan(a/2 + (b*x)/2)^2/(4*b) + (15*log(tan(a/2
+ (b*x)/2)))/(8*b) + ((15*tan(a/2 + (b*x)/2)^2)/64 + (9*tan(a/2 + (b*x)/2)^
4)/4 - 1/64)/(b*(tan(a/2 + (b*x)/2)^4 + tan(a/2 + (b*x)/2)^6))
```

sympy [A] time = 6.83, size = 330, normalized size = 4.71

$$\begin{cases} \frac{120\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+64b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}+\frac{120\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+64b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}+\frac{\tan^{10}\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+64b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}-\frac{15\tan^8\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b\tan^6\left(\frac{a}{2}+\frac{bx}{2}\right)+64b\tan^4\left(\frac{a}{2}\right.} \\ \frac{x\cos^6(a)}{\sin^5(a)} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((120*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(64*b*tan(a/2 + b*
x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 120*log(tan(a/2 + b*x/2))*tan(a/2 + b
*x/2)**4/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + tan(a/2 +
b*x/2)**10/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) - 15*tan(a
/2 + b*x/2)**8/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) + 160*
tan(a/2 + b*x/2)**4/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4) +
 15*tan(a/2 + b*x/2)**2/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**
4) - 1/(64*b*tan(a/2 + b*x/2)**6 + 64*b*tan(a/2 + b*x/2)**4), Ne(b, 0)), (x
*cos(a)**6/sin(a)**5, True))
```

3.175 $\int \cot^5(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\cot^4(a+bx)}{4b}+\frac{\cot^2(a+bx)}{2b}+\frac{\log(\sin(a+bx))}{b}$$

```
[Out] 1/2*cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+ln(sin(b*x+a))/b
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3473, 3475}

$$-\frac{\cot^4(a+bx)}{4b}+\frac{\cot^2(a+bx)}{2b}+\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^5,x]
```

```
[Out] Cot[a + b*x]^2/(2*b) - Cot[a + b*x]^4/(4*b) + Log[Sin[a + b*x]]/b
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^5(a+bx)\,dx &= -\frac{\cot^4(a+bx)}{4b}-\int \cot^3(a+bx)\,dx\\ &= \frac{\cot^2(a+bx)}{2b}-\frac{\cot^4(a+bx)}{4b}+\int \cot(a+bx)\,dx\\ &= \frac{\cot^2(a+bx)}{2b}-\frac{\cot^4(a+bx)}{4b}+\frac{\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A] time = 0.10, size = 46, normalized size = 1.10

$$\frac{-\cot^4(a+bx)+2\cot^2(a+bx)+4\log(\tan(a+bx))+4\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^5,x]
```

```
[Out] (2*Cot[a + b*x]^2 - Cot[a + b*x]^4 + 4*Log[Cos[a + b*x]] + 4*Log[Tan[a + b*
x]])/(4*b)
```

fricas [A] time = 0.43, size = 70, normalized size = 1.67

$$-\frac{4\cos(bx+a)^2-4\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\sin(bx+a)\right)-3}{4\left(b\cos(bx+a)^4-2\,b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*cos(b*x + a)^2 - 4*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*
sin(b*x + a)) - 3)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

giac [B] time = 0.25, size = 164, normalized size = 3.90

$$-\frac{\frac{\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{48(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}+\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-32\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)+64\log\left(\left|-\frac{\cos}{\cos}\right.\right.}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 48*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 12*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) +
1)^2 - 32*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) + 64*log(abs(-(
cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

maple [A] time = 0.02, size = 39, normalized size = 0.93

$$\frac{\cot^2(bx+a)}{2b}-\frac{\cot^4(bx+a)}{4b}+\frac{\ln(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5/sin(b*x+a)^5,x)
```

```
[Out] 1/2*cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+ln(sin(b*x+a))/b
```

maxima [A] time = 0.35, size = 38, normalized size = 0.90

$$\frac{\frac{4\sin(bx+a)^2-1}{\sin(bx+a)^4} + 2\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/4*((4*sin(b*x + a)^2 - 1)/sin(b*x + a)^4 + 2*log(sin(b*x + a)^2))/b
```

mupad [B] time = 0.42, size = 52, normalized size = 1.24

$$\frac{\ln\left(\tan\left(a+b\,x\right)\right)}{b} - \frac{\ln\left(\tan\left(a+b\,x\right)^2+1\right)}{2\,b} + \frac{\frac{\tan(a+b\,x)^2}{2} - \frac{1}{4}}{b\tan\left(a+b\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5/sin(a + b*x)^5,x)
```

```
[Out] log(tan(a + b*x))/b - log(tan(a + b*x)^2 + 1)/(2*b) + (tan(a + b*x)^2/2 - 1
/4)/(b*tan(a + b*x)^4)
```

sympy [A] time = 3.04, size = 61, normalized size = 1.45

$$\begin{cases} \frac{\log\left(\sin\left(a+bx\right)\right)}{b} + \frac{\cos^2\left(a+bx\right)}{2b\sin^2\left(a+bx\right)} - \frac{\cos^4\left(a+bx\right)}{4b\sin^4\left(a+bx\right)} & \text{for } b \neq 0 \\ \frac{x\cos^5\left(a\right)}{\sin^5\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((log(sin(a + b*x))/b + cos(a + b*x)**2/(2*b*sin(a + b*x)**2) - co
s(a + b*x)**4/(4*b*sin(a + b*x)**4), Ne(b, 0)), (x*cos(a)**5/sin(a)**5, Tru
e))
```

3.176 $\int \cot^4(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{3\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}+\frac{3\cot(a+bx)\csc(a+bx)}{8b}$$

```
[Out] -3/8*arctanh(cos(b*x+a))/b+3/8*cot(b*x+a)*csc(b*x+a)/b-1/4*cot(b*x+a)^3*csc
(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2611, 3770}

$$-\frac{3\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\cot^3(a+bx)\csc(a+bx)}{4b}+\frac{3\cot(a+bx)\csc(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^4*Csc[a + b*x],x]
```

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(8*b) + (3*Cot[a + b*x]*Csc[a + b*x])/(8*b) - (C
ot[a + b*x]^3*Csc[a + b*x])/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^4(a+bx)\csc(a+bx)\,dx = -\frac{\cot^3(a+bx)\csc(a+bx)}{4b} - \frac{3}{4}\int \cot^2(a+bx)\csc(a+bx)\,dx$$
$$= \frac{3\cot(a+bx)\csc(a+bx)}{8b} - \frac{\cot^3(a+bx)\csc(a+bx)}{4b} + \frac{3}{8}\int \csc(a+bx)\,dx$$
$$= -\frac{3\tanh^{-1}(\cos(a+bx))}{8b} + \frac{3\cot(a+bx)\csc(a+bx)}{8b} - \frac{\cot^3(a+bx)\csc(a+bx)}{4b}$$

Mathematica [B] time = 0.03, size = 113, normalized size = 2.05

$$-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{5\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{5\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{3\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{8b}-\frac{3\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^4*Csc[a + b*x],x]
```

```
[Out] (5*Csc[(a + b*x)/2]^2)/(32*b) - Csc[(a + b*x)/2]^4/(64*b) - (3*Log[Cos[(a +
 b*x)/2]])/(8*b) + (3*Log[Sin[(a + b*x)/2]])/(8*b) - (5*Sec[(a + b*x)/2]^2)
/(32*b) + Sec[(a + b*x)/2]^4/(64*b)
```

fricas [B] time = 0.46, size = 112, normalized size = 2.04

$$-\frac{10\cos(bx+a)^3+3\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-3\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-6\cos(bx+a)}{16\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/16*(10*cos(b*x + a)^3 + 3*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/
2*cos(b*x + a) + 1/2) - 3*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/2*
cos(b*x + a) + 1/2) - 6*cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^
2 + b)
```

giac [B] time = 0.59, size = 139, normalized size = 2.53

$$-\frac{\frac{\left(\frac{8(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{18(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{8(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-12\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/64*((8*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 18*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 + 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 8*(cos
(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)
^2 - 12*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.02, size = 89, normalized size = 1.62

$$-\frac{\cos^5(bx+a)}{4b\sin(bx+a)^4}+\frac{\cos^5(bx+a)}{8b\sin(bx+a)^2}+\frac{\cos^3(bx+a)}{8b}+\frac{3\cos(bx+a)}{8b}+\frac{3\ln(\csc(bx+a)-\cot(bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b*cos(b*x+a)^5/sin(b*x+a)^4+1/8/b*cos(b*x+a)^5/sin(b*x+a)^2+1/8*cos(b*
x+a)^3/b+3/8*cos(b*x+a)/b+3/8/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [A] time = 0.35, size = 71, normalized size = 1.29

$$-\frac{\frac{2\left(5\cos(bx+a)^3-3\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1}+3\log(\cos(bx+a)+1)-3\log(\cos(bx+a)-1)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(5*cos(b*x + a)^3 - 3*cos(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x +
a)^2 + 1) + 3*log(cos(b*x + a) + 1) - 3*log(cos(b*x + a) - 1))/b
```

mupad [B] time = 0.49, size = 78, normalized size = 1.42

$$\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4}{64b}-\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{8b}+\frac{3\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{8b}+\frac{\cot\left(\frac{a}{2}+\frac{bx}{2}\right)^4\left(\frac{\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2}{8}-\frac{1}{64}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4/sin(a + b*x)^5,x)
```

```
[Out] tan(a/2 + (b*x)/2)^4/(64*b) - tan(a/2 + (b*x)/2)^2/(8*b) + (3*log(tan(a/2 +
 (b*x)/2)))/(8*b) + (cot(a/2 + (b*x)/2)^4*(tan(a/2 + (b*x)/2)^2/8 - 1/64))/
b
```

sympy [A] time = 4.16, size = 92, normalized size = 1.67

$$\begin{cases} \frac{3\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{8b}+\frac{\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b}-\frac{\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}+\frac{1}{8b\tan^2\left(\frac{a}{2}+\frac{bx}{2}\right)}-\frac{1}{64b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for } b\neq 0 \\ \frac{x\cos^4(a)}{\sin^5(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((3*log(tan(a/2 + b*x/2))/(8*b) + tan(a/2 + b*x/2)**4/(64*b) - tan
(a/2 + b*x/2)**2/(8*b) + 1/(8*b*tan(a/2 + b*x/2)**2) - 1/(64*b*tan(a/2 + b*
x/2)**4), Ne(b, 0)), (x*cos(a)**4/sin(a)**5, True))
```

3.177 $\int \cot^3(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cot^4(a+bx)}{4b}$$

```
[Out] -1/4*cot(b*x+a)^4/b
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2607, 30}

$$-\frac{\cot^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^3*Csc[a + b*x]^2,x]
```

```
[Out] -Cot[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \cot^3(a+bx)\csc^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^3\,dx, x, -\cot(a+bx)\right)}{b}\\ &= -\frac{\cot^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{\cot^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^3*Csc[a + b*x]^2,x]
```

```
[Out] -1/4*Cot[a + b*x]^4/b
```

fricas [B] time = 0.44, size = 39, normalized size = 2.60

$$-\frac{2\cos(bx+a)^2-1}{4\left(b\cos(bx+a)^4-2\,b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

giac [A] time = 0.28, size = 25, normalized size = 1.67

$$\frac{2\sin(bx+a)^2-1}{4\,b\sin(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/4*(2*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^4)
```

maple [A] time = 0.02, size = 22, normalized size = 1.47

$$-\frac{\cos^4(bx+a)}{4\sin(bx+a)^4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(b*x+a)^5,x)
```

```
[Out] -1/4*cos(b*x+a)^4/sin(b*x+a)^4/b
```

maxima [A] time = 0.78, size = 25, normalized size = 1.67

$$\frac{2\sin(bx+a)^2-1}{4\,b\sin(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sin(b*x + a)^2 - 1)/(b*sin(b*x + a)^4)
```

mupad [B] time = 0.39, size = 25, normalized size = 1.67

$$-\frac{\left(\sin\left(a+b\,x\right)^2-1\right)^2}{4\,b\,\sin\left(a+b\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(a + b*x)^5,x)
[Out] -(sin(a + b*x)^2 - 1)^2/(4*b*sin(a + b*x)^4)
```

sympy [A] time = 2.90, size = 44, normalized size = 2.93

$$\begin{cases} \frac{1}{4b\sin^2\left(a+bx\right)} - \frac{\cos^2\left(a+bx\right)}{4b\sin^4\left(a+bx\right)} & \text{for}\ b \neq 0 \\ \frac{x\cos^3\left(a\right)}{\sin^5\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(b*x+a)**5,x)
[Out] Piecewise((1/(4*b*sin(a + b*x)**2) - cos(a + b*x)**2/(4*b*sin(a + b*x)**4),
 Ne(b, 0)), (x*cos(a)**3/sin(a)**5, True))
```

3.178 $\int \cot^2(a+bx)\csc^3(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}+\frac{\cot(a+bx)\csc(a+bx)}{8b}$$

```
[Out] 1/8*arctanh(cos(b*x+a))/b+1/8*cot(b*x+a)*csc(b*x+a)/b-1/4*cot(b*x+a)*csc(b*
x+a)^3/b
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2611, 3768, 3770}

$$\frac{\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\cot(a+bx)\csc^3(a+bx)}{4b}+\frac{\cot(a+bx)\csc(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]^2*Csc[a + b*x]^3,x]
```

```
[Out] ArcTanh[Cos[a + b*x]]/(8*b) + (Cot[a + b*x]*Csc[a + b*x])/(8*b) - (Cot[a +
b*x]*Csc[a + b*x]^3)/(4*b)
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot^2(a+bx)\csc^3(a+bx)\,dx = -\frac{\cot(a+bx)\csc^3(a+bx)}{4b} - \frac{1}{4}\int \csc^3(a+bx)\,dx$$
$$= \frac{\cot(a+bx)\csc(a+bx)}{8b} - \frac{\cot(a+bx)\csc^3(a+bx)}{4b} - \frac{1}{8}\int \csc(a+bx)\,dx$$
$$= \frac{\tanh^{-1}(\cos(a+bx))}{8b} + \frac{\cot(a+bx)\csc(a+bx)}{8b} - \frac{\cot(a+bx)\csc^3(a+bx)}{4b}$$

Mathematica [B] time = 0.03, size = 113, normalized size = 2.05

$$-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{64b}+\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{32b}+\frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{64b}-\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{32b}-\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{8b}+\frac{\log\left(\cos\left(\frac{1}{2}(a\right.\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]^2*Csc[a + b*x]^3,x]
```

```
[Out] Csc[(a + b*x)/2]^2/(32*b) - Csc[(a + b*x)/2]^4/(64*b) + Log[Cos[(a + b*x)/2
]]/(8*b) - Log[Sin[(a + b*x)/2]]/(8*b) - Sec[(a + b*x)/2]^2/(32*b) + Sec[(a
 + b*x)/2]^4/(64*b)
```

fricas [B] time = 0.45, size = 111, normalized size = 2.02

$$-\frac{2\cos(bx+a)^3-\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)+\left(\cos(bx+a)^4-2\cos(bx+a\right.}{16\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/16*(2*cos(b*x + a)^3 - (cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(1/2*c
os(b*x + a) + 1/2) + (cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/2*cos(b
*x + a) + 1/2) + 2*cos(b*x + a))/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b
)
```

giac [A] time = 0.31, size = 98, normalized size = 1.78

$$\frac{\frac{\left(\frac{2(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-4\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*((2*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^
2/(cos(b*x + a) - 1)^2 + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 - 4*log(
abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.02, size = 76, normalized size = 1.38

$$-\frac{\cos^3(bx+a)}{4b\sin(bx+a)^4}-\frac{\cos^3(bx+a)}{8b\sin(bx+a)^2}-\frac{\cos(bx+a)}{8b}-\frac{\ln(\csc(bx+a)-\cot(bx+a))}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b*cos(b*x+a)^3/sin(b*x+a)^4-1/8/b*cos(b*x+a)^3/sin(b*x+a)^2-1/8*cos(b*
x+a)/b-1/8/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [A] time = 0.34, size = 65, normalized size = 1.18

$$-\frac{\frac{2\left(\cos(bx+a)^3+\cos(bx+a)\right)}{\cos(bx+a)^4-2\cos(bx+a)^2+1}-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/16*(2*(cos(b*x + a)^3 + cos(b*x + a))/(cos(b*x + a)^4 - 2*cos(b*x + a)^2
 + 1) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
```

mupad [B] time = 0.47, size = 48, normalized size = 0.87

$$\frac{\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}{64\,b}-\frac{1}{64\,b\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4}-\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(a + b*x)^5,x)
```

```
[Out] tan(a/2 + (b*x)/2)^4/(64*b) - 1/(64*b*tan(a/2 + (b*x)/2)^4) - log(tan(a/2 +
 (b*x)/2))/(8*b)
```

sympy [A] time = 3.95, size = 58, normalized size = 1.05

$$\begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{8b}+\frac{\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)}{64b}-\frac{1}{64b\tan^4\left(\frac{a}{2}+\frac{bx}{2}\right)} & \text{for } b\neq 0 \\ \frac{x\cos^2(a)}{\sin^5(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(b*x+a)**5,x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2))/(8*b) + tan(a/2 + b*x/2)**4/(64*b) - 1/(6
4*b*tan(a/2 + b*x/2)**4), Ne(b, 0)), (x*cos(a)**2/sin(a)**5, True))
```

3.179 $\int \cot(a+bx)\csc^4(a+bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\csc^4(a+bx)}{4b}$$

```
[Out] -1/4*csc(b*x+a)^4/b
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2606, 30}

$$-\frac{\csc^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x]*Csc[a + b*x]^4,x]
```

```
[Out] -Csc[a + b*x]^4/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot(a+bx)\csc^4(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^3\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{\csc^4(a+bx)}{4b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\csc^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x]*Csc[a + b*x]^4,x]
```

```
[Out] -1/4*Csc[a + b*x]^4/b
```

fricas [B] time = 0.42, size = 27, normalized size = 1.80

$$-\frac{1}{4\left(b\cos\left(bx+a\right)^{4}-2\,b\cos\left(bx+a\right)^{2}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

giac [A] time = 0.30, size = 13, normalized size = 0.87

$$-\frac{1}{4\,b\sin\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4/(b*sin(b*x + a)^4)
```

maple [A] time = 0.01, size = 14, normalized size = 0.93

$$-\frac{1}{4\sin\left(bx+a\right)^{4}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+a)^5,x)
```

```
[Out] -1/4/sin(b*x+a)^4/b
```

maxima [A] time = 0.37, size = 13, normalized size = 0.87

$$-\frac{1}{4\,b\sin\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4/(b*sin(b*x + a)^4)
```

mupad [B] time = 0.41, size = 23, normalized size = 1.53

$$-\frac{\cot(a+b\,x)^2\,\left(\cot(a+b\,x)^2+2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(a + b*x)^5,x)
[Out] -(cot(a + b*x)^2*(cot(a + b*x)^2 + 2))/(4*b)
```

sympy [A] time = 2.68, size = 24, normalized size = 1.60

$$\begin{cases} -\frac{1}{4b\sin^4(a+bx)} & \text{for } b \neq 0 \\ \frac{x\cos(a)}{\sin^5(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+a)**5,x)
[Out] Piecewise((-1/(4*b*sin(a + b*x)**4), Ne(b, 0)), (x*cos(a)/sin(a)**5, True))
```

3.180 $\int \csc^5(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=40

$$-\frac{\cot^4(a+bx)}{4b}-\frac{\cot^2(a+bx)}{b}+\frac{\log(\tan(a+bx))}{b}$$

```
[Out] -cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+ln(tan(b*x+a))/b
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2620, 266, 43}

$$-\frac{\cot^4(a+bx)}{4b}-\frac{\cot^2(a+bx)}{b}+\frac{\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x],x]
```

```
[Out] -(Cot[a + b*x]^2/b) - Cot[a + b*x]^4/(4*b) + Log[Tan[a + b*x]]/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \csc^5(a+bx)\sec(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^5}\,dx,x,\tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3}\,dx,x,\tan^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{2}{x^2}+\frac{1}{x}\right)dx,x,\tan^2(a+bx)\right)}{2b}$$
$$= -\frac{\cot^2(a+bx)}{b}-\frac{\cot^4(a+bx)}{4b}+\frac{\log(\tan(a+bx))}{b}$$

Mathematica [A] time = 0.11, size = 44, normalized size = 1.10

$$-\frac{\csc^4(a+bx)+2\csc^2(a+bx)-4\log(\sin(a+bx))+4\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x],x]
```

```
[Out] -1/4*(2*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 4*Log[Cos[a + b*x]] - 4*Log[Sin[a
 + b*x]])/b
```

fricas [B] time = 0.44, size = 105, normalized size = 2.62

$$\frac{2\cos(bx+a)^2-2\left(\cos(bx+a)^4-2\cos(bx+a)^2+1\right)\log\left(\cos(bx+a)^2\right)+2\left(\cos(bx+a)^4-2\cos(bx+a)^2\right.}{4\left(b\cos(bx+a)^4-2b\cos(bx+a)^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*cos(b*x + a)^2 - 2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(cos(b
*x + a)^2) + 2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*log(-1/4*cos(b*x + a
)^2 + 1/4) - 3)/(b*cos(b*x + a)^4 - 2*b*cos(b*x + a)^2 + b)
```

giac [B] time = 0.33, size = 165, normalized size = 4.12

$$\frac{\frac{\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{48(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}+\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+32\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)-64\log\left(\left|-\frac{\text{co}}{\text{co}}\right.\right.}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*((12*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 48*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 12*(co
s(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1
)^2 + 32*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 64*log(abs(-(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

maple [A] time = 0.03, size = 39, normalized size = 0.98

$$-\frac{1}{4\sin(bx+a)^4 b}-\frac{1}{2\sin(bx+a)^2 b}+\frac{\ln(\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/sin(b*x+a)^5,x)
```

```
[Out] -1/4/sin(b*x+a)^4/b-1/2/sin(b*x+a)^2/b+ln(tan(b*x+a))/b
```

maxima [A] time = 0.43, size = 51, normalized size = 1.28

$$-\frac{\frac{2\sin(bx+a)^2+1}{\sin(bx+a)^4}+2\log\left(\sin(bx+a)^2-1\right)-2\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*((2*sin(b*x + a)^2 + 1)/sin(b*x + a)^4 + 2*log(sin(b*x + a)^2 - 1) - 2
*log(sin(b*x + a)^2))/b
```

mupad [B] time = 0.44, size = 79, normalized size = 1.98

$$\frac{\ln\left(\frac{\cos(2a+2bx)}{2}-\frac{1}{2}\right)}{2b}-\frac{\ln(\cos(a+bx))}{b}-\frac{\frac{\cos(2a+2bx)}{4}-\frac{1}{2}}{b\left(\cos(2a+2bx)-\left(\frac{\cos(2a+2bx)}{2}+\frac{1}{2}\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^5),x)
```

```
[Out] log(cos(2*a + 2*b*x)/2 - 1/2)/(2*b) - log(cos(a + b*x))/b - (cos(2*a + 2*b*
x)/4 - 1/2)/(b*(cos(2*a + 2*b*x) - (cos(2*a + 2*b*x)/2 + 1/2)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)}{\sin^5(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/sin(b*x+a)**5,x)
```

```
[Out] Integral(sec(a + b*x)/sin(a + b*x)**5, x)
```

3.181 $\int \csc^5(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{15\sec(a+bx)}{8b}-\frac{15\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^4(a+bx)\sec(a+bx)}{4b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{8b}$$

```
[Out] -15/8*arctanh(cos(b*x+a))/b+15/8*sec(b*x+a)/b-5/8*csc(b*x+a)^2*sec(b*x+a)/b
-1/4*csc(b*x+a)^4*sec(b*x+a)/b
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 321, 207}

$$\frac{15\sec(a+bx)}{8b}-\frac{15\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^4(a+bx)\sec(a+bx)}{4b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x]^2,x]
```

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(8*b) + (15*Sec[a + b*x])/(8*b) - (5*Csc[a + b*
x]^2*Sec[a + b*x])/(8*b) - (Csc[a + b*x]^4*Sec[a + b*x])/(4*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^5(a+bx)\sec^2(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^3}\,dx, x, \sec(a+bx)\right)}{b}$$
$$= -\frac{\csc^4(a+bx)\sec(a+bx)}{4b} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{4b}$$
$$= -\frac{5\csc^2(a+bx)\sec(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec(a+bx)}{4b} + \frac{15\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,d$$
$$= \frac{15\sec(a+bx)}{8b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec(a+bx)}{4b} + \frac{15\,S$$
$$= -\frac{15\tanh^{-1}(\cos(a+bx))}{8b} + \frac{15\sec(a+bx)}{8b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{8b} - \frac{\csc$$

Mathematica [A] time = 3.99, size = 129, normalized size = 1.84

$$-\frac{\csc^4\left(\frac{1}{2}(a+bx)\right) + 14\csc^2\left(\frac{1}{2}(a+bx)\right) + \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)\left(-14\tan^2\left(\frac{1}{2}(a+bx)\right)+\cos(a+bx)\left(\sec^4\left(\frac{1}{2}(a+bx)\right)-8\left(-15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)+\right.\right.\right.}{\tan^2\left(\frac{1}{2}(a+bx)\right)-1}}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x]^2,x]
```

```
[Out] -1/64*(14*Csc[(a + b*x)/2]^2 + Csc[(a + b*x)/2]^4 + (Sec[(a + b*x)/2]^2*(78
 + Cos[a + b*x]*(-8*(8 + 15*Log[Cos[(a + b*x)/2]] - 15*Log[Sin[(a + b*x)/2]
]) + Sec[(a + b*x)/2]^4) - 14*Tan[(a + b*x)/2]^2))/(-1 + Tan[(a + b*x)/2]^2
))/b
```

fricas [B] time = 0.46, size = 132, normalized size = 1.89

$$\frac{30\,\cos\left(bx+a\right)^{4}-50\,\cos\left(bx+a\right)^{2}-15\left(\cos\left(bx+a\right)^{5}-2\,\cos\left(bx+a\right)^{3}+\cos\left(bx+a\right)\right)\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)}{16\left(b\cos\left(bx+a\right)^{5}-2\,b\cos\left(bx+a\right)^{3}+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(b*
x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 -
 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b*cos
(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

giac [B] time = 0.23, size = 163, normalized size = 2.33

$$\frac{\frac{\left(\frac{16\,(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{90\,(\cos(bx+a)-1)^{2}}{(\cos(bx+a)+1)^{2}}-1\right)(\cos(bx+a)+1)^{2}}{(\cos(bx+a)-1)^{2}}-\frac{16\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^{2}}{(\cos(bx+a)+1)^{2}}+\frac{128}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1}+60\,\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*((16*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 90*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 16*(co
s(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1
)^2 + 128/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + 60*log(abs(-cos(b*x
 + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.03, size = 78, normalized size = 1.11

$$-\frac{1}{4b\sin\left(bx+a\right)^{4}\cos\left(bx+a\right)}-\frac{5}{8b\sin\left(bx+a\right)^{2}\cos\left(bx+a\right)}+\frac{15}{8b\cos\left(bx+a\right)}+\frac{15\ln\left(\csc\left(bx+a\right)-\cot\left(bx+a\right)\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b/sin(b*x+a)^4/cos(b*x+a)-5/8/b/sin(b*x+a)^2/cos(b*x+a)+15/8/b/cos(b*x
+a)+15/8/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [A] time = 0.34, size = 79, normalized size = 1.13

$$\frac{\frac{2\left(15\,\cos(bx+a)^{4}-25\,\cos(bx+a)^{2}+8\right)}{\cos(bx+a)^{5}-2\,\cos(bx+a)^{3}+\cos(bx+a)}-15\,\log\left(\cos\left(bx+a\right)+1\right)+15\,\log\left(\cos\left(bx+a\right)-1\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/16*(2*(15*cos(b*x + a)^4 - 25*cos(b*x + a)^2 + 8)/(cos(b*x + a)^5 - 2*cos
(b*x + a)^3 + cos(b*x + a)) - 15*log(cos(b*x + a) + 1) + 15*log(cos(b*x + a
) - 1))/b
```

mupad [B] time = 0.39, size = 66, normalized size = 0.94

$$\frac{\frac{15\cos(a+b\,x)^4}{8}-\frac{25\cos(a+b\,x)^2}{8}+1}{b\,\left(\cos\left(a+b\,x\right)^5-2\cos\left(a+b\,x\right)^3+\cos\left(a+b\,x\right)\right)}-\frac{15\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^5),x)
```

```
[Out] ((15*cos(a + b*x)^4)/8 - (25*cos(a + b*x)^2)/8 + 1)/(b*(cos(a + b*x) - 2*co
s(a + b*x)^3 + cos(a + b*x)^5)) - (15*atanh(cos(a + b*x)))/(8*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)}{\sin^5(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/sin(b*x+a)**5,x)
```

```
[Out] Integral(sec(a + b*x)**2/sin(a + b*x)**5, x)
```

3.182 $\int \csc^5(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=58

$$\frac{\tan^2(a+bx)}{2b}-\frac{\cot^4(a+bx)}{4b}-\frac{3\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}$$

```
[Out] -3/2*cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+3*ln(tan(b*x+a))/b+1/2*tan(b*x+a)^2/
b
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tan^2(a+bx)}{2b}-\frac{\cot^4(a+bx)}{4b}-\frac{3\cot^2(a+bx)}{2b}+\frac{3\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x]^3,x]
```

```
[Out] (-3*Cot[a + b*x]^2)/(2*b) - Cot[a + b*x]^4/(4*b) + (3*Log[Tan[a + b*x]])/b
+ Tan[a + b*x]^2/(2*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \csc^5(a+bx)\sec^3(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^3}\,dx, x, \tan^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^3}+\frac{3}{x^2}+\frac{3}{x}\right)dx, x, \tan^2(a+bx)\right)}{2b}$$
$$= -\frac{3\cot^2(a+bx)}{2b} - \frac{\cot^4(a+bx)}{4b} + \frac{3\log(\tan(a+bx))}{b} + \frac{\tan^2(a+bx)}{2b}$$

Mathematica [A] time = 0.30, size = 54, normalized size = 0.93

$$-\frac{\csc^4(a+bx)+4\csc^2(a+bx)-2\sec^2(a+bx)-12\log(\sin(a+bx))+12\log(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x]^3,x]
```

```
[Out] -1/4*(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[Sin
[a + b*x]] - 2*Sec[a + b*x]^2)/b
```

fricas [B] time = 0.45, size = 138, normalized size = 2.38

$$\frac{6\cos(bx+a)^4 - 9\cos(bx+a)^2 - 6\left(\cos(bx+a)^6 - 2\cos(bx+a)^4 + \cos(bx+a)^2\right)\log\left(\cos(bx+a)^2\right) + 6\left(c\right.}{4\left(b\cos(bx+a)^6 - 2b\cos(bx+a)^4 + b\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x +
a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(b*x
+ a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*x + a
)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

giac [B] time = 0.63, size = 232, normalized size = 4.00

$$\frac{\frac{20(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{\frac{18(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{111(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{36(\cos(bx+a)-1)^3}{(\cos(bx+a)+1)^3}+\frac{72(\cos(bx+a)-1)^4}{(\cos(bx+a)+1)^4}-1}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}\right)^2} + 96\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) -}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*(20*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(
b*x + a) + 1)^2 + (18*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 111*(cos(b*x
+ a) - 1)^2/(cos(b*x + a) + 1)^2 + 36*(cos(b*x + a) - 1)^3/(cos(b*x + a) +
1)^3 + 72*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 1)/((cos(b*x + a) - 1
)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2)^2 + 96*lo
g(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 192*log(abs(-(cos(b*x + a
) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

maple [A] time = 0.05, size = 69, normalized size = 1.19

$$-\frac{1}{4b\sin(bx+a)^4\cos(bx+a)^2}+\frac{3}{4b\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{2\sin(bx+a)^2 b}+\frac{3\ln(\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b/sin(b*x+a)^4/cos(b*x+a)^2+3/4/b/sin(b*x+a)^2/cos(b*x+a)^2-3/2/sin(b*
x+a)^2/b+3*ln(tan(b*x+a))/b
```

maxima [A] time = 0.50, size = 74, normalized size = 1.28

$$-\frac{\frac{6\sin(bx+a)^4-3\sin(bx+a)^2-1}{\sin(bx+a)^6-\sin(bx+a)^4}+6\log\left(\sin(bx+a)^2-1\right)-6\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*((6*sin(b*x + a)^4 - 3*sin(b*x + a)^2 - 1)/(sin(b*x + a)^6 - sin(b*x +
 a)^4) + 6*log(sin(b*x + a)^2 - 1) - 6*log(sin(b*x + a)^2))/b
```

mupad [B] time = 0.47, size = 82, normalized size = 1.41

$$\frac{3\ln\left(\sin(a+b\,x)^2\right)}{2\,b}-\frac{3\ln(\cos(a+b\,x))}{b}+\frac{\frac{3\cos(a+b\,x)^4}{2}-\frac{9\cos(a+b\,x)^2}{4}+\frac{1}{2}}{b\,\left(\cos(a+b\,x)^6-2\cos(a+b\,x)^4+\cos(a+b\,x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^5),x)
```

```
[Out] (3*log(sin(a + b*x)^2))/(2*b) - (3*log(cos(a + b*x)))/b + ((3*cos(a + b*x)^
4)/2 - (9*cos(a + b*x)^2)/4 + 1/2)/(b*(cos(a + b*x)^2 - 2*cos(a + b*x)^4 +
cos(a + b*x)^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(a+bx)}{\sin^5(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/sin(b*x+a)**5,x)
```

```
[Out] Integral(sec(a + b*x)**3/sin(a + b*x)**5, x)
```

3.183 $\int \csc^5(a+bx)\sec^4(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{35\sec^3(a+bx)}{24b}+\frac{35\sec(a+bx)}{8b}-\frac{35\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{4b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b}$$

```
[Out] -35/8*arctanh(cos(b*x+a))/b+35/8*sec(b*x+a)/b+35/24*sec(b*x+a)^3/b-7/8*csc(
b*x+a)^2*sec(b*x+a)^3/b-1/4*csc(b*x+a)^4*sec(b*x+a)^3/b
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2622, 288, 302, 207}

$$\frac{35\sec^3(a+bx)}{24b}+\frac{35\sec(a+bx)}{8b}-\frac{35\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{4b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x]^4,x]
```

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(8*b) + (35*Sec[a + b*x])/(8*b) + (35*Sec[a + b
*x]^3)/(24*b) - (7*Csc[a + b*x]^2*Sec[a + b*x]^3)/(8*b) - (Csc[a + b*x]^4*S
ec[a + b*x]^3)/(4*b)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^5(a+bx)\sec^4(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{\left(-1+x^2\right)^3}\,dx, x, \sec(a+bx)\right)}{b}$$
$$= -\frac{\csc^4(a+bx)\sec^3(a+bx)}{4b} + \frac{7\,\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \sec(a+bx)\right)}{4b}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{4b} + \frac{35\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\right.}{}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{4b} + \frac{35\,\text{Subst}\left(\int \left(1+\right.\right.}{}$$
$$= \frac{35\sec(a+bx)}{8b} + \frac{35\sec^3(a+bx)}{24b} - \frac{7\csc^2(a+bx)\sec^3(a+bx)}{8b} - \frac{\csc^4(a+bx}{}$$
$$= -\frac{35\tanh^{-1}(\cos(a+bx))}{8b} + \frac{35\sec(a+bx)}{8b} + \frac{35\sec^3(a+bx)}{24b} - \frac{7\csc^2(a+bx}{8}$$

Mathematica [B] time = 0.44, size = 268, normalized size = 3.01

$$-\frac{\csc^{10}(a+bx)\left(658\cos(2(a+bx)) - 228\cos(3(a+bx)) + 140\cos(4(a+bx)) - 76\cos(5(a+bx)) - 210\cos(6(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x]^4,x]
```

```
[Out] -1/24*(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)]
+ 140*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76*Co
s[7*(a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5*(a
+ b*x)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]]
+ 3*Cos[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b*x)/2]
```

```
]) + 315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x)]*Log[
Sin[(a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(b*(Csc[(a
 + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

fricas [A] time = 0.45, size = 148, normalized size = 1.66

$$\frac{210 \cos(bx+a)^6 - 350 \cos(bx+a)^4 + 112 \cos(bx+a)^2 - 105\left(\cos(bx+a)^7 - 2\cos(bx+a)^5 + \cos(bx+a)^3\right)}{48\left(b\cos(bx+a)^7 - 2b\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/48*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105*(c
os(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a) + 1
/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/2*cos
(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x +
 a)^3)
```

giac [B] time = 0.38, size = 209, normalized size = 2.35

$$\frac{\frac{3\left(\frac{24(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{210(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - 1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} - \frac{72(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{256\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{6(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + 5\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + 1\right)^3} + 42}{192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/192*(3*(24*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 210*(cos(b*x + a) - 1)
^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 72
*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*x +
a) + 1)^2 + 256*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 6*(cos(b*x + a)
- 1)^2/(cos(b*x + a) + 1)^2 + 5)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1
)^3 + 420*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)))/b
```

maple [A] time = 0.05, size = 99, normalized size = 1.11

$$-\frac{1}{4b\sin(bx+a)^4\cos(bx+a)^3} + \frac{7}{12b\sin(bx+a)^2\cos(bx+a)^3} - \frac{35}{24b\sin(bx+a)^2\cos(bx+a)} + \frac{35}{8b\cos(bx+a)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/sin(b*x+a)^5,x)
```

```
[Out] -1/4/b/sin(b*x+a)^4/cos(b*x+a)^3+7/12/b/sin(b*x+a)^2/cos(b*x+a)^3-35/24/b/s
in(b*x+a)^2/cos(b*x+a)+35/8/b/cos(b*x+a)+35/8/b*ln(csc(b*x+a)-cot(b*x+a))
```

maxima [A] time = 0.37, size = 91, normalized size = 1.02

$$\frac{\frac{2\left(105\,\cos\left(bx+a\right)^{6}-175\,\cos\left(bx+a\right)^{4}+56\,\cos\left(bx+a\right)^{2}+8\right)}{\cos\left(bx+a\right)^{7}-2\,\cos\left(bx+a\right)^{5}+\cos\left(bx+a\right)^{3}}-105\,\log\left(\cos\left(bx+a\right)+1\right)+105\,\log\left(\cos\left(bx+a\right)-1\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/48*(2*(105*cos(b*x + a)^6 - 175*cos(b*x + a)^4 + 56*cos(b*x + a)^2 + 8)/(
cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3) - 105*log(cos(b*x + a)
+ 1) + 105*log(cos(b*x + a) - 1))/b
```

mupad [B] time = 0.07, size = 78, normalized size = 0.88

$$\frac{\frac{35\cos\left(a+b\,x\right)^6}{8}-\frac{175\cos\left(a+b\,x\right)^4}{24}+\frac{7\cos\left(a+b\,x\right)^2}{3}+\frac{1}{3}}{b\,\left(\cos\left(a+b\,x\right)^7-2\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)}-\frac{35\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^4*sin(a + b*x)^5),x)
```

```
[Out] ((7*cos(a + b*x)^2)/3 - (175*cos(a + b*x)^4)/24 + (35*cos(a + b*x)^6)/8 + 1
/3)/(b*(cos(a + b*x)^3 - 2*cos(a + b*x)^5 + cos(a + b*x)^7)) - (35*atanh(co
s(a + b*x)))/(8*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4\left(a+bx\right)}{\sin^5\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/sin(b*x+a)**5,x)
```

```
[Out] Integral(sec(a + b*x)**4/sin(a + b*x)**5, x)
```

3.184 $\int \csc^5(a+bx)\sec^5(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{\tan^4(a+bx)}{4b}+\frac{2\tan^2(a+bx)}{b}-\frac{\cot^4(a+bx)}{4b}-\frac{2\cot^2(a+bx)}{b}+\frac{6\log(\tan(a+bx))}{b}$$

```
[Out] -2*cot(b*x+a)^2/b-1/4*cot(b*x+a)^4/b+6*ln(tan(b*x+a))/b+2*tan(b*x+a)^2/b+1/
4*tan(b*x+a)^4/b
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2620, 266, 43}

$$\frac{\tan^4(a+bx)}{4b}+\frac{2\tan^2(a+bx)}{b}-\frac{\cot^4(a+bx)}{4b}-\frac{2\cot^2(a+bx)}{b}+\frac{6\log(\tan(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^5*Sec[a + b*x]^5,x]
```

```
[Out] (-2*Cot[a + b*x]^2)/b - Cot[a + b*x]^4/(4*b) + (6*Log[Tan[a + b*x]])/b + (2
*Tan[a + b*x]^2)/b + Tan[a + b*x]^4/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rubi steps

$$\int \csc^5(a+bx)\sec^5(a+bx)\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^5}\,dx, x, \tan(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^4}{x^3}\,dx, x, \tan^2(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(4+\frac{1}{x^3}+\frac{4}{x^2}+\frac{6}{x}+x\right)\,dx, x, \tan^2(a+bx)\right)}{2b}$$
$$= -\frac{2\cot^2(a+bx)}{b} - \frac{\cot^4(a+bx)}{4b} + \frac{6\log(\tan(a+bx))}{b} + \frac{2\tan^2(a+bx)}{b} + \text{tan}$$

Mathematica [A] time = 0.01, size = 91, normalized size = 1.32

$$32\left(-\frac{\csc^4(a+bx)}{128b} - \frac{3\csc^2(a+bx)}{64b} + \frac{\sec^4(a+bx)}{128b} + \frac{3\sec^2(a+bx)}{64b} + \frac{3\log(\sin(a+bx))}{16b} - \frac{3\log(\cos(a+bx))}{16b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^5*Sec[a + b*x]^5,x]
```

```
[Out] 32*((-3*Csc[a + b*x]^2)/(64*b) - Csc[a + b*x]^4/(128*b) - (3*Log[Cos[a + b*
x]])/(16*b) + (3*Log[Sin[a + b*x]])/(16*b) + (3*Sec[a + b*x]^2)/(64*b) + Se
c[a + b*x]^4/(128*b))
```

fricas [B] time = 0.41, size = 148, normalized size = 2.14

$$\frac{12\cos(bx+a)^6 - 18\cos(bx+a)^4 + 4\cos(bx+a)^2 - 12\left(\cos(bx+a)^8 - 2\cos(bx+a)^6 + \cos(bx+a)^4\right)\log}{4\left(b\cos(bx+a)^8 - 2b\cos(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(12*cos(b*x + a)^6 - 18*cos(b*x + a)^4 + 4*cos(b*x + a)^2 - 12*(cos(b*x
 + a)^8 - 2*cos(b*x + a)^6 + cos(b*x + a)^4)*log(cos(b*x + a)^2) + 12*(cos(
b*x + a)^8 - 2*cos(b*x + a)^6 + cos(b*x + a)^4)*log(-1/4*cos(b*x + a)^2 + 1
/4) + 1)/(b*cos(b*x + a)^8 - 2*b*cos(b*x + a)^6 + b*cos(b*x + a)^4)
```

giac [B] time = 0.38, size = 278, normalized size = 4.03

$$\frac{\frac{\left(\frac{28(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{288(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - 1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} + \frac{28(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{32\left(\frac{84(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{126(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{84(\cos(bx}{(\cos(bx+}\right.}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^4}}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/64*((28*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 288*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 + 28*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) - (cos(b*x + a) - 1)^2/(cos(b*x + a) +
1)^2 + 32*(84*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 126*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 + 84*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 2
5*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 25)/((cos(b*x + a) - 1)/(cos(
b*x + a) + 1) + 1)^4 + 192*log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)
) - 384*log(abs(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)))/b
```

maple [A] time = 0.05, size = 90, normalized size = 1.30

$$\frac{1}{4b\sin(bx+a)^4\cos(bx+a)^4}-\frac{1}{2b\sin(bx+a)^4\cos(bx+a)^2}+\frac{3}{2b\sin(bx+a)^2\cos(bx+a)^2}-\frac{3}{\sin(bx+a)^2 b}+\frac{6\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^5/sin(b*x+a)^5,x)
```

```
[Out] 1/4/b/sin(b*x+a)^4/cos(b*x+a)^4-1/2/b/sin(b*x+a)^4/cos(b*x+a)^2+3/2/b/sin(b
*x+a)^2/cos(b*x+a)^2-3/sin(b*x+a)^2/b+6*ln(tan(b*x+a))/b
```

maxima [A] time = 0.52, size = 92, normalized size = 1.33

$$-\frac{\frac{12\sin(bx+a)^6-18\sin(bx+a)^4+4\sin(bx+a)^2+1}{\sin(bx+a)^8-2\sin(bx+a)^6+\sin(bx+a)^4}+12\log\left(\sin(bx+a)^2-1\right)-12\log\left(\sin(bx+a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^5/sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*((12*sin(b*x + a)^6 - 18*sin(b*x + a)^4 + 4*sin(b*x + a)^2 + 1)/(sin(b
*x + a)^8 - 2*sin(b*x + a)^6 + sin(b*x + a)^4) + 12*log(sin(b*x + a)^2 - 1)
 - 12*log(sin(b*x + a)^2))/b
```

mupad [B] time = 0.41, size = 64, normalized size = 0.93

$$\frac{2\tan(a+bx)^2}{b}+\frac{\tan(a+bx)^4}{4b}+\frac{6\ln(\tan(a+bx))}{b}-\frac{\cot(a+bx)^4\left(2\tan(a+bx)^2+\frac{1}{4}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^5*sin(a + b*x)^5),x)
```

```
[Out] (2*tan(a + b*x)^2)/b + tan(a + b*x)^4/(4*b) + (6*log(tan(a + b*x)))/b - (co
t(a + b*x)^4*(2*tan(a + b*x)^2 + 1/4))/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(a+bx)}{\sin^5(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**5/sin(b*x+a)**5,x)
```

```
[Out] Integral(sec(a + b*x)**5/sin(a + b*x)**5, x)
```

3.185 $\int \cot^2(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{5}\cot^5(x)-\frac{\cot^3(x)}{3}$$

```
[Out] -1/3*cot(x)^3-1/5*cot(x)^5
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 14}

$$-\frac{1}{5}\cot^5(x)-\frac{\cot^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*Csc[x]^4,x]
```

```
[Out] -Cot[x]^3/3 - Cot[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\begin{aligned}\int \cot^2(x)\csc^4(x)\,dx &= \text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,-\cot(x)\right)\\ &= \text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,-\cot(x)\right)\\ &= -\frac{1}{3}\cot^3(x)-\frac{\cot^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.59

$$\frac{2\cot(x)}{15} - \frac{1}{5}\cot(x)\csc^4(x) + \frac{1}{15}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*Csc[x]^4,x]
[Out] (2*Cot[x])/15 + (Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

fricas [B] time = 0.41, size = 33, normalized size = 1.94

$$\frac{2\cos(x)^5 - 5\cos(x)^3}{15\left(\cos(x)^4 - 2\cos(x)^2 + 1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/sin(x)^6,x, algorithm="fricas")
[Out] 1/15*(2*cos(x)^5 - 5*cos(x)^3)/((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x))
```

giac [A] time = 0.23, size = 14, normalized size = 0.82

$$-\frac{5\tan(x)^2 + 3}{15\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/sin(x)^6,x, algorithm="giac")
[Out] -1/15*(5*tan(x)^2 + 3)/tan(x)^5
```

maple [A] time = 0.02, size = 22, normalized size = 1.29

$$-\frac{\cos^3(x)}{5\sin(x)^5} - \frac{2\left(\cos^3(x)\right)}{15\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/sin(x)^6,x)
[Out] -1/5*cos(x)^3/sin(x)^5-2/15*cos(x)^3/sin(x)^3
```

maxima [A] time = 0.31, size = 14, normalized size = 0.82

$$-\frac{5\tan(x)^2 + 3}{15\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/sin(x)^6,x, algorithm="maxima")
```

```
[Out] -1/15*(5*tan(x)^2 + 3)/tan(x)^5
```

mupad [B] time = 0.08, size = 19, normalized size = 1.12

$$-\cos(x)^3\left(\frac{2}{15\sin(x)^3}+\frac{1}{5\sin(x)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/sin(x)^6,x)
```

```
[Out] -cos(x)^3*(2/(15*sin(x)^3) + 1/(5*sin(x)^5))
```

sympy [B] time = 0.07, size = 29, normalized size = 1.71

$$\frac{2\cos{(x)}}{15\sin{(x)}}+\frac{\cos{(x)}}{15\sin^3{(x)}}-\frac{\cos{(x)}}{5\sin^5{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/sin(x)**6,x)
```

```
[Out] 2*cos(x)/(15*sin(x)) + cos(x)/(15*sin(x)**3) - cos(x)/(5*sin(x)**5)
```

3.186 $\int \cot^3(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

```
[Out] 1/4*csc(x)^4-1/6*csc(x)^6
```

Rubi [A] time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
\int \cot^3(x)\csc^4(x)\,dx &= -\text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx,x,\csc(x)\right)\\
&= -\text{Subst}\left(\int \left(-x^3+x^5\right)\,dx,x,\csc(x)\right)\\
&= \frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

fricas [B] time = 0.43, size = 30, normalized size = 1.76

$$\frac{3 \cos(x)^2 - 1}{12 \left(\cos(x)^6 - 3 \cos(x)^4 + 3 \cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

giac [A] time = 0.20, size = 14, normalized size = 0.82

$$\frac{3 \sin(x)^2 - 2}{12 \sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="giac")
```

```
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

maple [A] time = 0.02, size = 22, normalized size = 1.29

$$-\frac{\cos^4(x)}{6 \sin(x)^6} - \frac{\cos^4(x)}{12 \sin(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^7,x)
```

```
[Out] -1/6/sin(x)^6*cos(x)^4-1/12*cos(x)^4/sin(x)^4
```

maxima [A] time = 0.31, size = 14, normalized size = 0.82

$$\frac{3 \sin(x)^2 - 2}{12 \sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="maxima")
```

```
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

mupad [B] time = 0.44, size = 13, normalized size = 0.76

$$\frac{\frac{\sin(x)^2}{4} - \frac{1}{6}}{\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^7,x)
```

```
[Out] (sin(x)^2/4 - 1/6)/sin(x)^6
```

sympy [A] time = 0.11, size = 15, normalized size = 0.88

$$-\frac{2 - 3\sin^2(x)}{12\sin^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/sin(x)**7,x)
```

```
[Out] -(2 - 3*sin(x)**2)/(12*sin(x)**6)
```

3.187 $\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx$

Optimal. Leaf size=22

$$-\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

```
[Out] -2/5*(d*cos(b*x+a))^(5/2)/b/d
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$-\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x],x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(5/2))/(5*b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{3/2}\sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^{3/2}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{2(d\cos(a+bx))^{5/2}}{5bd}\end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 1.00

$$-\frac{2(d\cos(a+bx))^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x],x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(5/2))/(5*b*d)
```

fricas [A] time = 0.43, size = 24, normalized size = 1.09

$$-\frac{2\sqrt{d\cos(bx+a)}\,d\cos(bx+a)^2}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(d*cos(b*x + a))*d*cos(b*x + a)^2/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a), x)
```

maple [A] time = 0.02, size = 19, normalized size = 0.86

$$-\frac{2(d\cos(bx+a))^{\frac{5}{2}}}{5bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*sin(b*x+a),x)
```

```
[Out] -2/5*(d*cos(b*x+a))^(5/2)/b/d
```

maxima [A] time = 0.54, size = 18, normalized size = 0.82

$$-\frac{2\,(d\cos(bx+a))^{\frac{5}{2}}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/5*(d*cos(b*x + a))^(5/2)/(b*d)
```

mupad [B] time = 0.13, size = 18, normalized size = 0.82

$$-\frac{2\,{\left(d\,\cos\left(a+b\,x\right)\right)}^{5/2}}{5\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(d*cos(a + b*x))^(3/2),x)
```

```
[Out] -(2*(d*cos(a + b*x))^(5/2))/(5*b*d)
```

sympy [A] time = 52.07, size = 34, normalized size = 1.55

$$\begin{cases} -\frac{2d^{\frac{3}{2}}\cos^{\frac{5}{2}}(a+bx)}{5b} & \text{for}\ b \neq 0 \\ x\left(d\cos\left(a\right)\right)^{\frac{3}{2}}\sin\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*sin(b*x+a),x)
```

```
[Out] Piecewise((-2*d**(3/2)*cos(a + b*x)**(5/2)/(5*b), Ne(b, 0)), (x*(d*cos(a))*
*(3/2)*sin(a), True))
```

3.188 $\int \sqrt{d\cos(a+bx)}\,\sin(a+bx)\,dx$

Optimal. Leaf size=22

$$-\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

```
[Out] -2/3*(d*cos(b*x+a))^(3/2)/b/d
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$-\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(3/2))/(3*b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\cos(a+bx)}\,\sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \sqrt{x}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{2(d\cos(a+bx))^{3/2}}{3bd}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$-\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(3/2))/(3*b*d)
```

fricas [A] time = 0.44, size = 21, normalized size = 0.95

$$-\frac{2\sqrt{d\cos(bx+a)}\cos(bx+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(d*cos(b*x + a))*cos(b*x + a)/b
```

giac [A] time = 1.34, size = 18, normalized size = 0.82

$$-\frac{2\left(d\cos(bx+a)\right)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -2/3*(d*cos(b*x + a))^(3/2)/(b*d)
```

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$-\frac{2\left(d\cos(bx+a)\right)^{\frac{3}{2}}}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*sin(b*x+a),x)
```

```
[Out] -2/3*(d*cos(b*x+a))^(3/2)/b/d
```

maxima [A] time = 0.46, size = 18, normalized size = 0.82

$$-\frac{2\left(d\cos(bx+a)\right)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/3*(d*cos(b*x + a))^(3/2)/(b*d)
```

mupad [B] time = 0.42, size = 18, normalized size = 0.82

$$-\frac{2\,{\left(d\,\cos\left(a+b\,x\right)\right)}^{3/2}}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(d*cos(a + b*x))^(1/2),x)
[Out] -(2*(d*cos(a + b*x))^(3/2))/(3*b*d)
```

sympy [A] time = 1.67, size = 34, normalized size = 1.55

$$\begin{cases} -\frac{2\sqrt{d}\cos^{\frac{3}{2}}{\left(a+bx\right)}}{3b} & \text{for}\: b \neq 0 \\ x\sqrt{d\cos{\left(a\right)}}\sin{\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a),x)
[Out] Piecewise((-2*sqrt(d)*cos(a + b*x)**(3/2)/(3*b), Ne(b, 0)), (x*sqrt(d*cos(a
))*sin(a), True))
```

3.189 $\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=20

$$-\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

```
[Out] -2*(d*cos(b*x+a))^(1/2)/b/d
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$-\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-2*Sqrt[d*Cos[a + b*x]])/(b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= -\frac{2\sqrt{d\cos(a+bx)}}{bd}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$-\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
[Out] (-2*Sqrt[d*Cos[a + b*x]])/(b*d)
```

fricas [A] time = 0.42, size = 18, normalized size = 0.90

$$-\frac{2\sqrt{d\cos(bx+a)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(d*cos(b*x + a))/(b*d)
```

giac [A] time = 0.19, size = 18, normalized size = 0.90

$$-\frac{2\sqrt{d\cos(bx+a)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] -2*sqrt(d*cos(b*x + a))/(b*d)
```

maple [A] time = 0.02, size = 19, normalized size = 0.95

$$-\frac{2\sqrt{d\cos(bx+a)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x)
[Out] -2*(d*cos(b*x+a))^(1/2)/b/d
```

maxima [A] time = 0.40, size = 18, normalized size = 0.90

$$-\frac{2\sqrt{d\cos(bx+a)}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(d*cos(b*x + a))/(b*d)
```

mupad [B] time = 0.53, size = 18, normalized size = 0.90

$$-\frac{2\,\sqrt{d\,\cos\left(a+b\,x\right)}}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(d*cos(a + b*x))^(1/2),x)
```

```
[Out] -(2*(d*cos(a + b*x))^(1/2))/(b*d)
```

sympy [A] time = 1.55, size = 32, normalized size = 1.60

$$\begin{cases} -\frac{2\sqrt{\cos\left(a+bx\right)}}{b\sqrt{d}} & \text{for}\ b\neq 0 \\ \frac{x\sin\left(a\right)}{\sqrt{d\cos\left(a\right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(cos(a + b*x))/(b*sqrt(d)), Ne(b, 0)), (x*sin(a)/sqrt(d*c
os(a)), True))
```

3.190 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=20

$$\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/b/d/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] 2/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{bd\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
[Out] 2/(b*d*Sqrt[d*Cos[a + b*x]])
```

fricas [A] time = 0.42, size = 26, normalized size = 1.30

$$\frac{2\sqrt{d\cos(bx+a)}}{bd^2\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 2*sqrt(d*cos(b*x + a))/(b*d^2*cos(b*x + a))
```

giac [A] time = 0.84, size = 18, normalized size = 0.90

$$\frac{2}{\sqrt{d\cos(bx+a)}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
[Out] 2/(sqrt(d*cos(b*x + a))*b*d)
```

maple [A] time = 0.01, size = 19, normalized size = 0.95

$$\frac{2}{bd\sqrt{d\cos(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x)
[Out] 2/b/d/(d*cos(b*x+a))^(1/2)
```

maxima [A] time = 0.53, size = 18, normalized size = 0.90

$$\frac{2}{\sqrt{d\cos(bx+a)}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/(sqrt(d*cos(b*x + a))*b*d)
```

mupad [B] time = 0.18, size = 37, normalized size = 1.85

$$\frac{4\,\cos\left(a+b\,x\right)\,\sqrt{d\,\cos\left(a+b\,x\right)}}{b\,d^2\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(d*cos(a + b*x))^(3/2),x)
```

```
[Out] (4*cos(a + b*x)*(d*cos(a + b*x))^(1/2))/(b*d^2*(cos(2*a + 2*b*x) + 1))
```

sympy [A] time = 5.96, size = 31, normalized size = 1.55

$$\begin{cases} \frac{2}{bd^{\frac{3}{2}}\sqrt{\cos\left(a+bx\right)}} & \text{for}\ b\neq 0 \\ \frac{x\sin\left(a\right)}{\left(d\cos\left(a\right)\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((2/(b*d**(3/2)*sqrt(cos(a + b*x))), Ne(b, 0)), (x*sin(a)/(d*cos(a
))**(3/2), True))
```

3.191 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/3/b/d/(d*cos(b*x+a))^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] 2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{3bd(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] 2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

fricas [A] time = 0.41, size = 26, normalized size = 1.18

$$\frac{2\sqrt{d\cos\left(bx+a\right)}}{3\,bd^3\cos\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(d*cos(b*x + a))/(b*d^3*cos(b*x + a)^2)
```

giac [A] time = 0.81, size = 26, normalized size = 1.18

$$\frac{2}{3\sqrt{d\cos\left(bx+a\right)}\,bd^2\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3/(sqrt(d*cos(b*x + a))*b*d^2*cos(b*x + a))
```

maple [A] time = 0.02, size = 19, normalized size = 0.86

$$\frac{2}{3bd\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 2/3/b/d/(d*cos(b*x+a))^(3/2)
```

maxima [A] time = 0.54, size = 18, normalized size = 0.82

$$\frac{2}{3\,\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3/((d*cos(b*x + a))^(3/2)*b*d)
```

mupad [B] time = 0.81, size = 53, normalized size = 2.41

$$\frac{8\ (\cos(2\,a+2\,b\,x)+1)\ \sqrt{d\ \cos(a+b\,x)}}{3\,b\,d^3\ (4\ \cos(2\,a+2\,b\,x)+\cos(4\,a+4\,b\,x)+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(d*cos(a + b*x))^(5/2),x)
```

```
[Out] (8*(cos(2*a + 2*b*x) + 1)*(d*cos(a + b*x))^(1/2))/(3*b*d^3*(4*cos(2*a + 2*b
*x) + cos(4*a + 4*b*x) + 3))
```

sympy [A] time = 56.23, size = 32, normalized size = 1.45

$$\begin{cases} \frac{2}{3bd^{\frac{5}{2}}\cos^{\frac{3}{2}}(a+bx)} & \text{for } b \neq 0 \\ \frac{x\sin(a)}{(d\cos(a))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((2/(3*b*d**(5/2)*cos(a + b*x)**(3/2)), Ne(b, 0)), (x*sin(a)/(d*co
s(a))**(5/2), True))
```

3.192 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/5/b/d/(d*cos(b*x+a))^(5/2)
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$\frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] 2/(5*b*d*(d*Cos[a + b*x])^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{5bd(d\cos(a+bx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 22, normalized size = 1.00

$$\frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] 2/(5*b*d*(d*Cos[a + b*x])^(5/2))
```

fricas [A] time = 0.43, size = 26, normalized size = 1.18

$$\frac{2\sqrt{d\cos\left(bx+a\right)}}{5\,bd^4\cos\left(bx+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(d*cos(b*x + a))/(b*d^4*cos(b*x + a)^3)
```

giac [A] time = 0.87, size = 26, normalized size = 1.18

$$\frac{2}{5\sqrt{d\cos\left(bx+a\right)}\,bd^3\cos\left(bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] 2/5/(sqrt(d*cos(b*x + a))*b*d^3*cos(b*x + a)^2)
```

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$\frac{2}{5bd\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] 2/5/b/d/(d*cos(b*x+a))^(5/2)
```

maxima [A] time = 0.47, size = 18, normalized size = 0.82

$$\frac{2}{5\,\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5/((d*cos(b*x + a))^(5/2)*b*d)
```

mupad [B] time = 6.71, size = 65, normalized size = 2.95

$$\frac{16\,e^{a\,3i+b\,x\,3i}\,\sqrt{d\,\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}}{5\,b\,d^4\,\left(e^{a\,2i+b\,x\,2i}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(d*cos(a + b*x))^(7/2),x)
```

```
[Out] (16*exp(a*3i + b*x*3i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^
(1/2))/(5*b*d^4*(exp(a*2i + b*x*2i) + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

3.193 $\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

```
[Out] 2/7/b/d/(d*cos(b*x+a))^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$\frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] 2/(7*b*d*(d*Cos[a + b*x])^(7/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= \frac{2}{7bd(d\cos(a+bx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 22, normalized size = 1.00

$$\frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
[Out] 2/(7*b*d*(d*Cos[a + b*x])^(7/2))
```

fricas [A] time = 0.44, size = 26, normalized size = 1.18

$$\frac{2\sqrt{d\cos(bx+a)}}{7\,bd^5\cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
[Out] 2/7*sqrt(d*cos(b*x + a))/(b*d^5*cos(b*x + a)^4)
```

giac [A] time = 0.99, size = 26, normalized size = 1.18

$$\frac{2}{7\sqrt{d\cos(bx+a)}\,bd^4\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
[Out] 2/7/(sqrt(d*cos(b*x + a))*b*d^4*cos(b*x + a)^3)
```

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$\frac{2}{7bd\,(d\cos(bx+a))^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x)
[Out] 2/7/b/d/(d*cos(b*x+a))^(7/2)
```

maxima [A] time = 0.30, size = 18, normalized size = 0.82

$$\frac{2}{7\,(d\cos(bx+a))^{\frac{7}{2}}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] 2/7/((d*cos(b*x + a))^(7/2)*b*d)
```

mupad [B] time = 4.04, size = 65, normalized size = 2.95

$$\frac{32\,{\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,\sqrt{d\,\left(\frac{{\mathrm{e}}^{-a\,1{}\mathrm{i}-b\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}}{2}\right)}}{7\,b\,d^5\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(d*cos(a + b*x))^(9/2),x)
```

```
[Out] (32*exp(a*4i + b*x*4i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^
(1/2))/(7*b*d^5*(exp(a*2i + b*x*2i) + 1)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

3.194 $\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx$

Optimal. Leaf size=126

$$\frac{28d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{195b\sqrt{\cos(a+bx)}}+\frac{28d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{585b}-\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}+\frac{4d}{}$$

```
[Out] 28/585*d^3*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b+4/117*d*(d*cos(b*x+a))^(7/2)*s
in(b*x+a)/b-2/13*(d*cos(b*x+a))^(11/2)*sin(b*x+a)/b/d+28/195*d^4*(cos(1/2*a
+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))
*(d*cos(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2568, 2635, 2640, 2639}

$$\frac{28d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{585b}+\frac{28d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{195b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{11/2}}{13bd}+\frac{4d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^2,x]
```

```
[Out] (28*d^4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(195*b*Sqrt[Cos[a +
 b*x]]) + (28*d^3*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(585*b) + (4*d*(d*Co
s[a + b*x])^(7/2)*Sin[a + b*x])/(117*b) - (2*(d*Cos[a + b*x])^(11/2)*Sin[a
+ b*x])/(13*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{11/2}\sin(a+bx)}{13bd} + \frac{2}{13}\int (d\cos(a+bx))^{9/2}\,dx$$
$$= \frac{4d(d\cos(a+bx))^{7/2}\sin(a+bx)}{117b} - \frac{2(d\cos(a+bx))^{11/2}\sin(a+bx)}{13bd} + \frac{1}{117}($$
$$= \frac{28d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{585b} + \frac{4d(d\cos(a+bx))^{7/2}\sin(a+bx)}{117b} - 2($$
$$= \frac{28d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{585b} + \frac{4d(d\cos(a+bx))^{7/2}\sin(a+bx)}{117b} - 2($$
$$= \frac{28d^4\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{195b\sqrt{\cos(a+bx)}} + \frac{28d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{585b}$$

Mathematica [C] time = 0.13, size = 60, normalized size = 0.48

$$\frac{d^2\sqrt[4]{\cos^2(a+bx)}\,\tan^3(a+bx)(d\cos(a+bx))^{5/2}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^2,x]
```

```
[Out] (d^2*(d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4,
3/2, 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(d^4\cos(bx+a)^6 - d^4\cos(bx+a)^4\right)\sqrt{d\cos(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="fricas")

[Out] integral(-(d^4*cos(b*x + a)^6 - d^4*cos(b*x + a)^4)*sqrt(d*cos(b*x + a)), x
)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{9}{2}} \sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^2, x)

maple [A] time = 0.14, size = 249, normalized size = 1.98

$$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d^5\left(2880\left(\cos^{15}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-11520\left(\cos^{13}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+19280\left(\cos^{1}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x)

[Out] 4/585*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^5*(2880*c
os(1/2*b*x+1/2*a)^15-11520*cos(1/2*b*x+1/2*a)^13+19280*cos(1/2*b*x+1/2*a)^1
1-17520*cos(1/2*b*x+1/2*a)^9+9284*cos(1/2*b*x+1/2*a)^7-2808*cos(1/2*b*x+1/2
*a)^5+425*cos(1/2*b*x+1/2*a)^3+21*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*
b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-21*cos(1/2*b*x+
1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*
x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{9}{2}} \sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^2,x, algorithm="maxima")

[Out] integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(d*cos(a + b*x))^(9/2),x)
```

```
[Out] int(sin(a + b*x)^2*(d*cos(a + b*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.195 $\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx$

Optimal. Leaf size=126

$$\frac{20d^4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{231b\sqrt{d\cos(a+bx)}}+\frac{20d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{231b}-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}+\frac{4d\sin(a}{}$$

```
[Out] 4/77*d*(d*cos(b*x+a))^(5/2)*sin(b*x+a)/b-2/11*(d*cos(b*x+a))^(9/2)*sin(b*x+
a)/b/d+20/231*d^4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF
(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)+20/231
*d^3*sin(b*x+a)*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2568, 2635, 2642, 2641}

$$\frac{20d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{231b}+\frac{20d^4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{231b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{9/2}}{11bd}+\frac{4d\sin(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^2,x]
```

```
[Out] (20*d^4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(231*b*Sqrt[d*Cos[a +
 b*x]]) + (20*d^3*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(231*b) + (4*d*(d*Cos[
a + b*x])^(5/2)*Sin[a + b*x])/(77*b) - (2*(d*Cos[a + b*x])^(9/2)*Sin[a + b*
x])/(11*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{9/2}\sin(a+bx)}{11bd}+\frac{2}{11}\int (d\cos(a+bx))^{7/2}\,dx$$
$$=\frac{4d(d\cos(a+bx))^{5/2}\sin(a+bx)}{77b}-\frac{2(d\cos(a+bx))^{9/2}\sin(a+bx)}{11bd}+\frac{1}{77}\Big(10$$
$$=\frac{20d^3\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{231b}+\frac{4d(d\cos(a+bx))^{5/2}\sin(a+bx)}{77b}-\frac{2(d}{}$$
$$=\frac{20d^3\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{231b}+\frac{4d(d\cos(a+bx))^{5/2}\sin(a+bx)}{77b}-\frac{2(d}{}$$
$$=\frac{20d^4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{231b\sqrt{d\cos(a+bx)}}+\frac{20d^3\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{231b}+\frac{4}{}$$

Mathematica [C] time = 0.13, size = 60, normalized size = 0.48

$$\frac{d^2\cos^2(a+bx)^{3/4}\tan^3(a+bx)(d\cos(a+bx))^{3/2}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^2,x]
```

```
[Out] (d^2*(d*Cos[a + b*x])^(3/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4,
3/2, 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(d^3\cos(bx+a)^5-d^3\cos(bx+a)^3\right)\sqrt{d\cos(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(d^3*cos(b*x + a)^5 - d^3*cos(b*x + a)^3)*sqrt(d*cos(b*x + a)), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^2, x)
```

maple [A] time = 0.08, size = 236, normalized size = 1.87

$$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^4\left(672\left(\cos^{13}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2352\left(\cos^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+3312\left(\cos^{9}\left(\frac{bx}{2}\right.\right.\right.}{231\sqrt{-d\left(2\left(\sin\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x)
```

```
[Out] 4/231*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^4*(672*co
s(1/2*b*x+1/2*a)^13-2352*cos(1/2*b*x+1/2*a)^11+3312*cos(1/2*b*x+1/2*a)^9-24
00*cos(1/2*b*x+1/2*a)^7+922*cos(1/2*b*x+1/2*a)^5-159*cos(1/2*b*x+1/2*a)^3-5
*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(c
os(1/2*b*x+1/2*a),2^(1/2))+5*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^
4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^
2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(d*cos(a + b*x))^(7/2),x)
```

```
[Out] int(sin(a + b*x)^2*(d*cos(a + b*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(7/2)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.196 $\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx$

Optimal. Leaf size=98

$$\frac{4d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{15b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}+\frac{4d\sin(a+bx)(d\cos(a+bx))^{3/2}}{45b}$$

```
[Out] 4/45*d*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b-2/9*(d*cos(b*x+a))^(7/2)*sin(b*x+a
)/b/d+4/15*d^2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(si
n(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2568, 2635, 2640, 2639}

$$\frac{4d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{15b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{7/2}}{9bd}+\frac{4d\sin(a+bx)(d\cos(a+bx))^{3/2}}{45b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^2,x]
```

```
[Out] (4*d^2*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(15*b*Sqrt[Cos[a + b
*x]]) + (4*d*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(45*b) - (2*(d*Cos[a + b*
x])^(7/2)*Sin[a + b*x])/(9*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,dx &= -\frac{2(d\cos(a+bx))^{7/2}\sin(a+bx)}{9bd}+\frac{2}{9}\int (d\cos(a+bx))^{5/2}\,dx\\ &= \frac{4d(d\cos(a+bx))^{3/2}\sin(a+bx)}{45b}-\frac{2(d\cos(a+bx))^{7/2}\sin(a+bx)}{9bd}+\frac{1}{15}\Big(2d\\ &= \frac{4d(d\cos(a+bx))^{3/2}\sin(a+bx)}{45b}-\frac{2(d\cos(a+bx))^{7/2}\sin(a+bx)}{9bd}+\frac{\big(2d^2\sqrt{}}{}\\ &= \frac{4d^2\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b\sqrt{\cos(a+bx)}}+\frac{4d(d\cos(a+bx))^{3/2}\sin(a+bx)}{45b}-\frac{2}{}\end{aligned}$$

Mathematica [C] time = 0.05, size = 57, normalized size = 0.58

$$\frac{\sqrt[4]{\cos^2(a+bx)}\,\tan^3(a+bx)(d\cos(a+bx))^{5/2}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^2,x]
```

```
[Out] ((d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, 3/2,
 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(d^2\cos(bx+a)^4-d^2\cos(bx+a)^2\right)\sqrt{d\cos(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*sqrt(d*cos(b*x + a)), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^2, x)
```

maple [B] time = 0.09, size = 223, normalized size = 2.28

$$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^3\left(80\left(\cos^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-240\left(\cos^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+272\left(\cos^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right.\right.}{45\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x)
```

```
[Out] 4/45*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^3*(80*cos(
1/2*b*x+1/2*a)^11-240*cos(1/2*b*x+1/2*a)^9+272*cos(1/2*b*x+1/2*a)^7-144*cos
(1/2*b*x+1/2*a)^5+35*cos(1/2*b*x+1/2*a)^3+3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-
2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-3*cos
(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/s
in(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(d*cos(a + b*x))^(5/2),x)
```

```
[Out] int(sin(a + b*x)^2*(d*cos(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.197 $\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx$

Optimal. Leaf size=98

$$\frac{4d^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}+\frac{4d\sin(a+bx)\sqrt{d\cos(a+bx)}}{21b}$$

```
[Out] -2/7*(d*cos(b*x+a))^(5/2)*sin(b*x+a)/b/d+4/21*d^2*(cos(1/2*a+1/2*b*x)^2)^(1
/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/
2)/b/(d*cos(b*x+a))^(1/2)+4/21*d*sin(b*x+a)*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2568, 2635, 2642, 2641}

$$\frac{4d^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{5/2}}{7bd}+\frac{4d\sin(a+bx)\sqrt{d\cos(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^2,x]
```

```
[Out] (4*d^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(21*b*Sqrt[d*Cos[a + b
*x]]) + (4*d*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(21*b) - (2*(d*Cos[a + b*x]
)^(5/2)*Sin[a + b*x])/(7*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,dx &= -\frac{2(d\cos(a+bx))^{5/2}\sin(a+bx)}{7bd}+\frac{2}{7}\int (d\cos(a+bx))^{3/2}\,dx\\ &= \frac{4d\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{21b}-\frac{2(d\cos(a+bx))^{5/2}\sin(a+bx)}{7bd}+\frac{1}{21}\Big(2d^2\\ &= \frac{4d\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{21b}-\frac{2(d\cos(a+bx))^{5/2}\sin(a+bx)}{7bd}+\frac{\Big(2d^2\sqrt{c}}{}\\ &= \frac{4d^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b\sqrt{d\cos(a+bx)}}+\frac{4d\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{21b}-\frac{2(d\,c}{}\end{aligned}$$

Mathematica [C] time = 0.06, size = 57, normalized size = 0.58

$$\frac{\cos^2(a+bx)^{3/4}\tan^3(a+bx)(d\cos(a+bx))^{3/2}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^2,x]
```

```
[Out] ((d*Cos[a + b*x])^(3/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 3/2,
 5/2, Sin[a + b*x]^2]*Tan[a + b*x]^3)/(3*b)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(d\cos\left(bx+a\right)^3-d\cos\left(bx+a\right)\right)\sqrt{d\cos\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*cos(b*x + a)^3 - d*cos(b*x + a))*sqrt(d*cos(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^2, x)
```

maple [A] time = 0.08, size = 208, normalized size = 2.12

$$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^2\left(24\left(\cos^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-60\left(\cos^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+50\left(\cos^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cdots\right.}{21\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x)
```

```
[Out] 4/21*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^2*(24*cos(
1/2*b*x+1/2*a)^9-60*cos(1/2*b*x+1/2*a)^7+50*cos(1/2*b*x+1/2*a)^5-15*cos(1/2
*b*x+1/2*a)^3-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2
)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*
b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*
b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(d*cos(a + b*x))^(3/2),x)
```

```
[Out] int(sin(a + b*x)^2*(d*cos(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.198 $\int \sqrt{d\cos(a+bx)}\,\sin^2(a+bx)\,dx$

Optimal. Leaf size=69

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}$$

```
[Out] -2/5*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b/d+4/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/c
os(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2
)/b/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2568, 2640, 2639}

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}-\frac{2\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^2,x]
```

```
[Out] (4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]])
 - (2*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int \sqrt{d\cos(a+bx)}\,\sin^2(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd} + \frac{2}{5}\int \sqrt{d\cos(a+bx)}\,dx$$
$$= -\frac{2(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd} + \frac{\left(2\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{5\sqrt{\cos(a+bx)}}$$
$$= \frac{4\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}} - \frac{2(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd}$$

Mathematica [C] time = 0.09, size = 58, normalized size = 0.84

$$\frac{d\sin^3(a+bx)\sqrt[4]{\cos^2(a+bx)}\;{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^2,x]
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 3/2, 5/2, Sin[a + b*x]^2]*
Sin[a + b*x]^3)/(3*b*Sqrt[d*Cos[a + b*x]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\sqrt{d\cos\left(bx+a\right)}\left(\cos\left(bx+a\right)^2-1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="fricas")
[Out] integral(-sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos\left(bx+a\right)}\,\sin\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^2, x)
```

maple [B] time = 0.08, size = 194, normalized size = 2.81

$$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\; d\left(4\left(\cos^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8\left(\cos^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+5\left(\cos^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{\cdots}\right.}{5\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\;\sin\left(\frac{bx}{2}+\frac{a}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x)
```

```
[Out] 4/5*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d*(4*cos(1/2*
b*x+1/2*a)^7-8*cos(1/2*b*x+1/2*a)^5+5*cos(1/2*b*x+1/2*a)^3+(sin(1/2*b*x+1/2
*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a)
,2^(1/2))-cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a
)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos\left(bx+a\right)}\,\sin\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^2\,\sqrt{d\,\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(d*cos(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^2*(d*cos(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos\left(a+bx\right)}\,\sin^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(d*cos(a + b*x))*sin(a + b*x)**2, x)
```

3.199 $\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=69

$$\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}$$

```
[Out] 4/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2
*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)-2/3*sin(b*x+a)*(d*co
s(b*x+a))^(1/2)/b/d
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2568, 2642, 2641}

$$\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{d\cos(a+bx)}}-\frac{2\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]])
 - (2*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(3*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{3bd} + \frac{2}{3}\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx$$
$$= -\frac{2\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{3bd} + \frac{\left(2\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3\sqrt{d\cos(a+bx)}}$$
$$= \frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{d\cos(a+bx)}} - \frac{2\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{3bd}$$

Mathematica [C] time = 0.10, size = 58, normalized size = 0.84

$$\frac{d\sin^3(a+bx)\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3b(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/2, 5/2, Sin[a + b*x]^2]*
Sin[a + b*x]^3)/(3*b*(d*Cos[a + b*x])^(3/2))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2-1\right)}{d\cos(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 1)/(d*cos(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^2}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

maple [B]   time = 0.07, size = 188, normalized size = 2.72

$$\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}\cdots\right.}{3\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\left(\cdots\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] 4/3*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*cos(1/2*b*
x+1/2*a)*sin(1/2*b*x+1/2*a)^4-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1
/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))-sin(1/2*b*x+1/2*a)^2
*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/
2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)^2}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^2}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(d*cos(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^2/(d*cos(a + b*x))^(1/2), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.200 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}$$

```
[Out] 2*sin(b*x+a)/b/d/(d*cos(b*x+a))^(1/2)-4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/
2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/d
^2/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2566, 2640, 2639}

$$\frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*d^2*Sqrt[Cos[a + b*x
]]) + (2*Sin[a + b*x])/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx &= \frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{2\int\sqrt{d\cos(a+bx)}\,dx}{d^2}\\ &= \frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{\left(2\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,dx}{d^2\sqrt{\cos(a+bx)}}\\ &= -\frac{4\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bd^2\sqrt{\cos(a+bx)}} + \frac{2\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.08, size = 60, normalized size = 0.88

$$\frac{\sin^3(a+bx)\sqrt[4]{\cos^2(a+bx)}\;{}_2F_1\left(\frac{5}{4},\frac{3}{2};\frac{5}{2};\sin^2(a+bx)\right)}{3bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 3/2, 5/2, Sin[a + b*x]^2]*Si
n[a + b*x]^3)/(3*b*d*Sqrt[d*Cos[a + b*x]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2-1\right)}{d^2\cos(bx+a)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 1)/(d^2*cos(b*x + a)^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

maple [A] time = 0.09, size = 168, normalized size = 2.47

$$-\frac{4\sqrt{-2\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d}\ \left(\sqrt{2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ \sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\ \operatorname{EllipticE}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\right.\right.}{d\sqrt{-d\left(2\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\ \sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d\left(2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] -4/d*(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(1/2)*((2*sin(1/2*b
*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2
*a),2^(1/2))-sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/
2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/
2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^{2}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^{2}}{(d\,\cos(a+b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(d*cos(a + b*x))^(3/2),x)
```

```
[Out] int(sin(a + b*x)^2/(d*cos(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

3.201 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/3*sin(b*x+a)/b/d/(d*cos(b*x+a))^(3/2)-4/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/co
s(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/d
^2/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2566, 2642, 2641}

$$\frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b
*x]]) + (2*Sin[a + b*x])/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{3d^2}$$
$$= \frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{\left(2\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3d^2\sqrt{d\cos(a+bx)}}$$
$$= -\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}} + \frac{2\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

Mathematica [C] time = 0.08, size = 60, normalized size = 0.83

$$\frac{\sin^3(a+bx)\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{2},\frac{7}{4};\frac{5}{2};\sin^2(a+bx)\right)}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/2, 7/4, 5/2, Sin[a + b*x]^2]*Si
n[a + b*x]^3)/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2-1\right)}{d^3\cos(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 1)/(d^3*cos(b*x + a)^3), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

maple [B] time = 0.10, size = 242, normalized size = 3.36

$$-\frac{4\left(2\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\cdots}\right.}{3d^2\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\cdots\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] -4/3*(2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ellip
ticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2-(sin(1/2*b*x+1/2*a)^2
)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/
2))-sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/d^2*(d*(2*cos(1/2*b*x+1/2*a)^2
-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)/(-d*(2*sin(1/2*b
*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b
*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)^2}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^2}{\left(d\,\cos\left(a+b\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(d*cos(a + b*x))^(5/2),x)
```

```
[Out] int(sin(a + b*x)^2/(d*cos(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.202 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}}-\frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/5*sin(b*x+a)/b/d/(d*cos(b*x+a))^(5/2)-4/5*sin(b*x+a)/b/d^3/(d*cos(b*x+a))
^(1/2)+4/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/
2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/d^4/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2566, 2636, 2640, 2639}

$$-\frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}}+\frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*d^4*Sqrt[Cos[a + b*
x]]) + (2*Sin[a + b*x])/(5*b*d*(d*Cos[a + b*x])^(5/2)) - (4*Sin[a + b*x])/(
5*b*d^3*Sqrt[d*Cos[a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx &= \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{2\int \frac{1}{(d\cos(a+bx))^{3/2}}\,dx}{5d^2}\\
&= \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\int \sqrt{d\cos(a+bx)}\,dx}{5d^4}\\
&= \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{\left(2\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{5d^4\sqrt{\cos(a+bx)}}\\
&= \frac{4\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}} + \frac{2\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{4\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 59, normalized size = 0.59

$$\frac{\sin^3(2(a+bx))\sqrt[4]{\cos^2(a+bx)}\;{}_2F_1\left(\frac{3}{2},\frac{9}{4};\frac{5}{2};\sin^2(a+bx)\right)}{24b(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[3/2, 9/4, 5/2, Sin[a + b*x]^2]*Si
n[2*(a + b*x)]^3)/(24*b*(d*Cos[a + b*x])^(7/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2-1\right)}{d^4\cos(bx+a)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 1)/(d^4*cos(b*x + a)^4), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

maple [B] time = 0.13, size = 365, normalized size = 3.65

$$-\frac{4\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(4\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\ \mathrm{EllipticE}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] -4/5*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^4/sin(1/2*
b*x+1/2*a)^3/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+
1/2*a)^2-1)*(4*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2
)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^4-8*sin(1/2*b*x+
1/2*a)^6*cos(1/2*b*x+1/2*a)-4*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x
+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2
+8*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)
*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-sin(1/2
*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))*(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/
2*a)^2*d)^(1/2)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^2}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(d*cos(a + b*x))^(7/2),x)
[Out] int(sin(a + b*x)^2/(d*cos(a + b*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(7/2),x)
[Out] Timed out
```

3.203 $\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21bd^4\sqrt{d\cos(a+bx)}}-\frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}}+\frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

```
[Out] 2/7*sin(b*x+a)/b/d/(d*cos(b*x+a))^(7/2)-4/21*sin(b*x+a)/b/d^3/(d*cos(b*x+a)
)^(3/2)-4/21*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(
1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/d^4/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2566, 2636, 2642, 2641}

$$-\frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}}-\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21bd^4\sqrt{d\cos(a+bx)}}+\frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(21*b*d^4*Sqrt[d*Cos[a +
b*x]]) + (2*Sin[a + b*x])/(7*b*d*(d*Cos[a + b*x])^(7/2)) - (4*Sin[a + b*x])
/(21*b*d^3*(d*Cos[a + b*x])^(3/2))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx &= \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{2\int \frac{1}{(d\cos(a+bx))^{5/2}}\,dx}{7d^2}\\ &= \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{21d^4}\\ &= \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}} - \frac{\left(2\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{21d^4\sqrt{d\cos(a+bx)}}\\ &= -\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21bd^4\sqrt{d\cos(a+bx)}} + \frac{2\sin(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{4\sin(a+bx)}{21bd^3(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.06, size = 59, normalized size = 0.59

$$\frac{\sin^3(2(a+bx))\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{2},\frac{11}{4};\frac{5}{2};\sin^2(a+bx)\right)}{24b(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/2, 11/4, 5/2, Sin[a + b*x]^2]*S
in[2*(a + b*x)]^3)/(24*b*(d*Cos[a + b*x])^(9/2))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2-1\right)}{d^5\cos(bx+a)^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 1)/(d^5*cos(b*x + a)^5), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(9/2), x)
```

maple [B] time = 0.10, size = 396, normalized size = 3.96

$$4\left(-8\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+12\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 4/21*(-8*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Elli
pticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^6+12*(sin(1/2*b*x+1/2*
a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2
^(1/2))*sin(1/2*b*x+1/2*a)^4-8*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)-6*(s
in(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1
/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+8*cos(1/2*b*x+1/2*a)*sin(1/2*b*
x+1/2*a)^4+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*El
lipticF(cos(1/2*b*x+1/2*a),2^(1/2))+sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)
)/d^4*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-d*(2*sin(
1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^3/
sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^2}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/(d*cos(b*x + a))^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^2}{(d\,\cos(a+b\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(d*cos(a + b*x))^(9/2),x)
```

```
[Out] int(sin(a + b*x)^2/(d*cos(a + b*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

3.204 $\int \sqrt{d\cos(a+bx)}\,\sin^3(a+bx)\,dx$

Optimal. Leaf size=45

$$\frac{2(d\cos(a+bx))^{7/2}}{7bd^3}-\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

```
[Out] -2/3*(d*cos(b*x+a))^(3/2)/b/d+2/7*(d*cos(b*x+a))^(7/2)/b/d^3
```

Rubi [A] time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2(d\cos(a+bx))^{7/2}}{7bd^3}-\frac{2(d\cos(a+bx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^3,x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(3/2))/(3*b*d) + (2*(d*Cos[a + b*x])^(7/2))/(7*b*d^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\cos(a+bx)}\,\sin^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \sqrt{x}\left(1-\frac{x^2}{d^2}\right)dx,x,d\cos(a+bx)\right)}{bd}\\ &= -\frac{\text{Subst}\left(\int \left(\sqrt{x}-\frac{x^{5/2}}{d^2}\right)dx,x,d\cos(a+bx)\right)}{bd}\\ &= -\frac{2(d\cos(a+bx))^{3/2}}{3bd}+\frac{2(d\cos(a+bx))^{7/2}}{7bd^3}\end{aligned}$$

Mathematica [A] time = 0.31, size = 57, normalized size = 1.27

$$-\frac{d\left(3\sin^2(2(a+bx))+16\cos^2(a+bx)-16\sqrt[4]{\cos^2(a+bx)}\right)}{42b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^3,x]
```

```
[Out] -1/42*(d*(16*Cos[a + b*x]^2 - 16*(Cos[a + b*x]^2)^(1/4) + 3*Sin[2*(a + b*x)
]^2))/(b*Sqrt[d*Cos[a + b*x]])
```

fricas [A] time = 0.43, size = 34, normalized size = 0.76

$$\frac{2\left(3\cos(bx+a)^3-7\cos(bx+a)\right)\sqrt{d\cos(bx+a)}}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 2/21*(3*cos(b*x + a)^3 - 7*cos(b*x + a))*sqrt(d*cos(b*x + a))/b
```

giac [A] time = 21.88, size = 37, normalized size = 0.82

$$\frac{2\,(d\cos(bx+a))^{\frac{7}{2}}}{7\,bd^3}-\frac{2\,(d\cos(bx+a))^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2/7*(d*cos(b*x + a))^(7/2)/(b*d^3) - 2/3*(d*cos(b*x + a))^(3/2)/(b*d)
```

maple [A] time = 0.06, size = 63, normalized size = 1.40

$$-\frac{8\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}\,\left(6\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-9\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{21b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x)
```

```
[Out] -8/21*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(6*sin(1/2*b*x+1/2*a)^6-9*sin(1/2
*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2+1)/b
```

maxima [A] time = 0.31, size = 36, normalized size = 0.80

$$\frac{2\left(3\,\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}-7\,\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}d^{2}\right)}{21\,bd^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/21*(3*(d*cos(b*x + a))^(7/2) - 7*(d*cos(b*x + a))^(3/2)*d^2)/(b*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(a+b\,x\right)^3\,\sqrt{d\,\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(d*cos(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^3*(d*cos(a + b*x))^(1/2), x)
```

sympy [A] time = 12.46, size = 65, normalized size = 1.44

$$\begin{cases} -\frac{2\sqrt{d}\sin^{2}\left(a+bx\right)\cos^{\frac{3}{2}}\left(a+bx\right)}{3b}-\frac{8\sqrt{d}\cos^{\frac{7}{2}}\left(a+bx\right)}{21b} & \text{for } b\neq 0 \\ x\sqrt{d\cos\left(a\right)}\sin^{3}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-2*sqrt(d)*sin(a + b*x)**2*cos(a + b*x)**(3/2)/(3*b) - 8*sqrt(d)
*cos(a + b*x)**(7/2)/(21*b), Ne(b, 0)), (x*sqrt(d*cos(a))*sin(a)**3, True))
```

3.205 $\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=43

$$\frac{2(d\cos(a+bx))^{5/2}}{5bd^3}-\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

```
[Out] 2/5*(d*cos(b*x+a))^(5/2)/b/d^3-2*(d*cos(b*x+a))^(1/2)/b/d
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2(d\cos(a+bx))^{5/2}}{5bd^3}-\frac{2\sqrt{d\cos(a+bx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-2*Sqrt[d*Cos[a + b*x]])/(b*d) + (2*(d*Cos[a + b*x])^(5/2))/(5*b*d^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{\sqrt{x}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{\sqrt{x}}-\frac{x^{3/2}}{d^2}\right)dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{2\sqrt{d\cos(a+bx)}}{bd}+\frac{2(d\cos(a+bx))^{5/2}}{5bd^3}$$

Mathematica [A] time = 0.18, size = 57, normalized size = 1.33

$$\frac{\cos(a+bx)(\cos(2(a+bx))-9)+8\cos^2(a+bx)^{3/4}\sec(a+bx)}{5b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
[Out] (Cos[a + b*x]*(-9 + Cos[2*(a + b*x)]) + 8*(Cos[a + b*x]^2)^(3/4)*Sec[a + b*
x])/(5*b*Sqrt[d*Cos[a + b*x]])
```

fricas [A] time = 0.44, size = 28, normalized size = 0.65

$$\frac{2\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2-5\right)}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2/5*sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 - 5)/(b*d)
```

giac [A] time = 1.03, size = 40, normalized size = 0.93

$$\frac{2\left(\sqrt{d\cos(bx+a)}\cos(bx+a)^2-5\sqrt{d\cos(bx+a)}\right)}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] 2/5*(sqrt(d*cos(b*x + a))*cos(b*x + a)^2 - 5*sqrt(d*cos(b*x + a)))/(b*d)
```

maple [B] time = 0.05, size = 92, normalized size = 2.14

$$\frac{8\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}\,\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}\,\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\right.\right.}}{5db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] 1/5*(8*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^4-8*(-2*sin(1
/2*b*x+1/2*a)^2*d+d)^(1/2)*sin(1/2*b*x+1/2*a)^2-8*(-2*sin(1/2*b*x+1/2*a)^2*
d+d)^(1/2))/d/b
```

maxima [A] time = 0.41, size = 36, normalized size = 0.84

$$-\frac{2\left(5\sqrt{d\cos\left(bx+a\right)}-\frac{\left(d\cos(bx+a)\right)^{\frac{5}{2}}}{d^2}\right)}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*sqrt(d*cos(b*x + a)) - (d*cos(b*x + a))^(5/2)/d^2)/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(a+b\,x\right)^3}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(d*cos(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^3/(d*cos(a + b*x))^(1/2), x)
```

sympy [A] time = 6.17, size = 63, normalized size = 1.47

$$\begin{cases} -\frac{2\sin^2\left(a+bx\right)\sqrt{\cos\left(a+bx\right)}}{b\sqrt{d}}-\frac{8\cos^{\frac{5}{2}}\left(a+bx\right)}{5b\sqrt{d}} & \text{for } b\neq 0 \\ \frac{x\sin^3\left(a\right)}{\sqrt{d\cos\left(a\right)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((-2*sin(a + b*x)**2*sqrt(cos(a + b*x))/(b*sqrt(d)) - 8*cos(a + b*
x)**(5/2)/(5*b*sqrt(d)), Ne(b, 0)), (x*sin(a)**3/sqrt(d*cos(a)), True))
```

3.206 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2(d\cos(a+bx))^{3/2}}{3bd^3}+\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/3*(d*cos(b*x+a))^(3/2)/b/d^3+2/b/d/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2(d\cos(a+bx))^{3/2}}{3bd^3}+\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] 2/(b*d*Sqrt[d*Cos[a + b*x]]) + (2*(d*Cos[a + b*x])^(3/2))/(3*b*d^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{3/2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{3/2}}-\frac{\sqrt{x}}{d^2}\right)dx, x, d\cos(a+bx)\right)}{bd}$$
$$= \frac{2}{bd\sqrt{d\cos(a+bx)}}+\frac{2(d\cos(a+bx))^{3/2}}{3bd^3}$$

Mathematica [A] time = 0.07, size = 46, normalized size = 1.07

$$-\frac{2\left(\sin^2(a+bx)+4\sqrt[4]{\cos^2(a+bx)}-4\right)}{3bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-2*(-4 + 4*(Cos[a + b*x]^2)^(1/4) + Sin[a + b*x]^2))/(3*b*d*Sqrt[d*Cos[a +
 b*x]])
```

fricas [A] time = 0.42, size = 36, normalized size = 0.84

$$\frac{2\sqrt{d\cos(bx+a)}\left(\cos(bx+a)^2+3\right)}{3\,bd^2\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(d*cos(b*x + a))*(cos(b*x + a)^2 + 3)/(b*d^2*cos(b*x + a))
```

giac [A] time = 1.32, size = 41, normalized size = 0.95

$$\frac{2\left(\frac{\sqrt{d\cos(bx+a)}\,\cos(bx+a)}{d}+\frac{3}{\sqrt{d\cos(bx+a)}}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(d*cos(b*x + a))*cos(b*x + a)/d + 3/sqrt(d*cos(b*x + a)))/(b*d)
```

maple [A] time = 0.12, size = 70, normalized size = 1.63

$$-\frac{8\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}\ \left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)}{3d^2\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] -8/3/d^2*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(sin(1/2*b*x+1/2*a)^4-sin(1/2*
b*x+1/2*a)^2+1)/(2*sin(1/2*b*x+1/2*a)^2-1)/b
```

maxima [A] time = 0.38, size = 35, normalized size = 0.81

$$\frac{2\left(\frac{3}{\sqrt{d\cos(bx+a)}}+\frac{(d\cos(bx+a))^{\frac{3}{2}}}{d^2}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3/sqrt(d*cos(b*x + a)) + (d*cos(b*x + a))^(3/2)/d^2)/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(a+b\,x)^3}{(d\ \cos(a+b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(d*cos(a + b*x))^(3/2),x)
```

```
[Out] int(sin(a + b*x)^3/(d*cos(a + b*x))^(3/2), x)
```

sympy [A] time = 6.51, size = 61, normalized size = 1.42

$$\begin{cases} \frac{2\sin^2(a+bx)}{bd^{\frac{3}{2}}\sqrt{\cos(a+bx)}}+\frac{8\cos^{\frac{3}{2}}(a+bx)}{3bd^{\frac{3}{2}}} & \text{for } b\neq 0 \\ \frac{x\sin^3(a)}{(d\cos(a))^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((2*sin(a + b*x)**2/(b*d**(3/2)*sqrt(cos(a + b*x))) + 8*cos(a + b*
x)**(3/2)/(3*b*d**(3/2)), Ne(b, 0)), (x*sin(a)**3/(d*cos(a))**(3/2), True))
```

3.207 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{d\cos(a+bx)}}{bd^3}+\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/3/b/d/(d*cos(b*x+a))^(3/2)+2*(d*cos(b*x+a))^(1/2)/b/d^3
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2\sqrt{d\cos(a+bx)}}{bd^3}+\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] 2/(3*b*d*(d*Cos[a + b*x])^(3/2)) + (2*Sqrt[d*Cos[a + b*x]])/(b*d^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{5/2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{5/2}} - \frac{1}{d^2\sqrt{x}}\right)dx, x, d\cos(a+bx)\right)}{bd}$$
$$= \frac{2}{3bd(d\cos(a+bx))^{3/2}} + \frac{2\sqrt{d\cos(a+bx)}}{bd^3}$$

Mathematica [A] time = 0.09, size = 48, normalized size = 1.12

$$-\frac{2\left(3\sin^2(a+bx)+4\cos^2(a+bx)^{3/4}-4\right)}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (-2*(-4 + 4*(Cos[a + b*x]^2)^(3/4) + 3*Sin[a + b*x]^2))/(3*b*d*(d*Cos[a + b
*x])^(3/2))
```

fricas [A] time = 0.44, size = 38, normalized size = 0.88

$$\frac{2\sqrt{d\cos(bx+a)}\left(3\cos(bx+a)^2+1\right)}{3\,bd^3\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(d*cos(b*x + a))*(3*cos(b*x + a)^2 + 1)/(b*d^3*cos(b*x + a)^2)
```

giac [A] time = 1.30, size = 47, normalized size = 1.09

$$\frac{2\left(3\sqrt{d\cos(bx+a)}\,b^2+\frac{b^2d}{\sqrt{d\cos(bx+a)}\cos(bx+a)}\right)}{3\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(d*cos(b*x + a))*b^2 + b^2*d/(sqrt(d*cos(b*x + a))*cos(b*x + a))
)/(b^3*d^3)
```

maple [B] time = 0.15, size = 85, normalized size = 1.98

$$\frac{8\sqrt{-2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}\,\left(3\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-3\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)}{3d^{3}\left(4\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-4\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 8/3/d^3/(4*sin(1/2*b*x+1/2*a)^4-4*sin(1/2*b*x+1/2*a)^2+1)*(-2*sin(1/2*b*x+1
/2*a)^2*d+d)^(1/2)*(3*sin(1/2*b*x+1/2*a)^4-3*sin(1/2*b*x+1/2*a)^2+1)/b
```

maxima [A] time = 0.41, size = 34, normalized size = 0.79

$$\frac{2\left(\frac{1}{(d\cos(bx+a))^{\frac{3}{2}}}+\frac{3\sqrt{d\cos(bx+a)}}{d^{2}}\right)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(1/(d*cos(b*x + a))^(3/2) + 3*sqrt(d*cos(b*x + a))/d^2)/(b*d)
```

mupad [B] time = 1.06, size = 66, normalized size = 1.53

$$\frac{2\sqrt{d\,\cos\left(a+b\,x\right)}\,\left(16\,\cos\left(2\,a+2\,b\,x\right)+3\,\cos\left(4\,a+4\,b\,x\right)+13\right)}{3\,b\,d^{3}\,\left(4\,\cos\left(2\,a+2\,b\,x\right)+\cos\left(4\,a+4\,b\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(d*cos(a + b*x))^(5/2),x)
```

```
[Out] (2*(d*cos(a + b*x))^(1/2)*(16*cos(2*a + 2*b*x) + 3*cos(4*a + 4*b*x) + 13))/
(3*b*d^3*(4*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 3))
```

sympy [A] time = 55.34, size = 63, normalized size = 1.47

$$\begin{cases} \frac{2\sin^{2}{(a+bx)}}{3bd^{\frac{5}{2}}\cos^{\frac{3}{2}}{(a+bx)}}+\frac{8\sqrt{\cos{(a+bx)}}}{3bd^{\frac{5}{2}}} & \text{for}\: b \neq 0 \\ \frac{x\sin^{3}{(a)}}{(d\cos{(a)})^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((2*sin(a + b*x)**2/(3*b*d**(5/2)*cos(a + b*x)**(3/2)) + 8*sqrt(co
s(a + b*x))/(3*b*d**(5/2)), Ne(b, 0)), (x*sin(a)**3/(d*cos(a))**(5/2), True
))
```

3.208 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2}{5bd(d\cos(a+bx))^{5/2}}-\frac{2}{bd^3\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/5/b/d/(d*cos(b*x+a))^(5/2)-2/b/d^3/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2}{5bd(d\cos(a+bx))^{5/2}}-\frac{2}{bd^3\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] 2/(5*b*d*(d*Cos[a + b*x])^(5/2)) - 2/(b*d^3*Sqrt[d*Cos[a + b*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{7/2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{7/2}} - \frac{1}{d^2x^{3/2}}\right)dx, x, d\cos(a+bx)\right)}{bd}$$
$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} - \frac{2}{bd^3\sqrt{d\cos(a+bx)}}$$

Mathematica [A] time = 0.25, size = 70, normalized size = 1.63

$$\frac{2\tan^2(a+bx)\left(-4\sqrt[4]{\cos^2(a+bx)} + 4\left(\sqrt[4]{\cos^2(a+bx)} - 1\right)\csc^2(a+bx) + 5\right)}{5bd^3\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*(5 - 4*(Cos[a + b*x]^2)^(1/4) + 4*(-1 + (Cos[a + b*x]^2)^(1/4))*Csc[a +
b*x]^2)*Tan[a + b*x]^2)/(5*b*d^3*Sqrt[d*Cos[a + b*x]])
```

fricas [A] time = 0.43, size = 38, normalized size = 0.88

$$-\frac{2\sqrt{d\cos(bx+a)}\left(5\cos(bx+a)^2 - 1\right)}{5\,bd^4\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 - 1)/(b*d^4*cos(b*x + a)^3)
```

giac [A] time = 1.36, size = 51, normalized size = 1.19

$$-\frac{2\left(5\,b^3d^3\cos(bx+a)^2 - b^3d^3\right)}{5\sqrt{d\cos(bx+a)}\,b^4d^6\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] -2/5*(5*b^3*d^3*cos(b*x + a)^2 - b^3*d^3)/(sqrt(d*cos(b*x + a))*b^4*d^6*cos
(b*x + a)^2)
```

maple [B] time = 0.24, size = 98, normalized size = 2.28

$$\frac{8\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}\,\left(5\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-5\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)}{5d^4\left(8\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-12\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+6\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] 8/5/d^4/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a
)^2-1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(5*sin(1/2*b*x+1/2*a)^4-5*sin(1/
2*b*x+1/2*a)^2+1)/b
```

maxima [A] time = 0.46, size = 37, normalized size = 0.86

$$-\frac{2\left(5\,d^2\cos\left(bx+a\right)^2-d^2\right)}{5\,\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*d^2*cos(b*x + a)^2 - d^2)/((d*cos(b*x + a))^(5/2)*b*d^3)
```

mupad [B] time = 3.67, size = 93, normalized size = 2.16

$$-\frac{4\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{d\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}\,\left(6\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+5\,\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+5\right)}{5\,b\,d^4\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(d*cos(a + b*x))^(7/2),x)
```

```
[Out] -(4*exp(a*1i + b*x*1i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^
(1/2)*(6*exp(a*2i + b*x*2i) + 5*exp(a*4i + b*x*4i) + 5))/(5*b*d^4*(exp(a*2i
 + b*x*2i) + 1)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

3.209 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=45

$$\frac{2}{7bd(d\cos(a+bx))^{7/2}} - \frac{2}{3bd^3(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/7/b/d/(d*cos(b*x+a))^(7/2)-2/3/b/d^3/(d*cos(b*x+a))^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2}{7bd(d\cos(a+bx))^{7/2}} - \frac{2}{3bd^3(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] 2/(7*b*d*(d*Cos[a + b*x])^(7/2)) - 2/(3*b*d^3*(d*Cos[a + b*x])^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{9/2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{9/2}} - \frac{1}{d^2x^{5/2}}\right)dx, x, d\cos(a+bx)\right)}{bd}$$
$$= \frac{2}{7bd(d\cos(a+bx))^{7/2}} - \frac{2}{3bd^3(d\cos(a+bx))^{3/2}}$$

Mathematica [A] time = 0.27, size = 70, normalized size = 1.56

$$\frac{2\tan^2(a+bx)\left(-4\cos^2(a+bx)^{3/4} + 4\left(\cos^2(a+bx)^{3/4} - 1\right)\csc^2(a+bx) + 7\right)}{21bd^3(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(9/2),x]
[Out] (2*(7 - 4*(Cos[a + b*x]^2)^(3/4) + 4*(-1 + (Cos[a + b*x]^2)^(3/4))*Csc[a +
b*x]^2)*Tan[a + b*x]^2)/(21*b*d^3*(d*Cos[a + b*x])^(3/2))
```

fricas [A] time = 0.41, size = 38, normalized size = 0.84

$$-\frac{2\sqrt{d\cos(bx+a)}\left(7\cos(bx+a)^2 - 3\right)}{21\,bd^5\cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
[Out] -2/21*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 - 3)/(b*d^5*cos(b*x + a)^4)
```

giac [A] time = 1.58, size = 51, normalized size = 1.13

$$-\frac{2\left(7\,b^4d^4\cos(bx+a)^2 - 3\,b^4d^4\right)}{21\sqrt{d\cos(bx+a)}\,b^5d^8\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] -2/21*(7*b^4*d^4*cos(b*x + a)^2 - 3*b^4*d^4)/(sqrt(d*cos(b*x + a))*b^5*d^8*
cos(b*x + a)^3)
```

maple [B] time = 0.25, size = 111, normalized size = 2.47

$$-\frac{8\sqrt{-2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}\ \left(7\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-7\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)}{21d^{5}\left(16\left(\sin^{8}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-32\left(\sin^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+24\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-8\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] -8/21/d^5/(16*sin(1/2*b*x+1/2*a)^8-32*sin(1/2*b*x+1/2*a)^6+24*sin(1/2*b*x+1
/2*a)^4-8*sin(1/2*b*x+1/2*a)^2+1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*(7*si
n(1/2*b*x+1/2*a)^4-7*sin(1/2*b*x+1/2*a)^2+1)/b
```

maxima [A] time = 0.30, size = 37, normalized size = 0.82

$$-\frac{2\left(7\,d^{2}\cos\left(bx+a\right)^{2}-3\,d^{2}\right)}{21\,\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}\,bd^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] -2/21*(7*d^2*cos(b*x + a)^2 - 3*d^2)/((d*cos(b*x + a))^(7/2)*b*d^3)
```

mupad [B] time = 3.78, size = 93, normalized size = 2.07

$$-\frac{8\,{\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,\sqrt{d\,\left(\frac{{\mathrm{e}}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}\,\left(2\,{\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+7\,{\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+7\right)}{21\,b\,d^{5}\,{\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(d*cos(a + b*x))^(9/2),x)
```

```
[Out] -(8*exp(a*2i + b*x*2i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^
(1/2)*(2*exp(a*2i + b*x*2i) + 7*exp(a*4i + b*x*4i) + 7))/(21*b*d^5*(exp(a*2
i + b*x*2i) + 1)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

3.210 $\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx$

Optimal. Leaf size=45

$$\frac{2}{9bd(d\cos(a+bx))^{9/2}}-\frac{2}{5bd^3(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/9/b/d/(d*cos(b*x+a))^(9/2)-2/5/b/d^3/(d*cos(b*x+a))^(5/2)
```

Rubi [A] time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2565, 14}

$$\frac{2}{9bd(d\cos(a+bx))^{9/2}}-\frac{2}{5bd^3(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] 2/(9*b*d*(d*Cos[a + b*x])^(9/2)) - 2/(5*b*d^3*(d*Cos[a + b*x])^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{(d\cos(a+bx))^{11/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{d^2}}{x^{11/2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{x^{11/2}} - \frac{1}{d^2x^{7/2}}\right)dx, x, d\cos(a+bx)\right)}{bd}$$
$$= \frac{2}{9bd(d\cos(a+bx))^{9/2}} - \frac{2}{5bd^3(d\cos(a+bx))^{5/2}}$$

Mathematica [B] time = 0.53, size = 94, normalized size = 2.09

$$\frac{2\tan^4(a+bx)\left(4\sqrt[4]{\cos^2(a+bx)}+4\left(\sqrt[4]{\cos^2(a+bx)}-1\right)\csc^4(a+bx)+\left(9-8\sqrt[4]{\cos^2(a+bx)}\right)\csc^2(a+bx)\right)}{45bd^5\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*(4*(Cos[a + b*x]^2)^(1/4) + (9 - 8*(Cos[a + b*x]^2)^(1/4))*Csc[a + b*x]^
2 + 4*(-1 + (Cos[a + b*x]^2)^(1/4))*Csc[a + b*x]^4)*Tan[a + b*x]^4)/(45*b*d
^5*Sqrt[d*Cos[a + b*x]])
```

fricas [A] time = 0.43, size = 38, normalized size = 0.84

$$-\frac{2\sqrt{d\cos(bx+a)}\left(9\cos(bx+a)^2-5\right)}{45\,bd^6\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="fricas")
```

```
[Out] -2/45*sqrt(d*cos(b*x + a))*(9*cos(b*x + a)^2 - 5)/(b*d^6*cos(b*x + a)^5)
```

giac [A] time = 1.40, size = 51, normalized size = 1.13

$$-\frac{2\left(9\,b^5d^5\cos(bx+a)^2-5\,b^5d^5\right)}{45\sqrt{d\cos(bx+a)}\,b^6d^{10}\cos(bx+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
```

```
[Out] -2/45*(9*b^5*d^5*cos(b*x + a)^2 - 5*b^5*d^5)/(sqrt(d*cos(b*x + a))*b^6*d^10
*cos(b*x + a)^4)
```

maple [B] time = 0.29, size = 124, normalized size = 2.76

$$\frac{8\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}\,\left(9\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-9\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)}{45d^6\left(32\left(\sin^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-80\left(\sin^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+80\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-40\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+10\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x)
```

```
[Out] 8/45/d^6/(32*sin(1/2*b*x+1/2*a)^10-80*sin(1/2*b*x+1/2*a)^8+80*sin(1/2*b*x+1
/2*a)^6-40*sin(1/2*b*x+1/2*a)^4+10*sin(1/2*b*x+1/2*a)^2-1)*(-2*sin(1/2*b*x+
1/2*a)^2*d+d)^(1/2)*(9*sin(1/2*b*x+1/2*a)^4-9*sin(1/2*b*x+1/2*a)^2+1)/b
```

maxima [A] time = 0.54, size = 37, normalized size = 0.82

$$-\frac{2\left(9\,d^2\cos\left(bx+a\right)^2-5\,d^2\right)}{45\,\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}\,bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/(d*cos(b*x+a))^(11/2),x, algorithm="maxima")
```

```
[Out] -2/45*(9*d^2*cos(b*x + a)^2 - 5*d^2)/((d*cos(b*x + a))^(9/2)*b*d^3)
```

mupad [B] time = 5.33, size = 279, normalized size = 6.20

$$\frac{16\,e^{a\,1i+b\,x\,1i}\sqrt{d\,\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}}{5\,b\,d^6\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^2}-\frac{e^{a\,1i+b\,x\,1i}\sqrt{d\,\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}\,464i}{45\,b\,d^6\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)^3}-\frac{128\,e^{a\,1i+b\,x\,1i}\sqrt{d\,\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\right.}}{9\,b\,d^6\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(d*cos(a + b*x))^(11/2),x)
```

```
[Out] (16*exp(a*1i + b*x*1i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^
(1/2))/(5*b*d^6*(exp(a*2i + b*x*2i)*1i + 1i)^2) - (exp(a*1i + b*x*1i)*(d*(e
xp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))^(1/2)*464i)/(45*b*d^6*(exp(a
*2i + b*x*2i)*1i + 1i)^3) - (128*exp(a*1i + b*x*1i)*(d*(exp(- a*1i - b*x*1i
)/2 + exp(a*1i + b*x*1i)/2))^(1/2))/(9*b*d^6*(exp(a*2i + b*x*2i)*1i + 1i)^4
) + (exp(a*1i + b*x*1i)*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)/2))
^(1/2)*64i)/(9*b*d^6*(exp(a*2i + b*x*2i)*1i + 1i)^5)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/(d*cos(b*x+a))**(11/2),x)
```

```
[Out] Timed out
```

3.211 $\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx$

Optimal. Leaf size=156

$$\frac{56d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{1105b\sqrt{\cos(a+bx)}}+\frac{56d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{3315b}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}-\frac{12\text{s}}{}$$

```
[Out] 56/3315*d^3*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b+8/663*d*(d*cos(b*x+a))^(7/2)*
sin(b*x+a)/b-12/221*(d*cos(b*x+a))^(11/2)*sin(b*x+a)/b/d-2/17*(d*cos(b*x+a)
)^(11/2)*sin(b*x+a)^3/b/d+56/1105*d^4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*
a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/cos
(b*x+a)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2568, 2635, 2640, 2639}

$$\frac{56d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{3315b}+\frac{56d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{1105b\sqrt{\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{11/2}}{17bd}-\frac{12\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^4,x]
```

```
[Out] (56*d^4*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(1105*b*Sqrt[Cos[a
+ b*x]]) + (56*d^3*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(3315*b) + (8*d*(d*
Cos[a + b*x])^(7/2)*Sin[a + b*x])/(663*b) - (12*(d*Cos[a + b*x])^(11/2)*Sin
[a + b*x])/(221*b*d) - (2*(d*Cos[a + b*x])^(11/2)*Sin[a + b*x]^3)/(17*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{9/2}\sin^4(a+bx)\,dx &= -\frac{2(d\cos(a+bx))^{11/2}\sin^3(a+bx)}{17bd}+\frac{6}{17}\int (d\cos(a+bx))^{9/2}\sin^2(a+bx)\\ &= -\frac{12(d\cos(a+bx))^{11/2}\sin(a+bx)}{221bd}-\frac{2(d\cos(a+bx))^{11/2}\sin^3(a+bx)}{17bd}+\\ &= \frac{8d(d\cos(a+bx))^{7/2}\sin(a+bx)}{663b}-\frac{12(d\cos(a+bx))^{11/2}\sin(a+bx)}{221bd}-\frac{2(\\ &= \frac{56d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{3315b}+\frac{8d(d\cos(a+bx))^{7/2}\sin(a+bx)}{663b}-\\ &= \frac{56d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{3315b}+\frac{8d(d\cos(a+bx))^{7/2}\sin(a+bx)}{663b}-\\ &= \frac{56d^4\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{1105b\sqrt{\cos(a+bx)}}+\frac{56d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{3315b}\end{aligned}$$

Mathematica [C] time = 0.13, size = 57, normalized size = 0.37

$$\frac{\sqrt[4]{\cos^2(a+bx)}\,\tan^5(a+bx)(d\cos(a+bx))^{9/2}\,{}_2F_1\left(-\frac{7}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^4,x]
```

```
[Out] ((d*Cos[a + b*x])^(9/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4, 5/2,
 7/2, Sin[a + b*x]^2]*Tan[a + b*x]^5)/(5*b)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^4\cos\left(bx+a\right)^8-2\,d^4\cos\left(bx+a\right)^6+d^4\cos\left(bx+a\right)^4\right)\sqrt{d\cos\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="fricas")

[Out] integral((d^4*cos(b*x + a)^8 - 2*d^4*cos(b*x + a)^6 + d^4*cos(b*x + a)^4)*sqrt(d*cos(b*x + a)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{9}{2}} \sin(bx+a)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="giac")

[Out] integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^4, x)

maple [A] time = 0.12, size = 275, normalized size = 1.76

$$-\frac{8\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d^5\left(24960\left(\cos^{19}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-124800\left(\cos^{17}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+265440\left(\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x)

[Out] -8/3315*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^5*(24960*cos(1/2*b*x+1/2*a)^19-124800*cos(1/2*b*x+1/2*a)^17+265440*cos(1/2*b*x+1/2*a)^15-312960*cos(1/2*b*x+1/2*a)^13+222520*cos(1/2*b*x+1/2*a)^11-96360*cos(1/2*b*x+1/2*a)^9+23866*cos(1/2*b*x+1/2*a)^7-2652*cos(1/2*b*x+1/2*a)^5-35*cos(1/2*b*x+1/2*a)^3-21*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))+21*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{9}{2}} \sin(bx+a)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*cos(b*x+a))^(9/2)*sin(b*x+a)^4,x, algorithm="maxima")

[Out] integrate((d*cos(b*x + a))^(9/2)*sin(b*x + a)^4, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+bx)^4 (d\cos(a+bx))^{9/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4*(d*cos(a + b*x))^(9/2),x)
[Out] int(sin(a + b*x)^4*(d*cos(a + b*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*sin(b*x+a)**4,x)
[Out] Timed out
```

3.212 $\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx$

Optimal. Leaf size=156

$$\frac{8d^4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{231b\sqrt{d\cos(a+bx)}}+\frac{8d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{231b}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}-\frac{4\sin(a+b}{}$$

```
[Out] 8/385*d*(d*cos(b*x+a))^(5/2)*sin(b*x+a)/b-4/55*(d*cos(b*x+a))^(9/2)*sin(b*x
+a)/b/d-2/15*(d*cos(b*x+a))^(9/2)*sin(b*x+a)^3/b/d+8/231*d^4*(cos(1/2*a+1/2
*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos
(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)+8/231*d^3*sin(b*x+a)*(d*cos(b*x+a))^(1
/2)/b
```

Rubi [A] time = 0.15, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2568, 2635, 2642, 2641}

$$\frac{8d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{231b}+\frac{8d^4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{231b\sqrt{d\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{9/2}}{15bd}-\frac{4\sin(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^4,x]
```

```
[Out] (8*d^4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(231*b*Sqrt[d*Cos[a +
b*x]]) + (8*d^3*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(231*b) + (8*d*(d*Cos[a
+ b*x])^(5/2)*Sin[a + b*x])/(385*b) - (4*(d*Cos[a + b*x])^(9/2)*Sin[a + b*x
])/(55*b*d) - (2*(d*Cos[a + b*x])^(9/2)*Sin[a + b*x]^3)/(15*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{7/2}\sin^4(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{9/2}\sin^3(a+bx)}{15bd}+\frac{2}{5}\int (d\cos(a+bx))^{7/2}\sin^2(a+bx)\,dx$$

$$= -\frac{4(d\cos(a+bx))^{9/2}\sin(a+bx)}{55bd}-\frac{2(d\cos(a+bx))^{9/2}\sin^3(a+bx)}{15bd}+\frac{4}{55}$$

$$= \frac{8d(d\cos(a+bx))^{5/2}\sin(a+bx)}{385b}-\frac{4(d\cos(a+bx))^{9/2}\sin(a+bx)}{55bd}-\frac{2(d\,c}{}$$

$$= \frac{8d^3\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{231b}+\frac{8d(d\cos(a+bx))^{5/2}\sin(a+bx)}{385b}-\frac{4(d}{}$$

$$= \frac{8d^3\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{231b}+\frac{8d(d\cos(a+bx))^{5/2}\sin(a+bx)}{385b}-\frac{4(d}{}$$

$$= \frac{8d^4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{231b\sqrt{d\cos(a+bx)}}+\frac{8d^3\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{231b}+\frac{8d}{}$$

Mathematica [C] time = 0.09, size = 57, normalized size = 0.37

$$\frac{\cos^2(a+bx)^{3/4}\tan^5(a+bx)(d\cos(a+bx))^{7/2}\,{}_2F_1\left(-\frac{5}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^4,x]
```

```
[Out] ((d*Cos[a + b*x])^(7/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, 5/2,
 7/2, Sin[a + b*x]^2]*Tan[a + b*x]^5)/(5*b)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^3\cos(bx+a)^7-2\,d^3\cos(bx+a)^5+d^3\cos(bx+a)^3\right)\sqrt{d\cos(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((d^3*cos(b*x + a)^7 - 2*d^3*cos(b*x + a)^5 + d^3*cos(b*x + a)^3)*s
qrt(d*cos(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^4, x)
```

maple [A] time = 0.11, size = 262, normalized size = 1.68

$$-\frac{8\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^4\left(4928\left(\cos^{17}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-22176\left(\cos^{15}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+41216\left(\cos^1\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x)
```

```
[Out] -8/1155*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^4*(4928
*cos(1/2*b*x+1/2*a)^17-22176*cos(1/2*b*x+1/2*a)^15+41216*cos(1/2*b*x+1/2*a)
^13-40768*cos(1/2*b*x+1/2*a)^11+22868*cos(1/2*b*x+1/2*a)^9-6994*cos(1/2*b*x
+1/2*a)^7+926*cos(1/2*b*x+1/2*a)^5+5*cos(1/2*b*x+1/2*a)^3+5*(sin(1/2*b*x+1/
2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a
),2^(1/2))-5*cos(1/2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/
2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{7}{2}}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*sin(b*x + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+bx)^4 (d\cos(a+bx))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4*(d*cos(a + b*x))^(7/2),x)
[Out] int(sin(a + b*x)^4*(d*cos(a + b*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(7/2)*sin(b*x+a)**4,x)
[Out] Timed out
```

3.213 $\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx$

Optimal. Leaf size=128

$$\frac{8d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{65b\sqrt{\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}-\frac{4\sin(a+bx)(d\cos(a+bx))^{7/2}}{39bd}+\frac{8d\sin(a-$$

```
[Out] 8/195*d*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b-4/39*(d*cos(b*x+a))^(7/2)*sin(b*x
+a)/b/d-2/13*(d*cos(b*x+a))^(7/2)*sin(b*x+a)^3/b/d+8/65*d^2*(cos(1/2*a+1/2*
b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*c
os(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2568, 2635, 2640, 2639}

$$\frac{8d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{65b\sqrt{\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{7/2}}{13bd}-\frac{4\sin(a+bx)(d\cos(a+bx))^{7/2}}{39bd}+\frac{8d\sin(a-$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^4,x]
```

```
[Out] (8*d^2*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(65*b*Sqrt[Cos[a + b
*x]]) + (8*d*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(195*b) - (4*(d*Cos[a + b
*x])^(7/2)*Sin[a + b*x])/(39*b*d) - (2*(d*Cos[a + b*x])^(7/2)*Sin[a + b*x]^
3)/(13*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{5/2}\sin^4(a+bx)\,dx &= -\frac{2(d\cos(a+bx))^{7/2}\sin^3(a+bx)}{13bd}+\frac{6}{13}\int (d\cos(a+bx))^{5/2}\sin^2(a+bx)\,d\\ &= -\frac{4(d\cos(a+bx))^{7/2}\sin(a+bx)}{39bd}-\frac{2(d\cos(a+bx))^{7/2}\sin^3(a+bx)}{13bd}+\frac{4}{39}\\ &= \frac{8d(d\cos(a+bx))^{3/2}\sin(a+bx)}{195b}-\frac{4(d\cos(a+bx))^{7/2}\sin(a+bx)}{39bd}-\frac{2(d\,c}{}\\ &= \frac{8d(d\cos(a+bx))^{3/2}\sin(a+bx)}{195b}-\frac{4(d\cos(a+bx))^{7/2}\sin(a+bx)}{39bd}-\frac{2(d\,c}{}\\ &= \frac{8d^2\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{65b\sqrt{\cos(a+bx)}}+\frac{8d(d\cos(a+bx))^{3/2}\sin(a+bx)}{195b}-\end{aligned}$$

Mathematica [C] time = 0.08, size = 65, normalized size = 0.51

$$\frac{\sin^2(a+bx)\sqrt[4]{\cos^2(a+bx)}\,\tan^3(a+bx)(d\cos(a+bx))^{5/2}\,{}_2F_1\left(-\frac{3}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^4,x]
[Out] ((d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, 5/2,
 7/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Tan[a + b*x]^3)/(5*b)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2\cos(bx+a)^6-2\,d^2\cos(bx+a)^4+d^2\cos(bx+a)^2\right)\sqrt{d\cos(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((d^2*cos(b*x + a)^6 - 2*d^2*cos(b*x + a)^4 + d^2*cos(b*x + a)^2)*s
qrt(d*cos(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^4, x)
```

maple [A] time = 0.11, size = 249, normalized size = 1.95

$$-\frac{8\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^3\left(480\left(\cos^{15}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1920\left(\cos^{13}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+3040\left(\cos^{11}\left(\frac{b}{}\right.\right.\right.}{195\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x)
```

```
[Out] -8/195*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^3*(480*c
os(1/2*b*x+1/2*a)^15-1920*cos(1/2*b*x+1/2*a)^13+3040*cos(1/2*b*x+1/2*a)^11-
2400*cos(1/2*b*x+1/2*a)^9+958*cos(1/2*b*x+1/2*a)^7-156*cos(1/2*b*x+1/2*a)^5
-5*cos(1/2*b*x+1/2*a)^3-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*
a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))+3*cos(1/2*b*x+1/2*a))/(
-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/
(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{5}{2}}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sin(b*x + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)^4\,(d\,\cos(a+b\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4*(d*cos(a + b*x))^(5/2),x)
```

```
[Out] int(sin(a + b*x)^4*(d*cos(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

3.214 $\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx$

Optimal. Leaf size=128

$$\frac{8d^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{77b\sqrt{d\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}-\frac{12\sin(a+bx)(d\cos(a+bx))^{5/2}}{77bd}+\frac{8d\sin(a+$$

```
[Out] -12/77*(d*cos(b*x+a))^(5/2)*sin(b*x+a)/b/d-2/11*(d*cos(b*x+a))^(5/2)*sin(b*
x+a)^3/b/d+8/77*d^2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*Ellipti
cF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)+8/77
*d*sin(b*x+a)*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2568, 2635, 2642, 2641}

$$\frac{8d^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{77b\sqrt{d\cos(a+bx)}}-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{5/2}}{11bd}-\frac{12\sin(a+bx)(d\cos(a+bx))^{5/2}}{77bd}+\frac{8d\sin(a+$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^4,x]
```

```
[Out] (8*d^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(77*b*Sqrt[d*Cos[a + b
*x]]) + (8*d*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(77*b) - (12*(d*Cos[a + b*x
])^(5/2)*Sin[a + b*x])/(77*b*d) - (2*(d*Cos[a + b*x])^(5/2)*Sin[a + b*x]^3)
/(11*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{3/2}\sin^4(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{5/2}\sin^3(a+bx)}{11bd} + \frac{6}{11}\int (d\cos(a+bx))^{3/2}\sin^2(a+bx)\,d$$
$$= -\frac{12(d\cos(a+bx))^{5/2}\sin(a+bx)}{77bd} - \frac{2(d\cos(a+bx))^{5/2}\sin^3(a+bx)}{11bd} + \frac{12}{77}$$
$$= \frac{8d\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{77b} - \frac{12(d\cos(a+bx))^{5/2}\sin(a+bx)}{77bd} - \frac{2(d\,c}{}$$
$$= \frac{8d\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{77b} - \frac{12(d\cos(a+bx))^{5/2}\sin(a+bx)}{77bd} - \frac{2(d\,c}{}$$
$$= \frac{8d^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{77b\sqrt{d\cos(a+bx)}} + \frac{8d\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{77b} - \frac{12(}{}$$

Mathematica [C] time = 0.11, size = 65, normalized size = 0.51

$$\frac{\sin^2(a+bx)\cos^2(a+bx)^{3/4}\tan^3(a+bx)(d\cos(a+bx))^{3/2}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Sin[a + b*x]^4,x]
```

```
[Out] ((d*Cos[a + b*x])^(3/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 5/2,
 7/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Tan[a + b*x]^3)/(5*b)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(d\cos(bx+a)^5 - 2\,d\cos(bx+a)^3 + d\cos(bx+a)\right)\sqrt{d\cos(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a)^5 - 2*d*cos(b*x + a)^3 + d*cos(b*x + a))*sqrt(d*co
s(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^4, x)
```

maple [A] time = 0.11, size = 255, normalized size = 1.99

$$-\frac{8\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^2\left(112\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-280\left(\sin^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\right.\right.}{77\sqrt{-d\left(2\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x)
```

```
[Out] -8/77*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^2*(112*co
s(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^12-280*sin(1/2*b*x+1/2*a)^10*cos(1/2*b*
x+1/2*a)+228*sin(1/2*b*x+1/2*a)^8*cos(1/2*b*x+1/2*a)-62*sin(1/2*b*x+1/2*a)^
6*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1
)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+sin(1/2*b*x+1/2*a)^2*cos(1/2*
b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/
2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{3}{2}}\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*sin(b*x + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)^4\,(d\,\cos(a+b\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4*(d*cos(a + b*x))^(3/2),x)
```

```
[Out] int(sin(a + b*x)^4*(d*cos(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

3.215 $\int \sqrt{d\cos(a+bx)}\,\sin^4(a+bx)\,dx$

Optimal. Leaf size=99

$$-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{3/2}}{9bd}-\frac{4\sin(a+bx)(d\cos(a+bx))^{3/2}}{15bd}+\frac{8E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{15b\sqrt{\cos(a+bx)}}$$

```
[Out] -4/15*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b/d-2/9*(d*cos(b*x+a))^(3/2)*sin(b*x+
a)^3/b/d+8/15*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin
(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2568, 2640, 2639}

$$-\frac{2\sin^3(a+bx)(d\cos(a+bx))^{3/2}}{9bd}-\frac{4\sin(a+bx)(d\cos(a+bx))^{3/2}}{15bd}+\frac{8E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{15b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^4,x]
```

```
[Out] (8*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(15*b*Sqrt[Cos[a + b*x]]
) - (4*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(15*b*d) - (2*(d*Cos[a + b*x])^
(3/2)*Sin[a + b*x]^3)/(9*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

x]

Rubi steps

$$\int \sqrt{d\cos(a+bx)}\,\sin^4(a+bx)\,dx = -\frac{2(d\cos(a+bx))^{3/2}\sin^3(a+bx)}{9bd} + \frac{2}{3}\int \sqrt{d\cos(a+bx)}\,\sin^2(a+bx)\,dx$$
$$= -\frac{4(d\cos(a+bx))^{3/2}\sin(a+bx)}{15bd} - \frac{2(d\cos(a+bx))^{3/2}\sin^3(a+bx)}{9bd} + \frac{4}{15}\int$$
$$= -\frac{4(d\cos(a+bx))^{3/2}\sin(a+bx)}{15bd} - \frac{2(d\cos(a+bx))^{3/2}\sin^3(a+bx)}{9bd} + \frac{(4\sqrt{}}{}$$
$$= \frac{8\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b\sqrt{\cos(a+bx)}} - \frac{4(d\cos(a+bx))^{3/2}\sin(a+bx)}{15bd} - \frac{2(d\,\mathrm{c}}{}$$

Mathematica [C] time = 0.06, size = 58, normalized size = 0.59

$$\frac{d\sin^5(a+bx)\sqrt[4]{\cos^2(a+bx)}\;{}_2F_1\left(\frac{1}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sin[a + b*x]^4,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/2, 7/2, Sin[a + b*x]^2]*
Sin[a + b*x]^5)/(5*b*Sqrt[d*Cos[a + b*x]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\sqrt{d\cos(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*cos(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos(bx+a)}\,\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="giac")
[Out] integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^4, x)
```

maple [A] time = 0.10, size = 221, normalized size = 2.23

$$-\frac{8\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d\left(40\left(\cos^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-120\left(\cos^{9}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+118\left(\cos^{7}\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{45\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x)
[Out] -8/45*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d*(40*cos(1
/2*b*x+1/2*a)^11-120*cos(1/2*b*x+1/2*a)^9+118*cos(1/2*b*x+1/2*a)^7-36*cos(1
/2*b*x+1/2*a)^5-5*cos(1/2*b*x+1/2*a)^3-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*c
os(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))+3*cos(1/
2*b*x+1/2*a))/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(
1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos(bx+a)}\,\sin(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*sin(b*x+a)^4,x, algorithm="maxima")
[Out] integrate(sqrt(d*cos(b*x + a))*sin(b*x + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)^4\,\sqrt{d\,\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4*(d*cos(a + b*x))^(1/2),x)
[Out] int(sin(a + b*x)^4*(d*cos(a + b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*sin(b*x+a)**4,x)

[Out] Timed out
```

3.216 $\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=99

$$-\frac{2\sin^3(a+bx)\sqrt{d\cos(a+bx)}}{7bd}-\frac{4\sin(a+bx)\sqrt{d\cos(a+bx)}}{7bd}+\frac{8\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7b\sqrt{d\cos(a+bx)}}$$

```
[Out] 8/7*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2
*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)-4/7*sin(b*x+a)*(d*co
s(b*x+a))^(1/2)/b/d-2/7*sin(b*x+a)^3*(d*cos(b*x+a))^(1/2)/b/d
```

Rubi [A] time = 0.10, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2568, 2642, 2641}

$$-\frac{2\sin^3(a+bx)\sqrt{d\cos(a+bx)}}{7bd}-\frac{4\sin(a+bx)\sqrt{d\cos(a+bx)}}{7bd}+\frac{8\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (8*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(7*b*Sqrt[d*Cos[a + b*x]])
 - (4*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(7*b*d) - (2*Sqrt[d*Cos[a + b*x]]*
Sin[a + b*x]^3)/(7*b*d)
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

d}, x]

Rubi steps

$$\int \frac{\sin^4(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{2\sqrt{d\cos(a+bx)}\,\sin^3(a+bx)}{7bd} + \frac{6}{7}\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$
$$= -\frac{4\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{7bd} - \frac{2\sqrt{d\cos(a+bx)}\,\sin^3(a+bx)}{7bd} + \frac{4}{7}\int \frac{1}{\sqrt{d\cos(a+bx)}}$$
$$= -\frac{4\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{7bd} - \frac{2\sqrt{d\cos(a+bx)}\,\sin^3(a+bx)}{7bd} + \frac{\left(4\sqrt{\cos(a+bx)}\right)\int}{7\sqrt{d\cos(a+}}$$
$$= \frac{8\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7b\sqrt{d\cos(a+bx)}} - \frac{4\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{7bd} - \frac{2\sqrt{d\cos(a+bx)}\,\sin}{7bd}$$

Mathematica [C] time = 0.07, size = 58, normalized size = 0.59

$$\frac{d\sin^5(a+bx)\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 5/2, 7/2, Sin[a + b*x]^2]*
Sin[a + b*x]^5)/(5*b*(d*Cos[a + b*x])^(3/2))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\sqrt{d\cos(bx+a)}}{d\cos(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*cos(b*x + a))/(d*co
s(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^4}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^4/sqrt(d*cos(b*x + a)), x)
```

maple [A] time = 0.10, size = 208, normalized size = 2.10

$$-\frac{8\sqrt{d\left(2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\left(4\left(\sin^{8}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-6\left(\sin^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\cdots}{7\sqrt{-d\left(2\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] -8/7*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*b
*x+1/2*a)^8*cos(1/2*b*x+1/2*a)-6*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+(s
in(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1
/2*b*x+1/2*a),2^(1/2))+sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/(-d*(2*sin(
1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2*cos(
1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)^{4}}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^4/sqrt(d*cos(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(a+b\,x\right)}^{4}}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/(d*cos(a + b*x))^(1/2),x)
```

```
[Out] int(sin(a + b*x)^4/(d*cos(a + b*x))^(1/2), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.217 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{12\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd^3}-\frac{24E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^2\sqrt{\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] 12/5*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/b/d^3+2*sin(b*x+a)^3/b/d/(d*cos(b*x+a)
)^(1/2)-24/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(
1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/d^2/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2566, 2568, 2640, 2639}

$$\frac{12\sin(a+bx)(d\cos(a+bx))^{3/2}}{5bd^3}-\frac{24E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^2\sqrt{\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-24*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*d^2*Sqrt[Cos[a +
b*x]]) + (12*(d*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b*d^3) + (2*Sin[a + b*
x]^3)/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx &= \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{6\int \sqrt{d\cos(a+bx)}\,\sin^2(a+bx)\,dx}{d^2}\\
&= \frac{12(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd^3} + \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{12\int \sqrt{d\cos(a+bx)}\,dx}{5d^2}\\
&= \frac{12(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd^3} + \frac{2\sin^3(a+bx)}{bd\sqrt{d\cos(a+bx)}} - \frac{\left(12\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(}}{5d^2\sqrt{\cos(a+bx)}}\\
&= -\frac{24\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^2\sqrt{\cos(a+bx)}} + \frac{12(d\cos(a+bx))^{3/2}\sin(a+bx)}{5bd^3} + \frac{2\sin^3(a}{bd\sqrt{d\cos(}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 60, normalized size = 0.60

$$\frac{\sin^5(a+bx)\sqrt[4]{\cos^2(a+bx)}\;{}_2F_1\left(\frac{5}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/2, 7/2, Sin[a + b*x]^2]*Si
n[a + b*x]^5)/(5*b*d*Sqrt[d*Cos[a + b*x]])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\sqrt{d\cos(bx+a)}}{d^2\cos(bx+a)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*cos(b*x + a))/(d^2*
cos(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin (bx+a)^4}{(d\cos (bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(3/2), x)
```

maple [A] time = 0.11, size = 213, normalized size = 2.13

$$-\frac{8\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d}\,\left(-2\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cdots\right.}{5d\sqrt{-d\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] -8/5/d*(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(1/2)*(-2*sin(1/2
*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+2*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+
3*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(c
os(1/2*b*x+1/2*a),2^(1/2))-3*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/(-d*(
2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(
2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin (bx+a)^4}{(d\cos (bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+bx)^4}{(d\cos(a+bx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/(d*cos(a + b*x))^(3/2),x)
[Out] int(sin(a + b*x)^4/(d*cos(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(3/2),x)
[Out] Timed out
```

3.218 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=102

$$\frac{4\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd^3}-\frac{8\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/3*sin(b*x+a)^3/b/d/(d*cos(b*x+a))^(3/2)-8/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/
cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b
/d^2/(d*cos(b*x+a))^(1/2)+4/3*sin(b*x+a)*(d*cos(b*x+a))^(1/2)/b/d^3
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2566, 2568, 2642, 2641}

$$\frac{4\sin(a+bx)\sqrt{d\cos(a+bx)}}{3bd^3}-\frac{8\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (-8*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b
*x]]) + (4*Sqrt[d*Cos[a + b*x]]*Sin[a + b*x])/(3*b*d^3) + (2*Sin[a + b*x]^3
)/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx &= \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{2\int \frac{\sin^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx}{d^2}\\ &= \frac{4\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{3bd^3} + \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{4\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{3d^2}\\ &= \frac{4\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{3bd^3} + \frac{2\sin^3(a+bx)}{3bd(d\cos(a+bx))^{3/2}} - \frac{\left(4\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx}}}{3d^2\sqrt{d\cos(a+bx)}}\\ &= -\frac{8\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}} + \frac{4\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{3bd^3} + \frac{2\sin^3(a+bx}{3bd(d\cos(a+bx}\end{aligned}$$

Mathematica [C] time = 0.07, size = 60, normalized size = 0.59

$$\frac{\sin^5(a+bx)\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{7}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 5/2, 7/2, Sin[a + b*x]^2]*Si
n[a + b*x]^5)/(5*b*d*(d*Cos[a + b*x])^(3/2))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\sqrt{d\cos(bx+a)}}{d^3\cos(bx+a)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*cos(b*x + a))/(d^3*
cos(b*x + a)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(5/2), x)
```

maple [B] time = 0.11, size = 286, normalized size = 2.80

$$-\frac{8\left(-2\left(\sin^{6}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+2\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\;\mathrm{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sin\right.\right.}{3d^{2}\left(2\left(\cos^{2}\left(\frac{bx}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] -8/3*(-2*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+2*(sin(1/2*b*x+1/2*a)^2)^(
1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))
*sin(1/2*b*x+1/2*a)^2+2*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4-(sin(1/2*b*
x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/
2*a),2^(1/2))-sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/d^2*(d*(2*cos(1/2*b*
x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)/(-d*(2
*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(d*(2
*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(5/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+bx)^4}{(d\cos(a+bx))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/(d*cos(a + b*x))^(5/2),x)
```

```
[Out] int(sin(a + b*x)^4/(d*cos(a + b*x))^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.219 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=102

$$\frac{24E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}}-\frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/5*sin(b*x+a)^3/b/d/(d*cos(b*x+a))^(5/2)-12/5*sin(b*x+a)/b/d^3/(d*cos(b*x+
a))^(1/2)+24/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(si
n(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/d^4/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2566, 2640, 2639}

$$-\frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{24E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (24*Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*d^4*Sqrt[Cos[a + b
*x]]) - (12*Sin[a + b*x])/(5*b*d^3*Sqrt[d*Cos[a + b*x]]) + (2*Sin[a + b*x]^
3)/(5*b*d*(d*Cos[a + b*x])^(5/2))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

x]

Rubi steps

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}} - \frac{6\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx}{5d^2}$$
$$= -\frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}} + \frac{12\int \sqrt{d\cos(a+bx)}\,dx}{5d^4}$$
$$= -\frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}} + \frac{\left(12\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx}}{5d^4\sqrt{\cos(a+bx)}}$$
$$= \frac{24\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}} - \frac{12\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{2\sin^3(a+bx)}{5bd(d\cos(a+bx))^{5/2}}$$

Mathematica [C] time = 0.06, size = 65, normalized size = 0.64

$$\frac{\sin^5(a+bx)\cos^3(a+bx)\sqrt[4]{\cos^2(a+bx)}\;{}_2F_1\left(\frac{9}{4},\frac{5}{2};\frac{7}{2};\sin^2(a+bx)\right)}{5b(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (Cos[a + b*x]^3*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 5/2, 7/2, Sin
[a + b*x]^2]*Sin[a + b*x]^5)/(5*b*(d*Cos[a + b*x])^(7/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\sqrt{d\cos(bx+a)}}{d^4\cos(bx+a)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*cos(b*x + a))/(d^4*
cos(b*x + a)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(7/2), x)
```

maple [B] time = 0.16, size = 366, normalized size = 3.59

$$-\frac{8\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(12\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\mathrm{EllipticE}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] -8/5*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^4/sin(1/2*
b*x+1/2*a)^3/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+
1/2*a)^2-1)*(12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/
2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^4-14*sin(1/2*b*
x+1/2*a)^6*cos(1/2*b*x+1/2*a)-12*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*
b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a
)^2+14*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+3*(2*sin(1/2*b*x+1/2*a)^2-1)
^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))-3
*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))*(-2*sin(1/2*b*x+1/2*a)^4*d+sin(1/
2*b*x+1/2*a)^2*d)^(1/2)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^4}{(d\,\cos(a+b\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/(d*cos(a + b*x))^(7/2),x)
```

```
[Out] int(sin(a + b*x)^4/(d*cos(a + b*x))^(7/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

3.220 $\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=102

$$\frac{8\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7bd^4\sqrt{d\cos(a+bx)}}-\frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}}+\frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

```
[Out] -4/7*sin(b*x+a)/b/d^3/(d*cos(b*x+a))^(3/2)+2/7*sin(b*x+a)^3/b/d/(d*cos(b*x+
a))^(7/2)+8/7*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin
(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/d^4/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2566, 2642, 2641}

$$-\frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}}+\frac{8\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7bd^4\sqrt{d\cos(a+bx)}}+\frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^4/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (8*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(7*b*d^4*Sqrt[d*Cos[a + b*
x]]) - (4*Sin[a + b*x])/(7*b*d^3*(d*Cos[a + b*x])^(3/2)) + (2*Sin[a + b*x]^
3)/(7*b*d*(d*Cos[a + b*x])^(7/2))
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rubi steps

$$\int \frac{\sin^4(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}} - \frac{6\int \frac{\sin^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx}{7d^2}$$
$$= -\frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}} + \frac{4\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{7d^4}$$
$$= -\frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}} + \frac{\left(4\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{7d^4\sqrt{d\cos(a+bx)}}$$
$$= \frac{8\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7bd^4\sqrt{d\cos(a+bx)}} - \frac{4\sin(a+bx)}{7bd^3(d\cos(a+bx))^{3/2}} + \frac{2\sin^3(a+bx)}{7bd(d\cos(a+bx))^{7/2}}$$

Mathematica [C] time = 0.06, size = 65, normalized size = 0.64

$$\frac{\sin^5(a+bx)\cos^3(a+bx)\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{5}{2},\frac{11}{4};\frac{7}{2};\sin^2(a+bx)\right)}{5b(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^4/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (Cos[a + b*x]^3*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[5/2, 11/4, 7/2, Si
n[a + b*x]^2]*Sin[a + b*x]^5)/(5*b*(d*Cos[a + b*x])^(9/2))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos(bx+a)^4 - 2\cos(bx+a)^2 + 1\right)\sqrt{d\cos(bx+a)}}{d^5\cos(bx+a)^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sqrt(d*cos(b*x + a))/(d^5*
cos(b*x + a)^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(9/2), x)
```

maple [B] time = 0.12, size = 398, normalized size = 3.90

$$8\left(8\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,\operatorname{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-12\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 8/7*(8*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Ellipt
icF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^6-12*(sin(1/2*b*x+1/2*a)
^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(
1/2))*sin(1/2*b*x+1/2*a)^4-6*sin(1/2*b*x+1/2*a)^6*cos(1/2*b*x+1/2*a)+6*(sin
(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2
*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+6*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+
1/2*a)^4-(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Elli
pticF(cos(1/2*b*x+1/2*a),2^(1/2))-sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a))/
d^4*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(2*cos(1/2*b*
x+1/2*a)^2-1)^3/(-d*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/si
n(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^4}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^4/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^4/(d*cos(b*x + a))^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^4}{(d\,\cos(a+b\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4/(d*cos(a + b*x))^(9/2),x)
[Out] int(sin(a + b*x)^4/(d*cos(a + b*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**4/(d*cos(b*x+a))**(9/2),x)
[Out] Timed out
```

3.221 $\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx$

Optimal. Leaf size=52

$$-\frac{2\cos^{\frac{13}{2}}(a+bx)}{13b}+\frac{4\cos^{\frac{9}{2}}(a+bx)}{9b}-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] -2/5*cos(b*x+a)^(5/2)/b+4/9*cos(b*x+a)^(9/2)/b-2/13*cos(b*x+a)^(13/2)/b
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 270}

$$-\frac{2\cos^{\frac{13}{2}}(a+bx)}{13b}+\frac{4\cos^{\frac{9}{2}}(a+bx)}{9b}-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(3/2)*Sin[a + b*x]^5,x]
```

```
[Out] (-2*Cos[a + b*x]^(5/2))/(5*b) + (4*Cos[a + b*x]^(9/2))/(9*b) - (2*Cos[a + b
*x]^(13/2))/(13*b)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(a+bx)\sin^5(a+bx)\,dx = -\frac{\text{Subst}\left(\int x^{3/2}\left(1-x^2\right)^2\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \left(x^{3/2}-2x^{7/2}+x^{11/2}\right)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b} + \frac{4\cos^{\frac{9}{2}}(a+bx)}{9b} - \frac{2\cos^{\frac{13}{2}}(a+bx)}{13b}$$

Mathematica [B] time = 0.26, size = 111, normalized size = 2.13

$$\frac{2\sqrt{\cos(a+bx)}\left(-32\sqrt[4]{\cos^2(a+bx)}+45\sin^6(a+bx)\sqrt[4]{\cos^2(a+bx)}-5\sin^4(a+bx)\sqrt[4]{\cos^2(a+bx)}-8\sin^2(a\right.}{585b\sqrt[4]{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(3/2)*Sin[a + b*x]^5,x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*(32 - 32*(Cos[a + b*x]^2)^(1/4) - 8*(Cos[a + b*x]^2)^
(1/4)*Sin[a + b*x]^2 - 5*(Cos[a + b*x]^2)^(1/4)*Sin[a + b*x]^4 + 45*(Cos[a
+ b*x]^2)^(1/4)*Sin[a + b*x]^6))/(585*b*(Cos[a + b*x]^2)^(1/4))
```

fricas [A] time = 0.45, size = 44, normalized size = 0.85

$$-\frac{2\left(45\cos\left(bx+a\right)^{6}-130\cos\left(bx+a\right)^{4}+117\cos\left(bx+a\right)^{2}\right)\sqrt{\cos\left(bx+a\right)}}{585\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -2/585*(45*cos(b*x + a)^6 - 130*cos(b*x + a)^4 + 117*cos(b*x + a)^2)*sqrt(c
os(b*x + a))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)^{\frac{3}{2}}\sin\left(bx+a\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(3/2)*sin(b*x + a)^5, x)
```

maple [B] time = 0.13, size = 103, normalized size = 1.98

$$-\frac{32\sqrt{-2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\,\left(180\left(\sin^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-540\left(\sin^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+545\left(\sin^{8}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-190\left(\sin^{6}\left(\frac{bx}{2}\right.\right.\right.}{585b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x)
```

```
[Out] -32/585*(-2*sin(1/2*b*x+1/2*a)^2+1)^(1/2)*(180*sin(1/2*b*x+1/2*a)^12-540*si
n(1/2*b*x+1/2*a)^10+545*sin(1/2*b*x+1/2*a)^8-190*sin(1/2*b*x+1/2*a)^6+3*sin
(1/2*b*x+1/2*a)^4+2*sin(1/2*b*x+1/2*a)^2+2)/b
```

maxima [A] time = 0.31, size = 36, normalized size = 0.69

$$-\frac{2\left(45\cos\left(bx+a\right)^{\frac{13}{2}}-130\cos\left(bx+a\right)^{\frac{9}{2}}+117\cos\left(bx+a\right)^{\frac{5}{2}}\right)}{585\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -2/585*(45*cos(b*x + a)^(13/2) - 130*cos(b*x + a)^(9/2) + 117*cos(b*x + a)^
(5/2))/b
```

mupad [B] time = 0.64, size = 35, normalized size = 0.67

$$-\frac{2\cos\left(a+b\,x\right)^{5/2}\,\left(\frac{5\cos(a+b\,x)^4}{13}-\frac{10\cos(a+b\,x)^2}{9}+1\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(3/2)*sin(a + b*x)^5,x)
```

```
[Out] -(2*cos(a + b*x)^(5/2)*((5*cos(a + b*x)^4)/13 - (10*cos(a + b*x)^2)/9 + 1))
/(5*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(3/2)*sin(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.222 $\int (d \cos(a+bx))^{9/2} \csc(a+bx)\,dx$

Optimal. Leaf size=100

$$\frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d^3(d\cos(a+bx))^{3/2}}{3b}+\frac{2d(d\cos(a+bx))^{7/2}}{7b}$$

```
[Out] d^(9/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(9/2)*arctanh((d*cos(b*x+a
))^(1/2)/d^(1/2))/b+2/3*d^3*(d*cos(b*x+a))^(3/2)/b+2/7*d*(d*cos(b*x+a))^(7/
2)/b
```

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 321, 329, 298, 203, 206}

$$\frac{2d^3(d\cos(a+bx))^{3/2}}{3b}+\frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d(d\cos(a+bx))^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x],x]
```

```
[Out] (d^(9/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b - (d^(9/2)*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/b + (2*d^3*(d*Cos[a + b*x])^(3/2))/(3*b) + (2*d*(d*
Cos[a + b*x])^(7/2))/(7*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^{9/2}\csc(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{9/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2d(d\cos(a+bx))^{7/2}}{7b} - \frac{d\,\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{b}$$

$$= \frac{2d^3(d\cos(a+bx))^{3/2}}{3b} + \frac{2d(d\cos(a+bx))^{7/2}}{7b} - \frac{d^3\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\,\text{co}\right.}{b}$$

$$= \frac{2d^3(d\cos(a+bx))^{3/2}}{3b} + \frac{2d(d\cos(a+bx))^{7/2}}{7b} - \frac{\left(2d^3\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x,\right.}{b}$$

$$= \frac{2d^3(d\cos(a+bx))^{3/2}}{3b} + \frac{2d(d\cos(a+bx))^{7/2}}{7b} - \frac{d^5\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d}\right.}{b}$$

$$= \frac{d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d^3(d\cos(a+bx))^3}{3b}$$

Mathematica [A] time = 0.19, size = 83, normalized size = 0.83

$$\frac{d^4\sqrt{d\cos(a+bx)}\left(2\left(3\cos^2(a+bx)+7\right)\cos^{\frac{3}{2}}(a+bx)+21\tan^{-1}\left(\sqrt{\cos(a+bx)}\right)-21\tanh^{-1}\left(\sqrt{\cos(a+bx)}\right)\right)}{21b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x],x]
```

```
[Out] (d^4*Sqrt[d*Cos[a + b*x]]*(21*ArcTan[Sqrt[Cos[a + b*x]]] - 21*ArcTanh[Sqrt[
Cos[a + b*x]]] + 2*Cos[a + b*x]^(3/2)*(7 + 3*Cos[a + b*x]^2)))/(21*b*Sqrt[C
os[a + b*x]])
```

fricas [A] time = 0.59, size = 313, normalized size = 3.13

$$\left[\frac{42\sqrt{-d}\,d^4\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right) + 21\sqrt{-d}\,d^4\log\left(-\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}\,(\cos(bx+a)-1)-6\,d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right) +}{84\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/84*(42*sqrt(-d)*d^4*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x +
a) + d)) + 21*sqrt(-d)*d^4*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*
sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos
(b*x + a) + 1)) + 8*(3*d^4*cos(b*x + a)^3 + 7*d^4*cos(b*x + a))*sqrt(d*cos(
b*x + a)))/b, -1/84*(42*d^(9/2)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*co
s(b*x + a) - d)) - 21*d^(9/2)*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a
))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*c
os(b*x + a) + 1)) - 8*(3*d^4*cos(b*x + a)^3 + 7*d^4*cos(b*x + a))*sqrt(d*co
s(b*x + a)))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a), x)
```

maple [B] time = 0.20, size = 318, normalized size = 3.18

$$-\frac{16\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}\; d^4\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{7b}-\frac{d^{\frac{9}{2}}\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}-4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)}{2b}-\frac{d^{\frac{9}{2}}\ln\left(\frac{2\sqrt{d}\,\sqrt{-2}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(9/2)*csc(b*x+a),x)
```

```
[Out] -16/7/b*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^4*sin(1/2*b*x+1/2*a)^6-1/2/b*
d^(9/2)*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^
(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))-1/2/b*d^(9/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*
(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))+24/
7/b*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^4*sin(1/2*b*x+1/2*a)^4-64/21/b*(-
2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^4*sin(1/2*b*x+1/2*a)^2-1/(-d)^(1/2)/b*d
^5*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-
d))+20/21/b*d^4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
```

maxima [A] time = 0.43, size = 98, normalized size = 0.98

$$\frac{42\,d^{\frac{11}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)+21\,d^{\frac{11}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)+12\,\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}d^{2}+28\,\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}d^{4}}{42\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/42*(42*d^(11/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) + 21*d^(11/2)*log((s
qrt(d*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) + 12*(d*co
s(b*x + a))^(7/2)*d^2 + 28*(d*cos(b*x + a))^(3/2)*d^4)/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d\,\cos\left(a+b\,x\right)\right)^{9/2}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(9/2)/sin(a + b*x),x)
```

```
[Out] int((d*cos(a + b*x))^(9/2)/sin(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*csc(b*x+a),x)
```

```
[Out] Timed out
```

3.223 $\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx$

Optimal. Leaf size=99

$$-\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d^3\sqrt{d\cos(a+bx)}}{b}+\frac{2d(d\cos(a+bx))^{5/2}}{5b}$$

```
[Out] -d^(7/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(7/2)*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b+2/5*d*(d*cos(b*x+a))^(5/2)/b+2*d^3*(d*cos(b*x+a))^(1/2
)/b
```

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 321, 329, 212, 206, 203}

$$\frac{2d^3\sqrt{d\cos(a+bx)}}{b}-\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d(d\cos(a+bx))^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x],x]
```

```
[Out] -((d^(7/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b) - (d^(7/2)*ArcTanh[Sqrt
[d*Cos[a + b*x]]/Sqrt[d]])/b + (2*d^3*Sqrt[d*Cos[a + b*x]])/b + (2*d*(d*Cos
[a + b*x])^(5/2))/(5*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^{7/2}\csc(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{7/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2d(d\cos(a+bx))^{5/2}}{5b} - \frac{d\,\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{b}$$

$$= \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b} - \frac{d^3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos\right.}{b}$$

$$= \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b} - \frac{(2d^3)\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d}\right.}{b}$$

$$= \frac{2d^3\sqrt{d\cos(a+bx)}}{b} + \frac{2d(d\cos(a+bx))^{5/2}}{5b} - \frac{d^4\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos}\right.}{b}$$

$$= -\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d^3\sqrt{d\cos(a+bx)}}{b}$$

Mathematica [A] time = 0.18, size = 80, normalized size = 0.81

$$\frac{d^3\sqrt{d\cos(a+bx)}\left(\sqrt{\cos(a+bx)}(\cos(2(a+bx))+11) - 5\tan^{-1}\left(\sqrt{\cos(a+bx)}\right) - 5\tanh^{-1}\left(\sqrt{\cos(a+bx)}\right)\right)}{5b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x],x]
```

```
[Out] (d^3*Sqrt[d*Cos[a + b*x]]*(-5*ArcTan[Sqrt[Cos[a + b*x]]] - 5*ArcTanh[Sqrt[C
os[a + b*x]]] + Sqrt[Cos[a + b*x]]*(11 + Cos[2*(a + b*x)])))/(5*b*Sqrt[Cos[
a + b*x]])
```

fricas [A] time = 0.63, size = 299, normalized size = 3.02

$$\left[\frac{10\sqrt{-d}\,d^3\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right) + 5\sqrt{-d}\,d^3\log\left(-\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right) + 8\right.}{20b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/20*(10*sqrt(-d)*d^3*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x +
a) + d)) + 5*sqrt(-d)*d^3*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*s
qrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(
b*x + a) + 1)) + 8*(d^3*cos(b*x + a)^2 + 5*d^3)*sqrt(d*cos(b*x + a)))/b, 1/
20*(10*d^(7/2)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d))
+ 5*d^(7/2)*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*
x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1))
+ 8*(d^3*cos(b*x + a)^2 + 5*d^3)*sqrt(d*cos(b*x + a)))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a), x)
```

maple [B] time = 0.18, size = 280, normalized size = 2.83

$$\frac{8\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}\,d^3\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{5b}-\frac{d^{\frac{7}{2}}\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}+4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{2b}-\frac{d^{\frac{7}{2}}\ln\left(\frac{2\sqrt{d}\,\sqrt{-2(}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(7/2)*csc(b*x+a),x)
```

```
[Out] 8/5/b*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^3*sin(1/2*b*x+1/2*a)^4-1/2/b*d^
(7/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1
/2)+2*d*cos(1/2*b*x+1/2*a)-d))-1/2/b*d^(7/2)*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d
^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))-8/5/b
*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^3*sin(1/2*b*x+1/2*a)^2+12/5/b*d^3*(-
2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+1/(-d)^(1/2)/b*d^4*ln(2/cos(1/2*b*x+1/2*a
)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))
```

maxima [A] time = 0.43, size = 98, normalized size = 0.99

$$-\frac{10\,d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)-5\,d^{\frac{9}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)-4\,(d\cos(bx+a))^{\frac{5}{2}}d^2-20\sqrt{d\cos(bx+a)}\,d^4}{10\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/10*(10*d^(9/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 5*d^(9/2)*log((sqr
t(d*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 4*(d*cos(b
*x + a))^(5/2)*d^2 - 20*sqrt(d*cos(b*x + a))*d^4)/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(7/2)/sin(a + b*x),x)
```

```
[Out] int((d*cos(a + b*x))^(7/2)/sin(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(7/2)*csc(b*x+a),x)
```

```
[Out] Timed out
```

3.224 $\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx$

Optimal. Leaf size=78

$$\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d(d\cos(a+bx))^{3/2}}{3b}$$

```
[Out] d^(5/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(5/2)*arctanh((d*cos(b*x+a
))^(1/2)/d^(1/2))/b+2/3*d*(d*cos(b*x+a))^(3/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 321, 329, 298, 203, 206}

$$\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d(d\cos(a+bx))^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x],x]
```

```
[Out] (d^(5/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b - (d^(5/2)*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/b + (2*d*(d*Cos[a + b*x])^(3/2))/(3*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^{5/2}\csc(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2d(d\cos(a+bx))^{3/2}}{3b} - \frac{d\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{b}$$

$$= \frac{2d(d\cos(a+bx))^{3/2}}{3b} - \frac{(2d)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$

$$= \frac{2d(d\cos(a+bx))^{3/2}}{3b} - \frac{d^3\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} + \frac{d^3\,\text{Subst}(

Mathematica [A] time = 0.10, size = 68, normalized size = 0.87

$$\frac{(d\cos(a+bx))^{5/2}\left(2\cos^{\frac{3}{2}}(a+bx)+3\tan^{-1}\left(\sqrt{\cos(a+bx)}\right)-3\tanh^{-1}\left(\sqrt{\cos(a+bx)}\right)\right)}{3b\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x],x]
```

```
[Out] ((d*Cos[a + b*x])^(5/2)*(3*ArcTan[Sqrt[Cos[a + b*x]]] - 3*ArcTanh[Sqrt[Cos[
a + b*x]]] + 2*Cos[a + b*x]^(3/2)))/(3*b*Cos[a + b*x]^(5/2))
```

fricas [B] time = 0.60, size = 281, normalized size = 3.60

$$\left[\frac{6\sqrt{-d}\,d^2\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)+8\sqrt{d\cos(bx+a)}\,d^2\cos(bx+a)+3\sqrt{-d}\,d^2\log\left(-\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}}{\cos(bx+a)}\right.}{12\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(-d)*d^2*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a
) + d)) + 8*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a) + 3*sqrt(-d)*d^2*log(-(d*
cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*c
os(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)))/b, -1/12*(6*d^(5/2
)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d)) - 8*sqrt(d*co
s(b*x + a))*d^2*cos(b*x + a) - 3*d^(5/2)*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*
cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x +
 a)^2 - 2*cos(b*x + a) + 1)))/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a), x)
```

maple [B] time = 0.18, size = 244, normalized size = 3.13

$$-\frac{d^{\frac{5}{2}}\ln\left(\frac{2\sqrt{d}\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}-4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)}{2b}-\frac{d^{\frac{5}{2}}\ln\left(\frac{2\sqrt{d}\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}+4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{2b}-\frac{4\sqrt{-2\left(\sin^2\left(\frac{bx}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*csc(b*x+a),x)
```

```
[Out] -1/2/b*d^(5/2)*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))-1/2/b*d^(5/2)*ln(2/(cos(1/2*b*x+1/2
*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-
d))-4/3/b*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^2*sin(1/2*b*x+1/2*a)^2+2/3/
b*d^2*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-1/(-d)^(1/2)/b*d^3*ln(2/cos(1/2*b
*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))
```

maxima [A] time = 0.60, size = 83, normalized size = 1.06

$$\frac{6\,d^{\frac{7}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)+3\,d^{\frac{7}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)+4\,\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}d^2}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(6*d^(7/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) + 3*d^(7/2)*log((sqrt(d
*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) + 4*(d*cos(b*x
+ a))^(3/2)*d^2)/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,\cos\left(a+b\,x\right)\right)}^{5/2}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(5/2)/sin(a + b*x),x)
```

```
[Out] int((d*cos(a + b*x))^(5/2)/sin(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*csc(b*x+a),x)
```

```
[Out] Timed out
```

3.225 $\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx$

Optimal. Leaf size=77

$$-\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d\sqrt{d\cos(a+bx)}}{b}$$

```
[Out] -d^(3/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b-d^(3/2)*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b+2*d*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 321, 329, 212, 206, 203}

$$-\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}+\frac{2d\sqrt{d\cos(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x],x]
```

```
[Out] -((d^(3/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b) - (d^(3/2)*ArcTanh[Sqrt
[d*Cos[a + b*x]]/Sqrt[d]])/b + (2*d*Sqrt[d*Cos[a + b*x]])/b
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^{3/2}\csc(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2d\sqrt{d\cos(a+bx)}}{b} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{b}$$

$$= \frac{2d\sqrt{d\cos(a+bx)}}{b} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}$$

$$= \frac{2d\sqrt{d\cos(a+bx)}}{b} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} - \frac{d^2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} + \frac{2d\sqrt{d\cos(a+bx)}}{b}$$

Mathematica [A] time = 0.06, size = 65, normalized size = 0.84

$$\frac{(d\cos(a+bx))^{3/2}\left(2\sqrt{\cos(a+bx)}-\tan^{-1}\left(\sqrt{\cos(a+bx)}\right)-\tanh^{-1}\left(\sqrt{\cos(a+bx)}\right)\right)}{b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x],x]
```

```
[Out] ((-ArcTan[Sqrt[Cos[a + b*x]]] - ArcTanh[Sqrt[Cos[a + b*x]]] + 2*Sqrt[Cos[a
+ b*x]])*(d*Cos[a + b*x])^(3/2))/(b*Cos[a + b*x]^(3/2))
```

fricas [A] time = 0.58, size = 259, normalized size = 3.36

$$\left[\frac{2\sqrt{-d}\,d\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)+\sqrt{-d}\,d\log\left(-\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6\,d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right)+8\sqrt{d}}{4\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-d)*d*arctan(2*sqrt(d*cos(b*x + a))*sqrt(-d)/(d*cos(b*x + a) +
 d)) + sqrt(-d)*d*log(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*
(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a)
 + 1)) + 8*sqrt(d*cos(b*x + a))*d)/b, 1/4*(2*d^(3/2)*arctan(2*sqrt(d*cos(b*
x + a))*sqrt(d)/(d*cos(b*x + a) - d)) + d^(3/2)*log(-(d*cos(b*x + a)^2 - 4*
sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(co
s(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*d)/b]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a), x)
```

maple [B] time = 0.17, size = 204, normalized size = 2.65

$$-\frac{d^{\frac{3}{2}}\ln\left(\frac{2\sqrt{d}\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}+4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{2b}-\frac{d^{\frac{3}{2}}\ln\left(\frac{2\sqrt{d}\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}-4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)}{2b}+\frac{2d\sqrt{-2\left(\sin^2\left(\right.\right.}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*csc(b*x+a),x)
```

```
[Out] -1/2/b*d^(3/2)*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))-1/2/b*d^(3/2)*ln(2/(cos(1/2*b*x+1/2
*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-
d))+2/b*d*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+1/(-d)^(1/2)/b*d^2*ln(2/cos(1
/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))
```

maxima [A] time = 0.44, size = 83, normalized size = 1.08

$$-\frac{2\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)-d^{\frac{5}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)-4\sqrt{d\cos\left(bx+a\right)}\,d^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*d^(5/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - d^(5/2)*log((sqrt(d*
cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 4*sqrt(d*cos(b
*x + a))*d^2)/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,\cos\left(a+b\,x\right)\right)}^{3/2}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(3/2)/sin(a + b*x),x)
```

```
[Out] int((d*cos(a + b*x))^(3/2)/sin(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a),x)
```

```
[Out] Timed out
```

3.226 $\int \sqrt{d\cos(a+bx)}\csc(a+bx)\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}$$

```
[Out] arctan((d*cos(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b-arctanh((d*cos(b*x+a))^(1/2)
/d^(1/2))*d^(1/2)/b
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2565, 329, 298, 203, 206}

$$\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x],x]
```

```
[Out] (Sqrt[d]*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/b - (Sqrt[d]*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/b
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \sqrt{d\cos(a+bx)}\,\csc(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd}$$
$$= -\frac{d\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} + \frac{d\,\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+ \right.}{b}$$
$$= \frac{\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b} - \frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.88

$$\frac{\sqrt{d\cos(a+bx)}\left(\tan^{-1}\left(\sqrt{\cos(a+bx)}\right)-\tanh^{-1}\left(\sqrt{\cos(a+bx)}\right)\right)}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x],x]
```

```
[Out] ((ArcTan[Sqrt[Cos[a + b*x]]] - ArcTanh[Sqrt[Cos[a + b*x]]])*Sqrt[d*Cos[a +
b*x]])/(b*Sqrt[Cos[a + b*x]])
```

fricas [B] time = 0.51, size = 238, normalized size = 4.10

$$\left[\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2d\cos(bx+a)}\right)+\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right)}{4b}, \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) + 1
)/(d*cos(b*x + a))) + sqrt(-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a
))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*
cos(b*x + a) + 1)))/b, 1/4*(2*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(
b*x + a) - 1)/(sqrt(d)*cos(b*x + a))) + sqrt(d)*log((d*cos(b*x + a)^2 - 4*s
qrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos
(b*x + a)^2 - 2*cos(b*x + a) + 1)))/b]
```

giac [B] time = 1.15, size = 106, normalized size = 1.83

$$\frac{\frac{2d\arctan\left(-\frac{\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2-\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\sqrt{-d}\log\left(\left|-\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2+\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4+d}\right|\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*d*arctan(-(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1
/2*a)^4 + d))/sqrt(-d))/sqrt(-d) - sqrt(-d)*log(abs(-sqrt(-d)*tan(1/2*b*x +
 1/2*a)^2 + sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))))/b
```

maple [B] time = 0.17, size = 179, normalized size = 3.09

$$-\frac{\sqrt{d}\ln\left(\frac{2\sqrt{d}\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}-4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)}{2b}-\frac{\sqrt{d}\ln\left(\frac{2\sqrt{d}\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}+4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{2b}-\frac{d\ln\left(\frac{2\sqrt{-d}\sqrt{-}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*csc(b*x+a),x)
```

```
[Out] -1/2/b*d^(1/2)*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))-1/2/b*d^(1/2)*ln(2/(cos(1/2*b*x+1/2
*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-
d))-1/(-d)^(1/2)/b*d*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/
2*a)^2*d+d)^(1/2)-d))
```

maxima [A] time = 0.44, size = 67, normalized size = 1.16

$$\frac{2\,d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)+d^{\frac{3}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*d^(3/2)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) + d^(3/2)*log((sqrt(d*c
os(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,\cos\left(a+b\,x\right)}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(1/2)/sin(a + b*x),x)
```

```
[Out] int((d*cos(a + b*x))^(1/2)/sin(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos\left(a+bx\right)}\,\csc\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a),x)
```

```
[Out] Integral(sqrt(d*cos(a + b*x))*csc(a + b*x), x)
```

3.227 $\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=59

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}$$

```
[Out] -arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(1/2)-arctanh((d*cos(b*x+a))^(1/2
)/d^(1/2))/b/d^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2565, 329, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -(ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*Sqrt[d])) - ArcTanh[Sqrt[d*Cos[a
+ b*x]]/Sqrt[d]]/(b*Sqrt[d])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx &= -\frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd}\\
&= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd}\\
&= -\frac{\mathrm{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b} - \frac{\mathrm{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{b}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{b\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.85

$$-\frac{\sqrt{\cos(a+bx)}\left(\tan^{-1}\left(\sqrt{\cos(a+bx)}\right)+\tanh^{-1}\left(\sqrt{\cos(a+bx)}\right)\right)}{b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -(((ArcTan[Sqrt[Cos[a + b*x]]] + ArcTanh[Sqrt[Cos[a + b*x]]])*Sqrt[Cos[a +
b*x]])/(b*Sqrt[d*Cos[a + b*x]]))
```

fricas [B] time = 0.52, size = 246, normalized size = 4.17

$$\left[\frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)+1)}{2d\cos(bx+a)}\right)-\sqrt{-d}\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\sqrt{-d}(\cos(bx+a)-1)-6d\cos(bx+a)+d}{\cos(bx+a)^2+2\cos(bx+a)+1}\right)}{4bd},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) + 1
)/(d*cos(b*x + a))) - sqrt(-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a
))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*
cos(b*x + a) + 1)))/(b*d), -1/4*(2*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*
(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a))) - sqrt(d)*log((d*cos(b*x + a)^2
- 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)
/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)))/(b*d)]
```

giac [B] time = 1.59, size = 105, normalized size = 1.78

$$\frac{\frac{2\arctan\left(-\frac{\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2-\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{\log\left(\left|-\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2+\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4+d}\right|\right)}{\sqrt{-d}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*arctan(-(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2
*a)^4 + d))/sqrt(-d))/sqrt(-d) - log(abs(-sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 +
 sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)))/sqrt(-d))/b
```

maple [B] time = 0.20, size = 177, normalized size = 3.00

$$-\frac{\ln\left(\frac{2\sqrt{d}\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}+4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{2\sqrt{d}\,b}-\frac{\ln\left(\frac{2\sqrt{d}\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}-4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)}{2\sqrt{d}\,b}+\frac{\ln\left(\frac{2\sqrt{-d}\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right.}{\sqrt{-d}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] -1/2/d^(1/2)/b*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))-1/2/d^(1/2)/b*ln(2/(cos(1/2*b*x+1/2
*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-
d))+1/(-d)^(1/2)/b*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*
a)^2*d+d)^(1/2)-d))
```

maxima [A] time = 0.42, size = 68, normalized size = 1.15

$$-\frac{2\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)-\sqrt{d}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(d)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - sqrt(d)*log((sqrt(d*
cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(a+b\,x\right)\,\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(1/2)),x)
```

```
[Out] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(a+bx\right)}{\sqrt{d\cos\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)/sqrt(d*cos(a + b*x)), x)
```

3.228 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}}+\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)-arctanh((d*cos(b*x+a))^(1/2)
/d^(1/2))/b/d^(3/2)+2/b/d/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 325, 329, 298, 203, 206}

$$\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}}+\frac{2}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(3/2)) - ArcTanh[Sqrt[d*Cos[a + b
*x]]/Sqrt[d]]/(b*d^(3/2)) + 2/(b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2}{bd\sqrt{d\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd^3}$$

$$= \frac{2}{bd\sqrt{d\cos(a+bx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^3}$$

$$= \frac{2}{bd\sqrt{d\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd} + \frac{\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{d}\right.}{bd}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{3/2}} + \frac{2}{bd\sqrt{d\cos(a+bx)}}$$

Mathematica [C] time = 0.05, size = 36, normalized size = 0.46

$$\frac{2\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\cos^2(a+bx)\right)}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*Hypergeometric2F1[-1/4, 1, 3/4, Cos[a + b*x]^2])/(b*d*Sqrt[d*Cos[a + b*x
]])
```

fricas [B] time = 0.51, size = 309, normalized size = 3.96

$$\left[\frac{2\sqrt{-d}\,\arctan\left(\frac{\sqrt{d\cos(bx+a)}\,\sqrt{-d}\,(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)\cos(bx+a) - \sqrt{-d}\,\cos(bx+a)\log\left(\frac{d\cos(bx+a)^2-4\,\sqrt{d\cos(bx+a)}\,\sqrt{-d}\,(\cos}{\cos(bx+a)^2+2\,\cos(}\right.}{4\,bd^2\cos(bx+a)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) + 1
)/(d*cos(b*x + a)))*cos(b*x + a) - sqrt(-d)*cos(b*x + a)*log((d*cos(b*x + a
)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a)
 + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a)))/(b*d
^2*cos(b*x + a)), 1/4*(2*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x +
 a) - 1)/(sqrt(d)*cos(b*x + a)))*cos(b*x + a) + sqrt(d)*cos(b*x + a)*log((d
*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*c
os(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x
+ a)))/(b*d^2*cos(b*x + a))]
```

giac [B] time = 1.12, size = 156, normalized size = 2.00

$$\frac{\frac{2\arctan\left(-\frac{\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}+\frac{\log\left(\left|-\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}+\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}\right|\right)}{\sqrt{-d}}-\frac{8}{\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\sqrt{-d\tan\cdots}}}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*arctan(-(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2
*a)^4 + d))/sqrt(-d))/sqrt(-d) + log(abs(-sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 +
 sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)))/sqrt(-d) - 8/(sqrt(-d)*tan(1/2*b*x +
 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d) - sqrt(-d)))/(b*d)
```

maple [B] time = 0.33, size = 422, normalized size = 5.41

$$-\frac{\left(4\ln\left(\frac{2\sqrt{-d}\,\sqrt{-2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)d^{\frac{5}{2}}+2\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}+4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)\sqrt{-d}\,d^{2}+2\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin\cdots\right.}}{\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] 1/2*(-(4*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^
(1/2)-d))*d^(5/2)+2*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/
2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^2+2*ln(2/(cos(1/2
*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x
+1/2*a)-d))*(-d)^(1/2)*d^2)*sin(1/2*b*x+1/2*a)^2+2*ln(2/cos(1/2*b*x+1/2*a)*
((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(5/2)-4*(-2*sin(1/2*b
```

```
*x+1/2*a)^2*d+d)^(1/2)*d^(3/2)*(-d)^(1/2)+ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1
/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/
2)*d^2+ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(
1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^2)/d^(7/2)/(-d)^(1/2)/(2*sin(1
/2*b*x+1/2*a)^2-1)/b
```

maxima [A] time = 0.55, size = 79, normalized size = 1.01

$$\frac{\frac{2\arctan\left(\frac{\sqrt{d\cos\left(bx+a\right)}}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{\log\left(\frac{\sqrt{d\cos\left(bx+a\right)}-\sqrt{d}}{\sqrt{d\cos\left(bx+a\right)}+\sqrt{d}}\right)}{\sqrt{d}}+\frac{4}{\sqrt{d\cos\left(bx+a\right)}}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/sqrt(d) + log((sqrt(d*cos(b*x +
 a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/sqrt(d) + 4/sqrt(d*cos(b*
x + a)))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)\,{\left(d\,\cos\left(a+b\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(3/2)),x)
```

```
[Out] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc{\left(a + b x \right)}}{\left(d \cos{\left(a + b x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)/(d*cos(a + b*x))**(3/2), x)
```

3.229 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}}+\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

```
[Out] -arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)-arctanh((d*cos(b*x+a))^(1/2
)/d^(1/2))/b/d^(5/2)+2/3/b/d/(d*cos(b*x+a))^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 325, 329, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}}+\frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] -(ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(5/2))) - ArcTanh[Sqrt[d*Cos[a
+ b*x]]/Sqrt[d]]/(b*d^(5/2)) + 2/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2}{3bd(d\cos(a+bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd^3}$$

$$= \frac{2}{3bd(d\cos(a+bx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^3}$$

$$= \frac{2}{3bd(d\cos(a+bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^2} - \frac{\text{Subst}\left(\int \frac{1}{d+x^2}\,dx, x, \sqrt{}\right.}{bd^2}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{5/2}} + \frac{2}{3bd(d\cos(a+bx))^{3/2}}$$

Mathematica [C] time = 0.05, size = 38, normalized size = 0.47

$$\frac{2\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};\cos^2(a+bx)\right)}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*Hypergeometric2F1[-3/4, 1, 1/4, Cos[a + b*x]^2])/(3*b*d*(d*Cos[a + b*x])
^(3/2))
```

fricas [B] time = 0.51, size = 318, normalized size = 3.93

$$\left[\frac{6\sqrt{-d}\,\arctan\left(\frac{\sqrt{d\cos(bx+a)}\,\sqrt{-d}\,(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)\cos(bx+a)^2 - 3\sqrt{-d}\,\cos(bx+a)^2\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}\,\sqrt{-d}\,(}{\cos(bx+a)^2+2\,c}\right.}{12\,bd^3\cos(bx+a)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) +
1)/(d*cos(b*x + a)))*cos(b*x + a)^2 - 3*sqrt(-d)*cos(b*x + a)^2*log((d*cos(
b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b
*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a)
))/(b*d^3*cos(b*x + a)^2), -1/12*(6*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))
*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)))*cos(b*x + a)^2 - 3*sqrt(d)*cos(
b*x + a)^2*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x
+ a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) -
8*sqrt(d*cos(b*x + a)))/(b*d^3*cos(b*x + a)^2)]
```

giac [B] time = 1.29, size = 203, normalized size = 2.51

$$\frac{\frac{6\arctan\left(-\frac{\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{3\log\left(\left|-\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}+\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}\right|\right)}{\sqrt{-d}}+\frac{8\left(3\left(\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\cdots\right.\right.}{\left(\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\sqrt{\cdots}\right.}}{6bd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(6*arctan(-(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2
*a)^4 + d))/sqrt(-d))/sqrt(-d) - 3*log(abs(-sqrt(-d)*tan(1/2*b*x + 1/2*a)^2
 + sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)))/sqrt(-d) + 8*(3*(sqrt(-d)*tan(1/2*
b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^2 - d)/(sqrt(-d)*tan(
1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d) - sqrt(-d))^3)/(b*
d^2)
```

maple [B] time = 0.31, size = 624, normalized size = 7.70

$$\left(24\ln\left(\frac{2\sqrt{-d}\,\sqrt{-2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)d^{\frac{7}{2}}-12\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}+4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)\sqrt{-d}\,d^{3}-12\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\cdots}}{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 1/6*((24*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^
(1/2)-d))*d^(7/2)-12*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1
/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3-12*ln(2/(cos(1
```

```
/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b
*x+1/2*a)-d))*(-d)^(1/2)*d^3)*sin(1/2*b*x+1/2*a)^4+(-24*ln(2/cos(1/2*b*x+1/
2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(7/2)+12*ln(2/(c
os(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1
/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3+12*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-
2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3
)*sin(1/2*b*x+1/2*a)^2+6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*
x+1/2*a)^2*d+d)^(1/2)-d))*d^(7/2)+4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(
5/2)*(-d)^(1/2)-3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*
a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3-3*ln(2/(cos(1/2*b
*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1
/2*a)-d))*(-d)^(1/2)*d^3)/d^(11/2)/(-d)^(1/2)/(4*sin(1/2*b*x+1/2*a)^4-4*sin
(1/2*b*x+1/2*a)^2+1)/b
```

maxima [A] time = 0.47, size = 80, normalized size = 0.99

$$-\frac{\frac{6 \arctan\left(\frac{\sqrt{d}\cos(bx+a)}{\sqrt{d}}\right)}{d^{\frac{3}{2}}} - \frac{3 \log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{3}{2}}} - \frac{4}{(d\cos(bx+a))^{\frac{3}{2}}}}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*(6*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/d^(3/2) - 3*log((sqrt(d*cos(b*
x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/d^(3/2) - 4/(d*cos(b*x
 + a))^(3/2))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(a+b\,x)\,\left(d\,\cos(a+b\,x)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(5/2)),x)
```

```
[Out] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.230 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}}+\frac{2}{bd^3\sqrt{d\cos(a+bx)}}+\frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

```
[Out] arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(7/2)-arctanh((d*cos(b*x+a))^(1/2)
/d^(1/2))/b/d^(7/2)+2/5/b/d/(d*cos(b*x+a))^(5/2)+2/b/d^3/(d*cos(b*x+a))^(1/
2)
```

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 325, 329, 298, 203, 206}

$$\frac{2}{bd^3\sqrt{d\cos(a+bx)}}+\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}}+\frac{2}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(7/2)) - ArcTanh[Sqrt[d*Cos[a + b
*x]]/Sqrt[d]]/(b*d^(7/2)) + 2/(5*b*d*(d*Cos[a + b*x])^(5/2)) + 2/(b*d^3*Sqr
t[d*Cos[a + b*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^{7/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} - \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd^3}$$

$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd^5}$$

$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^5}$$

$$= \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{bd^3} -$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{7/2}} + \frac{2}{5bd(d\cos(a+bx))^{5/2}} + \frac{2}{bd^3\sqrt{d\cos(a-}}$$

Mathematica [C] time = 0.06, size = 38, normalized size = 0.38

$$\frac{2\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};\cos^2(a+bx)\right)}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*Hypergeometric2F1[-5/4, 1, -1/4, Cos[a + b*x]^2])/(5*b*d*(d*Cos[a + b*x]
)^(5/2))
```

fricas [B] time = 0.50, size = 342, normalized size = 3.42

$$\left[\frac{10\sqrt{-d}\,\arctan\left(\frac{\sqrt{d\cos(bx+a)}\,\sqrt{-d}\,(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)\cos(bx+a)^3 - 5\sqrt{-d}\,\cos(bx+a)^3\log\left(\frac{d\cos(bx+a)^2-4\sqrt{d\cos(bx+a)}\,\sqrt{-}}{\cos(bx+a)^2+}\right.}{20\,bd^4\cos(bx+a)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] [1/20*(10*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) +
 1)/(d*cos(b*x + a)))*cos(b*x + a)^3 - 5*sqrt(-d)*cos(b*x + a)^3*log((d*cos
(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(
b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a
))*(5*cos(b*x + a)^2 + 1))/(b*d^4*cos(b*x + a)^3), 1/20*(10*sqrt(d)*arctan(
1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)))*cos(b*x
 + a)^3 + 5*sqrt(d)*cos(b*x + a)^3*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x
 + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 -
 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 + 1))/(b*d
^4*cos(b*x + a)^3)]
```

giac [B] time = 1.46, size = 341, normalized size = 3.41

$$\frac{10\arctan\left(-\frac{\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2-\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}+\frac{5\log\left(\left|-\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2+\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4+d}\right|\right)}{\sqrt{-d}}-\frac{16\left(5\left(\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] 1/10*(10*arctan(-(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1
/2*a)^4 + d))/sqrt(-d))/sqrt(-d) + 5*log(abs(-sqrt(-d)*tan(1/2*b*x + 1/2*a)
^2 + sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)))/sqrt(-d) - 16*(5*(sqrt(-d)*tan(1
/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^4 - 10*(sqrt(-d)*t
an(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^3*sqrt(-d) - 2
0*(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^2
*d + 10*(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 +
 d))*sqrt(-d)*d + 3*d^2)/(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2
*b*x + 1/2*a)^4 + d) - sqrt(-d))^5)/(b*d^3)
```

maple [B] time = 0.32, size = 882, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] 1/10/d^(15/2)/(-d)^(1/2)/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*
sin(1/2*b*x+1/2*a)^2-1)*(10*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2
*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)-24*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
*d^(7/2)*(-d)^(1/2)+5*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+
1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4+5*ln(2/(cos(1
/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b
*x+1/2*a)-d))*(-d)^(1/2)*d^4-40*(2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*
sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)+ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^
(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(
1/2)*d^4+ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)
^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4)*sin(1/2*b*x+1/2*a)^6+20*(
6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d
))*d^(9/2)-4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(7/2)*(-d)^(1/2)+3*ln(2/
(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos
(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4+3*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^
4)*sin(1/2*b*x+1/2*a)^4-10*(6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1
/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)-8*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2
)*d^(7/2)*(-d)^(1/2)+3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x
+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4+3*ln(2/(cos(
1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*
b*x+1/2*a)-d))*(-d)^(1/2)*d^4)*sin(1/2*b*x+1/2*a)^2)/b
```

maxima [A] time = 0.47, size = 100, normalized size = 1.00

$$\frac{\frac{10\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{5\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{4\left(5\,d^2\cos(bx+a)^2+d^2\right)}{(d\cos(bx+a))^{\frac{5}{2}}d^2}}{10\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/10*(10*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/d^(5/2) + 5*log((sqrt(d*cos(b
*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/d^(5/2) + 4*(5*d^2*co
s(b*x + a)^2 + d^2)/((d*cos(b*x + a))^(5/2)*d^2))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)\,{\left(d\,\cos\left(a+b\,x\right)\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(7/2)),x)
```

```
[Out] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

3.231 $\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}}+\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}+\frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

```
[Out] -arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(9/2)-arctanh((d*cos(b*x+a))^(1/2
)/d^(1/2))/b/d^(9/2)+2/7/b/d/(d*cos(b*x+a))^(7/2)+2/3/b/d^3/(d*cos(b*x+a))^
(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2565, 325, 329, 212, 206, 203}

$$\frac{2}{3bd^3(d\cos(a+bx))^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}}+\frac{2}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] -(ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]]/(b*d^(9/2))) - ArcTanh[Sqrt[d*Cos[a
+ b*x]]/Sqrt[d]]/(b*d^(9/2)) + 2/(7*b*d*(d*Cos[a + b*x])^(7/2)) + 2/(3*b*d^
3*(d*Cos[a + b*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\csc(a+bx)}{(d\cos(a+bx))^{9/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^{9/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= \frac{2}{7bd(d\cos(a+bx))^{7/2}} - \frac{\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{bd^3}$$

$$= \frac{2}{7bd(d\cos(a+bx))^{7/2}} + \frac{2}{3bd^3(d\cos(a+bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+\right.}{bd^5}$$

$$= \frac{2}{7bd(d\cos(a+bx))^{7/2}} + \frac{2}{3bd^3(d\cos(a+bx))^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+b}\right.}{bd^5}$$

$$= \frac{2}{7bd(d\cos(a+bx))^{7/2}} + \frac{2}{3bd^3(d\cos(a+bx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right.}{bd^4}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{bd^{9/2}} + \frac{2}{7bd(d\cos(a+bx))^{7/2}} + \frac{2}{3bd^3(d\cos(}$$

Mathematica [C] time = 0.07, size = 38, normalized size = 0.37

$$\frac{2\,{}_2F_1\left(-\frac{7}{4},1;-\frac{3}{4};\cos^2(a+bx)\right)}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*Hypergeometric2F1[-7/4, 1, -3/4, Cos[a + b*x]^2])/(7*b*d*(d*Cos[a + b*x]
)^(7/2))
```

fricas [A] time = 0.52, size = 342, normalized size = 3.32

$$\left[\frac{42\sqrt{-d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}\,(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)\cos(bx+a)^4 - 21\sqrt{-d}\cos(bx+a)^4\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos(bx+a)}}{\cos(bx+a)^2}\right.}{84\,bd^5\cos(bx+a)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] [1/84*(42*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) +
 1)/(d*cos(b*x + a)))*cos(b*x + a)^4 - 21*sqrt(-d)*cos(b*x + a)^4*log((d*co
s(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos
(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x +
a))*(7*cos(b*x + a)^2 + 3))/(b*d^5*cos(b*x + a)^4), -1/84*(42*sqrt(d)*arcta
n(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)))*cos(b
*x + a)^4 - 21*sqrt(d)*cos(b*x + a)^4*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(
b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^
2 - 2*cos(b*x + a) + 1)) - 8*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 + 3))/(
b*d^5*cos(b*x + a)^4)]
```

giac [B] time = 1.38, size = 436, normalized size = 4.23

$$\frac{\frac{42\arctan\left(-\frac{\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{21\log\left(\left|-\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}+\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}\right|\right)}{\sqrt{-d}}+\frac{16\left(21\left(\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right.\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] 1/42*(42*arctan(-(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1
/2*a)^4 + d))/sqrt(-d))/sqrt(-d) - 21*log(abs(-sqrt(-d)*tan(1/2*b*x + 1/2*a
)^2 + sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)))/sqrt(-d) + 16*(21*(sqrt(-d)*tan
(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^6 - 42*(sqrt(-d)
*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^5*sqrt(-d) -
 119*(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)
)^4*d + 56*(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^
4 + d))^3*sqrt(-d)*d + 63*(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/
2*b*x + 1/2*a)^4 + d))^2*d^2 - 14*(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-
d*tan(1/2*b*x + 1/2*a)^4 + d))*sqrt(-d)*d^2 - 5*d^3)/(sqrt(-d)*tan(1/2*b*x
+ 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d) - sqrt(-d))^7)/(b*d^4)
```

maple [B] time = 0.37, size = 1086, normalized size = 10.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 1/42/d^(19/2)/(-d)^(1/2)/(16*sin(1/2*b*x+1/2*a)^8-32*sin(1/2*b*x+1/2*a)^6+2
4*sin(1/2*b*x+1/2*a)^4-8*sin(1/2*b*x+1/2*a)^2+1)*(42*ln(2/cos(1/2*b*x+1/2*a
)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(11/2)+40*(-2*sin(1
/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(9/2)*(-d)^(1/2)-21*ln(2/(cos(1/2*b*x+1/2*a)-1
)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(
-d)^(1/2)*d^5-21*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a
)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^5+336*(2*ln(2/cos(1/
2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(11/2)-l
n(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d
*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^5-ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)
*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*
d^5)*sin(1/2*b*x+1/2*a)^8-672*(2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*si
n(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(11/2)-ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(
1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1
/2)*d^5-ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^
(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^5)*sin(1/2*b*x+1/2*a)^6-56*(6
*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d)
)*d^(11/2)+2*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(9/2)*(-d)^(1/2)-3*ln(2/
(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos
(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^5-3*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^
5)*sin(1/2*b*x+1/2*a)^2+56*(18*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(
1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(11/2)+2*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1
/2)*d^(9/2)*(-d)^(1/2)-9*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b
*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^5-9*ln(2/(co
s(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/
2*b*x+1/2*a)-d))*(-d)^(1/2)*d^5)*sin(1/2*b*x+1/2*a)^4)/b
```

maxima [A] time = 0.58, size = 102, normalized size = 0.99

$$-\frac{\frac{42\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{7}{2}}}-\frac{21\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{7}{2}}}-\frac{4\left(7d^2\cos(bx+a)^2+3d^2\right)}{(d\cos(bx+a))^{\frac{7}{2}}d^2}}{42bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] -1/42*(42*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/d^(7/2) - 21*log((sqrt(d*cos
(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/d^(7/2) - 4*(7*d^2*
cos(b*x + a)^2 + 3*d^2)/((d*cos(b*x + a))^(7/2)*d^2))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(a+b\,x)\,\left(d\,\cos(a+b\,x)\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(9/2)),x)
[Out] int(1/(sin(a + b*x)*(d*cos(a + b*x))^(9/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/(d*cos(b*x+a))**(9/2),x)
[Out] Timed out
```

3.232 $\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx$

Optimal. Leaf size=124

$$-\frac{15d^6\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7b\sqrt{d\cos(a+bx)}}-\frac{15d^5\sin(a+bx)\sqrt{d\cos(a+bx)}}{7b}-\frac{9d^3\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}-\frac{d\csc$$

```
[Out] -d*(d*cos(b*x+a))^(9/2)*csc(b*x+a)/b-9/7*d^3*(d*cos(b*x+a))^(5/2)*sin(b*x+a
)/b-15/7*d^6*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(
1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)-15/7*d^5*si
n(b*x+a)*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2567, 2635, 2642, 2641}

$$-\frac{15d^5\sin(a+bx)\sqrt{d\cos(a+bx)}}{7b}-\frac{9d^3\sin(a+bx)(d\cos(a+bx))^{5/2}}{7b}-\frac{15d^6\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7b\sqrt{d\cos(a+bx)}}-\frac{d\csc$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(d*Cos[a + b*x])^(9/2)*Csc[a + b*x])/b) - (15*d^6*Sqrt[Cos[a + b*x]]*E
llipticF[(a + b*x)/2, 2])/(7*b*Sqrt[d*Cos[a + b*x]]) - (15*d^5*Sqrt[d*Cos[a
 + b*x]]*Sin[a + b*x])/(7*b) - (9*d^3*(d*Cos[a + b*x])^(5/2)*Sin[a + b*x])/
(7*b)
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{11/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{1}{2}\left(9d^2\right)\int (d\cos(a+bx))^{7/2}\,dx$$

$$= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{9d^3(d\cos(a+bx))^{5/2}\sin(a+bx)}{7b} - \frac{1}{14}$$

$$= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{15d^5\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{7b} - \frac{9d^3}{}$$

$$= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{15d^5\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{7b} - \frac{9d^3}{}$$

$$= -\frac{d(d\cos(a+bx))^{9/2}\csc(a+bx)}{b} - \frac{15d^6\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{7b\sqrt{d\cos(a+bx)}} - 1$$

Mathematica [A] time = 0.36, size = 89, normalized size = 0.72

$$\frac{d^5\csc(a+bx)\sqrt{d\cos(a+bx)}\left(\sqrt{\cos(a+bx)}\,(16\cos(2(a+bx))+\cos(4(a+bx))-45)-60\sin(a+bx)F\left(\frac{1}{2}(a+\right.\right.}{28b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^2,x]
```

```
[Out] (d^5*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]*(Sqrt[Cos[a + b*x]]*(-45 + 16*Cos[2*
(a + b*x)] + Cos[4*(a + b*x)]) - 60*EllipticF[(a + b*x)/2, 2]*Sin[a + b*x])
)/(28*b*Sqrt[Cos[a + b*x]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\cos(bx+a)}\,d^5\cos(bx+a)^5\csc(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*d^5*cos(b*x + a)^5*csc(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{11}{2}} \csc(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(11/2)*csc(b*x + a)^2, x)
```

maple [A] time = 0.28, size = 242, normalized size = 1.95

$$-\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d^7 \sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-128\left(\sin^{12}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+384\left(\sin^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-57\cdots\right.}{14\left(-2\left(\sin\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x)
```

```
[Out] -1/14*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^7/(-2*sin
(1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/
2*b*x+1/2*a)*(-128*sin(1/2*b*x+1/2*a)^12+384*sin(1/2*b*x+1/2*a)^10-576*sin(
1/2*b*x+1/2*a)^8+30*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)
^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)+512*sin(1/2
*b*x+1/2*a)^6-204*sin(1/2*b*x+1/2*a)^4+12*sin(1/2*b*x+1/2*a)^2+7)/(d*(2*cos
(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{11}{2}} \csc(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(11/2)*csc(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\cos(a+bx))^{11/2}}{\sin(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(11/2)/sin(a + b*x)^2,x)
[Out] int((d*cos(a + b*x))^(11/2)/sin(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(11/2)*csc(b*x+a)**2,x)
[Out] Timed out
```

3.233 $\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx$

Optimal. Leaf size=96

$$-\frac{21d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}-\frac{7d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}-\frac{d\csc(a+bx)(d\cos(a+bx))^{7/2}}{b}$$

```
[Out] -d*(d*cos(b*x+a))^(7/2)*csc(b*x+a)/b-7/5*d^3*(d*cos(b*x+a))^(3/2)*sin(b*x+a
)/b-21/5*d^4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(
1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2567, 2635, 2640, 2639}

$$-\frac{7d^3\sin(a+bx)(d\cos(a+bx))^{3/2}}{5b}-\frac{21d^4E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{7/2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(d*Cos[a + b*x])^(7/2)*Csc[a + b*x])/b) - (21*d^4*Sqrt[d*Cos[a + b*x]]
*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*x]]) - (7*d^3*(d*Cos[a + b*
x])^(3/2)*Sin[a + b*x])/(5*b)
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{9/2}\csc^2(a+bx)\,dx &= -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b}-\frac{1}{2}\left(7d^2\right)\int (d\cos(a+bx))^{5/2}\,dx\\ &= -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b}-\frac{7d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{5b}-\frac{1}{10}\Big(\\ &= -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b}-\frac{7d^3(d\cos(a+bx))^{3/2}\sin(a+bx)}{5b}-\frac{\big(21}{}\\ &= -\frac{d(d\cos(a+bx))^{7/2}\csc(a+bx)}{b}-\frac{21d^4\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}}-\end{aligned}$$

Mathematica [A] time = 0.24, size = 74, normalized size = 0.77

$$-\frac{d^4\sqrt{d\cos(a+bx)}\left(21E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\,(\sin(2(a+bx))+5\cot(a+bx))\right)}{5b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^2,x]
```

```
[Out] -1/5*(d^4*Sqrt[d*Cos[a + b*x]]*(21*EllipticE[(a + b*x)/2, 2] + Sqrt[Cos[a +
 b*x]]*(5*Cot[a + b*x] + Sin[2*(a + b*x)])))/(b*Sqrt[Cos[a + b*x]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{d\cos\left(bx+a\right)}\,d^4\cos\left(bx+a\right)^4\csc\left(bx+a\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*d^4*cos(b*x + a)^4*csc(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a)^2, x)
```

maple [B] time = 0.27, size = 229, normalized size = 2.39

$$\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^6\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-64\left(\sin^{10}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+160\left(\sin^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+42\left(2\right.\right.}{10\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x)
```

```
[Out] 1/10*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^6/(-2*sin(
1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2
*b*x+1/2*a)*(-64*sin(1/2*b*x+1/2*a)^10+160*sin(1/2*b*x+1/2*a)^8+42*(2*sin(1
/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x
+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)-104*sin(1/2*b*x+1/2*a)^6-4*sin(1/2*b*x+
1/2*a)^4+22*sin(1/2*b*x+1/2*a)^2-5)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\cos(a+b\,x))^{9/2}}{\sin(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(9/2)/sin(a + b*x)^2,x)
```

```
[Out] int((d*cos(a + b*x))^(9/2)/sin(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.234 $\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx$

Optimal. Leaf size=96

$$-\frac{5d^4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{d\cos(a+bx)}}-\frac{5d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}-\frac{d\csc(a+bx)(d\cos(a+bx))^{5/2}}{b}$$

```
[Out] -d*(d*cos(b*x+a))^(5/2)*csc(b*x+a)/b-5/3*d^4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/c
os(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/
(d*cos(b*x+a))^(1/2)-5/3*d^3*sin(b*x+a)*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2567, 2635, 2642, 2641}

$$-\frac{5d^3\sin(a+bx)\sqrt{d\cos(a+bx)}}{3b}-\frac{5d^4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{d\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{5/2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(d*Cos[a + b*x])^(5/2)*Csc[a + b*x])/b) - (5*d^4*Sqrt[Cos[a + b*x]]*El
lipticF[(a + b*x)/2, 2])/(3*b*Sqrt[d*Cos[a + b*x]]) - (5*d^3*Sqrt[d*Cos[a +
 b*x]]*Sin[a + b*x])/(3*b)
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{7/2}\csc^2(a+bx)\,dx &= -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b} - \frac{1}{2}\left(5d^2\right)\int (d\cos(a+bx))^{3/2}\,dx\\ &= -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b} - \frac{5d^3\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{3b} - \frac{1}{6}\Big(5d\\ &= -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b} - \frac{5d^3\sqrt{d\cos(a+bx)}\,\sin(a+bx)}{3b} - \frac{\big(5d^4}{}\\ &= -\frac{d(d\cos(a+bx))^{5/2}\csc(a+bx)}{b} - \frac{5d^4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{d\cos(a+bx)}} - \frac{5d^3}{}\end{aligned}$$

Mathematica [A] time = 0.22, size = 73, normalized size = 0.76

$$\frac{d^3\sqrt{d\cos(a+bx)}\left(\sqrt{\cos(a+bx)}\,(\cos(2(a+bx))-4)\csc(a+bx)-5F\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{3b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^2,x]
```

```
[Out] (d^3*Sqrt[d*Cos[a + b*x]]*(Sqrt[Cos[a + b*x]]*(-4 + Cos[2*(a + b*x)])*Csc[a
 + b*x] - 5*EllipticF[(a + b*x)/2, 2]))/(3*b*Sqrt[Cos[a + b*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\cos(bx+a)}\,d^3\cos(bx+a)^3\csc(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*d^3*cos(b*x + a)^3*csc(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a)^2, x)
```

maple [A] time = 0.24, size = 216, normalized size = 2.25

$$-\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,d^5\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(-32\left(\sin^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+10\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^{\frac{3}{2}}\right.}{6\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+\left(\sin^2\left(\frac{bx}{2}+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x)
```

```
[Out] -1/6*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^5/(-2*sin(
1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2
*b*x+1/2*a)*(-32*sin(1/2*b*x+1/2*a)^8+10*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(
sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b
*x+1/2*a)+64*sin(1/2*b*x+1/2*a)^6-28*sin(1/2*b*x+1/2*a)^4-4*sin(1/2*b*x+1/2
*a)^2+3)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\cos(a+b\,x))^{7/2}}{\sin(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(7/2)/sin(a + b*x)^2,x)
```

```
[Out] int((d*cos(a + b*x))^(7/2)/sin(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(7/2)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.235 $\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{3d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{3/2}}{b}$$

```
[Out] -d*(d*cos(b*x+a))^(3/2)*csc(b*x+a)/b-3*d^2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos
(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/
b/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2567, 2640, 2639}

$$-\frac{3d^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}-\frac{d\csc(a+bx)(d\cos(a+bx))^{3/2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(d*Cos[a + b*x])^(3/2)*Csc[a + b*x])/b) - (3*d^2*Sqrt[d*Cos[a + b*x]]*
EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\int (d\cos(a+bx))^{5/2}\csc^2(a+bx)\,dx = -\frac{d(d\cos(a+bx))^{3/2}\csc(a+bx)}{b} - \frac{1}{2}\left(3d^2\right)\int\sqrt{d\cos(a+bx)}\,dx$$
$$= -\frac{d(d\cos(a+bx))^{3/2}\csc(a+bx)}{b} - \frac{\left(3d^2\sqrt{d\cos(a+bx)}\right)\int\sqrt{\cos(a+bx)}\,d}{2\sqrt{\cos(a+bx)}}$$
$$= -\frac{d(d\cos(a+bx))^{3/2}\csc(a+bx)}{b} - \frac{3d^2\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}$$

Mathematica [A] time = 0.13, size = 58, normalized size = 0.88

$$-\frac{(d\cos(a+bx))^{5/2}\left(3E\left(\frac{1}{2}(a+bx)\middle|2\right)+\cos^{\frac{3}{2}}(a+bx)\csc(a+bx)\right)}{b\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^2,x]
```

```
[Out] -(((d*Cos[a + b*x])^(5/2)*(Cos[a + b*x]^(3/2)*Csc[a + b*x] + 3*EllipticE[(a
 + b*x)/2, 2]))/(b*Cos[a + b*x]^(5/2)))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\cos(bx+a)}\,d^2\cos(bx+a)^2\csc(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*d^2*cos(b*x + a)^2*csc(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a)^2, x)
```

maple [B] time = 0.25, size = 203, normalized size = 3.08

$$\frac{\sqrt{d\left(2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d^{4}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(6\left(2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^{\frac{3}{2}}\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\ \mathrm{EllipticE}}{2\left(-2\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d\right)^{\frac{3}{2}}\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x)
```

```
[Out] 1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^4/(-2*sin(1
/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2*
b*x+1/2*a)*(6*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)
*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)+8*sin(1/2*b*x+1/2
*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/(d*(2*cos(1/2*b*x+1
/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,\cos(a+b\,x))^{5/2}}{\sin(a+b\,x)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(5/2)/sin(a + b*x)^2,x)
```

```
[Out] int((d*cos(a + b*x))^(5/2)/sin(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.236 $\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{d^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{d\cos(a+bx)}}-\frac{d\csc(a+bx)\sqrt{d\cos(a+bx)}}{b}$$

```
[Out] -d^2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/
2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)-d*csc(b*x+a)*(d*cos
(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2567, 2642, 2641}

$$-\frac{d^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{d\cos(a+bx)}}-\frac{d\csc(a+bx)\sqrt{d\cos(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^2,x]
```

```
[Out] -((d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x])/b) - (d^2*Sqrt[Cos[a + b*x]]*Ellipt
icF[(a + b*x)/2, 2])/(b*Sqrt[d*Cos[a + b*x]])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{3/2}\csc^2(a+bx)\,dx = -\frac{d\sqrt{d\cos(a+bx)}\,\csc(a+bx)}{b} - \frac{1}{2}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx$$
$$= -\frac{d\sqrt{d\cos(a+bx)}\,\csc(a+bx)}{b} - \frac{\left(d^2\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{2\sqrt{d\cos(a+bx)}}$$
$$= -\frac{d\sqrt{d\cos(a+bx)}\,\csc(a+bx)}{b} - \frac{d^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{d\cos(a+bx)}}$$

Mathematica [A] time = 0.09, size = 56, normalized size = 0.85

$$-\frac{(d\cos(a+bx))^{3/2}\left(F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\,\csc(a+bx)\right)}{b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^2,x]
```

```
[Out] -(((d*Cos[a + b*x])^(3/2)*(Sqrt[Cos[a + b*x]]*Csc[a + b*x] + EllipticF[(a +
 b*x)/2, 2]))/(b*Cos[a + b*x]^(3/2)))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{d\cos(bx+a)}\,d\cos(bx+a)\csc(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*d*cos(b*x + a)*csc(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^2, x)
```

maple [B] time = 0.26, size = 190, normalized size = 2.88

$$-\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d^3 \sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^{\frac{3}{2}}\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\ \mathrm{EllipticF}}{2\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d\right)^{\frac{3}{2}}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x)
```

```
[Out] -1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^3/(-2*sin(
1/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2
*b*x+1/2*a)*(2*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2
)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)+4*sin(1/2*b*x+1/
2*a)^4-4*sin(1/2*b*x+1/2*a)^2+1)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,\cos(a+b\,x))^{3/2}}{\sin(a+b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(3/2)/sin(a + b*x)^2,x)
```

```
[Out] int((d*cos(a + b*x))^(3/2)/sin(a + b*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.237 $\int \sqrt{d\cos(a+bx)}\csc^2(a+bx)\,dx$

Optimal. Leaf size=65

$$-\frac{\csc(a+bx)(d\cos(a+bx))^{3/2}}{bd}-\frac{E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

[Out] -(d*cos(b*x+a))^(3/2)*csc(b*x+a)/b/d-(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2570, 2640, 2639}

$$-\frac{\csc(a+bx)(d\cos(a+bx))^{3/2}}{bd}-\frac{E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^2,x]

[Out] -(((d*Cos[a + b*x])^(3/2)*Csc[a + b*x])/(b*d)) - (Sqrt[d*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{d\cos(a+bx)}\,\csc^2(a+bx)\,dx &= -\frac{(d\cos(a+bx))^{3/2}\csc(a+bx)}{bd} - \frac{1}{2}\int \sqrt{d\cos(a+bx)}\,dx\\
&= -\frac{(d\cos(a+bx))^{3/2}\csc(a+bx)}{bd} - \frac{\sqrt{d\cos(a+bx)}\int \sqrt{\cos(a+bx)}\,dx}{2\sqrt{\cos(a+bx)}}\\
&= -\frac{(d\cos(a+bx))^{3/2}\csc(a+bx)}{bd} - \frac{\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 56, normalized size = 0.86

$$
-\frac{\sqrt{d\cos(a+bx)}\left(E\left(\frac{1}{2}(a+bx)\middle|2\right)+\cos^{\frac{3}{2}}(a+bx)\csc(a+bx)\right)}{b\sqrt{\cos(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^2,x]
```

```
[Out] -((Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^(3/2)*Csc[a + b*x] + EllipticE[(a + b
*x)/2, 2]))/(b*Sqrt[Cos[a + b*x]]))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\sqrt{d\cos(bx+a)}\,\csc(bx+a)^2, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sqrt{d\cos(bx+a)}\,\csc(bx+a)^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^2, x)
```

maple [B] time = 0.36, size = 203, normalized size = 3.12

$$\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d^2 \sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^{\frac{3}{2}}\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\ \mathrm{EllipticE}}{2\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d\right)^{\frac{3}{2}}\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x)
```

```
[Out] 1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*d^2/(-2*sin(1
/2*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*sin(1/2*
b*x+1/2*a)*(2*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)
*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)+8*sin(1/2*b*x+1/2
*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/(d*(2*cos(1/2*b*x+1
/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos(bx+a)}\,\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,\cos(a+b\,x)}}{\sin(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(1/2)/sin(a + b*x)^2,x)
```

```
[Out] int((d*cos(a + b*x))^(1/2)/sin(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos(a+bx)}\,\csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(d*cos(a + b*x))*csc(a + b*x)**2, x)
```

3.238 $\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{d\cos(a+bx)}}-\frac{\csc(a+bx)\sqrt{d\cos(a+bx)}}{bd}$$

```
[Out] (cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x
),2^(1/2))*cos(b*x+a)^(1/2)/b/(d*cos(b*x+a))^(1/2)-csc(b*x+a)*(d*cos(b*x+a)
)^(1/2)/b/d
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2570, 2642, 2641}

$$\frac{\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{d\cos(a+bx)}}-\frac{\csc(a+bx)\sqrt{d\cos(a+bx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -((Sqrt[d*Cos[a + b*x]]*Csc[a + b*x])/(b*d)) + (Sqrt[Cos[a + b*x]]*Elliptic
F[(a + b*x)/2, 2])/(b*Sqrt[d*Cos[a + b*x]])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx &= -\frac{\sqrt{d\cos(a+bx)}\,\csc(a+bx)}{bd}+\frac{1}{2}\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx\\
&= -\frac{\sqrt{d\cos(a+bx)}\,\csc(a+bx)}{bd}+\frac{\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{2\sqrt{d\cos(a+bx)}}\\
&= -\frac{\sqrt{d\cos(a+bx)}\,\csc(a+bx)}{bd}+\frac{\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{d\cos(a+bx)}}
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 47, normalized size = 0.73

$$
\frac{\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)-\cot(a+bx)}{b\sqrt{d\cos(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-Cot[a + b*x] + Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(b*Sqrt[d*Co
s[a + b*x]])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\,\csc\left(bx+a\right)^2}{d\cos\left(bx+a\right)}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^2/(d*cos(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\csc\left(bx+a\right)^2}{\sqrt{d\cos\left(bx+a\right)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

maple [B] time = 0.26, size = 188, normalized size = 2.94

$$\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, d\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\left(2\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^{\frac{3}{2}}\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\ \mathrm{EllipticF}\left(c\right.\right.}{2\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d\right)^{\frac{3}{2}}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] 1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-2*sin(1/2*b
*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*d*sin(1/2*b*
x+1/2*a)*(2*(2*sin(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*E
llipticF(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)-4*sin(1/2*b*x+1/2*a
)^4+4*sin(1/2*b*x+1/2*a)^2-1)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^2}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/sqrt(d*cos(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(a+b\,x\right)^2\,\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(1/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(a+bx\right)}{\sqrt{d\cos\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**2/sqrt(d*cos(a + b*x)), x)
```

3.239 $\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{3E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}+\frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

```
[Out] -csc(b*x+a)/b/d/(d*cos(b*x+a))^(1/2)+3*sin(b*x+a)/b/d/(d*cos(b*x+a))^(1/2)-
3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b
*x),2^(1/2))*(d*cos(b*x+a))^(1/2)/b/d^2/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2570, 2636, 2640, 2639}

$$-\frac{3E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\cos(a+bx)}}+\frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] -(Csc[a + b*x]/(b*d*Sqrt[d*Cos[a + b*x]])) - (3*Sqrt[d*Cos[a + b*x]]*Ellipt
icE[(a + b*x)/2, 2])/(b*d^2*Sqrt[Cos[a + b*x]]) + (3*Sin[a + b*x])/(b*d*Sqr
t[d*Cos[a + b*x]])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx &= -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}+\frac{3}{2}\int \frac{1}{(d\cos(a+bx))^{3/2}}\,dx\\ &= -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}+\frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{3\int \sqrt{d\cos(a+bx)}\,dx}{2d^2}\\ &= -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}+\frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{\left(3\sqrt{d\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{2d^2\sqrt{\cos(a+bx)}}\\ &= -\frac{\csc(a+bx)}{bd\sqrt{d\cos(a+bx)}}-\frac{3\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bd^2\sqrt{\cos(a+bx)}}+\frac{3\sin(a+bx)}{bd\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 65, normalized size = 0.69

$$\frac{2\sin(a+bx)-\cos(a+bx)\cot(a+bx)-3\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-(Cos[a + b*x]*Cot[a + b*x]) - 3*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2,
 2] + 2*Sin[a + b*x])/(b*d*Sqrt[d*Cos[a + b*x]])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\,\csc\left(bx+a\right)^2}{d^2\cos\left(bx+a\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^2/(d^2*cos(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

maple [A] time = 0.33, size = 209, normalized size = 2.22

$$-\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d\right)^{\frac{3}{2}}\left(6\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cdots}\right.}{2d^3\sin\left(\frac{bx}{2}+\frac{a}{2}\right)^5\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^2 \mathrm{co}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] -1/2*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^3/sin(1/2*
b*x+1/2*a)^5/(2*sin(1/2*b*x+1/2*a)^2-1)^2/cos(1/2*b*x+1/2*a)*(-2*sin(1/2*b*
x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)*(6*(2*sin(1/2*b*x+1/2*a)^2-1)^(1
/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*cos(
1/2*b*x+1/2*a)+12*sin(1/2*b*x+1/2*a)^4-12*sin(1/2*b*x+1/2*a)^2+1)/(d*(2*cos
(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(3/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**2/(d*cos(a + b*x))**(3/2), x)
```

3.240 $\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{5\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}$$

```
[Out] -csc(b*x+a)/b/d/(d*cos(b*x+a))^(3/2)+5/3*sin(b*x+a)/b/d/(d*cos(b*x+a))^(3/2
)+5/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1
/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/d^2/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2570, 2636, 2642, 2641}

$$\frac{5\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] -(Csc[a + b*x]/(b*d*(d*Cos[a + b*x])^(3/2))) + (5*Sqrt[Cos[a + b*x]]*Ellipt
icF[(a + b*x)/2, 2])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]) + (5*Sin[a + b*x])/(3*b
*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx &= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}+\frac{5}{2}\int \frac{1}{(d\cos(a+bx))^{5/2}}\,dx\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}+\frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}+\frac{5\int \frac{1}{\sqrt{d\cos(a+bx)}}\,dx}{6d^2}\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}+\frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}+\frac{\left(5\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{6d^2\sqrt{d\cos(a+bx)}}\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{3/2}}+\frac{5\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}+\frac{5\sin(a+bx)}{3bd(d\cos(a+bx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 62, normalized size = 0.63

$$\frac{2\tan(a+bx)-3\cot(a+bx)+5\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3bd^2\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (-3*Cot[a + b*x] + 5*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 2*Tan[a
 + b*x])/(3*b*d^2*Sqrt[d*Cos[a + b*x]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{d\cos(bx+a)}\,\csc(bx+a)^2}{d^3\cos(bx+a)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^2/(d^3*cos(b*x + a)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc (bx+a)^2}{(d\cos (bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

maple [A] time = 0.38, size = 190, normalized size = 1.94

$$\frac{\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(10\left(2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)^{\frac{3}{2}}\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\ \mathrm{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\right.\right.}{6d\left(-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d\right)^{\frac{3}{2}}\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 1/6*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d/(-2*sin(1/2
*b*x+1/2*a)^4*d+sin(1/2*b*x+1/2*a)^2*d)^(3/2)/cos(1/2*b*x+1/2*a)*(10*(2*sin
(1/2*b*x+1/2*a)^2-1)^(3/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cos(1/2*b
*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)-20*sin(1/2*b*x+1/2*a)^4+20*sin(1/2*b*
x+1/2*a)^2-3)*sin(1/2*b*x+1/2*a)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc (bx+a)^2}{(d\cos (bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin (a+b\,x)^2\,{\left(d\,\cos (a+b\,x)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(5/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.241 $\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{21E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}}+\frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

```
[Out] -csc(b*x+a)/b/d/(d*cos(b*x+a))^(5/2)+7/5*sin(b*x+a)/b/d/(d*cos(b*x+a))^(5/2
)+21/5*sin(b*x+a)/b/d^3/(d*cos(b*x+a))^(1/2)-21/5*(cos(1/2*a+1/2*b*x)^2)^(1
/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(d*cos(b*x+a))
^(1/2)/b/d^4/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2570, 2636, 2640, 2639}

$$\frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}-\frac{21E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\cos(a+bx)}}+\frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}-\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] -(Csc[a + b*x]/(b*d*(d*Cos[a + b*x])^(5/2))) - (21*Sqrt[d*Cos[a + b*x]]*Ell
ipticE[(a + b*x)/2, 2])/(5*b*d^4*Sqrt[Cos[a + b*x]]) + (7*Sin[a + b*x])/(5*
b*d*(d*Cos[a + b*x])^(5/2)) + (21*Sin[a + b*x])/(5*b*d^3*Sqrt[d*Cos[a + b*x
]])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx &= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}+\frac{7}{2}\int \frac{1}{(d\cos(a+bx))^{7/2}}\,dx\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}+\frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}+\frac{21\int \frac{1}{(d\cos(a+bx))^{3/2}}\,dx}{10d^2}\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}+\frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}+\frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}-\frac{21\int \sqrt{d\cos(}}{10d}\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}+\frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}+\frac{21\sin(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}-\frac{\left(21\sqrt{d\cos(a}\right.}{10}\\
&= -\frac{\csc(a+bx)}{bd(d\cos(a+bx))^{5/2}}-\frac{21\sqrt{d\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bd^4\sqrt{\cos(a+bx)}}+\frac{7\sin(a+bx)}{5bd(d\cos(a+bx))^{5/2}}+
\end{aligned}$$

Mathematica [A] time = 0.16, size = 82, normalized size = 0.65

$$\frac{16\sin(a+bx)-5\cos(a+bx)\cot(a+bx)-21\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)+2\tan(a+bx)\sec(a+bx)}{5bd^3\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (-5*Cos[a + b*x]*Cot[a + b*x] - 21*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2
, 2] + 16*Sin[a + b*x] + 2*Sec[a + b*x]*Tan[a + b*x])/(5*b*d^3*Sqrt[d*Cos[a
 + b*x]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\,\csc\left(bx+a\right)^{2}}{d^{4}\cos\left(bx+a\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^2/(d^4*cos(b*x + a)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^{2}}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

maple [B] time = 0.39, size = 408, normalized size = 3.24

$$-\frac{\sqrt{d\left(2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\left(168\sqrt{2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\text{EllipticE}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] -1/10*(d*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/d^5/(2*sin(
1/2*b*x+1/2*a)^2-1)/sin(1/2*b*x+1/2*a)^5/cos(1/2*b*x+1/2*a)/(8*sin(1/2*b*x+
1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)*(168*(2*sin(1/2*
b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/
2*a),2^(1/2))*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^4+336*sin(1/2*b*x+1/2*a
)^8-168*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*Ellip
ticE(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^2-67
2*sin(1/2*b*x+1/2*a)^6+42*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2
*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2))*cos(1/2*b*x+1/2*a)+448*s
in(1/2*b*x+1/2*a)^4-112*sin(1/2*b*x+1/2*a)^2+5)*(-2*sin(1/2*b*x+1/2*a)^4*d+
sin(1/2*b*x+1/2*a)^2*d)^(3/2)/(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^2/(d*cos(b*x + a))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(7/2)),x)
[Out] int(1/(sin(a + b*x)^2*(d*cos(a + b*x))^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/(d*cos(b*x+a))**(7/2),x)
[Out] Timed out
```

3.242 $\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx$

Optimal. Leaf size=135

$$\frac{9d^{11/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{9d^{11/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{9d^5\sqrt{d\cos(a+bx)}}{2b}-\frac{9d^3(d\cos(a+bx))^{5/2}}{10b}-\frac{d\csc^2(a+b}{}$$

```
[Out] 9/4*d^(11/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+9/4*d^(11/2)*arctanh((d
*cos(b*x+a))^(1/2)/d^(1/2))/b-9/10*d^3*(d*cos(b*x+a))^(5/2)/b-1/2*d*(d*cos(
b*x+a))^(9/2)*csc(b*x+a)^2/b-9/2*d^5*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2565, 288, 321, 329, 212, 206, 203}

$$-\frac{9d^5\sqrt{d\cos(a+bx)}}{2b}-\frac{9d^3(d\cos(a+bx))^{5/2}}{10b}+\frac{9d^{11/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{9d^{11/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^3,x]
```

```
[Out] (9*d^(11/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (9*d^(11/2)*ArcTa
nh[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (9*d^5*Sqrt[d*Cos[a + b*x]])/(2*b
) - (9*d^3*(d*Cos[a + b*x])^(5/2))/(10*b) - (d*(d*Cos[a + b*x])^(9/2)*Csc[a
 + b*x]^2)/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^{11/2}\csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{11/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b} + \frac{(9d)\,\text{Subst}\left(\int \frac{x^{7/2}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx\right.}{4b}$$

$$= -\frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a+bx)}{2b} + \frac{\left(9d^3\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a}{2b}$$

$$= -\frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a}{2b}$$

$$= -\frac{9d^5\sqrt{d\cos(a+bx)}}{2b} - \frac{9d^3(d\cos(a+bx))^{5/2}}{10b} - \frac{d(d\cos(a+bx))^{9/2}\csc^2(a}{2b}$$

$$= \frac{9d^{11/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{9d^{11/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{9d^5\sqrt{d\cos(a+}}{2b}$$

Mathematica [A] time = 2.17, size = 137, normalized size = 1.01

$$\frac{d(d\cos(a+bx))^{9/2}\left(-\frac{21}{2}\left(8\sqrt{\cos(a+bx)}+\log\left(1-\sqrt{\cos(a+bx)}\right)-\log\left(\sqrt{\cos(a+bx)}+1\right)\right)+45\tan^{-1}\left(\sqrt{\cos(\right.\right.}{20b\cos^{\frac{9}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(11/2)*Csc[a + b*x]^3,x]
```

```
[Out] (d*(d*Cos[a + b*x])^(9/2)*(45*ArcTan[Sqrt[Cos[a + b*x]]] + 24*ArcTanh[Sqrt[
Cos[a + b*x]]] - 2*Sqrt[Cos[a + b*x]]*(2*Cos[2*(a + b*x)] + 5*Csc[a + b*x]^
2) - (21*(8*Sqrt[Cos[a + b*x]] + Log[1 - Sqrt[Cos[a + b*x]]] - Log[1 + Sqrt
[Cos[a + b*x]]]))/2))/(20*b*Cos[a + b*x]^(9/2))
```

fricas [A] time = 0.68, size = 419, normalized size = 3.10

$$\left[-\frac{90\left(d^5\cos\left(bx+a\right)^2-d^5\right)\sqrt{-d}\,\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-45\left(d^5\cos\left(bx+a\right)^2-d^5\right)\sqrt{-d}\,\log\left(-\frac{d\cos(bx+a)^2+4}{\cdots}\right.}{80\left(b\cos\left(b\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/80*(90*(d^5*cos(b*x + a)^2 - d^5)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a)
)*sqrt(-d)/(d*cos(b*x + a) + d)) - 45*(d^5*cos(b*x + a)^2 - d^5)*sqrt(-d)*l
og(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1)
- 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*(4*d^5*c
os(b*x + a)^4 + 36*d^5*cos(b*x + a)^2 - 45*d^5)*sqrt(d*cos(b*x + a)))/(b*co
s(b*x + a)^2 - b), -1/80*(90*(d^5*cos(b*x + a)^2 - d^5)*sqrt(d)*arctan(2*sq
rt(d*cos(b*x + a))*sqrt(d)/(d*cos(b*x + a) - d)) - 45*(d^5*cos(b*x + a)^2 -
 d^5)*sqrt(d)*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(
b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)
) + 8*(4*d^5*cos(b*x + a)^4 + 36*d^5*cos(b*x + a)^2 - 45*d^5)*sqrt(d*cos(b*
x + a)))/(b*cos(b*x + a)^2 - b)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.30, size = 433, normalized size = 3.21

$$-\frac{8d^5\left(\cos^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{5b}+\frac{8d^5\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{5b}+\frac{8d^5\sqrt{2\left(\cos^2\right.\cdots}}{5\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x)
```

```
[Out] -8/5/b*d^5*cos(1/2*b*x+1/2*a)^4*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)+8/5/b*d^
5*cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)+8/5/b*d^5*(2*cos(
```

```
1/2*b*x+1/2*a)^2*d-d)^(1/2)-6/b*d^5*(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)+9/
8/b*d^(11/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*
d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))-1/16/b*d^5/(cos(1/2*b*x+1/2*a)+1)*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+9/8/b*d^(11/2)*ln((-4*d*cos(1/2*b*x+1/2*
a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1
))-9/4/b*d^6/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^
(1/2))/cos(1/2*b*x+1/2*a))-1/8/b*d^5/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/
2*a)^2*d-d)^(1/2)+1/16/b*d^5/(cos(1/2*b*x+1/2*a)-1)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)
```

maxima [A] time = 0.58, size = 133, normalized size = 0.99

$$\frac{\frac{20\sqrt{d\cos(bx+a)}\,d^8}{d^2\cos(bx+a)^2-d^2}+90\,d^{\frac{13}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)-45\,d^{\frac{13}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)-16\,\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}d^4-160\,\sqrt{d\cos}}{40\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(11/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/40*(20*sqrt(d*cos(b*x + a))*d^8/(d^2*cos(b*x + a)^2 - d^2) + 90*d^(13/2)*
arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 45*d^(13/2)*log((sqrt(d*cos(b*x + a)
) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 16*(d*cos(b*x + a))^(5/2)*
d^4 - 160*sqrt(d*cos(b*x + a))*d^6)/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,\cos\left(a+b\,x\right)\right)}^{11/2}}{{\sin\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(11/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*cos(a + b*x))^(11/2)/sin(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(11/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.243 $\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx$

Optimal. Leaf size=113

$$-\frac{7d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{7d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{7d^3(d\cos(a+bx))^{3/2}}{6b}-\frac{d\csc^2(a+bx)(d\cos(a+bx))^{7/2}}{2b}$$

```
[Out] -7/4*d^(9/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+7/4*d^(9/2)*arctanh((d*
cos(b*x+a))^(1/2)/d^(1/2))/b-7/6*d^3*(d*cos(b*x+a))^(3/2)/b-1/2*d*(d*cos(b*
x+a))^(7/2)*csc(b*x+a)^2/b
```

Rubi [A] time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2565, 288, 321, 329, 298, 203, 206}

$$-\frac{7d^3(d\cos(a+bx))^{3/2}}{6b}-\frac{7d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{7d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)(d\cos(a+bx))^{7/2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^3,x]
```

```
[Out] (-7*d^(9/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (7*d^(9/2)*ArcTan
h[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (7*d^3*(d*Cos[a + b*x])^(3/2))/(6*
b) - (d*(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^2)/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^{9/2}\csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{9/2}}{\left(1-\frac{x^2}{d^2}\right)^2}dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b} + \frac{(7d)\,\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{d^2}}dx, x, d\cos(a+b$$

$$= -\frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b} + \frac{\left(7d^3\right)\text{Subst}}{}$$

$$= -\frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b} + \frac{\left(7d^3\right)\text{Subst}}{}$$

$$= -\frac{7d^3(d\cos(a+bx))^{3/2}}{6b} - \frac{d(d\cos(a+bx))^{7/2}\csc^2(a+bx)}{2b} + \frac{\left(7d^5\right)\text{Subst}}{}$$

$$= -\frac{7d^{9/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{7d^{9/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{7d^3(d\cos(a+}{6b}$$

Mathematica [C] time = 0.64, size = 78, normalized size = 0.69

$$\frac{d^5\left(21\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\csc^2(a+bx)\right)+(2\cos(2(a+bx))-5)\cot^2(a+bx)\right)}{6b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Csc[a + b*x]^3,x]
```

```
[Out] (d^5*((-5 + 2*Cos[2*(a + b*x)])*Cot[a + b*x]^2 + 21*(-Cot[a + b*x]^2)^(1/4)
*Hypergeometric2F1[1/4, 1/4, 5/4, Csc[a + b*x]^2]))/(6*b*Sqrt[d*Cos[a + b*x
]])
```

fricas [B] time = 0.65, size = 405, normalized size = 3.58

$$\left[-\frac{42\left(d^4\cos(bx+a)^2-d^4\right)\sqrt{-d}\,\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right)-21\left(d^4\cos(bx+a)^2-d^4\right)\sqrt{-d}\,\log\left(-\frac{d\cos(bx+a)^2-4}{}\right.}{48\left(b\cos(bx+a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(42*(d^4*cos(b*x + a)^2 - d^4)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a)
)*sqrt(-d)/(d*cos(b*x + a) + d)) - 21*(d^4*cos(b*x + a)^2 - d^4)*sqrt(-d)*l
og(-(d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1)
- 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*(4*d^4*c
os(b*x + a)^3 - 7*d^4*cos(b*x + a))*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^2
 - b), 1/48*(42*(d^4*cos(b*x + a)^2 - d^4)*sqrt(d)*arctan(2*sqrt(d*cos(b*x
+ a))*sqrt(d)/(d*cos(b*x + a) - d)) + 21*(d^4*cos(b*x + a)^2 - d^4)*sqrt(d)
*log(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1)
 + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) - 8*(4*d^4*
cos(b*x + a)^3 - 7*d^4*cos(b*x + a))*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^
2 - b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{9}{2}}\csc(bx+a)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*csc(b*x + a)^3, x)
```

maple [B] time = 0.31, size = 394, normalized size = 3.49

$$-\frac{4d^{4}\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\sqrt{2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{3b}-\frac{4d^{4}\sqrt{2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{3b}+\frac{2d^{4}\sqrt{d\left(2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x)
```

```
[Out] -4/3/b*d^4*cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)-4/3/b*d^
4*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)+2/b*d^4*(d*(2*cos(1/2*b*x+1/2*a)^2-1))
^(1/2)+7/8/b*d^(9/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1
/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))-1/16/b*d^4/(cos(1/2*b*x+1/2
*a)+1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+7/8/b*d^(9/2)*ln((-4*d*cos(1/2*b
*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1
/2*a)+1))+7/4/b*d^5/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^
2*d-d)^(1/2))/cos(1/2*b*x+1/2*a))+1/8/b*d^4/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2
```

```
*b*x+1/2*a)^2*d-d)^(1/2)+1/16/b*d^4/(cos(1/2*b*x+1/2*a)-1)*(-2*sin(1/2*b*x+
1/2*a)^2*d+d)^(1/2)
```

maxima [A] time = 0.71, size = 118, normalized size = 1.04

$$\frac{\frac{12\,(d\cos(bx+a))^{\frac{3}{2}}d^6}{d^2\cos(bx+a)^2-d^2} - 42\,d^{\frac{11}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - 21\,d^{\frac{11}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right) - 16\,(d\cos(bx+a))^{\frac{3}{2}}d^4}{24\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(12*(d*cos(b*x + a))^(3/2)*d^6/(d^2*cos(b*x + a)^2 - d^2) - 42*d^(11/2
)*arctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 21*d^(11/2)*log((sqrt(d*cos(b*x +
a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 16*(d*cos(b*x + a))^(3/2
)*d^4)/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\cos(a+b\,x))^{9/2}}{\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(9/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*cos(a + b*x))^(9/2)/sin(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.244 $\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx$

Optimal. Leaf size=113

$$\frac{5d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{5d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{5d^3\sqrt{d\cos(a+bx)}}{2b}-\frac{d\csc^2(a+bx)(d\cos(a+bx))^{5/2}}{2b}$$

```
[Out] 5/4*d^(7/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+5/4*d^(7/2)*arctanh((d*c
os(b*x+a))^(1/2)/d^(1/2))/b-1/2*d*(d*cos(b*x+a))^(5/2)*csc(b*x+a)^2/b-5/2*d
^3*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2565, 288, 321, 329, 212, 206, 203}

$$-\frac{5d^3\sqrt{d\cos(a+bx)}}{2b}+\frac{5d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{5d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)(d\cos(a+bx))^{5/2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^3,x]
```

```
[Out] (5*d^(7/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (5*d^(7/2)*ArcTanh
[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (5*d^3*Sqrt[d*Cos[a + b*x]])/(2*b)
- (d*(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^2)/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^{7/2}\csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{7/2}}{\left(1-\frac{x^2}{d^2}\right)^2}dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b} + \frac{(5d)\,\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{d^2}}dx, x, d\cos(a+bx\right.}{4b}$$

$$= -\frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b} + \frac{\left(5d^3\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b} + \frac{\left(5d^3\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{5d^3\sqrt{d\cos(a+bx)}}{2b} - \frac{d(d\cos(a+bx))^{5/2}\csc^2(a+bx)}{2b} + \frac{\left(5d^4\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{5d^{7/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{5d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{5d^3\sqrt{d\cos(a+b}}{2b}$$

Mathematica [A] time = 1.16, size = 118, normalized size = 1.04

$$\frac{(d\cos(a+bx))^{7/2}\left(-8\sqrt{\cos(a+bx)} - \log\left(1-\sqrt{\cos(a+bx)}\right) + \log\left(\sqrt{\cos(a+bx)}+1\right) + 5\tan^{-1}\left(\sqrt{\cos(a+bx)}\right.\right.}{4b\cos^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)*Csc[a + b*x]^3,x]
```

```
[Out] ((d*Cos[a + b*x])^(7/2)*(5*ArcTan[Sqrt[Cos[a + b*x]]] + 3*ArcTanh[Sqrt[Cos[
a + b*x]]] - 8*Sqrt[Cos[a + b*x]] - 2*Sqrt[Cos[a + b*x]]*Csc[a + b*x]^2 - L
og[1 - Sqrt[Cos[a + b*x]]] + Log[1 + Sqrt[Cos[a + b*x]]]))/(4*b*Cos[a + b*x
]^(7/2))
```

fricas [B] time = 0.62, size = 393, normalized size = 3.48

$$\left[-\frac{10\left(d^3\cos(bx+a)^2-d^3\right)\sqrt{-d}\,\arctan\left(\frac{2\sqrt{d\cos(bx+a)}\sqrt{-d}}{d\cos(bx+a)+d}\right) - 5\left(d^3\cos(bx+a)^2-d^3\right)\sqrt{-d}\,\log\left(-\frac{d\cos(bx+a)^2+4\sqrt{d}}{}\right.}{16\left(b\cos(bx+a)^2-b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*(d^3*cos(b*x + a)^2 - d^3)*sqrt(-d)*arctan(2*sqrt(d*cos(b*x + a)
)*sqrt(-d)/(d*cos(b*x + a) + d)) - 5*(d^3*cos(b*x + a)^2 - d^3)*sqrt(-d)*lo
g(-(d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) -
 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*(4*d^3*co
s(b*x + a)^2 - 5*d^3)*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^2 - b), -1/16*(
10*(d^3*cos(b*x + a)^2 - d^3)*sqrt(d)*arctan(2*sqrt(d*cos(b*x + a))*sqrt(d)
/(d*cos(b*x + a) - d)) - 5*(d^3*cos(b*x + a)^2 - d^3)*sqrt(d)*log(-(d*cos(b
*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x
 + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*(4*d^3*cos(b*x + a)^2
 - 5*d^3)*sqrt(d*cos(b*x + a)))/(b*cos(b*x + a)^2 - b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{7}{2}}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)*csc(b*x + a)^3, x)
```

maple [B] time = 0.28, size = 327, normalized size = 2.89

$$-\frac{2d^3\sqrt{d\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}}{b}+\frac{5d^{\frac{7}{2}}\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}+4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{8b}-\frac{d^3\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+}}{16b\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x)
```

```
[Out] -2/b*d^3*(d*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)+5/8/b*d^(7/2)*ln((4*d*cos(1/2
*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x
+1/2*a)-1))-1/16/b*d^3/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)
^(1/2)+5/8/b*d^(7/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+
1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))-5/4/b*d^4/(-d)^(1/2)*ln((-
2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2))/cos(1/2*b*x+1/2*a))-1/
8/b*d^3/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)+1/16/b*d^3/
(cos(1/2*b*x+1/2*a)-1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
```

maxima [A] time = 2.31, size = 118, normalized size = 1.04

$$\frac{\frac{4\sqrt{d\cos(bx+a)}\,d^6}{d^2\cos(bx+a)^2-d^2} + 10\,d^{\frac{9}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right) - 5\,d^{\frac{9}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right) - 16\sqrt{d\cos(bx+a)}\,d^4}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*sqrt(d*cos(b*x + a))*d^6/(d^2*cos(b*x + a)^2 - d^2) + 10*d^(9/2)*arc
tan(sqrt(d*cos(b*x + a))/sqrt(d)) - 5*d^(9/2)*log((sqrt(d*cos(b*x + a)) - s
qrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))) - 16*sqrt(d*cos(b*x + a))*d^4)/(b
*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\cos(a+b\,x))^{7/2}}{\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(7/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*cos(a + b*x))^(7/2)/sin(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(7/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.245 $\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{3d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)(d\cos(a+bx))^{3/2}}{2b}$$

```
[Out] -3/4*d^(5/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+3/4*d^(5/2)*arctanh((d*
cos(b*x+a))^(1/2)/d^(1/2))/b-1/2*d*(d*cos(b*x+a))^(3/2)*csc(b*x+a)^2/b
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2565, 288, 329, 298, 203, 206}

$$-\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{3d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)(d\cos(a+bx))^{3/2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^3,x]
```

```
[Out] (-3*d^(5/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (3*d^(5/2)*ArcTan
h[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (d*(d*Cos[a + b*x])^(3/2)*Csc[a +
b*x]^2)/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^{5/2}\csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{5/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2b} + \frac{(3d)\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right.}{4b}$$

$$= -\frac{d(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2b} + \frac{(3d)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+b}\right.}{2b}$$

$$= -\frac{d(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2b} + \frac{\left(3d^3\right)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+}\right.}{4b}$$

$$= -\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{3d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{d(d\cos(a+bx)}{2}$$

Mathematica [C]   time = 0.29, size = 65, normalized size = 0.71

$$-\frac{d^3\left(\cot^2(a+bx)-3\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\csc^2(a+bx)\right)\right)}{2b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^3,x]
```

```
[Out] -1/2*(d^3*(Cot[a + b*x]^2 - 3*(-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4
, 1/4, 5/4, Csc[a + b*x]^2]))/(b*Sqrt[d*Cos[a + b*x]])
```

fricas [B]   time = 0.53, size = 380, normalized size = 4.18

$$\left[\frac{8\sqrt{d\cos(bx+a)}\,d^2\cos(bx+a)-6\left(d^2\cos(bx+a)^2-d^2\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{d\cos(bx+a)}\,\sqrt{-d}\,(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)+3\left(d^2\cos\ldots\right.}{16\left(b\cos(bx+a)^2-b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a) - 6*(d^2*cos(b*x + a)^2 - d^
2)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*
cos(b*x + a))) + 3*(d^2*cos(b*x + a)^2 - d^2)*sqrt(-d)*log((d*cos(b*x + a)^
2 - 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) +
 d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)))/(b*cos(b*x + a)^2 - b), 1/16*(8
*sqrt(d*cos(b*x + a))*d^2*cos(b*x + a) - 6*(d^2*cos(b*x + a)^2 - d^2)*sqrt(
d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x + a)
)) + 3*(d^2*cos(b*x + a)^2 - d^2)*sqrt(d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*
cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x +
 a)^2 - 2*cos(b*x + a) + 1)))/(b*cos(b*x + a)^2 - b)]
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{5}{2}}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*csc(b*x + a)^3, x)
```

maple [B] time = 0.41, size = 300, normalized size = 3.30

$$\frac{3d^{\frac{5}{2}}\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}+4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{8b}-\frac{d^2\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}}{16b\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}+\frac{3d^{\frac{5}{2}}\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}-4d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x)
```

```
[Out] 3/8/b*d^(5/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2
*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))-1/16/b*d^2/(cos(1/2*b*x+1/2*a)+1)*
(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+3/8/b*d^(5/2)*ln((-4*d*cos(1/2*b*x+1/2*
a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1
))+3/4/b*d^3/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^
(1/2))/cos(1/2*b*x+1/2*a))+1/8/b*d^2/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/
2*a)^2*d-d)^(1/2)+1/16/b*d^2/(cos(1/2*b*x+1/2*a)-1)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)
```

maxima [A] time = 0.70, size = 103, normalized size = 1.13

$$\frac{\frac{4\,(d\cos(bx+a))^{\frac{3}{2}}d^4}{d^2\cos(bx+a)^2-d^2}-6\,d^{\frac{7}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)-3\,d^{\frac{7}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*(d*cos(b*x + a))^(3/2)*d^4/(d^2*cos(b*x + a)^2 - d^2) - 6*d^(7/2)*ar
ctan(sqrt(d*cos(b*x + a))/sqrt(d)) - 3*d^(7/2)*log((sqrt(d*cos(b*x + a)) -
sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\cos(a+b\,x))^{5/2}}{\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(5/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*cos(a + b*x))^(5/2)/sin(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.246 $\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx$

Optimal. Leaf size=91

$$\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)\sqrt{d\cos(a+bx)}}{2b}$$

```
[Out] 1/4*d^(3/2)*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b+1/4*d^(3/2)*arctanh((d*c
os(b*x+a))^(1/2)/d^(1/2))/b-1/2*d*csc(b*x+a)^2*(d*cos(b*x+a))^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2565, 288, 329, 212, 206, 203}

$$\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{d\csc^2(a+bx)\sqrt{d\cos(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^3,x]
```

```
[Out] (d^(3/2)*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) + (d^(3/2)*ArcTanh[Sqr
t[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^2)
/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^{3/2}\csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^{3/2}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{d\sqrt{d\cos(a+bx)}\,\csc^2(a+bx)}{2b} + \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx\right.}{4b}$$

$$= -\frac{d\sqrt{d\cos(a+bx)}\,\csc^2(a+bx)}{2b} + \frac{d\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2b}$$

$$= -\frac{d\sqrt{d\cos(a+bx)}\,\csc^2(a+bx)}{2b} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right.}{4b}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} + \frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{d\sqrt{d\cos(a+bx)}\,\text{c}}{2b}$$

Mathematica [C] time = 0.18, size = 76, normalized size = 0.84

$$\frac{\left(-\cot^2(a+bx)\right)^{3/4}\sec^3(a+bx)(d\cos(a+bx))^{3/2}\left({}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\csc^2(a+bx)\right)+3\sqrt[4]{-\cot^2(a+bx)}\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^3,x]
```

```
[Out] ((d*Cos[a + b*x])^(3/2)*(-Cot[a + b*x]^2)^(3/4)*(3*(-Cot[a + b*x]^2)^(1/4)
+ Hypergeometric2F1[3/4, 3/4, 7/4, Csc[a + b*x]^2])*Sec[a + b*x]^3)/(6*b)
```

fricas [B] time = 0.52, size = 347, normalized size = 3.81

$$\left[-\frac{2\left(d\cos(bx+a)^2-d\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{d\cos(bx+a)}\,\sqrt{-d}\,(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-\left(d\cos(bx+a)^2-d\right)\sqrt{-d}\,\log\left(\frac{d\cos(bx+a)^2+4\,\sqrt{\cdots}}{\cdots}\right.}{16\left(b\cos(bx+a)^2-b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(d*cos(b*x + a)^2 - d)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*s
qrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - (d*cos(b*x + a)^2 - d)*sqrt(
-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) -
 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) - 8*sqrt
(d*cos(b*x + a))*d)/(b*cos(b*x + a)^2 - b), 1/16*(2*(d*cos(b*x + a)^2 - d)*
sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x
 + a))) + (d*cos(b*x + a)^2 - d)*sqrt(d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*c
os(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x +
a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*d)/(b*cos(b*x + a)^2 -
 b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{3}{2}}\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^3, x)
```

maple [B] time = 0.28, size = 294, normalized size = 3.23

$$\frac{d^{\frac{3}{2}}\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}+4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{8b}-\frac{d\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}}{16b\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}+\frac{d^{\frac{3}{2}}\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}-4d\cos}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x)
```

```
[Out] 1/8/b*d^(3/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2
*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))-1/16/b*d/(cos(1/2*b*x+1/2*a)+1)*(-
2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+1/8/b*d^(3/2)*ln((-4*d*cos(1/2*b*x+1/2*a)
+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))
-1/4/b*d^2/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1
/2))/cos(1/2*b*x+1/2*a))-1/8/b*d/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)
^2*d-d)^(1/2)+1/16/b*d/(cos(1/2*b*x+1/2*a)-1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)
^(1/2)
```

maxima [A] time = 0.63, size = 103, normalized size = 1.13

$$\frac{\frac{4\sqrt{d\cos(bx+a)}\,d^4}{d^2\cos(bx+a)^2-d^2}+2\,d^{\frac{5}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)-d^{\frac{5}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*sqrt(d*cos(b*x + a))*d^4/(d^2*cos(b*x + a)^2 - d^2) + 2*d^(5/2)*arct
an(sqrt(d*cos(b*x + a))/sqrt(d)) - d^(5/2)*log((sqrt(d*cos(b*x + a)) - sqrt
(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\cos(a+b\,x))^{3/2}}{\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(3/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*cos(a + b*x))^(3/2)/sin(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.247 $\int \sqrt{d \cos(a+bx)} \csc^3(a+bx)\, dx$

Optimal. Leaf size=93

$$\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{\csc^2(a+bx)(d\cos(a+bx))^{3/2}}{2bd}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}$$

```
[Out] -1/2*(d*cos(b*x+a))^(3/2)*csc(b*x+a)^2/b/d+1/4*arctan((d*cos(b*x+a))^(1/2)/
d^(1/2))*d^(1/2)/b-1/4*arctanh((d*cos(b*x+a))^(1/2)/d^(1/2))*d^(1/2)/b
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2565, 290, 329, 298, 203, 206}

$$\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}-\frac{\csc^2(a+bx)(d\cos(a+bx))^{3/2}}{2bd}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^3,x]
```

```
[Out] (Sqrt[d]*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - (Sqrt[d]*ArcTanh[Sqr
t[d*Cos[a + b*x]]/Sqrt[d]])/(4*b) - ((d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^2)
/(2*b*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \sqrt{d\cos(a+bx)}\,\csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= -\frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd} - \frac{\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2bd}$$

$$= -\frac{(d\cos(a+bx))^{3/2}\csc^2(a+bx)}{2bd} - \frac{d\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} +$$

$$= \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b} - \frac{(d\cos(a+bx))^{3/2}\csc}{2bd}$$

Mathematica [C] time = 0.25, size = 62, normalized size = 0.67

$$-\frac{d\left(\sqrt[4]{-\cot^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\csc^2(a+bx)\right)+\cot^2(a+bx)\right)}{2b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^3,x]
```

```
[Out] -1/2*(d*(Cot[a + b*x]^2 + (-Cot[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/
4, 5/4, Csc[a + b*x]^2]))/(b*Sqrt[d*Cos[a + b*x]])
```

fricas [B] time = 0.50, size = 340, normalized size = 3.66

$$\left[\frac{2\left(\cos(bx+a)^2-1\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{d\cos(bx+a)}\,\sqrt{-d}\,(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)+\left(\cos(bx+a)^2-1\right)\sqrt{-d}\,\log\left(\frac{d\cos(bx+a)^2+4\,\sqrt{d\cos}}{\cos}\right.}{16\left(b\cos(bx+a)^2-b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(cos(b*x + a)^2 - 1)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt
(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) + (cos(b*x + a)^2 - 1)*sqrt(-d)*l
og((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1) -
 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*co
s(b*x + a))*cos(b*x + a))/(b*cos(b*x + a)^2 - b), 1/16*(2*(cos(b*x + a)^2 -
 1)*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos
(b*x + a))) + (cos(b*x + a)^2 - 1)*sqrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d
*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x
+ a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*cos(b*x + a))/(b*cos
(b*x + a)^2 - b)]
```

giac [B] time = 1.55, size = 178, normalized size = 1.91

$$\frac{\frac{2\,d\arctan\left(-\frac{\sqrt{-d}\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2-\sqrt{-d\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^4+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\sqrt{-d}\,\log\left(\left|-\sqrt{-d}\,\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^2+\sqrt{-d\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^4+d}\right|\right.}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*d*arctan(-(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1
/2*a)^4 + d))/sqrt(-d))/sqrt(-d) - sqrt(-d)*log(abs(-sqrt(-d)*tan(1/2*b*x +
 1/2*a)^2 + sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))) + sqrt(-d*tan(1/2*b*x + 1
/2*a)^4 + d) + 2*sqrt(-d)*d/((sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan
(1/2*b*x + 1/2*a)^4 + d))^2 - d))/b
```

maple [B] time = 0.28, size = 289, normalized size = 3.11

$$-\frac{\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}}{16b\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}-\frac{\sqrt{d}\,\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}-4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)}{8b}+\frac{\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d-d}}{8b\cos\left(\frac{bx}{2}+\frac{a}{2}\right)^2}-d\ln\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x)
```

```
[Out] -1/16/b/(cos(1/2*b*x+1/2*a)+1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-1/8/b*d^
(1/2)*ln((-4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(
1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1))+1/8/b/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x
+1/2*a)^2*d-d)^(1/2)-1/4/b*d/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*
x+1/2*a)^2*d-d)^(1/2))/cos(1/2*b*x+1/2*a))+1/16/b/(cos(1/2*b*x+1/2*a)-1)*(-
2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-1/8/b*d^(1/2)*ln((4*d*cos(1/2*b*x+1/2*a)+
2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))
```

maxima [A] time = 0.60, size = 102, normalized size = 1.10

$$\frac{\frac{4\,(d\cos(bx+a))^{\frac{3}{2}}d^2}{d^2\cos(bx+a)^2-d^2}+2\,d^{\frac{3}{2}}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)+d^{\frac{3}{2}}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*(d*cos(b*x + a))^(3/2)*d^2/(d^2*cos(b*x + a)^2 - d^2) + 2*d^(3/2)*ar
ctan(sqrt(d*cos(b*x + a))/sqrt(d)) + d^(3/2)*log((sqrt(d*cos(b*x + a)) - sq
rt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,\cos\left(a+b\,x\right)}}{\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(1/2)/sin(a + b*x)^3,x)
```

```
[Out] int((d*cos(a + b*x))^(1/2)/sin(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos(a+bx)}\csc^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(d*cos(a + b*x))*csc(a + b*x)**3, x)
```

3.248 $\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=93

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}}-\frac{\csc^2(a+bx)\sqrt{d\cos(a+bx)}}{2bd}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}}$$

```
[Out] -3/4*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(1/2)-3/4*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b/d^(1/2)-1/2*csc(b*x+a)^2*(d*cos(b*x+a))^(1/2)/b/d
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2565, 290, 329, 212, 206, 203}

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}}-\frac{\csc^2(a+bx)\sqrt{d\cos(a+bx)}}{2bd}-\frac{3\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-3*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*Sqrt[d]) - (3*ArcTanh[Sqrt[d
*Cos[a + b*x]]/Sqrt[d]])/(4*b*Sqrt[d]) - (Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]
^2)/(2*b*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= -\frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2bd}$$

$$= -\frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd} - \frac{3\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{1}{d+\cdots}\right.}{}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}} - \frac{3\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4b\sqrt{d}} - \frac{\sqrt{d\cos(a+bx)}\csc^2(a+bx)}{2bd}$$

Mathematica [C] time = 0.22, size = 69, normalized size = 0.74

$$\frac{d\left(-\cot^2(a+bx)\right)^{3/4}\left(\sqrt[4]{-\cot^2(a+bx)} - {}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\csc^2(a+bx)\right)\right)}{2b(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (d*(-Cot[a + b*x]^2)^(3/4)*((-Cot[a + b*x]^2)^(1/4) - Hypergeometric2F1[3/4
, 3/4, 7/4, Csc[a + b*x]^2]))/(2*b*(d*Cos[a + b*x])^(3/2))
```

fricas [B] time = 0.52, size = 334, normalized size = 3.59

$$\left[\frac{6\left(\cos(bx+a)^2-1\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{-d}\,(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right) - 3\left(\cos(bx+a)^2-1\right)\sqrt{-d}\,\log\left(\frac{d\cos(bx+a)^2+4\sqrt{d\cos\cdots}}{\cos\cdots}\right.}{16\left(bd\cos(bx+a)^2-bd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(6*(cos(b*x + a)^2 - 1)*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*x + a))*sqrt
(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 3*(cos(b*x + a)^2 - 1)*sqrt(-d)
*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))*sqrt(-d)*(cos(b*x + a) - 1)
 - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x + a) + 1)) + 8*sqrt(d*
cos(b*x + a)))/(b*d*cos(b*x + a)^2 - b*d), -1/16*(6*(cos(b*x + a)^2 - 1)*sq
rt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x +
 a))) - 3*(cos(b*x + a)^2 - 1)*sqrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos
(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)
^2 - 2*cos(b*x + a) + 1)) - 8*sqrt(d*cos(b*x + a)))/(b*d*cos(b*x + a)^2 - b
*d)]
```

giac [B] time = 1.32, size = 181, normalized size = 1.95

$$\frac{\frac{6\arctan\left(-\frac{\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{3\log\left(\left|-\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}+\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}\right|\right)}{\sqrt{-d}}+\frac{2\sqrt{\cdot}}{\left(\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\sqrt{\cdot}\right.}}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(6*arctan(-(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2
*a)^4 + d))/sqrt(-d))/sqrt(-d) - 3*log(abs(-sqrt(-d)*tan(1/2*b*x + 1/2*a)^2
 + sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)))/sqrt(-d) + 2*sqrt(-d)/((sqrt(-d)*t
an(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^2 - d) - sqrt(
-d*tan(1/2*b*x + 1/2*a)^4 + d)/d)/b
```

maple [B] time = 0.28, size = 297, normalized size = 3.19

$$-\frac{3\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}+4d\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-1}\right)}{8\sqrt{d}\,b}-\frac{\sqrt{-2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}}{16bd\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}-\frac{3\ln\left(\frac{2\sqrt{d}\,\sqrt{-2\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}-4d\cos\left(\cdot\right.}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\right)}{8\sqrt{d}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] -3/8/b/d^(1/2)*ln((4*d*cos(1/2*b*x+1/2*a)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^
2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)-1))-1/16/b/d/(cos(1/2*b*x+1/2*a)+1)*(
-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-3/8/b/d^(1/2)*ln((-4*d*cos(1/2*b*x+1/2*a
```

```
)+2*d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d)/(cos(1/2*b*x+1/2*a)+1)
)+3/4/b/(-d)^(1/2)*ln((-2*d+2*(-d)^(1/2)*(2*cos(1/2*b*x+1/2*a)^2*d-d)^(1/2)
)/cos(1/2*b*x+1/2*a))-1/8/b/d/cos(1/2*b*x+1/2*a)^2*(2*cos(1/2*b*x+1/2*a)^2*
d-d)^(1/2)+1/16/b/d/(cos(1/2*b*x+1/2*a)-1)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1
/2)
```

maxima [A] time = 0.50, size = 103, normalized size = 1.11

$$\frac{\frac{4\sqrt{d\cos(bx+a)}\,d^2}{d^2\cos(bx+a)^2-d^2}-6\sqrt{d}\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)+3\sqrt{d}\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(4*sqrt(d*cos(b*x + a))*d^2/(d^2*cos(b*x + a)^2 - d^2) - 6*sqrt(d)*arct
an(sqrt(d*cos(b*x + a))/sqrt(d)) + 3*sqrt(d)*log((sqrt(d*cos(b*x + a)) - sq
rt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d))))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)^3\,\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(1/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**3/sqrt(d*cos(a + b*x)), x)
```

3.249 $\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=115

$$\frac{5\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}}+\frac{5}{2bd\sqrt{d\cos(a+bx)}}-\frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}}$$

```
[Out] 5/4*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(3/2)-5/4*arctanh((d*cos(b*x+a
))^(1/2)/d^(1/2))/b/d^(3/2)+5/2/b/d/(d*cos(b*x+a))^(1/2)-1/2*csc(b*x+a)^2/b
/d/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2565, 290, 325, 329, 298, 203, 206}

$$\frac{5\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}}+\frac{5}{2bd\sqrt{d\cos(a+bx)}}-\frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (5*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(3/2)) - (5*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(3/2)) + 5/(2*b*d*Sqrt[d*Cos[a + b*x]]) - Cs
c[a + b*x]^2/(2*b*d*Sqrt[d*Cos[a + b*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}} - \frac{5\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{4bd^3}$$

$$= \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}} - \frac{5\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{2bd^3}$$

$$= \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd\sqrt{d\cos(a+bx)}} - \frac{5\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx)}\right)}{4bd} -$$

$$= \frac{5\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}} - \frac{5\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{3/2}} + \frac{5}{2bd\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+}{2bd\sqrt{d\cos(}}$$

Mathematica [C] time = 0.24, size = 91, normalized size = 0.79

$$\frac{5\cot^2(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\csc^2(a+bx)\right) - \left(-\cot^2(a+bx)\right)^{3/4}\left(\cot^2(a+bx)-4\right)}{2bd\left(-\cot^2(a+bx)\right)^{3/4}\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (-((-Cot[a + b*x]^2)^(3/4)*(-4 + Cot[a + b*x]^2)) + 5*Cot[a + b*x]^2*Hyperg
eometric2F1[1/4, 1/4, 5/4, Csc[a + b*x]^2])/(2*b*d*Sqrt[d*Cos[a + b*x]]*(-C
ot[a + b*x]^2)^(3/4))
```

fricas [B] time = 0.50, size = 406, normalized size = 3.53

$$\left[\frac{10\left(\cos(bx+a)^{3}-\cos(bx+a)\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{d\cos(bx+a)}\,\sqrt{-d}\,(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-5\left(\cos(bx+a)^{3}-\cos(bx+a)\right)\sqrt{-d}}{16\left(bd^{2}\cos(bx+a)^{3}-bd^{2}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(10*(cos(b*x + a)^3 - cos(b*x + a))*sqrt(-d)*arctan(1/2*sqrt(d*cos(b*
x + a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 5*(cos(b*x + a)^3 -
 cos(b*x + a))*sqrt(-d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt
(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*cos(b*x
 + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(5*cos(b*x + a)^2 - 4))/(b*d^2*cos(b*x
 + a)^3 - b*d^2*cos(b*x + a)), 1/16*(10*(cos(b*x + a)^3 - cos(b*x + a))*sqr
t(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*cos(b*x +
a))) + 5*(cos(b*x + a)^3 - cos(b*x + a))*sqrt(d)*log((d*cos(b*x + a)^2 - 4*
sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x + a) + d)/(co
s(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(5*cos(b*x + a
)^2 - 4))/(b*d^2*cos(b*x + a)^3 - b*d^2*cos(b*x + a))]
```

giac [B] time = 1.18, size = 361, normalized size = 3.14

$$\frac{\frac{10\arctan\left(-\frac{\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}+\frac{5\log\left(\left|-\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}+\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}\right|\right)}{\sqrt{-d}}-\frac{2(1}{\left(\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\sqrt{-}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(10*arctan(-(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/
2*a)^4 + d))/sqrt(-d))/sqrt(-d) + 5*log(abs(-sqrt(-d)*tan(1/2*b*x + 1/2*a)^
2 + sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)))/sqrt(-d) - 2*(16*(sqrt(-d)*tan(1/
2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^2 - (sqrt(-d)*tan(1
/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))*sqrt(-d) - 17*d)/(
(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^3 -
 (sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^2*
sqrt(-d) - (sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^
4 + d))*d + sqrt(-d)*d) + sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)/d)/(b*d)
```

maple [B] time = 0.54, size = 705, normalized size = 6.13

$$\frac{-\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}\,d^{\frac{3}{2}}\sqrt{-d}-\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(20\ln\left(\frac{2\sqrt{-d}\,\sqrt{-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)d+d}-2d}{\cos\left(\frac{bx}{2}+\frac{a}{2}\right)}\right)d^{\frac{5}{2}}+10\ln\left(\frac{2\sqrt{d}\,\sqrt{-2(}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] 1/8/d^(7/2)/(-d)^(1/2)/sin(1/2*b*x+1/2*a)^2/(2*sin(1/2*b*x+1/2*a)^4-3*sin(1
/2*b*x+1/2*a)^2+1)*(-(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(3/2)*(-d)^(1/2)
-sin(1/2*b*x+1/2*a)^6*(20*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b
*x+1/2*a)^2*d+d)^(1/2)-d))*d^(5/2)+10*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*
(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d
^2+10*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1
/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^2)+5*(6*ln(2/cos(1/2*b*x+1/2*a)
*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(5/2)-4*(-2*sin(1/2*
b*x+1/2*a)^2*d+d)^(1/2)*d^(3/2)*(-d)^(1/2)+3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d
^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^
(1/2)*d^2+3*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d
+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^2)*sin(1/2*b*x+1/2*a)^4-5
*(2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)
-d))*d^(5/2)-4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(3/2)*(-d)^(1/2)+ln(2/
(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos
(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^2+ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2
*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^2)
*sin(1/2*b*x+1/2*a)^2)/b
```

maxima [A] time = 0.87, size = 117, normalized size = 1.02

$$\frac{\frac{10\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{5\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{\sqrt{d}}+\frac{4\left(5d^2\cos(bx+a)^2-4d^2\right)}{(d\cos(bx+a))^{\frac{5}{2}}-\sqrt{d\cos(bx+a)}\,d^2}}{8bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(10*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/sqrt(d) + 5*log((sqrt(d*cos(b*
x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/sqrt(d) + 4*(5*d^2*cos
(b*x + a)^2 - 4*d^2)/((d*cos(b*x + a))^(5/2) - sqrt(d*cos(b*x + a))*d^2))/(
b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)^3\,{\left(d\,\cos\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(3/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}{\left(a+bx\right)}}{\left(d\cos{\left(a+bx\right)}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**3/(d*cos(a + b*x))**(3/2), x)
```

3.250 $\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}}+\frac{7}{6bd(d\cos(a+bx))^{3/2}}-\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}}$$

```
[Out] -7/4*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(5/2)-7/4*arctanh((d*cos(b*x+
a))^(1/2)/d^(1/2))/b/d^(5/2)+7/6/b/d/(d*cos(b*x+a))^(3/2)-1/2*csc(b*x+a)^2/
b/d/(d*cos(b*x+a))^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2565, 290, 325, 329, 212, 206, 203}

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}}+\frac{7}{6bd(d\cos(a+bx))^{3/2}}-\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (-7*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(5/2)) - (7*ArcTanh[Sqrt[d
*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(5/2)) + 7/(6*b*d*(d*Cos[a + b*x])^(3/2)) -
 Csc[a + b*x]^2/(2*b*d*(d*Cos[a + b*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+\right.}{4bd^3}$$

$$= \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{d^2}}\,dx, x, \sqrt{d\cos(a+bx}\right.}{2bd^3}$$

$$= \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{3/2}} - \frac{7\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d\cos(a+bx}\right.}{4bd^2}$$

$$= -\frac{7\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}} - \frac{7\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{5/2}} + \frac{7}{6bd(d\cos(a+bx))^{3/2}} - \frac{\csc^2(}{2bd(d\cos}$$

Mathematica [C] time = 0.36, size = 92, normalized size = 0.80

$$\frac{7\cot^2(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\csc^2(a+bx)\right) + \sqrt[4]{-\cot^2(a+bx)}\left(4-3\cot^2(a+bx)\right)}{6bd\sqrt[4]{-\cot^2(a+bx)}\,(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((-Cot[a + b*x]^2)^(1/4)*(4 - 3*Cot[a + b*x]^2) + 7*Cot[a + b*x]^2*Hypergeo
metric2F1[3/4, 3/4, 7/4, Csc[a + b*x]^2])/(6*b*d*(d*Cos[a + b*x])^(3/2)*(-C
ot[a + b*x]^2)^(1/4))
```

fricas [B] time = 0.54, size = 418, normalized size = 3.63

$$\left[\frac{42\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{d\cos(bx+a)}\,\sqrt{-d}\,(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-21\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\sqrt{\cdots}}{48\left(bd^{3}\cos\left(bx+a\right)^{4}-b\cdots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(42*(cos(b*x + a)^4 - cos(b*x + a)^2)*sqrt(-d)*arctan(1/2*sqrt(d*cos(
b*x + a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 21*(cos(b*x + a)^
4 - cos(b*x + a)^2)*sqrt(-d)*log((d*cos(b*x + a)^2 + 4*sqrt(d*cos(b*x + a))
*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*co
s(b*x + a) + 1)) + 8*sqrt(d*cos(b*x + a))*(7*cos(b*x + a)^2 - 4))/(b*d^3*co
s(b*x + a)^4 - b*d^3*cos(b*x + a)^2), -1/48*(42*(cos(b*x + a)^4 - cos(b*x +
 a)^2)*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x + a) - 1)/(sqrt(d)*
cos(b*x + a))) - 21*(cos(b*x + a)^4 - cos(b*x + a)^2)*sqrt(d)*log((d*cos(b*
x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a) + 1) + 6*d*cos(b*x
+ a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) - 8*sqrt(d*cos(b*x + a))*(
7*cos(b*x + a)^2 - 4))/(b*d^3*cos(b*x + a)^4 - b*d^3*cos(b*x + a)^2)]
```

giac [B] time = 1.32, size = 279, normalized size = 2.43

$$\frac{\frac{42\arctan\left(-\frac{\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}-\frac{21\log\left(\left|-\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}+\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}\right|\right)}{\sqrt{-d}}+\frac{6\sqrt{\cdots}}{\left(\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\sqrt{\cdots}\right.}\cdots}{24\,bd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(42*arctan(-(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1
/2*a)^4 + d))/sqrt(-d))/sqrt(-d) - 21*log(abs(-sqrt(-d)*tan(1/2*b*x + 1/2*a
)^2 + sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)))/sqrt(-d) + 6*sqrt(-d)/((sqrt(-d
)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^2 - d) - 3*
sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)/d + 32*(3*(sqrt(-d)*tan(1/2*b*x + 1/2*a
)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^2 - d)/(sqrt(-d)*tan(1/2*b*x + 1
/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d) - sqrt(-d))^3)/(b*d^2)
```

maple [B] time = 0.46, size = 909, normalized size = 7.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 1/24/d^(11/2)/(-d)^(1/2)/sin(1/2*b*x+1/2*a)^2/(4*sin(1/2*b*x+1/2*a)^6-8*sin
(1/2*b*x+1/2*a)^4+5*sin(1/2*b*x+1/2*a)^2-1)*(3*(-2*sin(1/2*b*x+1/2*a)^2*d+d
)^(1/2)*d^(5/2)*(-d)^(1/2)+84*(2*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*si
n(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(7/2)-ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1
/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/
2)*d^3-ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(
1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3)*sin(1/2*b*x+1/2*a)^8-168*(2
*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d)
)*d^(7/2)-ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d
)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3-ln(2/(cos(1/2*b*x+1/2*a)-
1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*
(-d)^(1/2)*d^3)*sin(1/2*b*x+1/2*a)^6-7*(6*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/
2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(7/2)+4*(-2*sin(1/2*b*x+1/2*a)
^2*d+d)^(1/2)*d^(5/2)*(-d)^(1/2)-3*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2
*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3-
3*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+
2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3)*sin(1/2*b*x+1/2*a)^2+7*(30*ln(2/
cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(7
/2)+4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(5/2)*(-d)^(1/2)-15*ln(2/(cos(1
/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b
*x+1/2*a)-d))*(-d)^(1/2)*d^3-15*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*si
n(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^3)*si
n(1/2*b*x+1/2*a)^4)/b
```

maxima [A] time = 0.49, size = 117, normalized size = 1.02

$$\frac{\frac{4\left(7\,d^{2}\cos\left(bx+a\right)^{2}-4\,d^{2}\right)}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}-\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}d^{2}}-\frac{42\,\arctan\left(\frac{\sqrt{d\cos\left(bx+a\right)}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{21\,\log\left(\frac{\sqrt{d\cos\left(bx+a\right)}-\sqrt{d}}{\sqrt{d\cos\left(bx+a\right)}+\sqrt{d}}\right)}{d^{\frac{3}{2}}}}{24\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(4*(7*d^2*cos(b*x + a)^2 - 4*d^2)/((d*cos(b*x + a))^(7/2) - (d*cos(b*x
 + a))^(3/2)*d^2) - 42*arctan(sqrt(d*cos(b*x + a))/sqrt(d))/d^(3/2) + 21*lo
g((sqrt(d*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos(b*x + a)) + sqrt(d)))/d^(3/2
))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\sin\left(a+b\,x\right)}^3\,{\left(d\,\cos\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(5/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.251 $\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=137

$$\frac{9\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}}+\frac{9}{2bd^3\sqrt{d\cos(a+bx)}}+\frac{9}{10bd(d\cos(a+bx))^{5/2}}-\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx)}$$

```
[Out] 9/4*arctan((d*cos(b*x+a))^(1/2)/d^(1/2))/b/d^(7/2)-9/4*arctanh((d*cos(b*x+a
))^(1/2)/d^(1/2))/b/d^(7/2)+9/10/b/d/(d*cos(b*x+a))^(5/2)-1/2*csc(b*x+a)^2/
b/d/(d*cos(b*x+a))^(5/2)+9/2/b/d^3/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2565, 290, 325, 329, 298, 203, 206}

$$\frac{9}{2bd^3\sqrt{d\cos(a+bx)}}+\frac{9\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}}-\frac{9\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}}+\frac{9}{10bd(d\cos(a+bx))^{5/2}}-\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (9*ArcTan[Sqrt[d*Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(7/2)) - (9*ArcTanh[Sqrt[d*
Cos[a + b*x]]/Sqrt[d]])/(4*b*d^(7/2)) + 9/(10*b*d*(d*Cos[a + b*x])^(5/2)) +
 9/(2*b*d^3*Sqrt[d*Cos[a + b*x]]) - Csc[a + b*x]^2/(2*b*d*(d*Cos[a + b*x])^
(5/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
```

```
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{(d\cos(a+bx))^{7/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{x^{7/2}\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$

$$= -\frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{x^{7/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a+bx)\right)}{4bd}$$

$$= \frac{9}{10bd(d\cos(a+bx))^{5/2}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\,\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{d^2}\right)}\,dx, x, d\cos(a\right.}{4bd^3}$$

$$= \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{9}{2bd^3\sqrt{d\cos(a+bx)}} - \frac{\csc^2(a+bx)}{2bd(d\cos(a+bx))^{5/2}} - \frac{9\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{9\tan^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}} - \frac{9\tanh^{-1}\left(\frac{\sqrt{d\cos(a+bx)}}{\sqrt{d}}\right)}{4bd^{7/2}} + \frac{9}{10bd(d\cos(a+bx))^{5/2}} + \frac{}{2bd^3\sqrt{d\,c}}$$

Mathematica [C] time = 0.47, size = 102, normalized size = 0.74

$$\frac{45\cot^2(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\csc^2(a+bx)\right) + \left(-\cot^2(a+bx)\right)^{3/4}\left(-5\cot^2(a+bx) + 4\sec^2(a+bx) + 40\right)}{10bd^3\left(-\cot^2(a+bx)\right)^{3/4}\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (45*Cot[a + b*x]^2*Hypergeometric2F1[1/4, 1/4, 5/4, Csc[a + b*x]^2] + (-Cot
[a + b*x]^2)^(3/4)*(40 - 5*Cot[a + b*x]^2 + 4*Sec[a + b*x]^2))/(10*b*d^3*Sq
rt[d*Cos[a + b*x]]*(-Cot[a + b*x]^2)^(3/4))
```

fricas [A] time = 0.54, size = 438, normalized size = 3.20

$$\left[\frac{90\left(\cos\left(bx+a\right)^{5}-\cos\left(bx+a\right)^{3}\right)\sqrt{-d}\,\arctan\left(\frac{\sqrt{d\cos(bx+a)}\,\sqrt{-d}\,(\cos(bx+a)+1)}{2\,d\cos(bx+a)}\right)-45\left(\cos\left(bx+a\right)^{5}-\cos\left(bx+a\right)^{3}\right)\surd}{80\left(bd^{4}\cos\left(bx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] [1/80*(90*(cos(b*x + a)^5 - cos(b*x + a)^3)*sqrt(-d)*arctan(1/2*sqrt(d*cos(
b*x + a))*sqrt(-d)*(cos(b*x + a) + 1)/(d*cos(b*x + a))) - 45*(cos(b*x + a)^
5 - cos(b*x + a)^3)*sqrt(-d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))
*sqrt(-d)*(cos(b*x + a) - 1) - 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 + 2*co
s(b*x + a) + 1)) + 8*(45*cos(b*x + a)^4 - 36*cos(b*x + a)^2 - 4)*sqrt(d*cos
(b*x + a)))/(b*d^4*cos(b*x + a)^5 - b*d^4*cos(b*x + a)^3), 1/80*(90*(cos(b*
x + a)^5 - cos(b*x + a)^3)*sqrt(d)*arctan(1/2*sqrt(d*cos(b*x + a))*(cos(b*x
 + a) - 1)/(sqrt(d)*cos(b*x + a))) + 45*(cos(b*x + a)^5 - cos(b*x + a)^3)*s
qrt(d)*log((d*cos(b*x + a)^2 - 4*sqrt(d*cos(b*x + a))*sqrt(d)*(cos(b*x + a)
 + 1) + 6*d*cos(b*x + a) + d)/(cos(b*x + a)^2 - 2*cos(b*x + a) + 1)) + 8*(4
5*cos(b*x + a)^4 - 36*cos(b*x + a)^2 - 4)*sqrt(d*cos(b*x + a)))/(b*d^4*cos(
b*x + a)^5 - b*d^4*cos(b*x + a)^3)]
```

giac [B] time = 1.41, size = 417, normalized size = 3.04

$$\frac{\frac{90\arctan\left(-\frac{\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}+\frac{45\log\left(\left|-\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}+\sqrt{-d\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{4}+d}\right|\right)}{\sqrt{-d}}+\frac{10\surd}{\left(\sqrt{-d}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^{2}-\surd\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] 1/40*(90*arctan(-(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1
/2*a)^4 + d))/sqrt(-d))/sqrt(-d) + 45*log(abs(-sqrt(-d)*tan(1/2*b*x + 1/2*a
)^2 + sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)))/sqrt(-d) + 10*sqrt(-d)/((sqrt(-
d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^2 - d) + 5
*sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d)/d - 32*(15*(sqrt(-d)*tan(1/2*b*x + 1/2
*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^4 - 40*(sqrt(-d)*tan(1/2*b*x +
 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^3*sqrt(-d) - 70*(sqrt(-d)*
tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))^2*d + 40*(sqr
t(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*a)^4 + d))*sqrt(-d
```

```
)*d + 11*d^2)/(sqrt(-d)*tan(1/2*b*x + 1/2*a)^2 - sqrt(-d*tan(1/2*b*x + 1/2*
a)^4 + d) - sqrt(-d))^5)/(b*d^3)
```

maple [B] time = 0.52, size = 1165, normalized size = 8.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x)

[Out] 1/40/d^(15/2)/(-d)^(1/2)/sin(1/2*b*x+1/2*a)^2/(8*sin(1/2*b*x+1/2*a)^8-20*si
n(1/2*b*x+1/2*a)^6+18*sin(1/2*b*x+1/2*a)^4-7*sin(1/2*b*x+1/2*a)^2+1)*(-5*(-
2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(7/2)*(-d)^(1/2)-360*(2*ln(2/cos(1/2*b*
x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)+ln(2/(
cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(
1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4+ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*
sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4)*
sin(1/2*b*x+1/2*a)^10+180*(10*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1
/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)-4*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2
)*d^(7/2)*(-d)^(1/2)+5*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x
+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4+5*ln(2/(cos(
1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*
b*x+1/2*a)-d))*(-d)^(1/2)*d^4)*sin(1/2*b*x+1/2*a)^8-90*(18*ln(2/cos(1/2*b*x
+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)-16*(-2*
sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(7/2)*(-d)^(1/2)+9*ln(2/(cos(1/2*b*x+1/2*
a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d
))*(-d)^(1/2)*d^4+9*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/
2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4)*sin(1/2*b*x+1/
2*a)^6+9*(70*ln(2/cos(1/2*b*x+1/2*a)*((-d)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d
+d)^(1/2)-d))*d^(9/2)-104*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)*d^(7/2)*(-d)^
(1/2)+35*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)
^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4+35*ln(2/(cos(1/2*b*x+1/2*a
)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)+2*d*cos(1/2*b*x+1/2*a)-d)
)*(-d)^(1/2)*d^4)*sin(1/2*b*x+1/2*a)^4-9*(10*ln(2/cos(1/2*b*x+1/2*a)*((-d)^
(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-d))*d^(9/2)-24*(-2*sin(1/2*b*x+1/
2*a)^2*d+d)^(1/2)*d^(7/2)*(-d)^(1/2)+5*ln(2/(cos(1/2*b*x+1/2*a)+1)*(d^(1/2)
*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1/2)-2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*
d^4+5*ln(2/(cos(1/2*b*x+1/2*a)-1)*(d^(1/2)*(-2*sin(1/2*b*x+1/2*a)^2*d+d)^(1
/2)+2*d*cos(1/2*b*x+1/2*a)-d))*(-d)^(1/2)*d^4)*sin(1/2*b*x+1/2*a)^2)/b
```

maxima [A] time = 0.62, size = 134, normalized size = 0.98

$$\frac{\frac{4\left(45\,d^4\cos(bx+a)^4-36\,d^4\cos(bx+a)^2-4\,d^4\right)}{(d\cos(bx+a))^{\frac{9}{2}}d^2-(d\cos(bx+a))^{\frac{5}{2}}d^4}+\frac{90\,\arctan\left(\frac{\sqrt{d\cos(bx+a)}}{\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{45\,\log\left(\frac{\sqrt{d\cos(bx+a)}-\sqrt{d}}{\sqrt{d\cos(bx+a)}+\sqrt{d}}\right)}{d^{\frac{5}{2}}}}{40\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] 1/40*(4*(45*d^4*cos(b*x + a)^4 - 36*d^4*cos(b*x + a)^2 - 4*d^4)/((d*cos(b*x
 + a))^(9/2)*d^2 - (d*cos(b*x + a))^(5/2)*d^4) + 90*arctan(sqrt(d*cos(b*x +
 a))/sqrt(d))/d^(5/2) + 45*log((sqrt(d*cos(b*x + a)) - sqrt(d))/(sqrt(d*cos
(b*x + a)) + sqrt(d)))/d^(5/2))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)^3\,{\left(d\,\cos\left(a+b\,x\right)\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(7/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^3*(d*cos(a + b*x))^(7/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

3.252 $\int \sqrt[5]{d \cos(a+bx)} \sin(a+bx) \, dx$

Optimal. Leaf size=22

$$-\frac{5(d\cos(a+bx))^{6/5}}{6bd}$$

```
[Out] -5/6*(d*cos(b*x+a))^(6/5)/b/d
```

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 30}

$$-\frac{5(d\cos(a+bx))^{6/5}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(1/5)*Sin[a + b*x],x]
```

```
[Out] (-5*(d*Cos[a + b*x])^(6/5))/(6*b*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}
\int \sqrt[5]{d\cos(a+bx)} \sin(a+bx) \, dx &= -\frac{\operatorname{Subst}\left(\int \sqrt[5]{x} \, dx, x, d\cos(a+bx)\right)}{bd} \\
&= -\frac{5(d\cos(a+bx))^{6/5}}{6bd}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 1.00

$$-\frac{5(d\cos(a+bx))^{6/5}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(1/5)*Sin[a + b*x],x]
```

```
[Out] (-5*(d*Cos[a + b*x])^(6/5))/(6*b*d)
```

fricas [A] time = 0.44, size = 21, normalized size = 0.95

$$-\frac{5\,\left(d\cos\left(bx+a\right)\right)^{\frac{1}{5}}\cos\left(bx+a\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/5)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -5/6*(d*cos(b*x + a))^(1/5)*cos(b*x + a)/b
```

giac [A] time = 1.16, size = 21, normalized size = 0.95

$$-\frac{5\,\left(d\cos\left(bx+a\right)\right)^{\frac{1}{5}}\cos\left(bx+a\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/5)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -5/6*(d*cos(b*x + a))^(1/5)*cos(b*x + a)/b
```

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$-\frac{5\left(d\cos\left(bx+a\right)\right)^{\frac{6}{5}}}{6bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/5)*sin(b*x+a),x)
```

```
[Out] -5/6*(d*cos(b*x+a))^(6/5)/b/d
```

maxima [A] time = 1.06, size = 18, normalized size = 0.82

$$-\frac{5\,\left(d\cos\left(bx+a\right)\right)^{\frac{6}{5}}}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/5)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -5/6*(d*cos(b*x + a))^(6/5)/(b*d)
```

mupad [B] time = 0.10, size = 18, normalized size = 0.82

$$-\frac{5\,{\left(d\,\cos\left(a+b\,x\right)\right)}^{6/5}}{6\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(d*cos(a + b*x))^(1/5),x)
```

```
[Out] -(5*(d*cos(a + b*x))^(6/5))/(6*b*d)
```

sympy [A] time = 28.39, size = 34, normalized size = 1.55

$$\begin{cases} -\frac{5\sqrt[5]{d}\cos^{\frac{6}{5}}{\left(a+bx \right)}}{6b} & \text{for}\: b \neq 0 \\ x\sqrt[5]{d\cos{\left(a \right)}}\sin{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/5)*sin(b*x+a),x)
```

```
[Out] Piecewise((-5*d**(1/5)*cos(a + b*x)**(6/5)/(6*b), Ne(b, 0)), (x*(d*cos(a))*
*(1/5)*sin(a), True))
```

3.253 $\int \cos^3(x)\sqrt{\sin(x)}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)$$

```
[Out] 2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2564, 14}

$$\frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sqrt[Sin[x]],x]
```

```
[Out] (2*Sin[x]^(3/2))/3 - (2*Sin[x]^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\sqrt{\sin(x)}\,dx &= \text{Subst}\left(\int \sqrt{x}\left(1-x^2\right)dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(\sqrt{x}-x^{5/2}\right)dx,x,\sin(x)\right)\\ &= \frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.86

$$\frac{1}{21}\sin^{\frac{3}{2}}(x)(3\cos(2x)+11)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sqrt[Sin[x]],x]
[Out] ((11 + 3*Cos[2*x])*Sin[x]^(3/2))/21
```

fricas [A] time = 0.42, size = 14, normalized size = 0.67

$$\frac{2}{21}\left(3\cos(x)^2+4\right)\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="fricas")
[Out] 2/21*(3*cos(x)^2 + 4)*sin(x)^(3/2)
```

giac [A] time = 0.31, size = 13, normalized size = 0.62

$$-\frac{2}{7}\sin(x)^{\frac{7}{2}}+\frac{2}{3}\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="giac")
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

maple [A] time = 0.07, size = 14, normalized size = 0.67

$$\frac{2\left(\sin^{\frac{3}{2}}(x)\right)}{3}-\frac{2\left(\sin^{\frac{7}{2}}(x)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^(1/2),x)
[Out] 2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

maxima [A] time = 0.32, size = 13, normalized size = 0.62

$$-\frac{2}{7}\sin(x)^{\frac{7}{2}}+\frac{2}{3}\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

mupad [B] time = 0.47, size = 25, normalized size = 1.19

$$-\frac{\cos(x)^4 \sin(x)^{3/2} \,{}_2\mathrm{F}_1\left(\frac{1}{4}, 2;\; 3;\; \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^(1/2),x)
```

```
[Out] -(cos(x)^4*sin(x)^(3/2)*hypergeom([1/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(3/
4))
```

sympy [B] time = 37.16, size = 167, normalized size = 7.95

$$\frac{28\sqrt{2}\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^{\frac{11}{2}}\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21}+\frac{8\sqrt{2}\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^{\frac{7}{2}}\left(\frac{x}{2}\right)}{21\tan^6\left(\frac{x}{2}\right)+63\tan^4\left(\frac{x}{2}\right)+63\tan^2\left(\frac{x}{2}\right)+21}+\frac{28\sqrt{2}\sqrt{\frac{}{\tan}}}{21\tan^6\left(\frac{x}{2}\right)+63\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**(1/2),x)
```

```
[Out] 28*sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(11/2)/(21*tan(x/2)**6 + 63*
tan(x/2)**4 + 63*tan(x/2)**2 + 21) + 8*sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*ta
n(x/2)**(7/2)/(21*tan(x/2)**6 + 63*tan(x/2)**4 + 63*tan(x/2)**2 + 21) + 28*
sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(3/2)/(21*tan(x/2)**6 + 63*tan(
x/2)**4 + 63*tan(x/2)**2 + 21)
```

3.254 $\int \cos^3(x) \sin^{\frac{3}{2}}(x) \, dx$

Optimal. Leaf size=21

$$\frac{2}{5}\sin^{\frac{5}{2}}(x) - \frac{2}{9}\sin^{\frac{9}{2}}(x)$$

```
[Out] 2/5*sin(x)^(5/2)-2/9*sin(x)^(9/2)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2564, 14}

$$\frac{2}{5}\sin^{\frac{5}{2}}(x) - \frac{2}{9}\sin^{\frac{9}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sin[x]^(3/2),x]
```

```
[Out] (2*Sin[x]^(5/2))/5 - (2*Sin[x]^(9/2))/9
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(x)\sin^{\frac{3}{2}}(x)\,dx &= \text{Subst}\left(\int x^{3/2}\left(1-x^2\right)\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(x^{3/2}-x^{7/2}\right)\,dx, x, \sin(x)\right)\\
&= \frac{2}{5}\sin^{\frac{5}{2}}(x) - \frac{2}{9}\sin^{\frac{9}{2}}(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 0.86

$$\frac{1}{45}\sin^{\frac{5}{2}}(x)(5\cos(2x)+13)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sin[x]^(3/2),x]
[Out] ((13 + 5*Cos[2*x])*Sin[x]^(5/2))/45
```

fricas [A] time = 0.42, size = 20, normalized size = 0.95

$$-\frac{2}{45}\left(5\,\cos(x)^4-\cos(x)^2-4\right)\sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(3/2),x, algorithm="fricas")
[Out] -2/45*(5*cos(x)^4 - cos(x)^2 - 4)*sqrt(sin(x))
```

giac [A] time = 0.31, size = 13, normalized size = 0.62

$$-\frac{2}{9}\,\sin(x)^{\frac{9}{2}}+\frac{2}{5}\,\sin(x)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(3/2),x, algorithm="giac")
[Out] -2/9*sin(x)^(9/2) + 2/5*sin(x)^(5/2)
```

maple [A] time = 0.06, size = 14, normalized size = 0.67

$$\frac{2\left(\sin^{\frac{5}{2}}(x)\right)}{5}-\frac{2\left(\sin^{\frac{9}{2}}(x)\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^(3/2),x)
[Out] 2/5*sin(x)^(5/2)-2/9*sin(x)^(9/2)
```

maxima [A] time = 0.31, size = 13, normalized size = 0.62

$$-\frac{2}{9}\,\sin(x)^{\frac{9}{2}}+\frac{2}{5}\,\sin(x)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/9*sin(x)^(9/2) + 2/5*sin(x)^(5/2)
```

mupad [B] time = 0.47, size = 25, normalized size = 1.19

$$-\frac{\cos(x)^4 \sin(x)^{5/2}\, {}_2F_1\left(-\frac{1}{4}, 2;\ 3;\ \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^(3/2),x)
```

```
[Out] -(cos(x)^4*sin(x)^(5/2)*hypergeom([-1/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(5
/4))
```

sympy [A] time = 66.23, size = 24, normalized size = 1.14

$$\frac{8\sin^{\frac{9}{2}}(x)}{45} + \frac{2\sin^{\frac{5}{2}}(x)\cos^2(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**(3/2),x)
```

```
[Out] 8*sin(x)**(9/2)/45 + 2*sin(x)**(5/2)*cos(x)**2/5
```

3.255 $\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}\sin^{\frac{7}{2}}(x)-\frac{2}{11}\sin^{\frac{11}{2}}(x)$$

```
[Out] 2/7*sin(x)^(7/2)-2/11*sin(x)^(11/2)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2564, 14}

$$\frac{2}{7}\sin^{\frac{7}{2}}(x)-\frac{2}{11}\sin^{\frac{11}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sin[x]^(5/2),x]
```

```
[Out] (2*Sin[x]^(7/2))/7 - (2*Sin[x]^(11/2))/11
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\sin^{\frac{5}{2}}(x)\,dx&=\text{Subst}\left(\int x^{5/2}\left(1-x^2\right)\,dx,x,\sin(x)\right)\\&=\text{Subst}\left(\int \left(x^{5/2}-x^{9/2}\right)\,dx,x,\sin(x)\right)\\&=\frac{2}{7}\sin^{\frac{7}{2}}(x)-\frac{2}{11}\sin^{\frac{11}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.86

$$\frac{1}{77}\sin^{\frac{7}{2}}(x)(7\cos(2x)+15)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sin[x]^(5/2),x]
[Out] ((15 + 7*Cos[2*x])*Sin[x]^(7/2))/77
```

fricas [A] time = 0.45, size = 20, normalized size = 0.95

$$-\frac{2}{77}\left(7\,\cos(x)^4-3\,\cos(x)^2-4\right)\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(5/2),x, algorithm="fricas")
[Out] -2/77*(7*cos(x)^4 - 3*cos(x)^2 - 4)*sin(x)^(3/2)
```

giac [A] time = 0.57, size = 13, normalized size = 0.62

$$-\frac{2}{11}\,\sin(x)^{\frac{11}{2}}+\frac{2}{7}\,\sin(x)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(5/2),x, algorithm="giac")
[Out] -2/11*sin(x)^(11/2) + 2/7*sin(x)^(7/2)
```

maple [A] time = 0.05, size = 14, normalized size = 0.67

$$\frac{2\left(\sin^{\frac{7}{2}}(x)\right)}{7}-\frac{2\left(\sin^{\frac{11}{2}}(x)\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^(5/2),x)
[Out] 2/7*sin(x)^(7/2)-2/11*sin(x)^(11/2)
```

maxima [A] time = 0.32, size = 13, normalized size = 0.62

$$-\frac{2}{11}\,\sin(x)^{\frac{11}{2}}+\frac{2}{7}\,\sin(x)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/11*sin(x)^(11/2) + 2/7*sin(x)^(7/2)
```

mupad [B] time = 0.45, size = 25, normalized size = 1.19

$$-\frac{\cos(x)^4 \sin(x)^{7/2} {}_2F_1\left(-\frac{3}{4}, 2;\ 3;\ \cos(x)^2\right)}{4 \left(\sin(x)^2\right)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^(5/2),x)
```

```
[Out] -(cos(x)^4*sin(x)^(7/2)*hypergeom([-3/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(7
/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**(5/2),x)
```

```
[Out] Timed out
```

3.256 $\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx$

Optimal. Leaf size=19

$$2\sqrt{\sin(x)}-\frac{2}{5}\sin^{\frac{5}{2}}(x)$$

```
[Out] -2/5*sin(x)^(5/2)+2*sin(x)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2564, 14}

$$2\sqrt{\sin(x)}-\frac{2}{5}\sin^{\frac{5}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/Sqrt[Sin[x]],x]
```

```
[Out] 2*Sqrt[Sin[x]] - (2*Sin[x]^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{\sqrt{\sin(x)}}\,dx &= \text{Subst}\left(\int \frac{1-x^2}{\sqrt{x}}\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{\sqrt{x}}-x^{3/2}\right)dx,x,\sin(x)\right)\\ &= 2\sqrt{\sin(x)}-\frac{2}{5}\sin^{\frac{5}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{1}{5}\sqrt{\sin(x)}\,(\cos(2x)+9)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/Sqrt[Sin[x]],x]
[Out] ((9 + Cos[2*x])*Sqrt[Sin[x]])/5
```

fricas [A] time = 0.43, size = 12, normalized size = 0.63

$$\frac{2}{5}\left(\cos(x)^2+4\right)\sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^(1/2),x, algorithm="fricas")
[Out] 2/5*(cos(x)^2 + 4)*sqrt(sin(x))
```

giac [A] time = 0.35, size = 13, normalized size = 0.68

$$-\frac{2}{5}\sin(x)^{\frac{5}{2}}+2\sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^(1/2),x, algorithm="giac")
[Out] -2/5*sin(x)^(5/2) + 2*sqrt(sin(x))
```

maple [A] time = 0.06, size = 14, normalized size = 0.74

$$-\frac{2\left(\sin^{\frac{5}{2}}(x)\right)}{5}+2\left(\sqrt{\sin}\,(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^(1/2),x)
[Out] -2/5*sin(x)^(5/2)+2*sin(x)^(1/2)
```

maxima [A] time = 0.36, size = 13, normalized size = 0.68

$$-\frac{2}{5}\sin(x)^{\frac{5}{2}}+2\sqrt{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*sin(x)^(5/2) + 2*sqrt(sin(x))
```

mupad [B] time = 0.44, size = 25, normalized size = 1.32

$$-\frac{\cos(x)^4 \sqrt{\sin(x)}\ {}_2F_1\left(\frac{3}{4}, 2;\ 3;\ \cos(x)^2\right)}{4\left(\sin(x)^2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^(1/2),x)
```

```
[Out] -(cos(x)^4*sin(x)^(1/2)*hypergeom([3/4, 2], 3, cos(x)^2))/(4*(sin(x)^2)^(1/
4))
```

sympy [B] time = 32.71, size = 323, normalized size = 17.00

$$\frac{10\sqrt{2}\tan^5\left(\frac{x}{2}\right)}{5\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^{\frac{13}{2}}\left(\frac{x}{2}\right)+15\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^{\frac{9}{2}}\left(\frac{x}{2}\right)+15\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\tan^{\frac{5}{2}}\left(\frac{x}{2}\right)+5\sqrt{\frac{1}{\tan^2\left(\frac{x}{2}\right)+1}}\sqrt{\tan\left(\frac{x}{2}\right)}}+\frac{}{5\sqrt{\frac{}{\tan^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/sin(x)**(1/2),x)
```

```
[Out] 10*sqrt(2)*tan(x/2)**5/(5*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(13/2) + 15*s
qrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(9/2) + 15*sqrt(1/(tan(x/2)**2 + 1))*tan
(x/2)**(5/2) + 5*sqrt(1/(tan(x/2)**2 + 1))*sqrt(tan(x/2))) + 12*sqrt(2)*tan
(x/2)**3/(5*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(13/2) + 15*sqrt(1/(tan(x/2
)**2 + 1))*tan(x/2)**(9/2) + 15*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(5/2) +
 5*sqrt(1/(tan(x/2)**2 + 1))*sqrt(tan(x/2))) + 10*sqrt(2)*tan(x/2)/(5*sqrt(
1/(tan(x/2)**2 + 1))*tan(x/2)**(13/2) + 15*sqrt(1/(tan(x/2)**2 + 1))*tan(x/
2)**(9/2) + 15*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(5/2) + 5*sqrt(1/(tan(x/
2)**2 + 1))*sqrt(tan(x/2)))
```

3.257 $\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx$

Optimal. Leaf size=132

$$\frac{7d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}+\frac{7d^3(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{30bc}+\frac{d(c\sin(a+bx))^{3/2}(}{5bc}$$

```
[Out] 7/30*d^3*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b/c+1/5*d*(d*cos(b*x+a))
^(7/2)*(c*sin(b*x+a))^(3/2)/b/c-7/20*d^4*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+
1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*si
n(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2569, 2572, 2639}

$$\frac{7d^3(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{30bc}+\frac{7d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(}{5bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (7*d^3*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(30*b*c) + (d*(d*Cos[
a + b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(5*b*c) + (7*d^4*Sqrt[d*Cos[a + b*x
]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(20*b*Sqrt[Sin[2*a +
2*b*x]])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}\,dx = \frac{d(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bc} + \frac{1}{10}\left(7d^2\right)\int (d\cos(a+bx))^{5/2}\sqrt{c}$$

$$= \frac{7d^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{30bc} + \frac{d(d\cos(a+bx))^{7/2}(c\sin(a+b}{5bc}$$

$$= \frac{7d^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{30bc} + \frac{d(d\cos(a+bx))^{7/2}(c\sin(a+b}{5bc}$$

$$= \frac{7d^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{30bc} + \frac{d(d\cos(a+bx))^{7/2}(c\sin(a+b}{5bc}$$

Mathematica [C] time = 0.09, size = 70, normalized size = 0.53

$$\frac{2d^4\sqrt[4]{\cos^2(a+bx)}\,\tan(a+bx)\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}\,{}_2F_1\left(-\frac{7}{4},\frac{3}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*d^4*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4,
3/4, 7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}\,d^4\cos(bx+a)^4\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d^4*cos(b*x + a)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a)), x)
```

maple [B] time = 0.29, size = 532, normalized size = 4.03

$$-\frac{\sqrt{c\sin\left(bx+a\right)}\,\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}\left(12\left(\cos^{6}\left(bx+a\right)\right)\sqrt{2}+2\left(\cos^{4}\left(bx+a\right)\right)\sqrt{2}-21\cos\left(bx+a\right)\sqrt{\frac{1-\cos(bx+a)+\cdots}{\sin(bx+\cdots}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/120/b*(c*sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(9/2)*(12*cos(b*x+a)^6*2^(1/2)
+2*cos(b*x+a)^4*2^(1/2)-21*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(
b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^
(1/2))+42*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ell
ipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-21*((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+42*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*
2^(1/2))+7*cos(b*x+a)^2*2^(1/2)-21*cos(b*x+a)*2^(1/2))/sin(b*x+a)/cos(b*x+a
)^5*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}\sqrt{c\sin\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\cos\left(a+b\,x\right)\right)}^{9/2}\,\sqrt{c\,\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(1/2),x)
```

```
[Out] int((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.258 $\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx$

Optimal. Leaf size=95

$$\frac{d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}$$

```
[Out] 1/3*d*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b/c-1/2*d^2*(sin(a+1/4*Pi+b
*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(
b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2569, 2572, 2639}

$$\frac{d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}+\frac{d(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (d*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(3*b*c) + (d^2*Sqrt[d*Cos
[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqrt[Sin
[2*a + 2*b*x]])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}\,dx = \frac{d(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bc}+\frac{1}{2}d^2\int\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a$$

$$= \frac{d(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bc}+\frac{\left(d^2\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+b}{2\sqrt{\sin(2a+}$$

$$= \frac{d(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bc}+\frac{d^2\sqrt{d\cos(a+bx)}\,E\left(a-\frac{\pi}{4}+bx\right.}{2b\sqrt{\sin(2a+2b}$$

Mathematica [C] time = 0.08, size = 70, normalized size = 0.74

$$\frac{2d^2\sqrt[4]{\cos^2(a+bx)}\,\tan(a+bx)\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*d^2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4,
3/4, 7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}\,d^2\cos(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d^2*cos(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a)), x)
```

maple [B] time = 0.18, size = 518, normalized size = 5.45

$$-\frac{\sqrt{c\sin\left(bx+a\right)}\,\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}\left(2\left(\cos^{4}\left(bx+a\right)\right)\sqrt{2}-3\cos\left(bx+a\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin}{\sin(bx+a)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/12/b*(c*sin(b*x+a))^(1/2)*(d*cos(b*x+a))^(5/2)*(2*cos(b*x+a)^4*2^(1/2)-3
*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*cos(b*x+a)*((1-co
s(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*
2^(1/2))+6*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1
-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^(1/2)
-3*cos(b*x+a)*2^(1/2))/sin(b*x+a)/cos(b*x+a)^3*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}\sqrt{c\sin\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(d\,\cos\left(a+b\,x\right)\right)}^{5/2}\,\sqrt{c\,\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(1/2),x)
```

```
[Out] int((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.259 $\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx$

Optimal. Leaf size=53

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),
2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2572, 2639}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (Sqrt[d*Cos[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b
*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx &= \frac{\left(\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{\sqrt{\sin(2a+2bx)}}\\
&= \frac{\sqrt{d\cos(a+bx)}\,E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{b\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 67, normalized size = 1.26

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 3/4,
7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)), x)
```

maple [B] time = 0.14, size = 505, normalized size = 9.53

$$-\frac{\left(2\cos(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\text{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*(2*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ell
ipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)
*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/s
```

```
in(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*((1-cos(b*x+a)+sin(b*x+a))/s
in(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x
+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2),1/2*2^(1/2))-((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elliptic
F(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2*2^
(1/2)-cos(b*x+a)*2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/sin(b*x
+a)/cos(b*x+a)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{d\,\cos(a+b\,x)}\,\sqrt{c\,\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(1/2),x)
[Out] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**(1/2),x)
[Out] Integral(sqrt(c*sin(a + b*x))*sqrt(d*cos(a + b*x)), x)
```

3.260 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

```
[Out] 2*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(1/2)+2*(sin(a+1/4*Pi+b*x)^2)^(
1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^
(1/2)*(c*sin(b*x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2571, 2572, 2639}

$$\frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}}-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(b*c*d*Sqrt[d*Cos[a + b*x]]) - (2*Sqrt[d*Cos[a +
 b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt[Sin[2
*a + 2*b*x]])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}} - \frac{2\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx}{d^2}$$
$$= \frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}} - \frac{\left(2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}}$$
$$= \frac{2(c\sin(a+bx))^{3/2}}{bcd\sqrt{d\cos(a+bx)}} - \frac{2\sqrt{d\cos(a+bx)}\,E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] time = 0.10, size = 70, normalized size = 0.75

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\,\tan(a+bx)\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[3/4, 5/4,
7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*d^2)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{d^2\cos(bx+a)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(d^2*cos(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\sin(bx+a)}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(3/2), x)
```

maple [B] time = 0.17, size = 493, normalized size = 5.30

$$\frac{\left(2\cos\left(bx+a\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] 1/b*(2*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellipt
icE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)*((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
)/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),
1/2*2^(1/2))-((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF((
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)*2^(1/2)
+2^(1/2))*(c*sin(b*x+a))^(1/2)*cos(b*x+a)/(d*cos(b*x+a))^(3/2)/sin(b*x+a)*2
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,\sin\left(a+b\,x\right)}}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(3/2),x)
```

```
[Out] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(3/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Integral(sqrt(c*sin(a + b*x))/(d*cos(a + b*x))**(3/2), x)
```

3.261 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}+\frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/5*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(5/2)+4/5*(c*sin(b*x+a))^(3/2
)/b/c/d^3/(d*cos(b*x+a))^(1/2)+4/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi
+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+
a))^(1/2)/b/d^4/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2571, 2572, 2639}

$$\frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}}-\frac{4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}+\frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (4*(c*Sin[a +
 b*x])^(3/2))/(5*b*c*d^3*Sqrt[d*Cos[a + b*x]]) - (4*Sqrt[d*Cos[a + b*x]]*El
lipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*d^4*Sqrt[Sin[2*a + 2*
b*x]])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx}{5d^2}$$

$$= \frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}} - \frac{4\int \sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}\,d}{5d^4}$$

$$= \frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}} - \frac{\left(4\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}\right)\int}{5d^4\sqrt{\sin(2a+2bx}}$$

$$= \frac{2(c\sin(a+bx))^{3/2}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{4(c\sin(a+bx))^{3/2}}{5bcd^3\sqrt{d\cos(a+bx)}} - \frac{4\sqrt{d\cos(a+bx)}\,E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{}}{5bd^4\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] time = 0.13, size = 70, normalized size = 0.52

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\,\tan(a+bx)\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}\,{}_2F_1\left(\frac{3}{4},\frac{9}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3bd^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[3/4, 9/4,
7/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*d^4)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}}{d^4\cos(bx+a)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(d^4*cos(b*x + a)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\sin(bx+a)}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(7/2), x)
```

maple [B] time = 0.19, size = 528, normalized size = 3.94

$$\frac{\left(4\left(\cos^3(bx+a)\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] 1/5/b*(4*cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*El
lipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x
+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+4*cos(b*x+a)^2*((1-cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^2*((1-cos(b*x+a)+sin(b*x+a))
/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
1/2),1/2*2^(1/2))-2*cos(b*x+a)^3*2^(1/2)+cos(b*x+a)^2*2^(1/2)+2^(1/2))*(c*s
in(b*x+a))^(1/2)*cos(b*x+a)/(d*cos(b*x+a))^(7/2)/sin(b*x+a)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\sin(bx+a)}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,\sin(a+b\,x)}}{(d\,\cos(a+b\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(7/2),x)
```

```
[Out] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

3.262 $\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx$

Optimal. Leaf size=320

$$-\frac{\sqrt{c}\,d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}\,b}+\frac{\sqrt{c}\,d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{4\sqrt{2}\,b}+\frac{\sqrt{c}\,d^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{}\right.}{8\sqrt{2}\,b}$$

```
[Out] -1/8*d^(3/2)*arctan(1-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(d*cos(b
*x+a))^(1/2))*c^(1/2)/b*2^(1/2)+1/8*d^(3/2)*arctan(1+2^(1/2)*d^(1/2)*(c*sin
(b*x+a))^(1/2)/c^(1/2)/(d*cos(b*x+a))^(1/2))*c^(1/2)/b*2^(1/2)+1/16*d^(3/2)
*ln(c^(1/2)-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/
2)*tan(b*x+a))*c^(1/2)/b*2^(1/2)-1/16*d^(3/2)*ln(c^(1/2)+2^(1/2)*d^(1/2)*(c
*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/2)*tan(b*x+a))*c^(1/2)/b*2^(1/
2)+1/2*d*(c*sin(b*x+a))^(3/2)*(d*cos(b*x+a))^(1/2)/b/c
```

Rubi [A] time = 0.30, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2569, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{c}\,d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}\,b}+\frac{\sqrt{c}\,d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{4\sqrt{2}\,b}+\frac{\sqrt{c}\,d^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{}\right.}{8\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -(Sqrt[c]*d^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c
]*Sqrt[d*Cos[a + b*x]])])/(4*Sqrt[2]*b) + (Sqrt[c]*d^(3/2)*ArcTan[1 + (Sqrt
[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(4*Sqrt[
2]*b) + (Sqrt[c]*d^(3/2)*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]
])/Sqrt[d*Cos[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b) - (Sqrt[c]*d
^(3/2)*Log[Sqrt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a +
b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b) + (d*Sqrt[d*Cos[a + b*x]]*(c*S
in[a + b*x])^(3/2))/(2*b*c)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
```

```
& LtQ[m, 1]
```

Rubi steps

$$\int (d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}\,dx = \frac{d\sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{3/2}}{2bc}+\frac{1}{4}d^2\int\frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx$$
$$=\frac{d\sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{3/2}}{2bc}+\frac{(cd^3)\,\text{Subst}\left(\int\frac{x^2}{c^2+d^2x^4}\,dx,x,\frac{\sqrt{c\sin}}{\sqrt{d\cos}}\right.}{2b}$$
$$=\frac{d\sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{3/2}}{2bc}-\frac{(cd^2)\,\text{Subst}\left(\int\frac{c-dx^2}{c^2+d^2x^4}\,dx,x,\frac{\sqrt{c\sin}}{\sqrt{d\cos}}\right.}{4b}$$
$$=\frac{d\sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{3/2}}{2bc}+\frac{(cd)\,\text{Subst}\left(\int\frac{1}{\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx,x,\right.}{8b}$$
$$=\frac{\sqrt{c}\,d^{3/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}\,b}-\frac{\sqrt{c}\,d^{3/2}\log\left(\right.}{}$$
$$=-\frac{\sqrt{c}\,d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}\,b}+\frac{\sqrt{c}\,d^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(}}{\sqrt{c}\sqrt{d\cos(a+}}\right.}{4\sqrt{2}\,b}$$

Mathematica [C] time = 0.11, size = 70, normalized size = 0.22

$$\frac{2d^2\cos^2(a+bx)^{3/4}\tan(a+bx)\sqrt{c\sin(a+bx)}\;{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*d^2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, Sin[a + b*x
]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*Sqrt[d*Cos[a + b*x]])
```

fricas [B] time = 46.86, size = 2205, normalized size = 6.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")

[Out] 1/64*(32*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d*sin(b*x + a) + 4*sqrt(
2)*(c^2*d^6/b^4)^(1/4)*b*arctan(((sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c^3*d^8*cos
(b*x + a) + sqrt(2)*(c^2*d^6/b^4)^(3/4)*b^3*c^2*d^5*sin(b*x + a))*sqrt(d*co
s(b*x + a))*sqrt(c*sin(b*x + a)) + sqrt(4*sqrt(c^2*d^6/b^4)*b^2*c^3*d^7*cos
(b*x + a)*sin(b*x + a) + c^4*d^10 - 2*(sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c^3*d^
8*cos(b*x + a) + sqrt(2)*(c^2*d^6/b^4)^(3/4)*b^3*c^2*d^5*sin(b*x + a))*sqrt
(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(2*c^2*d^5*cos(b*x + a)*sin(b*x + a)
 + sqrt(c^2*d^6/b^4)*b^2*c*d^2 + (sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c*d^3*sin(b
*x + a) + sqrt(2)*(c^2*d^6/b^4)^(3/4)*b^3*cos(b*x + a))*sqrt(d*cos(b*x + a)
)*sqrt(c*sin(b*x + a))))/(2*c^4*d^10*cos(b*x + a)^2 - c^4*d^10)) + 4*sqrt(2
)*(c^2*d^6/b^4)^(1/4)*b*arctan(((sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c^3*d^8*cos(
b*x + a) + sqrt(2)*(c^2*d^6/b^4)^(3/4)*b^3*c^2*d^5*sin(b*x + a))*sqrt(d*cos
(b*x + a))*sqrt(c*sin(b*x + a)) - sqrt(4*sqrt(c^2*d^6/b^4)*b^2*c^3*d^7*cos(
b*x + a)*sin(b*x + a) + c^4*d^10 + 2*(sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c^3*d^8
*cos(b*x + a) + sqrt(2)*(c^2*d^6/b^4)^(3/4)*b^3*c^2*d^5*sin(b*x + a))*sqrt(
d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(2*c^2*d^5*cos(b*x + a)*sin(b*x + a)
+ sqrt(c^2*d^6/b^4)*b^2*c*d^2 - (sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c*d^3*sin(b*
x + a) + sqrt(2)*(c^2*d^6/b^4)^(3/4)*b^3*cos(b*x + a))*sqrt(d*cos(b*x + a))
*sqrt(c*sin(b*x + a))))/(2*c^4*d^10*cos(b*x + a)^2 - c^4*d^10)) + 4*sqrt(2)
*(c^2*d^6/b^4)^(1/4)*b*arctan(-1/2*(2*c^4*d^10*cos(b*x + a)*sin(b*x + a) -
sqrt(4*sqrt(c^2*d^6/b^4)*b^2*c^3*d^7*cos(b*x + a)*sin(b*x + a) + c^4*d^10 +
 2*(sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c^3*d^8*cos(b*x + a) + sqrt(2)*(c^2*d^6/b
^4)^(3/4)*b^3*c^2*d^5*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a
)))*(sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c*d^3*sin(b*x + a) + sqrt(2)*(c^2*d^6/b^
4)^(3/4)*b^3*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + (sqr
t(2)*(c^2*d^6/b^4)^(1/4)*b*c^3*d^8*sin(b*x + a) + sqrt(2)*(c^2*d^6/b^4)^(3/
4)*b^3*c^2*d^5*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - 4*
(b^2*c^3*d^7*cos(b*x + a)^4 - b^2*c^3*d^7*cos(b*x + a)^2)*sqrt(c^2*d^6/b^4)
)/((2*c^4*d^10*cos(b*x + a)^3 - c^4*d^10*cos(b*x + a))*sin(b*x + a))) + 4*s
qrt(2)*(c^2*d^6/b^4)^(1/4)*b*arctan(1/2*(2*c^4*d^10*cos(b*x + a)*sin(b*x +
a) + sqrt(4*sqrt(c^2*d^6/b^4)*b^2*c^3*d^7*cos(b*x + a)*sin(b*x + a) + c^4*d
^10 - 2*(sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c^3*d^8*cos(b*x + a) + sqrt(2)*(c^2*
d^6/b^4)^(3/4)*b^3*c^2*d^5*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*
x + a)))*(sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c*d^3*sin(b*x + a) + sqrt(2)*(c^2*d
^6/b^4)^(3/4)*b^3*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) -
 (sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c^3*d^8*sin(b*x + a) + sqrt(2)*(c^2*d^6/b^4
)^(3/4)*b^3*c^2*d^5*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))
 - 4*(b^2*c^3*d^7*cos(b*x + a)^4 - b^2*c^3*d^7*cos(b*x + a)^2)*sqrt(c^2*d^6
/b^4))/((2*c^4*d^10*cos(b*x + a)^3 - c^4*d^10*cos(b*x + a))*sin(b*x + a)))
- sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*log(4*sqrt(c^2*d^6/b^4)*b^2*c^3*d^7*cos(b*x
 + a)*sin(b*x + a) + c^4*d^10 + 2*(sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c^3*d^8*co
s(b*x + a) + sqrt(2)*(c^2*d^6/b^4)^(3/4)*b^3*c^2*d^5*sin(b*x + a))*sqrt(d*c
os(b*x + a))*sqrt(c*sin(b*x + a))) + sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*log(4*sq
```

```
rt(c^2*d^6/b^4)*b^2*c^3*d^7*cos(b*x + a)*sin(b*x + a) + c^4*d^10 - 2*(sqrt(
2)*(c^2*d^6/b^4)^(1/4)*b*c^3*d^8*cos(b*x + a) + sqrt(2)*(c^2*d^6/b^4)^(3/4)
*b^3*c^2*d^5*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) - sqr
t(2)*(c^2*d^6/b^4)^(1/4)*b*log(1/4*sqrt(c^2*d^6/b^4)*b^2*c^3*d^7*cos(b*x +
a)*sin(b*x + a) + 1/16*c^4*d^10 + 1/8*(sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c^3*d^
8*cos(b*x + a) + sqrt(2)*(c^2*d^6/b^4)^(3/4)*b^3*c^2*d^5*sin(b*x + a))*sqrt
(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*log(
1/4*sqrt(c^2*d^6/b^4)*b^2*c^3*d^7*cos(b*x + a)*sin(b*x + a) + 1/16*c^4*d^10
 - 1/8*(sqrt(2)*(c^2*d^6/b^4)^(1/4)*b*c^3*d^8*cos(b*x + a) + sqrt(2)*(c^2*d
^6/b^4)^(3/4)*b^3*c^2*d^5*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x
 + a))))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{3}{2}}\sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a)), x)
```

maple [C] time = 0.12, size = 514, normalized size = 1.61

$$\left(-i\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x)
```

```
[Out] 1/8/b*(-I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1
-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+I*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+((1-cos(b*x+a)+sin(b*x+a))/s
in(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x
+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2),1/2-1/2*I,1/2*2^(1/2))+((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-
1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/
2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^
(1/2))+2*cos(b*x+a)^2*2^(1/2)-2*cos(b*x+a)*2^(1/2))*(d*cos(b*x+a))^(3/2)*(c
*sin(b*x+a))^(1/2)*sin(b*x+a)/(-1+cos(b*x+a))/cos(b*x+a)^2*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{3}{2}}\sqrt{c\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d\cos(a+b\,x))^{3/2}\sqrt{c\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(1/2),x)
[Out] int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*(c*sin(b*x+a))**(1/2),x)
[Out] Timed out
```

3.263 $\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=280

$$-\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}b\sqrt{d}}+\frac{\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+\right.}{2\sqrt{2}b\sqrt{d}}$$

```
[Out] -1/2*arctan(1-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(d*cos(b*x+a))^(
1/2))*c^(1/2)/b*2^(1/2)/d^(1/2)+1/2*arctan(1+2^(1/2)*d^(1/2)*(c*sin(b*x+a))
^(1/2)/c^(1/2)/(d*cos(b*x+a))^(1/2))*c^(1/2)/b*2^(1/2)/d^(1/2)+1/4*ln(c^(1/
2)-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/2)*tan(b*
x+a))*c^(1/2)/b*2^(1/2)/d^(1/2)-1/4*ln(c^(1/2)+2^(1/2)*d^(1/2)*(c*sin(b*x+a
))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/2)*tan(b*x+a))*c^(1/2)/b*2^(1/2)/d^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}b\sqrt{d}}+\frac{\sqrt{c}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+\right.}{2\sqrt{2}b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -((Sqrt[c]*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[
d*Cos[a + b*x]])])/(Sqrt[2]*b*Sqrt[d])) + (Sqrt[c]*ArcTan[1 + (Sqrt[2]*Sqrt
[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(Sqrt[2]*b*Sqrt[
d]) + (Sqrt[c]*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*
Cos[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(2*Sqrt[2]*b*Sqrt[d]) - (Sqrt[c]*Log
[Sqrt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a + b*x]] + Sq
rt[c]*Tan[a + b*x]])/(2*Sqrt[2]*b*Sqrt[d])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx = \frac{(2cd)\,\text{Subst}\left(\int \frac{x^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{b}$$
$$= -\frac{c\,\text{Subst}\left(\int \frac{c-dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{b} + \frac{c\,\text{Subst}\left(\int \frac{c+dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{b}$$
$$= \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2bd} + \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{c}{d}+\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2bd}$$
$$= \frac{\sqrt{c}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}b\sqrt{d}} - \frac{\sqrt{c}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\right.}{2\sqrt{2}b\sqrt{d}}$$
$$= -\frac{\sqrt{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}} + \frac{\sqrt{c}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}b\sqrt{d}} + \frac{\sqrt{c}\log\left(\sqrt{c}-\right.}{}$$

Mathematica [C] time = 0.06, size = 67, normalized size = 0.24

$$\frac{2\cos^2(a+bx)^{3/4}\tan(a+bx)\sqrt{c\sin(a+bx)}\;{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\sin^2(a+bx)\right)}{3b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[a + b*x]^2]*
Sqrt[c*Sin[a + b*x]]*Tan[a + b*x])/(3*b*Sqrt[d*Cos[a + b*x]])
```

fricas [B] time = 49.21, size = 2003, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*(c^2/(b^4*d^2))^(1/4)*arctan(((sqrt(2)*b^3*c^2*d*(c^2/(b^4*d^2)
)^(3/4)*sin(b*x + a) + sqrt(2)*b*c^3*(c^2/(b^4*d^2))^(1/4)*cos(b*x + a))*sq
rt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + sqrt(4*b^2*c^3*d*sqrt(c^2/(b^4*d^
2))*cos(b*x + a)*sin(b*x + a) + c^4 - 2*(sqrt(2)*b^3*c^2*d*(c^2/(b^4*d^2))^
```

```
(3/4)*sin(b*x + a) + sqrt(2)*b*c^3*(c^2/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt
(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(b^2*c*d*sqrt(c^2/(b^4*d^2)) + 2*c^2
*cos(b*x + a)*sin(b*x + a) + (sqrt(2)*b^3*d*(c^2/(b^4*d^2))^(3/4)*cos(b*x +
 a) + sqrt(2)*b*c*(c^2/(b^4*d^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*
sqrt(c*sin(b*x + a))))/(2*c^4*cos(b*x + a)^2 - c^4)) + 1/4*sqrt(2)*(c^2/(b^
4*d^2))^(1/4)*arctan(((sqrt(2)*b^3*c^2*d*(c^2/(b^4*d^2))^(3/4)*sin(b*x + a)
 + sqrt(2)*b*c^3*(c^2/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*s
qrt(c*sin(b*x + a)) - sqrt(4*b^2*c^3*d*sqrt(c^2/(b^4*d^2))*cos(b*x + a)*sin
(b*x + a) + c^4 + 2*(sqrt(2)*b^3*c^2*d*(c^2/(b^4*d^2))^(3/4)*sin(b*x + a) +
 sqrt(2)*b*c^3*(c^2/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqr
t(c*sin(b*x + a)))*(b^2*c*d*sqrt(c^2/(b^4*d^2)) + 2*c^2*cos(b*x + a)*sin(b*
x + a) - (sqrt(2)*b^3*d*(c^2/(b^4*d^2))^(3/4)*cos(b*x + a) + sqrt(2)*b*c*(c
^2/(b^4*d^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))
))/(2*c^4*cos(b*x + a)^2 - c^4)) + 1/4*sqrt(2)*(c^2/(b^4*d^2))^(1/4)*arctan
(-1/2*(2*c^4*cos(b*x + a)*sin(b*x + a) - sqrt(4*b^2*c^3*d*sqrt(c^2/(b^4*d^2
))*cos(b*x + a)*sin(b*x + a) + c^4 + 2*(sqrt(2)*b^3*c^2*d*(c^2/(b^4*d^2))^(
3/4)*sin(b*x + a) + sqrt(2)*b*c^3*(c^2/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(
d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(sqrt(2)*b^3*d*(c^2/(b^4*d^2))^(3/4)*
cos(b*x + a) + sqrt(2)*b*c*(c^2/(b^4*d^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b
*x + a))*sqrt(c*sin(b*x + a)) + (sqrt(2)*b^3*c^2*d*(c^2/(b^4*d^2))^(3/4)*co
s(b*x + a) + sqrt(2)*b*c^3*(c^2/(b^4*d^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b
*x + a))*sqrt(c*sin(b*x + a)) - 4*(b^2*c^3*d*cos(b*x + a)^4 - b^2*c^3*d*cos
(b*x + a)^2)*sqrt(c^2/(b^4*d^2)))/((2*c^4*cos(b*x + a)^3 - c^4*cos(b*x + a)
)*sin(b*x + a))) + 1/4*sqrt(2)*(c^2/(b^4*d^2))^(1/4)*arctan(1/2*(2*c^4*cos(
b*x + a)*sin(b*x + a) + sqrt(4*b^2*c^3*d*sqrt(c^2/(b^4*d^2))*cos(b*x + a)*s
in(b*x + a) + c^4 - 2*(sqrt(2)*b^3*c^2*d*(c^2/(b^4*d^2))^(3/4)*sin(b*x + a)
 + sqrt(2)*b*c^3*(c^2/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*s
qrt(c*sin(b*x + a)))*(sqrt(2)*b^3*d*(c^2/(b^4*d^2))^(3/4)*cos(b*x + a) + sq
rt(2)*b*c*(c^2/(b^4*d^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*s
in(b*x + a)) - (sqrt(2)*b^3*c^2*d*(c^2/(b^4*d^2))^(3/4)*cos(b*x + a) + sqrt
(2)*b*c^3*(c^2/(b^4*d^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*s
in(b*x + a)) - 4*(b^2*c^3*d*cos(b*x + a)^4 - b^2*c^3*d*cos(b*x + a)^2)*sqrt
(c^2/(b^4*d^2)))/((2*c^4*cos(b*x + a)^3 - c^4*cos(b*x + a))*sin(b*x + a)))
- 1/16*sqrt(2)*(c^2/(b^4*d^2))^(1/4)*log(4*b^2*c^3*d*sqrt(c^2/(b^4*d^2))*co
s(b*x + a)*sin(b*x + a) + c^4 + 2*(sqrt(2)*b^3*c^2*d*(c^2/(b^4*d^2))^(3/4)*
sin(b*x + a) + sqrt(2)*b*c^3*(c^2/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*cos
(b*x + a))*sqrt(c*sin(b*x + a))) + 1/16*sqrt(2)*(c^2/(b^4*d^2))^(1/4)*log(4
*b^2*c^3*d*sqrt(c^2/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) + c^4 - 2*(sqrt(2)
*b^3*c^2*d*(c^2/(b^4*d^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^3*(c^2/(b^4*d^2
))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) - 1/16*sq
rt(2)*(c^2/(b^4*d^2))^(1/4)*log(1/4*b^2*c^3*d*sqrt(c^2/(b^4*d^2))*cos(b*x +
 a)*sin(b*x + a) + 1/16*c^4 + 1/8*(sqrt(2)*b^3*c^2*d*(c^2/(b^4*d^2))^(3/4)*
sin(b*x + a) + sqrt(2)*b*c^3*(c^2/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*cos
(b*x + a))*sqrt(c*sin(b*x + a))) + 1/16*sqrt(2)*(c^2/(b^4*d^2))^(1/4)*log(1
/4*b^2*c^3*d*sqrt(c^2/(b^4*d^2))*cos(b*x + a)*sin(b*x + a) + 1/16*c^4 - 1/8
```

```
*(sqrt(2)*b^3*c^2*d*(c^2/(b^4*d^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^3*(c^2
/(b^4*d^2))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\sin(bx+a)}}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sin(b*x + a))/sqrt(d*cos(b*x + a)), x)
```

maple [C] time = 0.11, size = 271, normalized size = 0.97

$$-\frac{\sqrt{c\sin(bx+a)}\,\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x)
[Out] -1/2/b*(c*sin(b*x+a))^(1/2)*(I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)/(-1+cos(b*x+a))/(d*cos(b*x+a))^(1/2)*2
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\sin(bx+a)}}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sin(b*x + a))/sqrt(d*cos(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,\sin(a+b\,x)}}{\sqrt{d\,\cos(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(1/2),x)
```

```
[Out] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c \sin (a+bx)}}{\sqrt{d \cos (a+bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*sin(a + b*x))/sqrt(d*cos(a + b*x)), x)
```

3.264 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/3*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2563}

$$\frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(3*b*c*d*(d*Cos[a + b*x])^(3/2))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

Mathematica [A] time = 0.08, size = 37, normalized size = 1.00

$$\frac{2(c\sin(a+bx))^{3/2}}{3bcd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(3*b*c*d*(d*Cos[a + b*x])^(3/2))
```

fricas [A] time = 0.46, size = 42, normalized size = 1.14

$$\frac{2 \sqrt{d \cos (bx + a)} \sqrt{c \sin (bx + a)} \sin (bx + a)}{3 \, bd^{3} \cos (bx + a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
[Out] 2/3*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sin(b*x + a)/(b*d^3*cos(b*x +
 a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c \sin (bx + a)}}{(d \cos (bx + a))^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(5/2), x)
```

maple [A] time = 0.14, size = 38, normalized size = 1.03

$$\frac{2 \sin (bx + a) \cos (bx + a) \sqrt{c \sin (bx + a)}}{3b \, (d \cos (bx + a))^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x)
[Out] 2/3/b*sin(b*x+a)*cos(b*x+a)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c \sin (bx + a)}}{(d \cos (bx + a))^{\frac{5}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(5/2), x)
```

mupad [B] time = 0.99, size = 50, normalized size = 1.35

$$\frac{2\,\sin\left(2\,a+2\,b\,x\right)\,\sqrt{c\,\sin\left(a+b\,x\right)}}{3\,b\,d^2\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)\,\sqrt{d\,\cos\left(a+b\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(5/2),x)
```

```
[Out] (2*sin(2*a + 2*b*x)*(c*sin(a + b*x))^(1/2))/(3*b*d^2*(cos(2*a + 2*b*x) + 1)
*(d*cos(a + b*x))^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.265 $\int \frac{\sqrt{c \sin(a+bx)}}{(d \cos(a+bx))^{9/2}} \, dx$

Optimal. Leaf size=75

$$\frac{8(c \sin(a+bx))^{3/2}}{21bcd^3(d \cos(a+bx))^{3/2}} + \frac{2(c \sin(a+bx))^{3/2}}{7bcd(d \cos(a+bx))^{7/2}}$$

```
[Out] 2/7*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(7/2)+8/21*(c*sin(b*x+a))^(3/
2)/b/c/d^3/(d*cos(b*x+a))^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2571, 2563}

$$\frac{8(c \sin(a+bx))^{3/2}}{21bcd^3(d \cos(a+bx))^{3/2}} + \frac{2(c \sin(a+bx))^{3/2}}{7bcd(d \cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (8*(c*Sin[a +
 b*x])^(3/2))/(21*b*c*d^3*(d*Cos[a + b*x])^(3/2))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{4\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx}{7d^2}$$
$$= \frac{2(c\sin(a+bx))^{3/2}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{8(c\sin(a+bx))^{3/2}}{21bcd^3(d\cos(a+bx))^{3/2}}$$

Mathematica [A] time = 0.22, size = 57, normalized size = 0.76

$$\frac{2(2\cos(2(a+bx))+5)\sec^4(a+bx)(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{21bcd^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(5 + 2*Cos[2*(a + b*x)])*Sec[a + b*x]^4*(c*Sin[a +
b*x])^(3/2))/(21*b*c*d^5)
```

fricas [A] time = 0.48, size = 54, normalized size = 0.72

$$\frac{2\,\sqrt{d\cos\left(bx+a\right)}\left(4\,\cos\left(bx+a\right)^2+3\right)\sqrt{c\sin\left(bx+a\right)}\,\sin\left(bx+a\right)}{21\,bd^5\cos\left(bx+a\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] 2/21*sqrt(d*cos(b*x + a))*(4*cos(b*x + a)^2 + 3)*sqrt(c*sin(b*x + a))*sin(b
*x + a)/(b*d^5*cos(b*x + a)^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(9/2), x)
```

maple [A] time = 0.14, size = 50, normalized size = 0.67

$$\frac{2\left(4\left(\cos^2\left(bx+a\right)\right)+3\right)\sqrt{c\sin\left(bx+a\right)}\,\cos\left(bx+a\right)\sin\left(bx+a\right)}{21b\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 2/21/b*(4*cos(b*x+a)^2+3)*(c*sin(b*x+a))^(1/2)*cos(b*x+a)*sin(b*x+a)/(d*cos
(b*x+a))^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\sin(bx+a)}}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(9/2), x)
```

mupad [B] time = 2.06, size = 95, normalized size = 1.27

$$\frac{8\sqrt{c\sin(a+b\,x)}\,\left(11\sin(2\,a+2\,b\,x)+7\sin(4\,a+4\,b\,x)+\sin(6\,a+6\,b\,x)\right)}{21\,b\,d^4\sqrt{d\cos(a+b\,x)}\,\left(15\cos(2\,a+2\,b\,x)+6\cos(4\,a+4\,b\,x)+\cos(6\,a+6\,b\,x)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(9/2),x)
```

```
[Out] (8*(c*sin(a + b*x))^(1/2)*(11*sin(2*a + 2*b*x) + 7*sin(4*a + 4*b*x) + sin(6
*a + 6*b*x)))/(21*b*d^4*(d*cos(a + b*x))^(1/2)*(15*cos(2*a + 2*b*x) + 6*cos
(4*a + 4*b*x) + cos(6*a + 6*b*x) + 10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

3.266 $\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx$

Optimal. Leaf size=112

$$\frac{64(c\sin(a+bx))^{3/2}}{231bcd^5(d\cos(a+bx))^{3/2}}+\frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}$$

```
[Out] 2/11*(c*sin(b*x+a))^(3/2)/b/c/d/(d*cos(b*x+a))^(11/2)+16/77*(c*sin(b*x+a))^
(3/2)/b/c/d^3/(d*cos(b*x+a))^(7/2)+64/231*(c*sin(b*x+a))^(3/2)/b/c/d^5/(d*c
os(b*x+a))^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2571, 2563}

$$\frac{64(c\sin(a+bx))^{3/2}}{231bcd^5(d\cos(a+bx))^{3/2}}+\frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}}+\frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(13/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(3/2))/(11*b*c*d*(d*Cos[a + b*x])^(11/2)) + (16*(c*Sin[
a + b*x])^(3/2))/(77*b*c*d^3*(d*Cos[a + b*x])^(7/2)) + (64*(c*Sin[a + b*x])
^(3/2))/(231*b*c*d^5*(d*Cos[a + b*x])^(3/2))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx = \frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}} + \frac{8\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx}{11d^2}$$
$$= \frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}} + \frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}} + \frac{32\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx}{77d^4}$$
$$= \frac{2(c\sin(a+bx))^{3/2}}{11bcd(d\cos(a+bx))^{11/2}} + \frac{16(c\sin(a+bx))^{3/2}}{77bcd^3(d\cos(a+bx))^{7/2}} + \frac{64(c\sin(a+bx))^{3/2}}{231bcd^5(d\cos(a+bx))^{3/2}}$$

Mathematica [A] time = 0.23, size = 67, normalized size = 0.60

$$\frac{2(28\cos(2(a+bx)) + 4\cos(4(a+bx)) + 45)\sec^6(a+bx)(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}{231bcd^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]]/(d*Cos[a + b*x])^(13/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(45 + 28*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Sec
[a + b*x]^6*(c*Sin[a + b*x])^(3/2))/(231*b*c*d^7)
```

fricas [A] time = 0.57, size = 64, normalized size = 0.57

$$\frac{2\left(32\cos(bx+a)^4 + 24\cos(bx+a)^2 + 21\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\sin(bx+a)}{231\,bd^7\cos(bx+a)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x, algorithm="fricas")
```

```
[Out] 2/231*(32*cos(b*x + a)^4 + 24*cos(b*x + a)^2 + 21)*sqrt(d*cos(b*x + a))*sqr
t(c*sin(b*x + a))*sin(b*x + a)/(b*d^7*cos(b*x + a)^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\sin(bx+a)}}{(d\cos(bx+a))^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(13/2), x)
```

maple [A] time = 0.21, size = 60, normalized size = 0.54

$$\frac{2\left(32\left(\cos^{4}\left(bx+a\right)\right)+24\left(\cos^{2}\left(bx+a\right)\right)+21\right)\sqrt{c\sin\left(bx+a\right)}\ \cos\left(bx+a\right)\sin\left(bx+a\right)}{231b\left(d\cos\left(bx+a\right)\right)^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x)
```

```
[Out] 2/231/b*(32*cos(b*x+a)^4+24*cos(b*x+a)^2+21)*(c*sin(b*x+a))^(1/2)*cos(b*x+a
)*sin(b*x+a)/(d*cos(b*x+a))^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c\sin\left(bx+a\right)}}{\left(d\cos\left(bx+a\right)\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(13/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))/(d*cos(b*x + a))^(13/2), x)
```

mupad [B] time = 6.21, size = 216, normalized size = 1.93

$$-\frac{\sqrt{c\,\sin\left(a+b\,x\right)}\,\left(2\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-1\right)\,\left(2\,\sin\left(\frac{5\,a}{2}+\frac{5\,b\,x}{2}\right)^2+\sin\left(5\,a+5\,b\,x\right)\,1{}\mathrm{i}-1\right)\,\left(\frac{1984\,\sin\left(a+b\,x\right)\,\left(-2\,\sin\left(\frac{5\,a}{2}+\frac{5\,b}{2}\right.\right.}{231\,b\,d^6}\right.}{32\,\left(\sin\left(a+b\,x\right)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(1/2)/(d*cos(a + b*x))^(13/2),x)
```

```
[Out] -((c*sin(a + b*x))^(1/2)*(2*sin(a/2 + (b*x)/2)^2 - 1)*(sin(5*a + 5*b*x)*1i
+ 2*sin((5*a)/2 + (5*b*x)/2)^2 - 1)*((1984*sin(a + b*x)*(sin(5*a + 5*b*x)*1
i - 2*sin((5*a)/2 + (5*b*x)/2)^2 + 1))/(231*b*d^6) + (256*sin(3*a + 3*b*x)*
(sin(5*a + 5*b*x)*1i - 2*sin((5*a)/2 + (5*b*x)/2)^2 + 1))/(77*b*d^6) + (128
*sin(5*a + 5*b*x)*(sin(5*a + 5*b*x)*1i - 2*sin((5*a)/2 + (5*b*x)/2)^2 + 1))
/(231*b*d^6)))/(32*(sin(a + b*x)^2 - 1)^3*(-d*(2*sin(a/2 + (b*x)/2)^2 - 1))
^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(1/2)/(d*cos(b*x+a))**(13/2),x)
```

```
[Out] Timed out
```

3.267 $\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx$

Optimal. Leaf size=131

$$\frac{c^2d^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{12b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bd}+\frac{cd\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{6b}$$

```
[Out] -1/3*c*(d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2)/b/d+1/6*c*d*(d*cos(b*x+a))
^(1/2)*(c*sin(b*x+a))^(1/2)/b-1/12*c^2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(
a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/(
d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2568, 2569, 2573, 2641}

$$\frac{c^2d^2\sqrt{\sin(2a+2bx)}F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{12b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{c\sqrt{c\sin(a+bx)}(d\cos(a+bx))^{5/2}}{3bd}+\frac{cd\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (c*d*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(6*b) - (c*(d*Cos[a + b*x])
^(5/2)*Sqrt[c*Sin[a + b*x]])/(3*b*d) + (c^2*d^2*EllipticF[a - Pi/4 + b*x, 2
]*Sqrt[Sin[2*a + 2*b*x]])/(12*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}\,dx &= -\frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd}+\frac{1}{6}c^2\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx\\ &= \frac{cd\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6b}-\frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd}\\ &= \frac{cd\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6b}-\frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd}\\ &= \frac{cd\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6b}-\frac{c(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bd}\end{aligned}$$

Mathematica [C] time = 0.12, size = 71, normalized size = 0.54

$$\frac{2cd\cos^2(a+bx)^{3/4}\tan^2(a+bx)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\frac{9}{4};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (2*c*d*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4,
5/4, 9/4, Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*Tan[a + b*x]^2)/(5*b)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\,cd\cos(bx+a)\sin(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*d*cos(b*x + a)*sin(b*x
 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{3}{2}} (c\sin(bx+a))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^(3/2), x)
```

maple [A] time = 0.20, size = 216, normalized size = 1.65

$$-\frac{\left(\sin(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}, \frac{\sqrt{2}}{2}\right.\right.}{12b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x)
```

```
[Out] -1/12/b*(sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elli
pticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a
)^4*2^(1/2)-2*cos(b*x+a)^3*2^(1/2)-cos(b*x+a)^2*2^(1/2)+cos(b*x+a)*2^(1/2))
*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/sin(b*x+a)/(-1+cos(b*x+a))/cos(b
*x+a)^2*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^{\frac{3}{2}} (c\sin(bx+a))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\ \cos(a+b\,x))^{3/2}\,(c\ \sin(a+b\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(3/2),x)
```

```
[Out] int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*(c*sin(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

3.268 $\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=93

$$\frac{c^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}-\frac{c\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}{bd}$$

```
[Out] -c*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/d-1/2*c^2*(sin(a+1/4*Pi+b*x)
^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+
2*a)^(1/2)/b/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2568, 2573, 2641}

$$\frac{c^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}-\frac{c\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] -((c*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(b*d)) + (c^2*EllipticF[a -
 Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Si
n[a + b*x]])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c\sin(a+bx))^{3/2}}{\sqrt{d\cos(a+bx)}}\,dx &= -\frac{c\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bd}+\frac{1}{2}c^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx\\ &= -\frac{c\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bd}+\frac{\left(c^2\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\\ &= -\frac{c\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{bd}+\frac{c^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.08, size = 67, normalized size = 0.72

$$\frac{2\cos^2(a+bx)^{3/4}\tan(a+bx)(c\sin(a+bx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{5}{4};\frac{9}{4};\sin^2(a+bx)\right)}{5b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 5/4, 9/4, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(3/2)*Tan[a + b*x])/(5*b*Sqrt[d*Cos[a + b*x]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\,c\sin(bx+a)}{d\cos(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*sin(b*x + a)/(d*cos(b*
x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/sqrt(d*cos(b*x + a)), x)
```

maple [A] time = 0.15, size = 182, normalized size = 1.96

$$-\frac{\left(\sin \left(bx+a\right) \sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\, \sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\, \sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\, \mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}, \frac{\sqrt{2}}{2}\right.\right.}{2b\left(-1+\cos\left(bx+a\right)\right)\sqrt{d\cos\left(bx+a\right)}\, \sin\left(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*(sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2
*2^(1/2)-cos(b*x+a)*2^(1/2))*(c*sin(b*x+a))^(3/2)/(-1+cos(b*x+a))/(d*cos(b*
x+a))^(1/2)/sin(b*x+a)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}}{\sqrt{d\cos\left(bx+a\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/sqrt(d*cos(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,\sin\left(a+b\,x\right)\right)}^{3/2}}{\sqrt{d\,\cos\left(a+b\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(1/2),x)
```

```
[Out] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(a+bx\right)\right)^{\frac{3}{2}}}{\sqrt{d\cos\left(a+bx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(1/2),x)
[Out] Integral((c*sin(a + b*x))**(3/2)/sqrt(d*cos(a + b*x)), x)
```

3.269 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3bd^2\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}$$

```
[Out] 2/3*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(3/2)+1/3*c^2*(sin(a+1/4*Pi+b
*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b
*x+2*a)^(1/2)/b/d^2/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2566, 2573, 2641}

$$\frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3bd^2\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(3*b*d*(d*Cos[a + b*x])^(3/2)) - (c^2*EllipticF[
a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]*Sq
rt[c*Sin[a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{5/2}}\,dx &= \frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx}{3d^2}\\ &= \frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{\left(c^2\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3d^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\\ &= \frac{2c\sqrt{c\sin(a+bx)}}{3bd(d\cos(a+bx))^{3/2}}-\frac{c^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.16, size = 67, normalized size = 0.68

$$\frac{2\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{9}{4};\sin^2(a+bx)\right)}{5bcd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[5/4, 7/4, 9/4, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(5/2))/(5*b*c*d*(d*Cos[a + b*x])^(3/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\,c\sin(bx+a)}{d^3\cos(bx+a)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*sin(b*x + a)/(d^3*cos(
b*x + a)^3), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.12, size = 186, normalized size = 1.90

$$\frac{\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sin(bx+a)}{3b\left(-1+\cos(bx+a)\right)\left(d\cos(bx+a)\right)^{\frac{5}{2}}\sin(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 1/3/b*(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)*cos(b*x+a)+co
s(b*x+a)*2^(1/2)-2^(1/2))*(c*sin(b*x+a))^(3/2)*cos(b*x+a)/(-1+cos(b*x+a))/(
d*cos(b*x+a))^(5/2)/sin(b*x+a)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,\sin\left(a+b\,x\right)\right)}^{3/2}}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(5/2),x)
```

```
[Out] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.270 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=133

$$-\frac{2c^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{21bd^4\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}}+\frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}$$

```
[Out] 2/7*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(7/2)-2/21*c*(c*sin(b*x+a))^(
1/2)/b/d^3/(d*cos(b*x+a))^(3/2)+2/21*c^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+
1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/d^4
/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2566, 2571, 2573, 2641}

$$-\frac{2c^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{21bd^4\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}-\frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}}+\frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(7*b*d*(d*Cos[a + b*x])^(7/2)) - (2*c*Sqrt[c*Sin
[a + b*x]])/(21*b*d^3*(d*Cos[a + b*x])^(3/2)) - (2*c^2*EllipticF[a - Pi/4 +
 b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(21*b*d^4*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[
a + b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{9/2}}\,dx &= \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}-\frac{c^2\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx}{7d^2}\\ &= \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}-\frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}}-\frac{\left(2c^2\right)\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx}{21d^4}\\ &= \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}-\frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}}-\frac{\left(2c^2\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}}{21d^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\\ &= \frac{2c\sqrt{c\sin(a+bx)}}{7bd(d\cos(a+bx))^{7/2}}-\frac{2c\sqrt{c\sin(a+bx)}}{21bd^3(d\cos(a+bx))^{3/2}}-\frac{2c^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{21bd^4\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.14, size = 70, normalized size = 0.53

$$\frac{2\cos^2(a+bx)^{7/4}\cot(a+bx)(c\sin(a+bx))^{7/2}\,{}_2F_1\left(\frac{5}{4},\frac{11}{4};\frac{9}{4};\sin^2(a+bx)\right)}{5bc^2(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(7/4)*Cot[a + b*x]*Hypergeometric2F1[5/4, 11/4, 9/4, Si
n[a + b*x]^2]*(c*Sin[a + b*x])^(7/2))/(5*b*c^2*(d*Cos[a + b*x])^(9/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\,c\sin(bx+a)}{d^5\cos(bx+a)^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*sin(b*x + a)/(d^5*cos(
b*x + a)^5), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.14, size = 215, normalized size = 1.62

$$\frac{\left(2\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\;\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sin(bx+a\right.}{21b\,(-1+\cos(bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 1/21/b*(2*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)*cos(b*x+a)
^3-cos(b*x+a)^3*2^(1/2)+cos(b*x+a)^2*2^(1/2)+3*cos(b*x+a)*2^(1/2)-3*2^(1/2)
)*(c*sin(b*x+a))^(3/2)*cos(b*x+a)/(-1+cos(b*x+a))/(d*cos(b*x+a))^(9/2)/sin(
b*x+a)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,\sin(a+b\,x))^{3/2}}{(d\,\cos(a+b\,x))^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(9/2),x)
[Out] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(9/2),x)
[Out] Timed out
```

3.271 $\int \sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{3/2}\,dx$

Optimal. Leaf size=320

$$\frac{c^{3/2}\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}\,b}-\frac{c^{3/2}\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{4\sqrt{2}\,b}-\frac{c^{3/2}\sqrt{d}\,\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\cdots\right.}{8\sqrt{2}\,b}$$

```
[Out] -1/8*c^(3/2)*arctan(-1+2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/d^(1/2)/(c*sin(
b*x+a))^(1/2))*d^(1/2)/b*2^(1/2)-1/8*c^(3/2)*arctan(1+2^(1/2)*c^(1/2)*(d*co
s(b*x+a))^(1/2)/d^(1/2)/(c*sin(b*x+a))^(1/2))*d^(1/2)/b*2^(1/2)-1/16*c^(3/2
)*ln(d^(1/2)+cot(b*x+a)*d^(1/2)-2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(c*sin
(b*x+a))^(1/2))*d^(1/2)/b*2^(1/2)+1/16*c^(3/2)*ln(d^(1/2)+cot(b*x+a)*d^(1/2
)+2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2))*d^(1/2)/b*2^(1
/2)-1/2*c*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(1/2)/b/d
```

Rubi [A] time = 0.28, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2568, 2575, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{3/2}\sqrt{d}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}\,b}-\frac{c^{3/2}\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{4\sqrt{2}\,b}-\frac{c^{3/2}\sqrt{d}\,\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\cdots\right.}{8\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (c^(3/2)*Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]
*Sqrt[c*Sin[a + b*x]])])/(4*Sqrt[2]*b) - (c^(3/2)*Sqrt[d]*ArcTan[1 + (Sqrt[
2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])])/(4*Sqrt[2
]*b) - (c^(3/2)*Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] - (Sqrt[2]*Sqrt[
c]*Sqrt[d*Cos[a + b*x]])/Sqrt[c*Sin[a + b*x]]])/(8*Sqrt[2]*b) + (c^(3/2)*Sq
rt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a +
b*x]])/Sqrt[c*Sin[a + b*x]]])/(8*Sqrt[2]*b) - (c*(d*Cos[a + b*x])^(3/2)*Sqr
t[c*Sin[a + b*x]])/(2*b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
```

```
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{3/2}\,dx = -\frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd} + \frac{1}{4}c^2\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx$$

$$= -\frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd} - \frac{\left(c^3d\right)\text{Subst}\left(\int \frac{x^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\text{c}}}{\sqrt{c\text{s}}}\right.}{2b}$$

$$= -\frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd} + \frac{\left(c^2d\right)\text{Subst}\left(\int \frac{d-cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\text{c}}}{\sqrt{c\text{s}}}\right.}{4b}$$

$$= -\frac{c(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}{2bd} - \frac{\left(c^{3/2}\sqrt{d}\right)\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}+2x}{-\frac{d}{c}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\right.}{8\sqrt{2}\,b}$$

$$= -\frac{c^{3/2}\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{8\sqrt{2}\,b} + \frac{c^{3/2}\sqrt{d}\log}{}$$

$$= \frac{c^{3/2}\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{4\sqrt{2}\,b} - \frac{c^{3/2}\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a\cdot}}{\sqrt{d}\sqrt{c\sin(a+b}}\right.}{4\sqrt{2}\,b}$$

Mathematica [C] time = 0.06, size = 67, normalized size = 0.21

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\sin^2(a+bx)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 5/4,
9/4, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(3/2)*Tan[a + b*x])/(5*b)
```

fricas [B] time = 27.20, size = 1868, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/32*(16*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c*cos(b*x + a) + 2*sqrt
(2)*(c^6*d^2/b^4)^(1/4)*b*arctan(1/2*(2*c^10*d^4*cos(b*x + a)*sin(b*x + a)
+ sqrt(4*sqrt(c^6*d^2/b^4)*b^2*c^7*d^3*cos(b*x + a)*sin(b*x + a) + c^10*d^4
 + 2*(sqrt(2)*(c^6*d^2/b^4)^(1/4)*b*c^8*d^3*sin(b*x + a) + sqrt(2)*(c^6*d^2
/b^4)^(3/4)*b^3*c^5*d^2*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
 a)))*(sqrt(2)*(c^6*d^2/b^4)^(1/4)*b*c^3*d*cos(b*x + a) + sqrt(2)*(c^6*d^2/
b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + (s
qrt(2)*(c^6*d^2/b^4)^(1/4)*b*c^8*d^3*cos(b*x + a) + sqrt(2)*(c^6*d^2/b^4)^(
3/4)*b^3*c^5*d^2*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) -
4*(b^2*c^7*d^3*cos(b*x + a)^4 - b^2*c^7*d^3*cos(b*x + a)^2)*sqrt(c^6*d^2/b^
4))/((2*c^10*d^4*cos(b*x + a)^3 - c^10*d^4*cos(b*x + a))*sin(b*x + a))) + 2
*sqrt(2)*(c^6*d^2/b^4)^(1/4)*b*arctan(-1/2*(2*c^10*d^4*cos(b*x + a)*sin(b*x
 + a) - sqrt(4*sqrt(c^6*d^2/b^4)*b^2*c^7*d^3*cos(b*x + a)*sin(b*x + a) + c^
10*d^4 - 2*(sqrt(2)*(c^6*d^2/b^4)^(1/4)*b*c^8*d^3*sin(b*x + a) + sqrt(2)*(c
^6*d^2/b^4)^(3/4)*b^3*c^5*d^2*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin
(b*x + a)))*(sqrt(2)*(c^6*d^2/b^4)^(1/4)*b*c^3*d*cos(b*x + a) + sqrt(2)*(c^
6*d^2/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)
) - (sqrt(2)*(c^6*d^2/b^4)^(1/4)*b*c^8*d^3*cos(b*x + a) + sqrt(2)*(c^6*d^2/
b^4)^(3/4)*b^3*c^5*d^2*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
a)) - 4*(b^2*c^7*d^3*cos(b*x + a)^4 - b^2*c^7*d^3*cos(b*x + a)^2)*sqrt(c^6*
d^2/b^4))/((2*c^10*d^4*cos(b*x + a)^3 - c^10*d^4*cos(b*x + a))*sin(b*x + a)
)) + 2*sqrt(2)*(c^6*d^2/b^4)^(1/4)*b*arctan(1/2*((sqrt(2)*(c^6*d^2/b^4)^(1/
4)*b*c^8*d^3*cos(b*x + a) - sqrt(2)*(c^6*d^2/b^4)^(3/4)*b^3*c^5*d^2*sin(b*x
 + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + sqrt(4*sqrt(c^6*d^2/b^4)
*b^2*c^7*d^3*cos(b*x + a)*sin(b*x + a) + c^10*d^4 - 2*(sqrt(2)*(c^6*d^2/b^4
)^(1/4)*b*c^8*d^3*sin(b*x + a) + sqrt(2)*(c^6*d^2/b^4)^(3/4)*b^3*c^5*d^2*co
s(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(2*c^5*d^2*cos(b*x +
 a)*sin(b*x + a) + (sqrt(2)*(c^6*d^2/b^4)^(1/4)*b*c^3*d*cos(b*x + a) + sqrt
(2)*(c^6*d^2/b^4)^(3/4)*b^3*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b
*x + a))))/(c^10*d^4*cos(b*x + a)*sin(b*x + a))) + 2*sqrt(2)*(c^6*d^2/b^4)^
(1/4)*b*arctan(1/2*((sqrt(2)*(c^6*d^2/b^4)^(1/4)*b*c^8*d^3*cos(b*x + a) - s
qrt(2)*(c^6*d^2/b^4)^(3/4)*b^3*c^5*d^2*sin(b*x + a))*sqrt(d*cos(b*x + a))*s
qrt(c*sin(b*x + a)) - sqrt(4*sqrt(c^6*d^2/b^4)*b^2*c^7*d^3*cos(b*x + a)*sin
(b*x + a) + c^10*d^4 + 2*(sqrt(2)*(c^6*d^2/b^4)^(1/4)*b*c^8*d^3*sin(b*x + a
) + sqrt(2)*(c^6*d^2/b^4)^(3/4)*b^3*c^5*d^2*cos(b*x + a))*sqrt(d*cos(b*x +
a))*sqrt(c*sin(b*x + a)))*(2*c^5*d^2*cos(b*x + a)*sin(b*x + a) - (sqrt(2)*(
c^6*d^2/b^4)^(1/4)*b*c^3*d*cos(b*x + a) + sqrt(2)*(c^6*d^2/b^4)^(3/4)*b^3*s
in(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))))/(c^10*d^4*cos(b*x
+ a)*sin(b*x + a))) - sqrt(2)*(c^6*d^2/b^4)^(1/4)*b*log(4*sqrt(c^6*d^2/b^4)
*b^2*c^7*d^3*cos(b*x + a)*sin(b*x + a) + c^10*d^4 + 2*(sqrt(2)*(c^6*d^2/b^4
)^(1/4)*b*c^8*d^3*sin(b*x + a) + sqrt(2)*(c^6*d^2/b^4)^(3/4)*b^3*c^5*d^2*co
```

```
s(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + sqrt(2)*(c^6*d^2/b
^4)^(1/4)*b*log(4*sqrt(c^6*d^2/b^4)*b^2*c^7*d^3*cos(b*x + a)*sin(b*x + a) +
 c^10*d^4 - 2*(sqrt(2)*(c^6*d^2/b^4)^(1/4)*b*c^8*d^3*sin(b*x + a) + sqrt(2)
*(c^6*d^2/b^4)^(3/4)*b^3*c^5*d^2*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*
sin(b*x + a))))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \cos (bx + a)}\, (c \sin (bx + a))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(3/2), x)
```

maple [C] time = 0.11, size = 656, normalized size = 2.05

$$\frac{(c \sin (bx + a))^{\frac{3}{2}} \sqrt{d \cos (bx + a)} \left(i \sin (bx + a) \sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}} \sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}} \sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{Ell}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x)
```

```
[Out] 1/8/b*(c*sin(b*x+a))^(3/2)*(d*cos(b*x+a))^(1/2)*(I*sin(b*x+a)*((1-cos(b*x+a
)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))
/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-I*sin(b*x+a)*((1-cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1
+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2),1/2+1/2*I,1/2*2^(1/2))+2*sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin
(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2
^(1/2))-sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-2
*cos(b*x+a)^3*2^(1/2)+2*cos(b*x+a)^2*2^(1/2))/sin(b*x+a)/cos(b*x+a)/(-1+cos
(b*x+a))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \cos (bx + a)}\, (c \sin (bx + a))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d\cos\left(a+b\,x\right)}\,{\left(c\sin\left(a+b\,x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(3/2),x)
[Out] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**(3/2),x)
[Out] Timed out
```

3.272 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=313

$$-\frac{c^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\,bd^{3/2}}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\,bd^{3/2}}+\frac{c^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+\right.}{2\sqrt{2}\,bd^{3/2}}$$

```
[Out] 1/2*c^(3/2)*arctan(-1+2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/d^(1/2)/(c*sin(b
*x+a))^(1/2))/b/d^(3/2)*2^(1/2)+1/2*c^(3/2)*arctan(1+2^(1/2)*c^(1/2)*(d*cos
(b*x+a))^(1/2)/d^(1/2)/(c*sin(b*x+a))^(1/2))/b/d^(3/2)*2^(1/2)+1/4*c^(3/2)*
ln(d^(1/2)+cot(b*x+a)*d^(1/2)-2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(c*sin(b
*x+a))^(1/2))/b/d^(3/2)*2^(1/2)-1/4*c^(3/2)*ln(d^(1/2)+cot(b*x+a)*d^(1/2)+2
^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2))/b/d^(3/2)*2^(1/2)
+2*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2566, 2575, 297, 1162, 617, 204, 1165, 628}

$$-\frac{c^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\,bd^{3/2}}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}\,bd^{3/2}}+\frac{c^{3/2}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+\right.}{2\sqrt{2}\,bd^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] -((c^(3/2)*ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[
c*Sin[a + b*x]])])/(Sqrt[2]*b*d^(3/2))) + (c^(3/2)*ArcTan[1 + (Sqrt[2]*Sqrt
[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])])/(Sqrt[2]*b*d^(3/
2)) + (c^(3/2)*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] - (Sqrt[2]*Sqrt[c]*Sqrt[d
*Cos[a + b*x]])/Sqrt[c*Sin[a + b*x]]])/(2*Sqrt[2]*b*d^(3/2)) - (c^(3/2)*Log
[Sqrt[d] + Sqrt[d]*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/Sq
rt[c*Sin[a + b*x]]])/(2*Sqrt[2]*b*d^(3/2)) + (2*c*Sqrt[c*Sin[a + b*x]])/(b*
d*Sqrt[d*Cos[a + b*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
```

```
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} - \frac{c^2\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx}{d^2}$$

$$= \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} + \frac{\left(2c^3\right)\text{Subst}\left(\int \frac{x^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{bd}$$

$$= \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} - \frac{c^2\,\text{Subst}\left(\int \frac{d-cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{bd} + \frac{c^2\,\text{Subst}\left(\int \frac{d+cx^2}{d^2+c^2x^4}\,dx,\right.}{bd}$$

$$= \frac{2c\sqrt{c\sin(a+bx)}}{bd\sqrt{d\cos(a+bx)}} + \frac{c^{3/2}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}+2x}{-\frac{d}{c}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}\,bd^{3/2}} + \frac{c^{3/2}\,\text{Subst}\left(\int -\right.}{}$$

$$= \frac{c^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}\,bd^{3/2}} - \frac{c^{3/2}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)\right.}{2\sqrt{2}\,bd^{3/2}}$$

$$= -\frac{c^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\,bd^{3/2}} + \frac{c^{3/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}\,bd^{3/2}} + \frac{c^{3/2}\log\left(\sqrt{d}\right.}{}$$

Mathematica [C] time = 0.15, size = 67, normalized size = 0.21

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\,(c\sin(a+bx))^{5/2}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\sin^2(a+bx)\right)}{5bcd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 5/4, 9/4, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(5/2))/(5*b*c*d*Sqrt[d*Cos[a + b*x]])
```

fricas [B] time = 27.41, size = 1865, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(2)*b*d^2*(c^6/(b^4*d^6))^(1/4)*arctan(1/2*(2*c^10*cos(b*x + a)*
sin(b*x + a) + sqrt(4*b^2*c^7*d^3*sqrt(c^6/(b^4*d^6))*cos(b*x + a)*sin(b*x
+ a) + c^10 + 2*(sqrt(2)*b^3*c^5*d^4*(c^6/(b^4*d^6))^(3/4)*cos(b*x + a) + s
qrt(2)*b*c^8*d*(c^6/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqr
t(c*sin(b*x + a)))*(sqrt(2)*b^3*d^4*(c^6/(b^4*d^6))^(3/4)*sin(b*x + a) + sq
rt(2)*b*c^3*d*(c^6/(b^4*d^6))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt
(c*sin(b*x + a)) + (sqrt(2)*b^3*c^5*d^4*(c^6/(b^4*d^6))^(3/4)*sin(b*x + a)
+ sqrt(2)*b*c^8*d*(c^6/(b^4*d^6))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*
sqrt(c*sin(b*x + a)) - 4*(b^2*c^7*d^3*cos(b*x + a)^4 - b^2*c^7*d^3*cos(b*x
+ a)^2)*sqrt(c^6/(b^4*d^6)))/((2*c^10*cos(b*x + a)^3 - c^10*cos(b*x + a))*s
in(b*x + a)))*cos(b*x + a) + 2*sqrt(2)*b*d^2*(c^6/(b^4*d^6))^(1/4)*arctan(-
1/2*(2*c^10*cos(b*x + a)*sin(b*x + a) - sqrt(4*b^2*c^7*d^3*sqrt(c^6/(b^4*d^
6))*cos(b*x + a)*sin(b*x + a) + c^10 - 2*(sqrt(2)*b^3*c^5*d^4*(c^6/(b^4*d^6
))^(3/4)*cos(b*x + a) + sqrt(2)*b*c^8*d*(c^6/(b^4*d^6))^(1/4)*sin(b*x + a))
*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(sqrt(2)*b^3*d^4*(c^6/(b^4*d^6)
)^(3/4)*sin(b*x + a) + sqrt(2)*b*c^3*d*(c^6/(b^4*d^6))^(1/4)*cos(b*x + a))*
sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - (sqrt(2)*b^3*c^5*d^4*(c^6/(b^4*
d^6))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^8*d*(c^6/(b^4*d^6))^(1/4)*cos(b*x +
a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - 4*(b^2*c^7*d^3*cos(b*x + a)
^4 - b^2*c^7*d^3*cos(b*x + a)^2)*sqrt(c^6/(b^4*d^6)))/((2*c^10*cos(b*x + a)
^3 - c^10*cos(b*x + a))*sin(b*x + a)))*cos(b*x + a) - 2*sqrt(2)*b*d^2*(c^6/
(b^4*d^6))^(1/4)*arctan(1/2*((sqrt(2)*b^3*c^5*d^4*(c^6/(b^4*d^6))^(3/4)*sin
(b*x + a) - sqrt(2)*b*c^8*d*(c^6/(b^4*d^6))^(1/4)*cos(b*x + a))*sqrt(d*cos(
b*x + a))*sqrt(c*sin(b*x + a)) + sqrt(4*b^2*c^7*d^3*sqrt(c^6/(b^4*d^6))*cos
(b*x + a)*sin(b*x + a) + c^10 - 2*(sqrt(2)*b^3*c^5*d^4*(c^6/(b^4*d^6))^(3/4
)*cos(b*x + a) + sqrt(2)*b*c^8*d*(c^6/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d
*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(2*c^5*cos(b*x + a)*sin(b*x + a) + (sq
rt(2)*b^3*d^4*(c^6/(b^4*d^6))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^3*d*(c^6/(b^
4*d^6))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))))/(c^
10*cos(b*x + a)*sin(b*x + a)))*cos(b*x + a) - 2*sqrt(2)*b*d^2*(c^6/(b^4*d^6
))^(1/4)*arctan(1/2*((sqrt(2)*b^3*c^5*d^4*(c^6/(b^4*d^6))^(3/4)*sin(b*x + a
) - sqrt(2)*b*c^8*d*(c^6/(b^4*d^6))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a)
)*sqrt(c*sin(b*x + a)) - sqrt(4*b^2*c^7*d^3*sqrt(c^6/(b^4*d^6))*cos(b*x + a
)*sin(b*x + a) + c^10 + 2*(sqrt(2)*b^3*c^5*d^4*(c^6/(b^4*d^6))^(3/4)*cos(b*
x + a) + sqrt(2)*b*c^8*d*(c^6/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x
 + a))*sqrt(c*sin(b*x + a)))*(2*c^5*cos(b*x + a)*sin(b*x + a) - (sqrt(2)*b^
3*d^4*(c^6/(b^4*d^6))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^3*d*(c^6/(b^4*d^6))^
(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))))/(c^10*cos(b
*x + a)*sin(b*x + a)))*cos(b*x + a) - sqrt(2)*b*d^2*(c^6/(b^4*d^6))^(1/4)*c
os(b*x + a)*log(4*b^2*c^7*d^3*sqrt(c^6/(b^4*d^6))*cos(b*x + a)*sin(b*x + a)
 + c^10 + 2*(sqrt(2)*b^3*c^5*d^4*(c^6/(b^4*d^6))^(3/4)*cos(b*x + a) + sqrt(
```

```
2)*b*c^8*d*(c^6/(b^4*d^6))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*
sin(b*x + a))) + sqrt(2)*b*d^2*(c^6/(b^4*d^6))^(1/4)*cos(b*x + a)*log(4*b^2
*c^7*d^3*sqrt(c^6/(b^4*d^6))*cos(b*x + a)*sin(b*x + a) + c^10 - 2*(sqrt(2)*
b^3*c^5*d^4*(c^6/(b^4*d^6))^(3/4)*cos(b*x + a) + sqrt(2)*b*c^8*d*(c^6/(b^4*
d^6))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + 16*s
qrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c)/(b*d^2*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^{\frac{3}{2}}}{(d\cos(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(3/2), x)
```

maple [C] time = 0.13, size = 642, normalized size = 2.05

$$\frac{\left(i\sin(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] 1/2/b*(I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)-
I*sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+sin(b*x
+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+sin(b*x+a)*((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*sin(b*x+a)*((1-cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))
/sin(b*x+a))^(1/2),1/2*2^(1/2))+2*cos(b*x+a)*2^(1/2)-2*2^(1/2))*(c*sin(b*x+
a))^(3/2)*cos(b*x+a)/(-1+cos(b*x+a))/(d*cos(b*x+a))^(3/2)/sin(b*x+a)*2^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c \sin\left(bx + a\right)\right)^{\frac{3}{2}}}{\left(d \cos\left(bx + a\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,\sin\left(a+b\,x\right)\right)}^{3/2}}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(3/2),x)
[Out] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c \sin{\left(a + bx \right)}\right)^{\frac{3}{2}}}{\left(d \cos{\left(a + bx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(3/2),x)
[Out] Integral((c*sin(a + b*x))**(3/2)/(d*cos(a + b*x))**(3/2), x)
```

3.273 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

[Out] 2/5*(c*sin(b*x+a))^(5/2)/b/c/d/(d*cos(b*x+a))^(5/2)

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2563}

$$\frac{2(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(7/2),x]

[Out] (2*(c*Sin[a + b*x])^(5/2))/(5*b*c*d*(d*Cos[a + b*x])^(5/2))

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2(c\sin(a+bx))^{5/2}}{5bcd(d\cos(a+bx))^{5/2}}$$

Mathematica [A] time = 0.11, size = 40, normalized size = 1.08

$$\frac{2\cot(a+bx)(c\sin(a+bx))^{7/2}}{5bc^2(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(7/2),x]

[Out] (2*Cot[a + b*x]*(c*Sin[a + b*x])^(7/2))/(5*b*c^2*(d*Cos[a + b*x])^(7/2))

fricas [A] time = 0.46, size = 50, normalized size = 1.35

$$-\frac{2\left(c\cos\left(bx+a\right)^{2}-c\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}{5\,bd^{4}\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
[Out] -2/5*(c*cos(b*x + a)^2 - c)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(b*d^
4*cos(b*x + a)^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.10, size = 38, normalized size = 1.03

$$\frac{2\sin\left(bx+a\right)\cos\left(bx+a\right)\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}}{5b\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x)
[Out] 2/5/b*sin(b*x+a)*cos(b*x+a)*(c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}}{\left(d\cos\left(bx+a\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(7/2), x)
```

mupad [B] time = 1.51, size = 64, normalized size = 1.73

$$-\frac{2\,c\,\left(\cos\left(4\,a+4\,b\,x\right)-1\right)\,\sqrt{c\,\sin\left(a+b\,x\right)}}{5\,b\,d^3\,\sqrt{d\,\cos\left(a+b\,x\right)}\,\left(4\,\cos\left(2\,a+2\,b\,x\right)+\cos\left(4\,a+4\,b\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(7/2),x)
```

```
[Out] -(2*c*(cos(4*a + 4*b*x) - 1)*(c*sin(a + b*x))^(1/2))/(5*b*d^3*(d*cos(a + b*
x))^(1/2)*(4*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(7/2),x)
```

```
[Out] Timed out
```

3.274 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{8c\sqrt{c\sin(a+bx)}}{45bd^5\sqrt{d\cos(a+bx)}}-\frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}}+\frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}}$$

```
[Out] 2/9*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(9/2)-2/45*c*(c*sin(b*x+a))^(
1/2)/b/d^3/(d*cos(b*x+a))^(5/2)-8/45*c*(c*sin(b*x+a))^(1/2)/b/d^5/(d*cos(b*
x+a))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2566, 2571, 2563}

$$-\frac{8c\sqrt{c\sin(a+bx)}}{45bd^5\sqrt{d\cos(a+bx)}}-\frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}}+\frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(9*b*d*(d*Cos[a + b*x])^(9/2)) - (2*c*Sqrt[c*Sin
[a + b*x]])/(45*b*d^3*(d*Cos[a + b*x])^(5/2)) - (8*c*Sqrt[c*Sin[a + b*x]])/
(45*b*d^5*Sqrt[d*Cos[a + b*x]])
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
```

```
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{11/2}}\,dx &= \frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}} - \frac{c^2\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx}{9d^2}\\
&= \frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}} - \frac{\left(4c^2\right)\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx}{45d^4}\\
&= \frac{2c\sqrt{c\sin(a+bx)}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{45bd^3(d\cos(a+bx))^{5/2}} - \frac{8c\sqrt{c\sin(a+bx)}}{45bd^5\sqrt{d\cos(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 57, normalized size = 0.54

$$\frac{2(2\cos(2(a+bx))+7)\sec^5(a+bx)(c\sin(a+bx))^{5/2}\sqrt{d\cos(a+bx)}}{45bcd^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(7 + 2*Cos[2*(a + b*x)])*Sec[a + b*x]^5*(c*Sin[a +
b*x])^(5/2))/(45*b*c*d^6)
```

fricas [A] time = 0.54, size = 61, normalized size = 0.58

$$-\frac{2\left(4\,c\cos\left(bx+a\right)^4+c\cos\left(bx+a\right)^2-5\,c\right)\sqrt{d\cos\left(bx+a\right)}\,\sqrt{c\sin\left(bx+a\right)}}{45\,bd^6\cos\left(bx+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x, algorithm="fricas")
```

```
[Out] -2/45*(4*c*cos(b*x + a)^4 + c*cos(b*x + a)^2 - 5*c)*sqrt(d*cos(b*x + a))*sq
rt(c*sin(b*x + a))/(b*d^6*cos(b*x + a)^5)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.11, size = 50, normalized size = 0.47

$$\frac{2\left(4\left(\cos^{2}\left(bx+a\right)\right)+5\right)\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}\cos\left(bx+a\right)\sin\left(bx+a\right)}{45b\left(d\cos\left(bx+a\right)\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x)
```

```
[Out] 2/45/b*(4*cos(b*x+a)^2+5)*(c*sin(b*x+a))^(3/2)*cos(b*x+a)*sin(b*x+a)/(d*cos
(b*x+a))^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}}{\left(d\cos\left(bx+a\right)\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(11/2), x)
```

mupad [B] time = 6.16, size = 207, normalized size = 1.95

$$-\frac{\sqrt{c\,\sin\left(a+b\,x\right)}\,\left(2\sin\left(2\,a+2\,b\,x\right)^2+\sin\left(4\,a+4\,b\,x\right)\,1\mathrm{i}-1\right)\left(\frac{32\,c\,\left(-2\sin(2\,a+2\,b\,x)^2+\sin(4\,a+4\,b\,x)\,1\mathrm{i}+1\right)}{15\,b\,d^5}+\frac{16\,c\,\left(2\sin(2\,a+\cdots\right.}{\cdots}\cdots\right.}{16\left(\sin\left(a+b\,x\right)^2-1\right)^2\sqrt{-d\left(2\sin\left(\cdots\right.\right.}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(11/2),x)
```

```
[Out] -((c*sin(a + b*x))^(1/2)*(sin(4*a + 4*b*x)*1i + 2*sin(2*a + 2*b*x)^2 - 1)*(
(32*c*(sin(4*a + 4*b*x)*1i - 2*sin(2*a + 2*b*x)^2 + 1))/(15*b*d^5) + (16*c*
(2*sin(2*a + 2*b*x)^2 - 1)*(sin(4*a + 4*b*x)*1i - 2*sin(2*a + 2*b*x)^2 + 1)
)/(45*b*d^5) + (16*c*(2*sin(a + b*x)^2 - 1)*(sin(4*a + 4*b*x)*1i - 2*sin(2*
```

```
a + 2*b*x)^2 + 1))/(9*b*d^5)))/(16*(sin(a + b*x)^2 - 1)^2*(-d*(2*sin(a/2 +
(b*x)/2)^2 - 1))^(1/2))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(11/2),x)
```

```
[Out] Timed out
```

3.275 $\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{64c\sqrt{c\sin(a+bx)}}{585bd^7\sqrt{d\cos(a+bx)}}-\frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}}-\frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}}+\frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}}$$

```
[Out] 2/13*c*(c*sin(b*x+a))^(1/2)/b/d/(d*cos(b*x+a))^(13/2)-2/117*c*(c*sin(b*x+a)
)^(1/2)/b/d^3/(d*cos(b*x+a))^(9/2)-16/585*c*(c*sin(b*x+a))^(1/2)/b/d^5/(d*c
os(b*x+a))^(5/2)-64/585*c*(c*sin(b*x+a))^(1/2)/b/d^7/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2566, 2571, 2563}

$$-\frac{64c\sqrt{c\sin(a+bx)}}{585bd^7\sqrt{d\cos(a+bx)}}-\frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}}-\frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}}+\frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(15/2),x]
```

```
[Out] (2*c*Sqrt[c*Sin[a + b*x]])/(13*b*d*(d*Cos[a + b*x])^(13/2)) - (2*c*Sqrt[c*S
in[a + b*x]])/(117*b*d^3*(d*Cos[a + b*x])^(9/2)) - (16*c*Sqrt[c*Sin[a + b*x
]])/(585*b*d^5*(d*Cos[a + b*x])^(5/2)) - (64*c*Sqrt[c*Sin[a + b*x]])/(585*b
*d^7*Sqrt[d*Cos[a + b*x]])
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{3/2}}{(d\cos(a+bx))^{15/2}}\,dx = \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{c^2\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx}{13d^2}$$

$$= \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}} - \frac{\left(8c^2\right)\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}}{117d^4}$$

$$= \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}} - \frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}} - \frac{(3}{}$$

$$= \frac{2c\sqrt{c\sin(a+bx)}}{13bd(d\cos(a+bx))^{13/2}} - \frac{2c\sqrt{c\sin(a+bx)}}{117bd^3(d\cos(a+bx))^{9/2}} - \frac{16c\sqrt{c\sin(a+bx)}}{585bd^5(d\cos(a+bx))^{5/2}} - \frac{6}{58}$$

Mathematica [A] time = 0.31, size = 67, normalized size = 0.48

$$\frac{2(36\cos(2(a+bx))+4\cos(4(a+bx))+77)\sec^7(a+bx)(c\sin(a+bx))^{5/2}\sqrt{d\cos(a+bx)}}{585bcd^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(3/2)/(d*Cos[a + b*x])^(15/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(77 + 36*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Sec
[a + b*x]^7*(c*Sin[a + b*x])^(5/2))/(585*b*c*d^8)
```

fricas [A] time = 0.67, size = 73, normalized size = 0.52

$$-\frac{2\left(32\,c\cos\left(bx+a\right)^6+8\,c\cos\left(bx+a\right)^4+5\,c\cos\left(bx+a\right)^2-45\,c\right)\sqrt{d\cos\left(bx+a\right)}\,\sqrt{c\sin\left(bx+a\right)}}{585\,bd^8\cos\left(bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x, algorithm="fricas")
```

```
[Out] -2/585*(32*c*cos(b*x + a)^6 + 8*c*cos(b*x + a)^4 + 5*c*cos(b*x + a)^2 - 45*
c)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(b*d^8*cos(b*x + a)^7)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.16, size = 60, normalized size = 0.43

$$\frac{2\left(32\left(\cos^{4}\left(bx+a\right)\right)+40\left(\cos^{2}\left(bx+a\right)\right)+45\right)\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}\cos\left(bx+a\right)\sin\left(bx+a\right)}{585b\left(d\cos\left(bx+a\right)\right)^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x)
[Out] 2/585/b*(32*cos(b*x+a)^4+40*cos(b*x+a)^2+45)*(c*sin(b*x+a))^(3/2)*cos(b*x+a
)*sin(b*x+a)/(d*cos(b*x+a))^(15/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}}{\left(d\cos\left(bx+a\right)\right)^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(3/2)/(d*cos(b*x+a))^(15/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(3/2)/(d*cos(b*x + a))^(15/2), x)
```

mupad [B] time = 6.67, size = 193, normalized size = 1.37

$$-\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\sqrt{c\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(-\frac{3776\,c\,\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}}{585\,b\,d^7}+\frac{2752\,c\,\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,\cos(2\,a+2\,b\,x)}{585\,b\,d^7}+\frac{896\,c\,\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,\cos(4\,a+4\,b\,x)}{585\,b\,d^7}+\cdots}{64\cos\left(a+b\,x\right)^6\,\sqrt{d\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(3/2)/(d*cos(a + b*x))^(15/2),x)
[Out] -(exp(- a*6i - b*x*6i)*(c*((exp(- a*1i - b*x*1i)*1i)/2 - (exp(a*1i + b*x*1i
)*1i)/2))^(1/2)*((2752*c*exp(a*6i + b*x*6i)*cos(2*a + 2*b*x))/(585*b*d^7) -
```

```
 (3776*c*exp(a*6i + b*x*6i))/(585*b*d^7) + (896*c*exp(a*6i + b*x*6i)*cos(4*
a + 4*b*x))/(585*b*d^7) + (128*c*exp(a*6i + b*x*6i)*cos(6*a + 6*b*x))/(585*
b*d^7)))/(64*cos(a + b*x)^6*(d*(exp(- a*1i - b*x*1i)/2 + exp(a*1i + b*x*1i)
/2))^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(3/2)/(d*cos(b*x+a))**(15/2),x)
```

```
[Out] Timed out
```

3.276 $\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx$

Optimal. Leaf size=166

$$\frac{3c^2d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{40b\sqrt{\sin(2a+2bx)}}+\frac{cd^3(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{20b}-\frac{c(c\sin(a+bx))^{3/2}}{7}$$

```
[Out] 1/20*c*d^3*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b+3/70*c*d*(d*cos(b*x+
a))^(7/2)*(c*sin(b*x+a))^(3/2)/b-1/7*c*(d*cos(b*x+a))^(11/2)*(c*sin(b*x+a))
^(3/2)/b/d-3/40*c^2*d^4*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Ellip
ticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b
/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2568, 2569, 2572, 2639}

$$\frac{3c^2d^4E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{40b\sqrt{\sin(2a+2bx)}}+\frac{cd^3(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{20b}-\frac{c(c\sin(a+bx))^{3/2}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(9/2)*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (c*d^3*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(20*b) + (3*c*d*(d*Co
s[a + b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(70*b) - (c*(d*Cos[a + b*x])^(11/
2)*(c*Sin[a + b*x])^(3/2))/(7*b*d) + (3*c^2*d^4*Sqrt[d*Cos[a + b*x]]*Ellipt
icE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(40*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx = -\frac{c(d\cos(a+bx))^{11/2}(c\sin(a+bx))^{3/2}}{7bd}+\frac{1}{14}\left(3c^2\right)\int (d\cos(a+bx))^{9/...}$$

$$= \frac{3cd(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{70b}-\frac{c(d\cos(a+bx))^{11/2}(c\sin(a+ \ldots}{7bd}$$

$$= \frac{cd^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{20b}+\frac{3cd(d\cos(a+bx))^{7/2}(c\sin(\ldots}{70b}$$

$$= \frac{cd^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{20b}+\frac{3cd(d\cos(a+bx))^{7/2}(c\sin(\ldots}{70b}$$

$$= \frac{cd^3(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{20b}+\frac{3cd(d\cos(a+bx))^{7/2}(c\sin(\ldots}{70b}$$

Mathematica [C] time = 0.17, size = 72, normalized size = 0.43

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\,\sec^5(a+bx)(c\sin(a+bx))^{7/2}(d\cos(a+bx))^{9/2}\,{}_2F_1\left(-\frac{7}{4},\frac{7}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(9/2)*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (2*(d*Cos[a + b*x])^(9/2)*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-7/4, 7/
4, 11/4, Sin[a + b*x]^2]*Sec[a + b*x]^5*(c*Sin[a + b*x])^(7/2))/(7*b*c)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2d^4\cos(bx+a)^6-c^2d^4\cos(bx+a)^4\right)\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*d^4*cos(b*x + a)^6 - c^2*d^4*cos(b*x + a)^4)*sqrt(d*cos(b*x
+ a))*sqrt(c*sin(b*x + a)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.16, size = 545, normalized size = 3.28

$$\frac{\left(40\left(\cos^{8}\left(bx+a\right)\right)\sqrt{2}-52\left(\cos^{6}\left(bx+a\right)\right)\sqrt{2}-2\left(\cos^{4}\left(bx+a\right)\right)\sqrt{2}+21\cos\left(bx+a\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x)
```

```
[Out] 1/560/b*(40*cos(b*x+a)^8*2^(1/2)-52*cos(b*x+a)^6*2^(1/2)-2*cos(b*x+a)^4*2^(
1/2)+21*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-42*cos(b*x+a
)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+21*((1-cos(b*x+a)+sin(b*x+a))
/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
1/2),1/2*2^(1/2))-42*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ell
ipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-7*cos(b*x+
a)^2*2^(1/2)+21*cos(b*x+a)*2^(1/2))*(d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/
2)/sin(b*x+a)^3/cos(b*x+a)^5*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(9/2)*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(9/2)*(c*sin(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cos(a+bx))^{9/2}(c\sin(a+bx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(5/2),x)
[Out] int((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(9/2)*(c*sin(b*x+a))**(5/2),x)
[Out] Timed out
```

3.277 $\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx$

Optimal. Leaf size=131

$$\frac{3c^2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bd}+\frac{cd(c\sin(a+bx))^{3/2}(}{10b}$$

```
[Out] 1/10*c*d*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b-1/5*c*(d*cos(b*x+a))^(
7/2)*(c*sin(b*x+a))^(3/2)/b/d-3/20*c^2*d^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(
a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*
sin(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2568, 2569, 2572, 2639}

$$\frac{3c^2d^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{20b\sqrt{\sin(2a+2bx)}}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{7/2}}{5bd}+\frac{cd(c\sin(a+bx))^{3/2}(}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(5/2)*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (c*d*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(10*b) - (c*(d*Cos[a +
b*x])^(7/2)*(c*Sin[a + b*x])^(3/2))/(5*b*d) + (3*c^2*d^2*Sqrt[d*Cos[a + b*x
]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(20*b*Sqrt[Sin[2*a +
2*b*x]])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx = -\frac{c(d\cos(a+bx))^{7/2}(c\sin(a+bx))^{3/2}}{5bd} + \frac{1}{10}\left(3c^2\right)\int (d\cos(a+bx))^{5/2}$$

$$= \frac{cd(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{10b} - \frac{c(d\cos(a+bx))^{7/2}(c\sin(a+}{5bd}$$

$$= \frac{cd(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{10b} - \frac{c(d\cos(a+bx))^{7/2}(c\sin(a+}{5bd}$$

$$= \frac{cd(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{10b} - \frac{c(d\cos(a+bx))^{7/2}(c\sin(a+}{5bd}$$

Mathematica [C] time = 0.18, size = 70, normalized size = 0.53

$$\frac{2d^2\sqrt[4]{\cos^2(a+bx)}\,\tan(a+bx)(c\sin(a+bx))^{5/2}\sqrt{d\cos(a+bx)}\;{}_2F_1\left(-\frac{3}{4},\frac{7}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(5/2)*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (2*d^2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4,
7/4, 11/4, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(5/2)*Tan[a + b*x])/(7*b)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2d^2\cos(bx+a)^4 - c^2d^2\cos(bx+a)^2\right)\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*d^2*cos(b*x + a)^4 - c^2*d^2*cos(b*x + a)^2)*sqrt(d*cos(b*x
+ a))*sqrt(c*sin(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d \cos(bx+a))^{\frac{5}{2}} (c \sin(bx+a))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*(c*sin(b*x + a))^(5/2), x)
```

maple [B] time = 0.11, size = 532, normalized size = 4.06

$$\frac{\left(4\left(\cos^{6}(bx+a)\right)\sqrt{2} - 6\left(\cos^{4}(bx+a)\right)\sqrt{2} - 6\cos(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cdots}{\sin\cdots}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x)
```

```
[Out] 1/40/b*(4*cos(b*x+a)^6*2^(1/2)-6*cos(b*x+a)^4*2^(1/2)-6*cos(b*x+a)*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+3*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))
/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
1/2),1/2*2^(1/2))-6*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elli
pticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+3*((1-cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)^2*2^(1/2)+3*cos(b*x+a)*2^(1/2
))*(d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2)/sin(b*x+a)^3/cos(b*x+a)^3*2^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d \cos(bx+a))^{\frac{5}{2}} (c \sin(bx+a))^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(5/2)*(c*sin(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cos(a+bx))^{5/2}(c\sin(a+bx))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(5/2),x)
[Out] int((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(5/2)*(c*sin(b*x+a))**(5/2),x)
[Out] Timed out
```

3.278 $\int \sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{5/2}\,dx$

Optimal. Leaf size=95

$$\frac{c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bd}$$

```
[Out] -1/3*c*(d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^(3/2)/b/d-1/2*c^2*(sin(a+1/4*Pi+
b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos
(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2568, 2572, 2639}

$$\frac{c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{2b\sqrt{\sin(2a+2bx)}}-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] -(c*(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^(3/2))/(3*b*d) + (c^2*Sqrt[d*Co
s[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(2*b*Sqrt[Si
n[2*a + 2*b*x]])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{5/2}\,dx = -\frac{c(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bd} + \frac{1}{2}c^2\int \sqrt{d\cos(a+bx)}\,\sqrt{c\sin(}$$
$$= -\frac{c(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bd} + \frac{\left(c^2\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+}\right.}{2\sqrt{\sin(2a}}$$
$$= -\frac{c(d\cos(a+bx))^{3/2}(c\sin(a+bx))^{3/2}}{3bd} + \frac{c^2\sqrt{d\cos(a+bx)}\,E\left(a-\frac{\pi}{4}+b\right.}{2b\sqrt{\sin(2a+2}}$$

Mathematica [C] time = 0.09, size = 67, normalized size = 0.71

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\,\tan(a+bx)(c\sin(a+bx))^{5/2}\sqrt{d\cos(a+bx)}\;{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 7/4,
11/4, Sin[a + b*x]^2]*(c*Sin[a + b*x])^(5/2)*Tan[a + b*x])/(7*b)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2\cos(bx+a)^2-c^2\right)\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos(bx+a)}\,(c\sin(bx+a))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(5/2), x)
```

maple [B] time = 0.15, size = 519, normalized size = 5.46

$$-\frac{\left(6\cos\left(bx+a\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\ \sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\ \sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x)
```

```
[Out] -1/12/b*(6*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*El
lipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*x
+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^4*2^(1/2)+6*((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
)/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),
1/2*2^(1/2))+5*cos(b*x+a)^2*2^(1/2)-3*cos(b*x+a)*2^(1/2))*(d*cos(b*x+a))^(1
/2)*(c*sin(b*x+a))^(5/2)/sin(b*x+a)^3/cos(b*x+a)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos\left(bx+a\right)}\ \left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,\cos\left(a+b\,x\right)}\,{\left(c\,\sin\left(a+b\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(5/2),x)
```

```
[Out] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**(5/2),x)
```

```
[Out] Timed out
```

3.279 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}}-\frac{3c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

```
[Out] 2*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(1/2)+3*c^2*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+
a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/d^2/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2566, 2572, 2639}

$$\frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}}-\frac{3c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(b*d*Sqrt[d*Cos[a + b*x]]) - (3*c^2*Sqrt[d*Cos
[a + b*x]]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(b*d^2*Sqrt[S
in[2*a + 2*b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{3/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}} - \frac{(3c^2)\int \sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\,dx}{d^2}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}} - \frac{\left(3c^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}\right)\int \sqrt{\sin(2a+2bx)}\,dx}{d^2\sqrt{\sin(2a+2bx)}}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{bd\sqrt{d\cos(a+bx)}} - \frac{3c^2\sqrt{d\cos(a+bx)}\,E\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{c\sin(a+bx)}}{bd^2\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] time = 0.12, size = 67, normalized size = 0.71

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\,(c\sin(a+bx))^{7/2}\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7bcd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, 7/4, 11/4, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*Sqrt[d*Cos[a + b*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^2\cos(bx+a)^2-c^2\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{d^2\cos(bx+a)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
a))/(d^2*cos(b*x + a)^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

[Out] Timed out

maple [B] time = 0.13, size = 508, normalized size = 5.40

$$\frac{\left(6\cos\left(bx+a\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] 1/2/b*(6*cos(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Elli
pticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*x+a
)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+
a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*((1-cos(b*x+a)+sin(b*x+a))/
sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2),1/2*2^(1/2))-3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+cos(b*x+a)^2
*2^(1/2)-3*cos(b*x+a)*2^(1/2)+2*2^(1/2))*(c*sin(b*x+a))^(5/2)*cos(b*x+a)/si
n(b*x+a)^3/(d*cos(b*x+a))^(3/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,\sin\left(a+b\,x\right)\right)}^{5/2}}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(3/2),x)
```

```
[Out] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

3.280 $\int \frac{(c \sin(a+bx))^{5/2}}{(d \cos(a+bx))^{7/2}} \, dx$

Optimal. Leaf size=133

$$\frac{6c^2 E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}-\frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/5*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(5/2)-6/5*c*(c*sin(b*x+a))^(3
/2)/b/d^3/(d*cos(b*x+a))^(1/2)-6/5*c^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/
4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*sin(
b*x+a))^(1/2)/b/d^4/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2566, 2571, 2572, 2639}

$$\frac{6c^2 E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{5bd^4\sqrt{\sin(2a+2bx)}}-\frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}}+\frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(5*b*d*(d*Cos[a + b*x])^(5/2)) - (6*c*(c*Sin[a
 + b*x])^(3/2))/(5*b*d^3*Sqrt[d*Cos[a + b*x]]) + (6*c^2*Sqrt[d*Cos[a + b*x]
]*EllipticE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(5*b*d^4*Sqrt[Sin[2*a
+ 2*b*x]])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{7/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{\left(3c^2\right)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx}{5d^2}$$
$$= \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{\left(6c^2\right)\int \sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx}}{5d^4}$$
$$= \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{\left(6c^2\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}\right)}{5d^4\sqrt{\sin(2a+2bx}}$$
$$= \frac{2c(c\sin(a+bx))^{3/2}}{5bd(d\cos(a+bx))^{5/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{5bd^3\sqrt{d\cos(a+bx)}} + \frac{6c^2\sqrt{d\cos(a+bx)}\,E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5bd^4\sqrt{\sin(2a+2bx)}}$$

Mathematica [C] time = 0.17, size = 70, normalized size = 0.53

$$\frac{2\cos^2(a+bx)^{5/4}\cot(a+bx)(c\sin(a+bx))^{9/2}\,{}_2F_1\left(\frac{7}{4},\frac{9}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7bc^2(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(7/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(5/4)*Cot[a + b*x]*Hypergeometric2F1[7/4, 9/4, 11/4, Si
n[a + b*x]^2]*(c*Sin[a + b*x])^(9/2))/(7*b*c^2*(d*Cos[a + b*x])^(7/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^2\cos(bx+a)^2-c^2\right)\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}}{d^4\cos(bx+a)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
a))/(d^4*cos(b*x + a)^4), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.14, size = 531, normalized size = 3.99

$$-\frac{\left(6\left(\cos^{3}(bx+a)\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\ \sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\ \sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x)
```

```
[Out] -1/5/b*(6*cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+co
s(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*E
llipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*
x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*cos(b*x+a)^2*((1-cos(b
*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-3*cos(b*x+a)^2*((1-cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(
b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2),1/2*2^(1/2))-3*cos(b*x+a)^3*2^(1/2)+4*cos(b*x+a)^2*2^(1/2)-2^(1/2))*(
c*sin(b*x+a))^(5/2)*cos(b*x+a)/sin(b*x+a)^3/(d*cos(b*x+a))^(7/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(7/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,\sin(a+b\,x))^{5/2}}{(d\,\cos(a+b\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(7/2),x)
[Out] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(7/2),x)
[Out] Timed out
```

3.281 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx$

Optimal. Leaf size=168

$$\frac{4c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{15bd^6\sqrt{\sin(2a+2bx)}}-\frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}}-\frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}}+\frac{2c(c\sin(a}{9bd(d\cos(}$$

```
[Out] 2/9*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(9/2)-2/15*c*(c*sin(b*x+a))^(
3/2)/b/d^3/(d*cos(b*x+a))^(5/2)-4/15*c*(c*sin(b*x+a))^(3/2)/b/d^5/(d*cos(b*
x+a))^(1/2)-4/15*c^2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*Elliptic
E(cos(a+1/4*Pi+b*x),2^(1/2))*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/d^
6/sin(2*b*x+2*a)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2566, 2571, 2572, 2639}

$$\frac{4c^2E\left(a+bx-\frac{\pi}{4}\middle|2\right)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{15bd^6\sqrt{\sin(2a+2bx)}}-\frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}}-\frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}}+\frac{2c(c\sin(a}{9bd(d\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(9*b*d*(d*Cos[a + b*x])^(9/2)) - (2*c*(c*Sin[a
 + b*x])^(3/2))/(15*b*d^3*(d*Cos[a + b*x])^(5/2)) - (4*c*(c*Sin[a + b*x])^(
3/2))/(15*b*d^5*Sqrt[d*Cos[a + b*x]]) + (4*c^2*Sqrt[d*Cos[a + b*x]]*Ellipti
cE[a - Pi/4 + b*x, 2]*Sqrt[c*Sin[a + b*x]])/(15*b*d^6*Sqrt[Sin[2*a + 2*b*x]
])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
```

```
1] && IntegersQ[2*m, 2*n]
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{11/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{c^2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{7/2}}\,dx}{3d^2}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{\left(2c^2\right)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{3/2}}\,dx}{15d^4}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}} + \frac{\left(4c^2\right)\int \sqrt{}}{}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}} + \frac{\left(4c^2\sqrt{d\,c}\right.}{}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{9bd(d\cos(a+bx))^{9/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{15bd^3(d\cos(a+bx))^{5/2}} - \frac{4c(c\sin(a+bx))^{3/2}}{15bd^5\sqrt{d\cos(a+bx)}} + \frac{4c^2\sqrt{d\,co}}{}$$

Mathematica [C] time = 0.17, size = 72, normalized size = 0.43

$$\frac{2\cos^5(a+bx)\sqrt[4]{\cos^2(a+bx)}\,(c\sin(a+bx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{13}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7bc(d\cos(a+bx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(11/2),x]
```

```
[Out] (2*Cos[a + b*x]^5*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[7/4, 13/4, 11/4,
 Sin[a + b*x]^2]*(c*Sin[a + b*x])^(7/2))/(7*b*c*(d*Cos[a + b*x])^(11/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^{2} \cos\left(bx+a\right)^{2}-c^{2}\right)\sqrt{d \cos\left(bx+a\right)}\,\sqrt{c \sin\left(bx+a\right)}}{d^{6} \cos\left(bx+a\right)^{6}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x +
a))/(d^6*cos(b*x + a)^6), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.12, size = 544, normalized size = 3.24

$$\frac{\left(-12\left(\cos^{5}\left(bx+a\right)\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\text{EllipticE}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x)
```

```
[Out] 1/45/b*(-12*cos(b*x+a)^5*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)
*EllipticE(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*cos(
b*x+a)^5*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-12*cos(b*x+a)^4*((1-co
s(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticE(((1-cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))+6*cos(b*x+a)^4*((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2),1/2*2^(1/2))+6*cos(b*x+a)^5*2^(1/2)-3*cos(b*x+a)^4*2^(1/2)-8*cos(b
*x+a)^2*2^(1/2)+5*2^(1/2))*(c*sin(b*x+a))^(5/2)*cos(b*x+a)/sin(b*x+a)^3/(d*
cos(b*x+a))^(11/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(11/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\ \sin(a+b\,x))^{5/2}}{(d\ \cos(a+b\,x))^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(11/2),x)
[Out] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(11/2),x)
[Out] Timed out
```

3.282 $\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=320

$$-\frac{3c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}}+\frac{3c^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{4\sqrt{2}b\sqrt{d}}+\frac{3c^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a\right.}{8\sqrt{2}b\sqrt{d}}$$

```
[Out] -3/8*c^(5/2)*arctan(1-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(d*cos(b
*x+a))^(1/2))/b*2^(1/2)/d^(1/2)+3/8*c^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(c*sin
(b*x+a))^(1/2)/c^(1/2)/(d*cos(b*x+a))^(1/2))/b*2^(1/2)/d^(1/2)+3/16*c^(5/2)
*ln(c^(1/2)-2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/
2)*tan(b*x+a))/b*2^(1/2)/d^(1/2)-3/16*c^(5/2)*ln(c^(1/2)+2^(1/2)*d^(1/2)*(c
*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/2)*tan(b*x+a))/b*2^(1/2)/d^(1/
2)-1/2*c*(c*sin(b*x+a))^(3/2)*(d*cos(b*x+a))^(1/2)/b/d
```

Rubi [A] time = 0.26, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2568, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}b\sqrt{d}}+\frac{3c^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{4\sqrt{2}b\sqrt{d}}+\frac{3c^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a\right.}{8\sqrt{2}b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (-3*c^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt
[d*Cos[a + b*x]])])/(4*Sqrt[2]*b*Sqrt[d]) + (3*c^(5/2)*ArcTan[1 + (Sqrt[2]*
Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(4*Sqrt[2]*b
*Sqrt[d]) + (3*c^(5/2)*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])
/Sqrt[d*Cos[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b*Sqrt[d]) - (3*c
^(5/2)*Log[Sqrt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a +
b*x]] + Sqrt[c]*Tan[a + b*x]])/(8*Sqrt[2]*b*Sqrt[d]) - (c*Sqrt[d*Cos[a + b*
x]]*(c*Sin[a + b*x])^(3/2))/(2*b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
```

```
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{\sqrt{d\cos(a+bx)}}\,dx = -\frac{c\sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{3/2}}{2bd} + \frac{1}{4}\left(3c^2\right)\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx$$

$$= -\frac{c\sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{3/2}}{2bd} + \frac{\left(3c^3d\right)\text{Subst}\left(\int \frac{x^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2b}$$

$$= -\frac{c\sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{3/2}}{2bd} - \frac{\left(3c^3\right)\text{Subst}\left(\int \frac{c-dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{4b} + \frac{(3c^3}{}$$

$$= -\frac{c\sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{3/2}}{2bd} + \frac{\left(3c^3\right)\text{Subst}\left(\int \frac{1}{\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{8bd} +$$

$$= \frac{3c^{5/2}\log\left(\sqrt{c} - \frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}} + \sqrt{c}\tan(a+bx)\right)}{8\sqrt{2}\,b\sqrt{d}} - \frac{3c^{5/2}\log\left(\sqrt{c} + \frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right.}{8\sqrt{2}\,b\sqrt{d}}$$

$$= -\frac{3c^{5/2}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}\,b\sqrt{d}} + \frac{3c^{5/2}\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{4\sqrt{2}\,b\sqrt{d}} + \frac{3c^{5/2}\log\left(\sqrt{\right.}}{}$$

Mathematica [C] time = 0.12, size = 67, normalized size = 0.21

$$\frac{2\cos^2(a+bx)^{3/4}\tan(a+bx)(c\sin(a+bx))^{5/2}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7b\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, 7/4, 11/4, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(5/2)*Tan[a + b*x])/(7*b*Sqrt[d*Cos[a + b*x]])
```

fricas [B] time = 55.06, size = 2074, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*(32*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c^2*sin(b*x + a) - 12*s
qrt(2)*(c^10/(b^4*d^2))^(1/4)*b*d*arctan(((sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b
*c^13*cos(b*x + a) + sqrt(2)*(c^10/(b^4*d^2))^(3/4)*b^3*c^8*d*sin(b*x + a))
*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + sqrt(4*sqrt(c^10/(b^4*d^2))*b^
2*c^11*d*cos(b*x + a)*sin(b*x + a) + c^16 - 2*(sqrt(2)*(c^10/(b^4*d^2))^(1/
4)*b*c^13*cos(b*x + a) + sqrt(2)*(c^10/(b^4*d^2))^(3/4)*b^3*c^8*d*sin(b*x +
 a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(2*c^8*cos(b*x + a)*sin(b*x
 + a) + sqrt(c^10/(b^4*d^2))*b^2*c^3*d + (sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*
c^5*sin(b*x + a) + sqrt(2)*(c^10/(b^4*d^2))^(3/4)*b^3*d*cos(b*x + a))*sqrt(
d*cos(b*x + a))*sqrt(c*sin(b*x + a))))/(2*c^16*cos(b*x + a)^2 - c^16)) - 12
*sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*d*arctan(((sqrt(2)*(c^10/(b^4*d^2))^(1/4)
*b*c^13*cos(b*x + a) + sqrt(2)*(c^10/(b^4*d^2))^(3/4)*b^3*c^8*d*sin(b*x + a
))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - sqrt(4*sqrt(c^10/(b^4*d^2))*
b^2*c^11*d*cos(b*x + a)*sin(b*x + a) + c^16 + 2*(sqrt(2)*(c^10/(b^4*d^2))^(
1/4)*b*c^13*cos(b*x + a) + sqrt(2)*(c^10/(b^4*d^2))^(3/4)*b^3*c^8*d*sin(b*x
 + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(2*c^8*cos(b*x + a)*sin(b
*x + a) + sqrt(c^10/(b^4*d^2))*b^2*c^3*d - (sqrt(2)*(c^10/(b^4*d^2))^(1/4)*
b*c^5*sin(b*x + a) + sqrt(2)*(c^10/(b^4*d^2))^(3/4)*b^3*d*cos(b*x + a))*sqr
t(d*cos(b*x + a))*sqrt(c*sin(b*x + a))))/(2*c^16*cos(b*x + a)^2 - c^16)) -
12*sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*d*arctan(-1/2*(2*c^16*cos(b*x + a)*sin(
b*x + a) - sqrt(4*sqrt(c^10/(b^4*d^2))*b^2*c^11*d*cos(b*x + a)*sin(b*x + a)
 + c^16 + 2*(sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*c^13*cos(b*x + a) + sqrt(2)*(
c^10/(b^4*d^2))^(3/4)*b^3*c^8*d*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*s
in(b*x + a)))*(sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*c^5*sin(b*x + a) + sqrt(2)*
(c^10/(b^4*d^2))^(3/4)*b^3*d*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(
b*x + a)) + (sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*c^13*sin(b*x + a) + sqrt(2)*(
c^10/(b^4*d^2))^(3/4)*b^3*c^8*d*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*s
in(b*x + a)) - 4*(b^2*c^11*d*cos(b*x + a)^4 - b^2*c^11*d*cos(b*x + a)^2)*sq
rt(c^10/(b^4*d^2)))/((2*c^16*cos(b*x + a)^3 - c^16*cos(b*x + a))*sin(b*x +
a))) - 12*sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*d*arctan(1/2*(2*c^16*cos(b*x + a
)*sin(b*x + a) + sqrt(4*sqrt(c^10/(b^4*d^2))*b^2*c^11*d*cos(b*x + a)*sin(b*
x + a) + c^16 - 2*(sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*c^13*cos(b*x + a) + sqr
t(2)*(c^10/(b^4*d^2))^(3/4)*b^3*c^8*d*sin(b*x + a))*sqrt(d*cos(b*x + a))*sq
rt(c*sin(b*x + a)))*(sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*c^5*sin(b*x + a) + sq
rt(2)*(c^10/(b^4*d^2))^(3/4)*b^3*d*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(
c*sin(b*x + a)) - (sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*c^13*sin(b*x + a) + sqr
t(2)*(c^10/(b^4*d^2))^(3/4)*b^3*c^8*d*cos(b*x + a))*sqrt(d*cos(b*x + a))*sq
rt(c*sin(b*x + a)) - 4*(b^2*c^11*d*cos(b*x + a)^4 - b^2*c^11*d*cos(b*x + a)
^2)*sqrt(c^10/(b^4*d^2)))/((2*c^16*cos(b*x + a)^3 - c^16*cos(b*x + a))*sin(
b*x + a))) + 3*sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*d*log(2916*sqrt(c^10/(b^4*d
^2))*b^2*c^11*d*cos(b*x + a)*sin(b*x + a) + 729*c^16 + 1458*(sqrt(2)*(c^10/
```

```
(b^4*d^2))^(1/4)*b*c^13*cos(b*x + a) + sqrt(2)*(c^10/(b^4*d^2))^(3/4)*b^3*c
^8*d*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) - 3*sqrt(2)*(
c^10/(b^4*d^2))^(1/4)*b*d*log(2916*sqrt(c^10/(b^4*d^2))*b^2*c^11*d*cos(b*x
+ a)*sin(b*x + a) + 729*c^16 - 1458*(sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*c^13*
cos(b*x + a) + sqrt(2)*(c^10/(b^4*d^2))^(3/4)*b^3*c^8*d*sin(b*x + a))*sqrt(
d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + 3*sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*
d*log(729/4*sqrt(c^10/(b^4*d^2))*b^2*c^11*d*cos(b*x + a)*sin(b*x + a) + 729
/16*c^16 + 729/8*(sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*c^13*cos(b*x + a) + sqrt
(2)*(c^10/(b^4*d^2))^(3/4)*b^3*c^8*d*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqr
t(c*sin(b*x + a))) - 3*sqrt(2)*(c^10/(b^4*d^2))^(1/4)*b*d*log(729/4*sqrt(c^
10/(b^4*d^2))*b^2*c^11*d*cos(b*x + a)*sin(b*x + a) + 729/16*c^16 - 729/8*(s
qrt(2)*(c^10/(b^4*d^2))^(1/4)*b*c^13*cos(b*x + a) + sqrt(2)*(c^10/(b^4*d^2)
)^(3/4)*b^3*c^8*d*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))))
/(b*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/sqrt(d*cos(b*x + a)), x)
```

maple [C] time = 0.15, size = 510, normalized size = 1.59

$$-\frac{\left(3i\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] -1/8/b*(3*I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi((
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
```

```
1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x
+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*
I,1/2*2^(1/2))+2*cos(b*x+a)^2*2^(1/2)-2*cos(b*x+a)*2^(1/2))*(c*sin(b*x+a))^
(5/2)/(-1+cos(b*x+a))/(d*cos(b*x+a))^(1/2)/sin(b*x+a)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{\sqrt{d\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)/sqrt(d*cos(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,\sin(a+b\,x))^{5/2}}{\sqrt{d\,\cos(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(1/2),x)
[Out] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(1/2),x)
[Out] Timed out
```

3.283 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=315

$$\frac{c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\,bd^{5/2}}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\,bd^{5/2}}-\frac{c^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right.}{2\sqrt{2}\,bd^{5/2}}$$

```
[Out] 2/3*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(3/2)+1/2*c^(5/2)*arctan(1-2^
(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(d*cos(b*x+a))^(1/2))/b/d^(5/2)*
2^(1/2)-1/2*c^(5/2)*arctan(1+2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/c^(1/2)/(
d*cos(b*x+a))^(1/2))/b/d^(5/2)*2^(1/2)-1/4*c^(5/2)*ln(c^(1/2)-2^(1/2)*d^(1/
2)*(c*sin(b*x+a))^(1/2)/(d*cos(b*x+a))^(1/2)+c^(1/2)*tan(b*x+a))/b/d^(5/2)*
2^(1/2)+1/4*c^(5/2)*ln(c^(1/2)+2^(1/2)*d^(1/2)*(c*sin(b*x+a))^(1/2)/(d*cos(
b*x+a))^(1/2)+c^(1/2)*tan(b*x+a))/b/d^(5/2)*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {2566, 2574, 297, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\,bd^{5/2}}-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}+1\right)}{\sqrt{2}\,bd^{5/2}}-\frac{c^{5/2}\log\left(-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right.}{2\sqrt{2}\,bd^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (c^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*
Cos[a + b*x]])])/(Sqrt[2]*b*d^(5/2)) - (c^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[d]
*Sqrt[c*Sin[a + b*x]])/(Sqrt[c]*Sqrt[d*Cos[a + b*x]])])/(Sqrt[2]*b*d^(5/2))
 - (c^(5/2)*Log[Sqrt[c] - (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos
[a + b*x]] + Sqrt[c]*Tan[a + b*x]])/(2*Sqrt[2]*b*d^(5/2)) + (c^(5/2)*Log[Sq
rt[c] + (Sqrt[2]*Sqrt[d]*Sqrt[c*Sin[a + b*x]])/Sqrt[d*Cos[a + b*x]] + Sqrt[
c]*Tan[a + b*x]])/(2*Sqrt[2]*b*d^(5/2)) + (2*c*(c*Sin[a + b*x])^(3/2))/(3*b
*d*(d*Cos[a + b*x])^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
```

```
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{5/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}} - \frac{c^2\int \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\,dx}{d^2}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}} - \frac{\left(2c^3\right)\text{Subst}\left(\int \frac{x^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{bd}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}} + \frac{c^3\,\text{Subst}\left(\int \frac{c-dx^2}{c^2+d^2x^4}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{bd^2} - \frac{c^3\,\text{Subst}\left(\int \frac{c+dx^2}{c^2+d^2x^4}\,dx\right.}{bd^2}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{3bd(d\cos(a+bx))^{3/2}} - \frac{c^3\,\text{Subst}\left(\int \frac{1}{\frac{c}{d}-\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{d}}+x^2}\,dx, x, \frac{\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right)}{2bd^3} - \frac{c^3\,\text{Subst}\left(\int \frac{}{\frac{c}{d}+\frac{\sqrt{}}{}}\right.}{}$$

$$= -\frac{c^{5/2}\log\left(\sqrt{c}-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}+\sqrt{c}\tan(a+bx)\right)}{2\sqrt{2}\,bd^{5/2}} + \frac{c^{5/2}\log\left(\sqrt{c}+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{d\cos(a+bx)}}\right.}{2\sqrt{2}\,bd^{5/2}}$$

$$= \frac{c^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\,bd^{5/2}} - \frac{c^{5/2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{d}\sqrt{c\sin(a+bx)}}{\sqrt{c}\sqrt{d\cos(a+bx)}}\right)}{\sqrt{2}\,bd^{5/2}} - \frac{c^{5/2}\log\left(\sqrt{c}-\right.}{}$$

Mathematica [C] time = 0.12, size = 67, normalized size = 0.21

$$\frac{2\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{7/2}\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};\sin^2(a+bx)\right)}{7bcd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, 7/4, 11/4, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*(d*Cos[a + b*x])^(3/2))
```

fricas [B] time = 54.20, size = 2268, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")

[Out] 1/48*(12*sqrt(2)*b*d^3*(c^10/(b^4*d^10))^(1/4)*arctan(-((sqrt(2)*b^3*c^8*d^7*(c^10/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^13*d^2*(c^10/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - sqrt(4*b^2*c^11*d^5*sqrt(c^10/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) + c^16 - 2*(sqrt(2)*b^3*c^8*d^7*(c^10/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^13*d^2*(c^10/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(b^2*c^3*d^5*sqrt(c^10/(b^4*d^10)) + 2*c^8*cos(b*x + a)*sin(b*x + a) + (sqrt(2)*b^3*d^7*(c^10/(b^4*d^10))^(3/4)*cos(b*x + a) + sqrt(2)*b*c^5*d^2*(c^10/(b^4*d^10))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))))/(2*c^16*cos(b*x + a)^2 - c^16))*cos(b*x + a)^2 + 12*sqrt(2)*b*d^3*(c^10/(b^4*d^10))^(1/4)*arctan(-((sqrt(2)*b^3*c^8*d^7*(c^10/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^13*d^2*(c^10/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + sqrt(4*b^2*c^11*d^5*sqrt(c^10/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) + c^16 + 2*(sqrt(2)*b^3*c^8*d^7*(c^10/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^13*d^2*(c^10/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(b^2*c^3*d^5*sqrt(c^10/(b^4*d^10)) + 2*c^8*cos(b*x + a)*sin(b*x + a) - (sqrt(2)*b^3*d^7*(c^10/(b^4*d^10))^(3/4)*cos(b*x + a) + sqrt(2)*b*c^5*d^2*(c^10/(b^4*d^10))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))))/(2*c^16*cos(b*x + a)^2 - c^16))*cos(b*x + a)^2 - 12*sqrt(2)*b*d^3*(c^10/(b^4*d^10))^(1/4)*arctan(-1/2*(2*c^16*cos(b*x + a)*sin(b*x + a) - sqrt(4*b^2*c^11*d^5*sqrt(c^10/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) + c^16 + 2*(sqrt(2)*b^3*c^8*d^7*(c^10/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^13*d^2*(c^10/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(sqrt(2)*b^3*d^7*(c^10/(b^4*d^10))^(3/4)*cos(b*x + a) + sqrt(2)*b*c^5*d^2*(c^10/(b^4*d^10))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + (sqrt(2)*b^3*c^8*d^7*(c^10/(b^4*d^10))^(3/4)*cos(b*x + a) + sqrt(2)*b*c^13*d^2*(c^10/(b^4*d^10))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - 4*(b^2*c^11*d^5*cos(b*x + a)^4 - b^2*c^11*d^5*cos(b*x + a)^2)*sqrt(c^10/(b^4*d^10)))/((2*c^16*cos(b*x + a)^3 - c^16*cos(b*x + a))*sin(b*x + a)))*cos(b*x + a)^2 - 12*sqrt(2)*b*d^3*(c^10/(b^4*d^10))^(1/4)*arctan(1/2*(2*c^16*cos(b*x + a)*sin(b*x + a) + sqrt(4*b^2*c^11*d^5*sqrt(c^10/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) + c^16 - 2*(sqrt(2)*b^3*c^8*d^7*(c^10/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^13*d^2*(c^10/(b^4*d^10))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(sqrt(2)*b^3*d^7*(c^10/(b^4*d^10))^(3/4)*cos(b*x + a) + sqrt(2)*b*c^5*d^2*(c^10/(b^4*d^10))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - (sqrt(2)*b^3*c^8*d^7*(c^10/(b^4*d^10))^(3/4)*cos(b*x + a) + sqrt(2)*b*c^13*d^2*(c^10/(b^4*d^10))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - 4*(b^2*c^11*d^5*cos(b*x + a)^4 - b^2*c^11*d^5*cos(b*x + a)^2)*sqrt(c^10/(b^4*d^10)))/((2*c^16*cos(b*x + a)^3 - c^16*cos(b*x + a))*sin(b*x + a)))*cos(b*x + a)^2 + 3*sq

```
rt(2)*b*d^3*(c^10/(b^4*d^10))^(1/4)*cos(b*x + a)^2*log(4*b^2*c^11*d^5*sqrt(
c^10/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) + c^16 + 2*(sqrt(2)*b^3*c^8*d^7*
(c^10/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^13*d^2*(c^10/(b^4*d^10))
^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) - 3*sqrt(2)
*b*d^3*(c^10/(b^4*d^10))^(1/4)*cos(b*x + a)^2*log(4*b^2*c^11*d^5*sqrt(c^10/
(b^4*d^10))*cos(b*x + a)*sin(b*x + a) + c^16 - 2*(sqrt(2)*b^3*c^8*d^7*(c^10
/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^13*d^2*(c^10/(b^4*d^10))^(1/4
)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + 3*sqrt(2)*b*d^
3*(c^10/(b^4*d^10))^(1/4)*cos(b*x + a)^2*log(1/4*b^2*c^11*d^5*sqrt(c^10/(b^
4*d^10))*cos(b*x + a)*sin(b*x + a) + 1/16*c^16 + 1/8*(sqrt(2)*b^3*c^8*d^7*(
c^10/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^13*d^2*(c^10/(b^4*d^10))^
(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) - 3*sqrt(2)*
b*d^3*(c^10/(b^4*d^10))^(1/4)*cos(b*x + a)^2*log(1/4*b^2*c^11*d^5*sqrt(c^10
/(b^4*d^10))*cos(b*x + a)*sin(b*x + a) + 1/16*c^16 - 1/8*(sqrt(2)*b^3*c^8*d
^7*(c^10/(b^4*d^10))^(3/4)*sin(b*x + a) + sqrt(2)*b*c^13*d^2*(c^10/(b^4*d^1
0))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) + 32*sqr
t(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*c^2*sin(b*x + a))/(b*d^3*cos(b*x + a
)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.13, size = 532, normalized size = 1.69

$$\frac{\left(3i\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] 1/6/b*(3*I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((
1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a
)-3*I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos
(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)-3*(
(1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin
```

```
(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)-3*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)+2*cos(b*x+a)*2^(1
/2)-2*2^(1/2))*(c*sin(b*x+a))^(5/2)*cos(b*x+a)/(-1+cos(b*x+a))/(d*cos(b*x+a
))^(5/2)/sin(b*x+a)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^{\frac{5}{2}}}{(d\cos(bx+a))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c\,\sin(a+b\,x))^{5/2}}{(d\,\cos(a+b\,x))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(5/2),x)
[Out] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(5/2),x)
[Out] Timed out
```

3.284 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

```
[Out] 2/7*(c*sin(b*x+a))^(7/2)/b/c/d/(d*cos(b*x+a))^(7/2)
```

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2563}

$$\frac{2(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*(c*Sin[a + b*x])^(7/2))/(7*b*c*d*(d*Cos[a + b*x])^(7/2))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{9/2}}\,dx = \frac{2(c\sin(a+bx))^{7/2}}{7bcd(d\cos(a+bx))^{7/2}}$$

Mathematica [A] time = 0.14, size = 40, normalized size = 1.08

$$\frac{2\cot(a+bx)(c\sin(a+bx))^{9/2}}{7bc^2(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(9/2),x]
```

```
[Out] (2*Cot[a + b*x]*(c*Sin[a + b*x])^(9/2))/(7*b*c^2*(d*Cos[a + b*x])^(9/2))
```

fricas [A] time = 0.54, size = 60, normalized size = 1.62

$$-\frac{2\left(c^{2}\cos\left(bx+a\right)^{2}-c^{2}\right)\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}\sin\left(bx+a\right)}{7\,bd^{5}\cos\left(bx+a\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x, algorithm="fricas")
```

```
[Out] -2/7*(c^2*cos(b*x + a)^2 - c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*s
in(b*x + a)/(b*d^5*cos(b*x + a)^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.10, size = 38, normalized size = 1.03

$$\frac{2\sin\left(bx+a\right)\cos\left(bx+a\right)\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}}{7b\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x)
```

```
[Out] 2/7/b*sin(b*x+a)*cos(b*x+a)*(c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}}{\left(d\cos\left(bx+a\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(9/2), x)
```

mupad [B] time = 1.78, size = 89, normalized size = 2.41

$$\frac{2\,c^2\,\sqrt{c\,\sin\left(a+b\,x\right)}\,\left(3\,\sin\left(2\,a+2\,b\,x\right)-\sin\left(6\,a+6\,b\,x\right)\right)}{7\,b\,d^4\,\sqrt{d\,\cos\left(a+b\,x\right)}\,\left(15\,\cos\left(2\,a+2\,b\,x\right)+6\,\cos\left(4\,a+4\,b\,x\right)+\cos\left(6\,a+6\,b\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(9/2),x)
```

```
[Out] (2*c^2*(c*sin(a + b*x))^(1/2)*(3*sin(2*a + 2*b*x) - sin(6*a + 6*b*x)))/(7*b
*d^4*(d*cos(a + b*x))^(1/2)*(15*cos(2*a + 2*b*x) + 6*cos(4*a + 4*b*x) + cos
(6*a + 6*b*x) + 10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(9/2),x)
```

```
[Out] Timed out
```

3.285 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{8c(c\sin(a+bx))^{3/2}}{77bd^5(d\cos(a+bx))^{3/2}}-\frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}}+\frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}}$$

```
[Out] 2/11*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(11/2)-6/77*c*(c*sin(b*x+a))
^(3/2)/b/d^3/(d*cos(b*x+a))^(7/2)-8/77*c*(c*sin(b*x+a))^(3/2)/b/d^5/(d*cos(
b*x+a))^(3/2)
```

Rubi [A] time = 0.17, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2566, 2571, 2563}

$$-\frac{8c(c\sin(a+bx))^{3/2}}{77bd^5(d\cos(a+bx))^{3/2}}-\frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}}+\frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(13/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(11*b*d*(d*Cos[a + b*x])^(11/2)) - (6*c*(c*Sin
[a + b*x])^(3/2))/(77*b*d^3*(d*Cos[a + b*x])^(7/2)) - (8*c*(c*Sin[a + b*x])
^(3/2))/(77*b*d^5*(d*Cos[a + b*x])^(3/2))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
```

```
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{13/2}}\,dx &= \frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}} - \frac{\left(3c^2\right)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx}{11d^2}\\ &= \frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}} - \frac{\left(12c^2\right)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{5/2}}\,dx}{77d^4}\\ &= \frac{2c(c\sin(a+bx))^{3/2}}{11bd(d\cos(a+bx))^{11/2}} - \frac{6c(c\sin(a+bx))^{3/2}}{77bd^3(d\cos(a+bx))^{7/2}} - \frac{8c(c\sin(a+bx))^{3/2}}{77bd^5(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.29, size = 57, normalized size = 0.54

$$\frac{2c^4(2\cos(2(a+bx))+9)\tan^5(a+bx)}{77bd^6(c\sin(a+bx))^{3/2}\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(13/2),x]
```

```
[Out] (2*c^4*(9 + 2*Cos[2*(a + b*x)])*Tan[a + b*x]^5)/(77*b*d^6*Sqrt[d*Cos[a + b*
x]]*(c*Sin[a + b*x])^(3/2))
```

fricas [A] time = 0.62, size = 74, normalized size = 0.70

$$-\frac{2\left(4\,c^2\cos\left(bx+a\right)^4+3\,c^2\cos\left(bx+a\right)^2-7\,c^2\right)\sqrt{d\cos\left(bx+a\right)}\,\sqrt{c\sin\left(bx+a\right)}\,\sin\left(bx+a\right)}{77\,bd^7\cos\left(bx+a\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x, algorithm="fricas")
```

```
[Out] -2/77*(4*c^2*cos(b*x + a)^4 + 3*c^2*cos(b*x + a)^2 - 7*c^2)*sqrt(d*cos(b*x
+ a))*sqrt(c*sin(b*x + a))*sin(b*x + a)/(b*d^7*cos(b*x + a)^6)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.10, size = 50, normalized size = 0.47

$$\frac{2\left(4\left(\cos^{2}\left(bx+a\right)\right)+7\right)\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}\cos\left(bx+a\right)\sin\left(bx+a\right)}{77b\left(d\cos\left(bx+a\right)\right)^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x)
```

```
[Out] 2/77/b*(4*cos(b*x+a)^2+7)*(c*sin(b*x+a))^(5/2)*cos(b*x+a)*sin(b*x+a)/(d*cos
(b*x+a))^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}}{\left(d\cos\left(bx+a\right)\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(13/2), x)
```

mupad [B] time = 6.34, size = 176, normalized size = 1.66

$$-\frac{e^{-a\,5i-b\,x\,5i}\sqrt{c\left(\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2}-\frac{e^{a\,1i+b\,x\,1i}\,1i}{2}\right)}\left(\frac{96\,c^2\,e^{a\,5i+b\,x\,5i}\,\sin(3\,a+3\,b\,x)}{77\,b\,d^6}+\frac{16\,c^2\,e^{a\,5i+b\,x\,5i}\,\sin(5\,a+5\,b\,x)}{77\,b\,d^6}-\frac{368\,c^2\,e^{a\,5i+b\,x\,5i}\,\sin(a+}{77\,b\,d^6}\right.}{32\cos\left(a+b\,x\right)^5\sqrt{d\left(\frac{e^{-a\,1i-b\,x\,1i}}{2}+\frac{e^{a\,1i+b\,x\,1i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(13/2),x)
```

```
[Out] -(exp(- a*5i - b*x*5i)*(c*((exp(- a*1i - b*x*1i)*1i)/2 - (exp(a*1i + b*x*1i
)*1i)/2))^(1/2)*((96*c^2*exp(a*5i + b*x*5i)*sin(3*a + 3*b*x))/(77*b*d^6) +
(16*c^2*exp(a*5i + b*x*5i)*sin(5*a + 5*b*x))/(77*b*d^6) - (368*c^2*exp(a*5i
 + b*x*5i)*sin(a + b*x))/(77*b*d^6)))/(32*cos(a + b*x)^5*(d*(exp(- a*1i - b
*x*1i)/2 + exp(a*1i + b*x*1i)/2))^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(13/2),x)
```

```
[Out] Timed out
```

3.286 $\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{64c(c\sin(a+bx))^{3/2}}{1155bd^7(d\cos(a+bx))^{3/2}}-\frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}}-\frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}}+\frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}}$$

```
[Out] 2/15*c*(c*sin(b*x+a))^(3/2)/b/d/(d*cos(b*x+a))^(15/2)-2/55*c*(c*sin(b*x+a))
^(3/2)/b/d^3/(d*cos(b*x+a))^(11/2)-16/385*c*(c*sin(b*x+a))^(3/2)/b/d^5/(d*c
os(b*x+a))^(7/2)-64/1155*c*(c*sin(b*x+a))^(3/2)/b/d^7/(d*cos(b*x+a))^(3/2)
```

Rubi [A] time = 0.23, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2566, 2571, 2563}

$$-\frac{64c(c\sin(a+bx))^{3/2}}{1155bd^7(d\cos(a+bx))^{3/2}}-\frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}}-\frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}}+\frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(17/2),x]
```

```
[Out] (2*c*(c*Sin[a + b*x])^(3/2))/(15*b*d*(d*Cos[a + b*x])^(15/2)) - (2*c*(c*Sin
[a + b*x])^(3/2))/(55*b*d^3*(d*Cos[a + b*x])^(11/2)) - (16*c*(c*Sin[a + b*x
])^(3/2))/(385*b*d^5*(d*Cos[a + b*x])^(7/2)) - (64*c*(c*Sin[a + b*x])^(3/2)
)/(1155*b*d^7*(d*Cos[a + b*x])^(3/2))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
```

```
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^{5/2}}{(d\cos(a+bx))^{17/2}}\,dx = \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{c^2\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{13/2}}\,dx}{5d^2}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}} - \frac{\left(8c^2\right)\int \frac{\sqrt{c\sin(a+bx)}}{(d\cos(a+bx))^{9/2}}\,dx}{55d^4}$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}} - \frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}} - \ldots$$

$$= \frac{2c(c\sin(a+bx))^{3/2}}{15bd(d\cos(a+bx))^{15/2}} - \frac{2c(c\sin(a+bx))^{3/2}}{55bd^3(d\cos(a+bx))^{11/2}} - \frac{16c(c\sin(a+bx))^{3/2}}{385bd^5(d\cos(a+bx))^{7/2}} - \ldots$$

Mathematica [A] time = 0.47, size = 67, normalized size = 0.48

$$\frac{2(44\cos(2(a+bx))+4\cos(4(a+bx))+117)\sec^8(a+bx)(c\sin(a+bx))^{7/2}\sqrt{d\cos(a+bx)}}{1155bcd^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^(5/2)/(d*Cos[a + b*x])^(17/2),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(117 + 44*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Se
c[a + b*x]^8*(c*Sin[a + b*x])^(7/2))/(1155*b*c*d^9)
```

fricas [A] time = 0.86, size = 87, normalized size = 0.62

$$-\frac{2\left(32\,c^2\cos(bx+a)^6+24\,c^2\cos(bx+a)^4+21\,c^2\cos(bx+a)^2-77\,c^2\right)\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}\,\sin(bx \ldots}{1155\,bd^9\cos(bx+a)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(32*c^2*cos(b*x + a)^6 + 24*c^2*cos(b*x + a)^4 + 21*c^2*cos(b*x + a
)^2 - 77*c^2)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*sin(b*x + a)/(b*d^9
*cos(b*x + a)^8)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 0.15, size = 60, normalized size = 0.43

$$\frac{2\left(32\left(\cos^{4}\left(bx+a\right)\right)+56\left(\cos^{2}\left(bx+a\right)\right)+77\right)\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}\cos\left(bx+a\right)\sin\left(bx+a\right)}{1155b\left(d\cos\left(bx+a\right)\right)^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x)
```

```
[Out] 2/1155/b*(32*cos(b*x+a)^4+56*cos(b*x+a)^2+77)*(c*sin(b*x+a))^(5/2)*cos(b*x+
a)*sin(b*x+a)/(d*cos(b*x+a))^(17/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}}{\left(d\cos\left(bx+a\right)\right)^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^(5/2)/(d*cos(b*x+a))^(17/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)/(d*cos(b*x + a))^(17/2), x)
```

mupad [**B**] time = 6.43, size = 207, normalized size = 1.47

$$-\frac{e^{-a\,7\mathrm{i}-b\,x\,7\mathrm{i}}\,\sqrt{c\,\left(\frac{e^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{e^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\,\left(\frac{1216\,c^2\,e^{a\,7\mathrm{i}+b\,x\,7\mathrm{i}}\,\sin(3\,a+3\,b\,x)}{385\,b\,d^8}+\frac{1024\,c^2\,e^{a\,7\mathrm{i}+b\,x\,7\mathrm{i}}\,\sin(5\,a+5\,b\,x)}{1155\,b\,d^8}+\frac{128\,c^2\,e^{a\,7\mathrm{i}+b\,x\,7\mathrm{i}}\,\mathrm{si}}{1155\,b\,d}\right.}{128\cos\left(a+b\,x\right)^7\,\sqrt{d\,\left(\frac{e^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}}{2}+\frac{e^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(5/2)/(d*cos(a + b*x))^(17/2),x)
```

```
[Out] -(exp(- a*7i - b*x*7i)*(c*((exp(- a*1i - b*x*1i)*1i)/2 - (exp(a*1i + b*x*1i
)*1i)/2))^(1/2)*((1216*c^2*exp(a*7i + b*x*7i)*sin(3*a + 3*b*x))/(385*b*d^8)
```

```
 + (1024*c^2*exp(a*7i + b*x*7i)*sin(5*a + 5*b*x))/(1155*b*d^8) + (128*c^2*e
xp(a*7i + b*x*7i)*sin(7*a + 7*b*x))/(1155*b*d^8) - (3392*c^2*exp(a*7i + b*x
*7i)*sin(a + b*x))/(231*b*d^8)))/(128*cos(a + b*x)^7*(d*(exp(- a*1i - b*x*1
i)/2 + exp(a*1i + b*x*1i)/2))^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**(5/2)/(d*cos(b*x+a))**(17/2),x)
```

```
[Out] Timed out
```

3.287 $\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=226

$$\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}\,b}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}\,b}-\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{}}{\sqrt{s}}\right.}{2\sqrt{2}\,b}$$

```
[Out] 2/5*sin(b*x+a)^(5/2)/b/cos(b*x+a)^(5/2)-1/2*arctan(-1+2^(1/2)*cos(b*x+a)^(1
/2)/sin(b*x+a)^(1/2))/b*2^(1/2)-1/2*arctan(1+2^(1/2)*cos(b*x+a)^(1/2)/sin(b
*x+a)^(1/2))/b*2^(1/2)-1/4*ln(1+cot(b*x+a)-2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x
+a)^(1/2))/b*2^(1/2)+1/4*ln(1+cot(b*x+a)+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a
)^(1/2))/b*2^(1/2)-2*sin(b*x+a)^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2566, 2575, 297, 1162, 617, 204, 1165, 628}

$$\frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}\,b}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}\,b}-\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{}}{\sqrt{s}}\right.}{2\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(7/2)/Cos[a + b*x]^(7/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - A
rcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - Lo
g[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(2*Sq
rt[2]*b) + Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a +
 b*x]]]/(2*Sqrt[2]*b) - (2*Sqrt[Sin[a + b*x]])/(b*Sqrt[Cos[a + b*x]]) + (2*
Sin[a + b*x]^(5/2))/(5*b*Cos[a + b*x]^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
```

```
&& LtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^{\frac{7}{2}}(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx &= \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} - \int \frac{\sin^{\frac{3}{2}}(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx \\
&= -\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} + \int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx \\
&= -\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{2\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b} \\
&= -\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} + \frac{\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b} - \frac{\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,d\right.}{b} \\
&= -\frac{2\sqrt{\sin(a+bx)}}{b\sqrt{\cos(a+bx)}} + \frac{2\sin^{\frac{5}{2}}(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} - \frac{\mathrm{Subst}\left(\int \frac{}{1}\right.}{} \\
&= -\frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}\,b} + \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}\,b} - \frac{2\sqrt{\sin(a}}{b\sqrt{\cos(a}} \\
&= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}\,b} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}\,b} - \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+b}}{\sqrt{\sin(a+bx)}}\right.}{2\sqrt{2}\,b}
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 57, normalized size = 0.25

$$
\frac{2\sin^{\frac{9}{2}}(a+bx)\sqrt[4]{\cos^2(a+bx)}\;{}_2F_1\left(\frac{9}{4},\frac{9}{4};\frac{13}{4};\sin^2(a+bx)\right)}{9b\sqrt{\cos(a+bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(7/2)/Cos[a + b*x]^(7/2),x]
[Out] (2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[9/4, 9/4, 13/4, Sin[a + b*x]^2]
*Sin[a + b*x]^(9/2))/(9*b*Sqrt[Cos[a + b*x]])
```

fricas [B] time = 26.09, size = 1282, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/40*(10*sqrt(2)*b*(b^(-4))^(1/4)*arctan(1/2*((sqrt(2)*b^3*(b^(-4))^(3/4)*
sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(4*b^2*sqrt(b^(-4
))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) +
 sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a
)) + 1)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + (sqrt(2)*b^3*(b^(-4))^(3/4)
*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*s
qrt(sin(b*x + a)) + 2*cos(b*x + a)*sin(b*x + a) - 4*(b^2*cos(b*x + a)^4 - b
^2*cos(b*x + a)^2)*sqrt(b^(-4)))/((2*cos(b*x + a)^3 - cos(b*x + a))*sin(b*x
 + a)))*cos(b*x + a)^3 + 10*sqrt(2)*b*(b^(-4))^(1/4)*arctan(1/2*((sqrt(2)*b
^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqr
t(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*(b^(-4))^(3
/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a)
)*sqrt(sin(b*x + a)) + 1)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + (sqrt(2)*
b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sq
rt(cos(b*x + a))*sqrt(sin(b*x + a)) - 2*cos(b*x + a)*sin(b*x + a) + 4*(b^2*
cos(b*x + a)^4 - b^2*cos(b*x + a)^2)*sqrt(b^(-4)))/((2*cos(b*x + a)^3 - cos
(b*x + a))*sin(b*x + a)))*cos(b*x + a)^3 + 10*sqrt(2)*b*(b^(-4))^(1/4)*arct
an(-1/2*((sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) - sqrt(2)*b*(b^(-4))^(1/4
)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - sqrt(4*b^2*sqrt(b^(
-4))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a)
 + sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x +
 a)) + 1)*((sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1
/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 2*cos(b*x + a)*si
n(b*x + a)))/(cos(b*x + a)*sin(b*x + a)))*cos(b*x + a)^3 + 10*sqrt(2)*b*(b^
(-4))^(1/4)*arctan(-1/2*((sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) - sqrt(2)
*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - sqr
t(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*(b^(-4))^(3
/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a)
)*sqrt(sin(b*x + a)) + 1)*((sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(
2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 2
*cos(b*x + a)*sin(b*x + a)))/(cos(b*x + a)*sin(b*x + a)))*cos(b*x + a)^3 -
5*sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a)^3*log(4*b^2*sqrt(b^(-4))*cos(b*x +
a)*sin(b*x + a) + 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b
^(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1) + 5*s
qrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a)^3*log(4*b^2*sqrt(b^(-4))*cos(b*x + a)*
sin(b*x + a) - 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-
4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1) + 16*(6*
```

```
cos(b*x + a)^2 - 1)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)))/(b*cos(b*x + a)^
3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.16, size = 692, normalized size = 3.06

$$-\frac{\left(5i\left(\cos^{2}\left(bx+a\right)\right)\sin\left(bx+a\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos}{}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x)
```

```
[Out] -1/10/b*(5*I*cos(b*x+a)^2*sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b
*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/
2*I,1/2*2^(1/2))-5*I*cos(b*x+a)^2*sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin
(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2
),1/2+1/2*I,1/2*2^(1/2))+5*cos(b*x+a)^2*sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos
(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))+5*cos(b*x+a)^2*sin(b*x+a)*((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*cos(b*x+a)^2*sin(b*x+a)*((1-cos(b*x
+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a)
)/sin(b*x+a))^(1/2),1/2*2^(1/2))+12*cos(b*x+a)^3*2^(1/2)-12*cos(b*x+a)^2*2^
(1/2)-2*cos(b*x+a)*2^(1/2)+2*2^(1/2))*sin(b*x+a)^(1/2)/(-1+cos(b*x+a))/cos(
b*x+a)^(5/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)^{\frac{7}{2}}}{\cos\left(bx+a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/2)/cos(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(7/2)/cos(b*x + a)^(7/2), x)
```

mupad [B] time = 2.05, size = 44, normalized size = 0.19

$$\frac{2\sin\left(a+b\,x\right)^{9/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},-\frac{5}{4};\ -\frac{1}{4};\ \cos\left(a+b\,x\right)^2\right)}{5\,b\cos\left(a+b\,x\right)^{5/2}\,{\left(\sin\left(a+b\,x\right)^2\right)}^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^(7/2)/cos(a + b*x)^(7/2),x)
```

```
[Out] (2*sin(a + b*x)^(9/2)*hypergeom([-5/4, -5/4], -1/4, cos(a + b*x)^2))/(5*b*c
os(a + b*x)^(5/2)*(sin(a + b*x)^2)^(9/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(7/2)/cos(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.288 $\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx$

Optimal. Leaf size=16

$$\frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

```
[Out] 2/5*sin(x)^(5/2)/cos(x)^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2563}

$$\frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^(3/2)/Cos[x]^(7/2),x]
```

```
[Out] (2*Sin[x]^(5/2))/(5*Cos[x]^(5/2))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^{\frac{3}{2}}(x)}{\cos^{\frac{7}{2}}(x)}\,dx = \frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$\frac{2\sin^{\frac{5}{2}}(x)}{5\cos^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^(3/2)/Cos[x]^(7/2),x]
```

```
[Out] (2*Sin[x]^(5/2))/(5*Cos[x]^(5/2))
```

fricas [A] time = 0.44, size = 16, normalized size = 1.00

$$-\frac{2\left(\cos(x)^{2}-1\right)\sqrt{\sin(x)}}{5\cos(x)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(3/2)/cos(x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(cos(x)^2 - 1)*sqrt(sin(x))/cos(x)^(5/2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)^{\frac{3}{2}}}{\cos(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(3/2)/cos(x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(x)^(3/2)/cos(x)^(7/2), x)
```

maple [B] time = 0.06, size = 33, normalized size = 2.06

$$\frac{\left(-\left(\sin^{2}(x)\right)+\cos^{2}(x)-2\cos(x)+1\right)\left(\sin^{\frac{5}{2}}(x)\right)}{5\left(-1+\cos(x)\right)\cos(x)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^(3/2)/cos(x)^(7/2),x)
```

```
[Out] 1/5*(-sin(x)^2+cos(x)^2-2*cos(x)+1)*sin(x)^(5/2)/(-1+cos(x))/cos(x)^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)^{\frac{3}{2}}}{\cos(x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(3/2)/cos(x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^(3/2)/cos(x)^(7/2), x)
```

mupad [B] time = 0.86, size = 53, normalized size = 3.31

$$-\frac{8\sqrt{2}\tan\left(\frac{x}{2}\right)^{5/2}\sqrt{1-\tan\left(\frac{x}{2}\right)^2}}{\tan\left(\frac{x}{2}\right)^2\left(\tan\left(\frac{x}{2}\right)^2\left(5\tan\left(\frac{x}{2}\right)^2-15\right)+15\right)-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^(3/2)/cos(x)^(7/2),x)
```

```
[Out] -(8*2^(1/2)*tan(x/2)^(5/2)*(1 - tan(x/2)^2)^(1/2))/(tan(x/2)^2*(tan(x/2)^2*
(5*tan(x/2)^2 - 15) + 15) - 5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**(3/2)/cos(x)**(7/2),x)
```

```
[Out] Timed out
```

3.289 $\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$

Optimal. Leaf size=122

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}$$

```
[Out] -1/2*arctan(1-2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/
2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+1/4*ln(1-2^(1/2)*sin(x)^(1/2)/cos(x)^
(1/2)+tan(x))*2^(1/2)-1/4*ln(1+2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2)+tan(x))*2^
(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[x]]/Sqrt[Cos[x]],x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2]) + ArcTan[1 + (Sq
rt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/Sqrt[2] + Log[1 - (Sqrt[2]*Sqrt[Sin[x]])/
Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Co
s[x]] + Tan[x]]/(2*Sqrt[2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx = 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)$$
$$= -\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) + \frac{\text{Su}}{}$$
$$= \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{\text{si}}}{\sqrt{\cos(}}\right.}{\sqrt{2}}$$
$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}} + \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\frac{\sqrt{2}}{\sqrt{}}\right.}{}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.31

$$\frac{2\sin^{\frac{3}{2}}(x)\cos^2(x)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\sin^2(x)\right)}{3\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[x]]/Sqrt[Cos[x]],x]
```

```
[Out] (2*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[x]^2]*Sin[x]^(3/2)
)/(3*Cos[x]^(3/2))
```

fricas [B] time = 0.81, size = 447, normalized size = 3.66

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{2\cos(x)^3-2\cos(x)^2\sin(x)+\sqrt{2}\sqrt{2\left(\sqrt{2}\cos(x)+\sqrt{2}\sin(x)\right)\sqrt{\cos(x)}\sqrt{\sin(x)}+4\cos(x)\sin(x}}{2\left(\cos(x)^3+\cos(x)^2\sin(x)-\cos(x)\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(1/2)/cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*(2*cos(x)^3 - 2*cos(x)^2*sin(x) + sqrt(2)*sqrt(2*(sq
rt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x)
+ 1)*sqrt(cos(x))*sqrt(sin(x)) - sqrt(2)*sqrt(cos(x))*sqrt(sin(x)) - 2*cos(
x))/(cos(x)^3 + cos(x)^2*sin(x) - cos(x))) + 1/4*sqrt(2)*arctan(-1/2*(2*cos
```

```
(x)^3 - 2*cos(x)^2*sin(x) - sqrt(2)*sqrt(-2*(sqrt(2)*cos(x) + sqrt(2)*sin(x
))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1)*sqrt(cos(x))*sqrt(sin(x
)) + sqrt(2)*sqrt(cos(x))*sqrt(sin(x)) - 2*cos(x))/(cos(x)^3 + cos(x)^2*sin
(x) - cos(x))) - 1/4*sqrt(2)*arctan(-(sqrt(-2*(sqrt(2)*cos(x) + sqrt(2)*sin
(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1)*(sqrt(2)*sqrt(cos(x))
*sqrt(sin(x)) + cos(x) + sin(x)) + sqrt(2)*sqrt(cos(x))*sqrt(sin(x)))/(cos(
x) - sin(x))) - 1/4*sqrt(2)*arctan(-(sqrt(2*(sqrt(2)*cos(x) + sqrt(2)*sin(x
))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1)*(sqrt(2)*sqrt(cos(x))*s
qrt(sin(x)) - cos(x) - sin(x)) + sqrt(2)*sqrt(cos(x))*sqrt(sin(x)))/(cos(x)
 - sin(x))) - 1/8*sqrt(2)*log(2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(
x))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1) + 1/8*sqrt(2)*log(-2*(sqrt(2)*cos(x
) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(1/2)/cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(x))/sqrt(cos(x)), x)
```

maple [C] time = 0.06, size = 166, normalized size = 1.36

$$-\frac{\left(\sin^{\frac{3}{2}}(x)\right)\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(x)+\sin(x)}{\sin(x)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(x)+\sin(x)}{\sin(x)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticPi}\left(\right.\right.}{2\,(-1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^(1/2)/cos(x)^(1/2),x)
```

```
[Out] -1/2*sin(x)^(3/2)*(I*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,
1/2*2^(1/2))-I*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^
(1/2))-EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-E
llipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*((-1+cos
(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((1-cos(x)+sin(x))/sin
(x))^(1/2)/(-1+cos(x))/cos(x)^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(1/2)/cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(x))/sqrt(cos(x)), x)
```

mupad [B] time = 0.72, size = 25, normalized size = 0.20

$$-\frac{2\sqrt{\cos(x)}\,\sin(x)^{3/2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \cos(x)^2\right)}{\left(\sin(x)^2\right)^{3/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^(1/2)/cos(x)^(1/2),x)
```

```
[Out] -(2*cos(x)^(1/2)*sin(x)^(3/2)*hypergeom([1/4, 1/4], 5/4, cos(x)^2))/(sin(x)
^2)^(3/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin{(x)}}}{\sqrt{\cos{(x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**(1/2)/cos(x)**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x))/sqrt(cos(x)), x)
```

3.290 $\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx$

Optimal. Leaf size=143

$$-\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{4\sqrt{2}}+\frac{3\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{8\sqrt{2}}-\frac{3\log\left(\right.}{}$$

```
[Out] -3/8*arctan(1-2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+3/8*arctan(1+2^(1/
2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+3/16*ln(1-2^(1/2)*sin(x)^(1/2)/cos(x)
^(1/2)+tan(x))*2^(1/2)-3/16*ln(1+2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2)+tan(x))*
2^(1/2)-1/2*sin(x)^(3/2)*cos(x)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {2568, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{1}{2}\sin^{\frac{3}{2}}(x)\sqrt{\cos(x)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{4\sqrt{2}}+\frac{3\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{8\sqrt{2}}-\frac{3\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^(5/2)/Sqrt[Cos[x]],x]
```

```
[Out] (-3*ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]])/(4*Sqrt[2]) + (3*ArcTa
n[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]])/(4*Sqrt[2]) + (3*Log[1 - (Sqrt[
2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]])/(8*Sqrt[2]) - (3*Log[1 + (Sqrt[2]*
Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]])/(8*Sqrt[2]) - (Sqrt[Cos[x]]*Sin[x]^(3
/2))/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sin^{\frac{5}{2}}(x)}{\sqrt{\cos(x)}}\,dx = -\frac{1}{2}\sqrt{\cos(x)}\,\sin^{\frac{3}{2}}(x) + \frac{3}{4}\int \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\,dx$$
$$= -\frac{1}{2}\sqrt{\cos(x)}\,\sin^{\frac{3}{2}}(x) + \frac{3}{2}\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)$$
$$= -\frac{1}{2}\sqrt{\cos(x)}\,\sin^{\frac{3}{2}}(x) - \frac{3}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) + \frac{3}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(}}{\sqrt{\cos(}}\right.$$
$$= -\frac{1}{2}\sqrt{\cos(x)}\,\sin^{\frac{3}{2}}(x) + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right) + \frac{3}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+}\right.$$
$$= \frac{3\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{8\sqrt{2}} - \frac{3\log\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{8\sqrt{2}} - \frac{1}{2}\sqrt{\cos(x)}\,\sin^{\frac{3}{2}}(x) + \frac{3\,\text{Sub}}{}$$
$$= -\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{4\sqrt{2}} + \frac{3\log\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+\tan(x)\right)}{8\sqrt{2}} - \frac{3\log}{}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.27

$$\frac{2\sin^{\frac{7}{2}}(x)\cos^2(x)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\sin^2(x)\right)}{7\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^(5/2)/Sqrt[Cos[x]],x]
```

```
[Out] (2*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 7/4, 11/4, Sin[x]^2]*Sin[x]^(7/2
))/(7*Cos[x]^(3/2))
```

fricas [B] time = 0.83, size = 457, normalized size = 3.20

$$-\frac{1}{2}\sqrt{\cos(x)}\,\sin(x)^{\frac{3}{2}} + \frac{3}{16}\sqrt{2}\arctan\left(\frac{2\cos(x)^3 - 2\cos(x)^2\sin(x) + \sqrt{2}\sqrt{2\left(\sqrt{2}\cos(x)+\sqrt{2}\sin(x)\right)\sqrt{\cos(x)}}}{2\left(\cos(x)^3 + \mathrm{co}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(5/2)/cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(cos(x))*sin(x)^(3/2) + 3/16*sqrt(2)*arctan(1/2*(2*cos(x)^3 - 2*co
s(x)^2*sin(x) + sqrt(2)*sqrt(2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x
```

```
))*sqrt(sin(x)) + 4*cos(x)*sin(x) + 1)*sqrt(cos(x))*sqrt(sin(x)) - sqrt(2)*
sqrt(cos(x))*sqrt(sin(x)) - 2*cos(x))/(cos(x)^3 + cos(x)^2*sin(x) - cos(x))
) + 3/16*sqrt(2)*arctan(-1/2*(2*cos(x)^3 - 2*cos(x)^2*sin(x) - sqrt(2)*sqrt
(-2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*
sin(x) + 1)*sqrt(cos(x))*sqrt(sin(x)) + sqrt(2)*sqrt(cos(x))*sqrt(sin(x)) -
 2*cos(x))/(cos(x)^3 + cos(x)^2*sin(x) - cos(x))) - 3/16*sqrt(2)*arctan(-(s
qrt(-2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(
x)*sin(x) + 1)*(sqrt(2)*sqrt(cos(x))*sqrt(sin(x)) + cos(x) + sin(x)) + sqrt
(2)*sqrt(cos(x))*sqrt(sin(x)))/(cos(x) - sin(x))) - 3/16*sqrt(2)*arctan(-(s
qrt(2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x
)*sin(x) + 1)*(sqrt(2)*sqrt(cos(x))*sqrt(sin(x)) - cos(x) - sin(x)) + sqrt(
2)*sqrt(cos(x))*sqrt(sin(x)))/(cos(x) - sin(x))) - 3/32*sqrt(2)*log(2*(sqrt
(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqrt(sin(x)) + 4*cos(x)*sin(x) +
1) + 3/32*sqrt(2)*log(-2*(sqrt(2)*cos(x) + sqrt(2)*sin(x))*sqrt(cos(x))*sqr
t(sin(x)) + 4*cos(x)*sin(x) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)^{\frac{5}{2}}}{\sqrt{\cos(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(5/2)/cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(x)^(5/2)/sqrt(cos(x)), x)
```

maple [C] time = 0.15, size = 2595, normalized size = 18.15

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^(5/2)/cos(x)^(1/2),x)
```

```
[Out] -1/32*sin(x)^(3/2)*(-6*cos(x)^2*sin(x)^2*((1-cos(x)+sin(x))/sin(x))^(1/2)*(
(-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-
cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-4*sin(x)^2*2^(1/2)-16*c
os(x)^3*2^(1/2)+4*cos(x)^4*2^(1/2)+24*cos(x)^2*2^(1/2)-16*cos(x)*2^(1/2)+4*
2^(1/2)-3*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2
)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/
2-1/2*I,1/2*2^(1/2))-3*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))
/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin
(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*cos(x)^2*sin(x)^2*((1-cos(x)+sin(x))/si
n(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*El
lipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+12*cos(x)*
```

```
sin(x)^2*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)
*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2
-1/2*I,1/2*2^(1/2))+12*cos(x)*sin(x)^2*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-
1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-co
s(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*sin(x)^4*((1-cos(x)+s
in(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))
^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3
*I*sin(x)^4*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1
/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),
1/2+1/2*I,1/2*2^(1/2))+3*I*cos(x)^4*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+c
os(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x
)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*cos(x)^4*((1-cos(x)+sin(
x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1
/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+6*I*
sin(x)^2*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)
*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2
-1/2*I,1/2*2^(1/2))+8*cos(x)*sin(x)^2*2^(1/2)-4*cos(x)^2*sin(x)^2*2^(1/2)+6
*I*cos(x)^2*sin(x)^2*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/s
in(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x
))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*I*cos(x)^2*sin(x)^2*((1-cos(x)+sin(x))/si
n(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*El
lipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-12*I*cos(x
)*sin(x)^2*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/
2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1
/2-1/2*I,1/2*2^(1/2))+12*I*cos(x)*sin(x)^2*((1-cos(x)+sin(x))/sin(x))^(1/2)
*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((
1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+12*((1-cos(x)+sin(x))
/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)
*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(x)+
12*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+
cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I
,1/2*2^(1/2))*cos(x)-3*sin(x)^4*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x
)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+si
n(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*sin(x)^4*((1-cos(x)+sin(x))/si
n(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*El
lipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*cos(x)^4
*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+co
s(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1
/2*2^(1/2))-3*cos(x)^4*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))
/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin
(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*sin(x)^2*((1-cos(x)+sin(x))/sin(x))^(1/
2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(
((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-6*sin(x)^2*((1-cos(
x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin
(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2
```

```
))+12*cos(x)^3*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))
^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/
2),1/2-1/2*I,1/2*2^(1/2))+12*cos(x)^3*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1
+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos
(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-18*cos(x)^2*((1-cos(x)+sin
(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(
1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-18*
cos(x)^2*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)
*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2
+1/2*I,1/2*2^(1/2))+3*I*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x)
)/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/si
n(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*I*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-
1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-co
s(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*I*sin(x)^2*((1-cos(x)+s
in(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))
^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-1
2*I*cos(x)^3*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(
1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))+12*I*cos(x)^3*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1
+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos
(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+18*I*cos(x)^2*((1-cos(x)+s
in(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))
^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-1
8*I*cos(x)^2*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(
1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2)
,1/2+1/2*I,1/2*2^(1/2))-12*I*cos(x)*((1-cos(x)+sin(x))/sin(x))^(1/2)*((-1+c
os(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x
)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))+12*I*cos(x)*((1-cos(x)+sin(x
))/sin(x))^(1/2)*((-1+cos(x)+sin(x))/sin(x))^(1/2)*((-1+cos(x))/sin(x))^(1/
2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2)))/(-1+
cos(x))^3/cos(x)^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)^{\frac{5}{2}}}{\sqrt{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(5/2)/cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^(5/2)/sqrt(cos(x)), x)
```

mupad [B] time = 0.81, size = 25, normalized size = 0.17

$$-\frac{2\sqrt{\cos(x)}\,\sin(x)^{7/2}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\ \frac{5}{4};\ \cos(x)^2\right)}{\left(\sin(x)^2\right)^{7/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^(5/2)/cos(x)^(1/2),x)
```

```
[Out] -(2*cos(x)^(1/2)*sin(x)^(7/2)*hypergeom([-3/4, 1/4], 5/4, cos(x)^2))/(sin(x
)^2)^(7/4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**(5/2)/cos(x)**(1/2),x)
```

```
[Out] Timed out
```

3.291 $\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=132

$$\frac{5d^4\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{12b\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}+\frac{5d^3\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}{6bc}+\frac{d\sqrt{c\sin(a+bx)}\,(d\cos(a+bx))^{5/2}}{3bc}$$

```
[Out] 1/3*d*(d*cos(b*x+a))^(5/2)*(c*sin(b*x+a))^(1/2)/b/c+5/6*d^3*(d*cos(b*x+a))^
(1/2)*(c*sin(b*x+a))^(1/2)/b/c-5/12*d^4*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1
/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/(d*c
os(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2569, 2573, 2641}

$$\frac{5d^3\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}{6bc}+\frac{5d^4\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{12b\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\,(d\cos(a+bx))^{5/2}}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(7/2)/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (5*d^3*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(6*b*c) + (d*(d*Cos[a + b
*x])^(5/2)*Sqrt[c*Sin[a + b*x]])/(3*b*c) + (5*d^4*EllipticF[a - Pi/4 + b*x,
 2]*Sqrt[Sin[2*a + 2*b*x]])/(12*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]]
)
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(d\cos(a+bx))^{7/2}}{\sqrt{c\sin(a+bx)}}\,dx = \frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc} + \frac{1}{6}\left(5d^2\right)\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx$$
$$= \frac{5d^3\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6bc} + \frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc} + \frac{1}{12}\left(5d^4\right)\int$$
$$= \frac{5d^3\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6bc} + \frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc} + \frac{\left(5d^4\sqrt{\sin(}\right.}{12\sqrt{d\,c}}$$
$$= \frac{5d^3\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}{6bc} + \frac{d(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}{3bc} + \frac{5d^4F\left(a-\right.}{12b\sqrt{d\,c}}$$

Mathematica [C] time = 0.10, size = 70, normalized size = 0.53

$$\frac{2\cos^2(a+bx)^{3/4}\sec^5(a+bx)\sqrt{c\sin(a+bx)}\,(d\cos(a+bx))^{7/2}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{4};\frac{5}{4};\sin^2(a+bx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(7/2)/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*(d*Cos[a + b*x])^(7/2)*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, 1/
4, 5/4, Sin[a + b*x]^2]*Sec[a + b*x]^5*Sqrt[c*Sin[a + b*x]])/(b*c)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}\,d^3\cos(bx+a)^3}{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d^3*cos(b*x + a)^3/(c*si
n(b*x + a)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.14, size = 216, normalized size = 1.64

$$\frac{\left(2\left(\cos^{4}(bx+a)\right)\sqrt{2}-5\sin(bx+a)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{1}{}}\right.\right.}{12b\left(-1+\cos(bx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] 1/12/b*(2*cos(b*x+a)^4*2^(1/2)-5*sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
)/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),
1/2*2^(1/2))-2*cos(b*x+a)^3*2^(1/2)+5*cos(b*x+a)^2*2^(1/2)-5*cos(b*x+a)*2^(
1/2))*(d*cos(b*x+a))^(7/2)*sin(b*x+a)/(-1+cos(b*x+a))/cos(b*x+a)^4/(c*sin(b
*x+a))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(bx+a))^{\frac{7}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(7/2)/sqrt(c*sin(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\cos(a+b\,x))^{7/2}}{\sqrt{c\,\sin(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(7/2)/(c*sin(a + b*x))^(1/2),x)
```

```
[Out] int((d*cos(a + b*x))^(7/2)/(c*sin(a + b*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(7/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.292 $\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=92

$$\frac{d^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}{bc}$$

```
[Out] d*(d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^(1/2)/b/c-1/2*d^2*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2
*a)^(1/2)/b/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2569, 2573, 2641}

$$\frac{d^2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}+\frac{d\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (d*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])/(b*c) + (d^2*EllipticF[a - Pi
/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(2*b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a
 + b*x]])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(d\cos(a+bx))^{3/2}}{\sqrt{c\sin(a+bx)}}\,dx &= \frac{d\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}}{bc}+\frac{1}{2}d^2\int \frac{1}{\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}}\,dx\\ &= \frac{d\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}}{bc}+\frac{\left(d^2\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{2\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}}\\ &= \frac{d\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}}{bc}+\frac{d^2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{2b\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.11, size = 68, normalized size = 0.74

$$\frac{2d^2\cos^2(a+bx)^{3/4}\tan(a+bx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{4};\frac{5}{4};\sin^2(a+bx)\right)}{b\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*d^2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, Sin[a + b*x
]^2]*Tan[a + b*x])/(b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{d\cos(bx+a)}\,\sqrt{c\sin(bx+a)}\,d\cos(bx+a)}{c\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))*d*cos(b*x + a)/(c*sin(b*
x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(bx+a))^{\frac{3}{2}}}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)/sqrt(c*sin(b*x + a)), x)
```

maple [A] time = 0.12, size = 188, normalized size = 2.04

$$-\frac{\left(\sin\left(bx+a\right)\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\right.}{2b\left(-1+\cos\left(bx+a\right)\right)\cos\left(bx+a\right)^{2}\sqrt{c\sin\left(\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*(sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ellip
ticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))-cos(b*x+a)^2
*2^(1/2)+cos(b*x+a)*2^(1/2))*(d*cos(b*x+a))^(3/2)*sin(b*x+a)/(-1+cos(b*x+a)
)/cos(b*x+a)^2/(c*sin(b*x+a))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}{\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)/sqrt(c*sin(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,\cos\left(a+b\,x\right)\right)}^{3/2}}{\sqrt{c\,\sin\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(3/2)/(c*sin(a + b*x))^(1/2),x)
```

```
[Out] int((d*cos(a + b*x))^(3/2)/(c*sin(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(a+bx\right)\right)^{\frac{3}{2}}}{\sqrt{c\sin\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)/(c*sin(b*x+a))**(1/2),x)
[Out] Integral((d*cos(a + b*x))**(3/2)/sqrt(c*sin(a + b*x)), x)
```

3.293 $\int \frac{1}{\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}$$

```
[Out] -(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),
2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2573, 2641}

$$\frac{\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(b*Sqrt[d*Cos[a + b*x
]]*Sqrt[c*Sin[a + b*x]])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}}\,dx &= \frac{\sqrt{\sin(2a+2bx)}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}}\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{b\sqrt{d\cos(a+bx)}\,\sqrt{c\sin(a+bx)}}\end{aligned}$$

Mathematica [C] time = 0.06, size = 65, normalized size = 1.23

$$\frac{2\cos^2(a+bx)^{3/4}\tan(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};\sin^2(a+bx)\right)}{b\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, Sin[a + b*x]^2]*
Tan[a + b*x])/(b*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{cd\cos(bx+a)\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(c*d*cos(b*x + a)*sin(b*
x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))), x)
```

maple [B] time = 0.10, size = 151, normalized size = 2.85

$$-\frac{\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\left(\sin^2(bx+\right.}{b\sqrt{c\sin(bx+a)}\,(-1+\cos(bx+a))\sqrt{d\cos(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/b*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*((
-1+cos(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*sin(b*x+a)^2/(c*sin(b*x+a)
)^(1/2)/(-1+cos(b*x+a))/(d*cos(b*x+a))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d\cos\left(bx+a\right)}\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{d\,\cos\left(a+b\,x\right)}\,\sqrt{c\,\sin\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(1/2)),x)
```

```
[Out] int(1/((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\sin\left(a+bx\right)}\sqrt{d\cos\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))**(1/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*sin(a + b*x))*sqrt(d*cos(a + b*x))), x)
```

3.294 $\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3bd^2\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}$$

```
[Out] 2/3*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(3/2)-2/3*(sin(a+1/4*Pi+b*x)^
2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2
*a)^(1/2)/b/d^2/(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2571, 2573, 2641}

$$\frac{2\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3bd^2\sqrt{c\sin(a+bx)}\,\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(3*b*c*d*(d*Cos[a + b*x])^(3/2)) + (2*EllipticF[a
- Pi/4 + b*x, 2]*Sqrt[Sin[2*a + 2*b*x]])/(3*b*d^2*Sqrt[d*Cos[a + b*x]]*Sqrt
[c*Sin[a + b*x]])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}\,dx}{3d^2}$$
$$= \frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}} + \frac{\left(2\sqrt{\sin(2a+2bx)}\right)\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx}{3d^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$
$$= \frac{2\sqrt{c\sin(a+bx)}}{3bcd(d\cos(a+bx))^{3/2}} + \frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2a+2bx)}}{3bd^2\sqrt{d\cos(a+bx)}\sqrt{c\sin(a+bx)}}$$

Mathematica [C] time = 0.10, size = 65, normalized size = 0.67

$$\frac{2\cos^2(a+bx)^{3/4}\sqrt{c\sin(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\sin^2(a+bx)\right)}{bcd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Cos[a + b*x])^(5/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/4, 7/4, 5/4, Sin[a + b*x]^2]*
Sqrt[c*Sin[a + b*x]])/(b*c*d*(d*Cos[a + b*x])^(3/2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{cd^3\cos(bx+a)^3\sin(bx+a)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(c*d^3*cos(b*x + a)^3*si
n(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\cos(bx+a))^{\frac{5}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a))), x)
```

maple [A] time = 0.13, size = 184, normalized size = 1.90

$$-\frac{\left(2\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sin(bx+\right.}{3b\left(-1+\cos(bx+a)\right)\left(d\cos(bx+a)\right)^{\frac{5}{2}}\sqrt{c\sin(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/3/b*(2*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)*cos(b*x+a)
-cos(b*x+a)*2^(1/2)+2^(1/2))*sin(b*x+a)*cos(b*x+a)/(-1+cos(b*x+a))/(d*cos(b
*x+a))^(5/2)/(c*sin(b*x+a))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(5/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*cos(b*x + a))^(5/2)*sqrt(c*sin(b*x + a))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{5/2}\,\sqrt{c\,\sin\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(1/2)),x)
```

```
[Out] int(1/((d*cos(a + b*x))^(5/2)*(c*sin(a + b*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))**(5/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.295 $\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=134

$$\frac{4\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{7bd^4\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}}+\frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}$$

```
[Out] 2/7*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(7/2)+4/7*(c*sin(b*x+a))^(1/2
)/b/c/d^3/(d*cos(b*x+a))^(3/2)-4/7*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi
+b*x)*EllipticF(cos(a+1/4*Pi+b*x),2^(1/2))*sin(2*b*x+2*a)^(1/2)/b/d^4/(d*co
s(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2571, 2573, 2641}

$$\frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}}+\frac{4\sqrt{\sin(2a+2bx)}\,F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{7bd^4\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(7*b*c*d*(d*Cos[a + b*x])^(7/2)) + (4*Sqrt[c*Sin[a
 + b*x]])/(7*b*c*d^3*(d*Cos[a + b*x])^(3/2)) + (4*EllipticF[a - Pi/4 + b*x,
 2]*Sqrt[Sin[2*a + 2*b*x]])/(7*b*d^4*Sqrt[d*Cos[a + b*x]]*Sqrt[c*Sin[a + b*
x]])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(d\cos(a+bx))^{9/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{6\int \frac{1}{(d\cos(a+bx))^{5/2}\sqrt{c\sin(a+bx)}}\,dx}{7d^2}$$

$$= \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}} + \frac{4\int \frac{1}{\sqrt{d\cos(a+bx)}\sqrt{c\sin}}}{7d^4}$$

$$= \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}} + \frac{\left(4\sqrt{\sin(2a+2bx)}\right)}{7d^4\sqrt{d\cos(a+bx)}}$$

$$= \frac{2\sqrt{c\sin(a+bx)}}{7bcd(d\cos(a+bx))^{7/2}} + \frac{4\sqrt{c\sin(a+bx)}}{7bcd^3(d\cos(a+bx))^{3/2}} + \frac{4F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{7bd^4\sqrt{d\cos(a+bx)}}$$

Mathematica [C] time = 0.12, size = 70, normalized size = 0.52

$$\frac{2\cos^3(a+bx)\cos^2(a+bx)^{3/4}\sqrt{c\sin(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{11}{4};\frac{5}{4};\sin^2(a+bx)\right)}{bc(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Cos[a + b*x])^(9/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Cos[a + b*x]^3*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[1/4, 11/4, 5/4,
Sin[a + b*x]^2]*Sqrt[c*Sin[a + b*x]])/(b*c*(d*Cos[a + b*x])^(9/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{cd^5\cos(bx+a)^5\sin(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(c*d^5*cos(b*x + a)^5*si
n(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a))), x)
```

maple [A] time = 0.16, size = 212, normalized size = 1.58

$$-\frac{\left(4\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{\sqrt{2}}{2}\right)\sin(bx+}{7b(-1+\cos(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/7/b*(4*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(
b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-
cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)*cos(b*x+a)
^3-2*cos(b*x+a)^3*2^(1/2)+2*cos(b*x+a)^2*2^(1/2)-cos(b*x+a)*2^(1/2)+2^(1/2)
)*sin(b*x+a)*cos(b*x+a)/(-1+cos(b*x+a))/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))
^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\cos(bx+a))^{\frac{9}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(9/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*cos(b*x + a))^(9/2)*sqrt(c*sin(b*x + a))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{(d\,\cos(a+b\,x))}^{9/2}\,\sqrt{c\,\sin(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(1/2)),x)
```

```
[Out] int(1/((d*cos(a + b*x))^(9/2)*(c*sin(a + b*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))**(9/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.296 $\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=280

$$\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}b\sqrt{c}}-\frac{\sqrt{d}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+b\right.}{2\sqrt{2}b\sqrt{c}}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/d^(1/2)/(c*sin(b*x+a))^
(1/2))*d^(1/2)/b*2^(1/2)/c^(1/2)-1/2*arctan(1+2^(1/2)*c^(1/2)*(d*cos(b*x+a)
)^(1/2)/d^(1/2)/(c*sin(b*x+a))^(1/2))*d^(1/2)/b*2^(1/2)/c^(1/2)-1/4*ln(d^(1
/2)+cot(b*x+a)*d^(1/2)-2^(1/2)*c^(1/2)*(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^
(1/2))*d^(1/2)/b*2^(1/2)/c^(1/2)+1/4*ln(d^(1/2)+cot(b*x+a)*d^(1/2)+2^(1/2)*
c^(1/2)*(d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2))*d^(1/2)/b*2^(1/2)/c^(1/2
)
```

Rubi [A] time = 0.18, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2575, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}+1\right)}{\sqrt{2}b\sqrt{c}}-\frac{\sqrt{d}\log\left(-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}+\sqrt{d}\cot(a+b\right.}{2\sqrt{2}b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (Sqrt[d]*ArcTan[1 - (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*
Sin[a + b*x]])])/(Sqrt[2]*b*Sqrt[c]) - (Sqrt[d]*ArcTan[1 + (Sqrt[2]*Sqrt[c]
*Sqrt[d*Cos[a + b*x]])/(Sqrt[d]*Sqrt[c*Sin[a + b*x]])])/(Sqrt[2]*b*Sqrt[c])
 - (Sqrt[d]*Log[Sqrt[d] + Sqrt[d]*Cot[a + b*x] - (Sqrt[2]*Sqrt[c]*Sqrt[d*Co
s[a + b*x]])/Sqrt[c*Sin[a + b*x]]])/(2*Sqrt[2]*b*Sqrt[c]) + (Sqrt[d]*Log[Sq
rt[d] + Sqrt[d]*Cot[a + b*x] + (Sqrt[2]*Sqrt[c]*Sqrt[d*Cos[a + b*x]])/Sqrt[
c*Sin[a + b*x]]])/(2*Sqrt[2]*b*Sqrt[c])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx = -\frac{(2cd)\,\text{Subst}\left(\int \frac{x^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{b}$$

$$= \frac{d\,\text{Subst}\left(\int \frac{d-cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{b} - \frac{d\,\text{Subst}\left(\int \frac{d+cx^2}{d^2+c^2x^4}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{b}$$

$$= -\frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}+2x}{-\frac{d}{c}-\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\,\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{c}}-2x}{-\frac{d}{c}+\frac{\sqrt{2}\sqrt{d}x}{\sqrt{c}}-x^2}\,dx, x, \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}}$$

$$= -\frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\right)}{2\sqrt{2}b\sqrt{c}} + \frac{\sqrt{d}\log\left(\sqrt{d}+\sqrt{d}\cot(a+bx)\right.}{2\sqrt{2}b\sqrt{c}}$$

$$= \frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{c}\sqrt{d\cos(a+bx)}}{\sqrt{d}\sqrt{c\sin(a+bx)}}\right)}{\sqrt{2}b\sqrt{c}} - \frac{\sqrt{d}\log\left(\sqrt{d}+\right.}{}$$

Mathematica [C] time = 0.06, size = 65, normalized size = 0.23

$$\frac{2\sqrt[4]{\cos^2(a+bx)}\tan(a+bx)\sqrt{d\cos(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\sin^2(a+bx)\right)}{b\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4,
5/4, Sin[a + b*x]^2]*Tan[a + b*x])/(b*Sqrt[c*Sin[a + b*x]])
```

fricas [B] time = 26.98, size = 1697, normalized size = 6.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*(d^2/(b^4*c^2))^(1/4)*arctan(1/2*(2*d^4*cos(b*x + a)*sin(b*x +
 a) + sqrt(4*b^2*c*d^3*sqrt(d^2/(b^4*c^2))*cos(b*x + a)*sin(b*x + a) + d^4
+ 2*(sqrt(2)*b^3*c*d^2*(d^2/(b^4*c^2))^(3/4)*cos(b*x + a) + sqrt(2)*b*d^3*(
d^2/(b^4*c^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)
```

```
))*(sqrt(2)*b^3*c*(d^2/(b^4*c^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*d*(d^2/(b^
4*c^2))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + (sq
rt(2)*b^3*c*d^2*(d^2/(b^4*c^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/(b^
4*c^2))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - 4*(
b^2*c*d^3*cos(b*x + a)^4 - b^2*c*d^3*cos(b*x + a)^2)*sqrt(d^2/(b^4*c^2)))/(
(2*d^4*cos(b*x + a)^3 - d^4*cos(b*x + a))*sin(b*x + a))) - 1/4*sqrt(2)*(d^2
/(b^4*c^2))^(1/4)*arctan(-1/2*(2*d^4*cos(b*x + a)*sin(b*x + a) - sqrt(4*b^2
*c*d^3*sqrt(d^2/(b^4*c^2))*cos(b*x + a)*sin(b*x + a) + d^4 - 2*(sqrt(2)*b^3
*c*d^2*(d^2/(b^4*c^2))^(3/4)*cos(b*x + a) + sqrt(2)*b*d^3*(d^2/(b^4*c^2))^(
1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(sqrt(2)*b^3*
c*(d^2/(b^4*c^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*d*(d^2/(b^4*c^2))^(1/4)*co
s(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - (sqrt(2)*b^3*c*d^2*
(d^2/(b^4*c^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*d^3*(d^2/(b^4*c^2))^(1/4)*co
s(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) - 4*(b^2*c*d^3*cos(b*
x + a)^4 - b^2*c*d^3*cos(b*x + a)^2)*sqrt(d^2/(b^4*c^2)))/((2*d^4*cos(b*x +
 a)^3 - d^4*cos(b*x + a))*sin(b*x + a))) - 1/4*sqrt(2)*(d^2/(b^4*c^2))^(1/4
)*arctan(-1/2*((sqrt(2)*b^3*c*d^2*(d^2/(b^4*c^2))^(3/4)*sin(b*x + a) - sqrt
(2)*b*d^3*(d^2/(b^4*c^2))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*s
in(b*x + a)) - sqrt(4*b^2*c*d^3*sqrt(d^2/(b^4*c^2))*cos(b*x + a)*sin(b*x +
a) + d^4 - 2*(sqrt(2)*b^3*c*d^2*(d^2/(b^4*c^2))^(3/4)*cos(b*x + a) + sqrt(2
)*b*d^3*(d^2/(b^4*c^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin
(b*x + a)))*(2*d^2*cos(b*x + a)*sin(b*x + a) + (sqrt(2)*b^3*c*(d^2/(b^4*c^2
))^(3/4)*sin(b*x + a) + sqrt(2)*b*d*(d^2/(b^4*c^2))^(1/4)*cos(b*x + a))*sqr
t(d*cos(b*x + a))*sqrt(c*sin(b*x + a))))/(d^4*cos(b*x + a)*sin(b*x + a))) -
 1/4*sqrt(2)*(d^2/(b^4*c^2))^(1/4)*arctan(-1/2*((sqrt(2)*b^3*c*d^2*(d^2/(b^
4*c^2))^(3/4)*sin(b*x + a) - sqrt(2)*b*d^3*(d^2/(b^4*c^2))^(1/4)*cos(b*x +
a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)) + sqrt(4*b^2*c*d^3*sqrt(d^2/(
b^4*c^2))*cos(b*x + a)*sin(b*x + a) + d^4 + 2*(sqrt(2)*b^3*c*d^2*(d^2/(b^4*
c^2))^(3/4)*cos(b*x + a) + sqrt(2)*b*d^3*(d^2/(b^4*c^2))^(1/4)*sin(b*x + a)
)*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a)))*(2*d^2*cos(b*x + a)*sin(b*x +
a) - (sqrt(2)*b^3*c*(d^2/(b^4*c^2))^(3/4)*sin(b*x + a) + sqrt(2)*b*d*(d^2/(
b^4*c^2))^(1/4)*cos(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))))/(
d^4*cos(b*x + a)*sin(b*x + a))) + 1/8*sqrt(2)*(d^2/(b^4*c^2))^(1/4)*log(4*b
^2*c*d^3*sqrt(d^2/(b^4*c^2))*cos(b*x + a)*sin(b*x + a) + d^4 + 2*(sqrt(2)*b
^3*c*d^2*(d^2/(b^4*c^2))^(3/4)*cos(b*x + a) + sqrt(2)*b*d^3*(d^2/(b^4*c^2))
^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))) - 1/8*sqrt(
2)*(d^2/(b^4*c^2))^(1/4)*log(4*b^2*c*d^3*sqrt(d^2/(b^4*c^2))*cos(b*x + a)*s
in(b*x + a) + d^4 - 2*(sqrt(2)*b^3*c*d^2*(d^2/(b^4*c^2))^(3/4)*cos(b*x + a)
 + sqrt(2)*b*d^3*(d^2/(b^4*c^2))^(1/4)*sin(b*x + a))*sqrt(d*cos(b*x + a))*s
qrt(c*sin(b*x + a)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\cos(bx+a)}}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))/sqrt(c*sin(b*x + a)), x)
```

maple [C] time = 0.11, size = 312, normalized size = 1.11

$$-\frac{\sqrt{d\cos\left(bx+a\right)}\,\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+}{\sin(bx+a)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] -1/2/b*(d*cos(b*x+a))^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((
-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1
/2)*(I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/
2*2^(1/2))-I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/
2*I,1/2*2^(1/2))+EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/
2-1/2*I,1/2*2^(1/2))+EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2
),1/2+1/2*I,1/2*2^(1/2))-2*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2),1/2*2^(1/2)))*sin(b*x+a)^2/(c*sin(b*x+a))^(1/2)/cos(b*x+a)/(-1+cos(b
*x+a))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\cos\left(bx+a\right)}}{\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))/sqrt(c*sin(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,\cos\left(a+b\,x\right)}}{\sqrt{c\,\sin\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(1/2)/(c*sin(a + b*x))^(1/2),x)
```

```
[Out] int((d*cos(a + b*x))^(1/2)/(c*sin(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\cos(a+bx)}}{\sqrt{c\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)/(c*sin(b*x+a))**(1/2),x)
[Out] Integral(sqrt(d*cos(a + b*x))/sqrt(c*sin(a + b*x)), x)
```

3.297 $\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=35

$$\frac{2\sqrt{c\sin(a+bx)}}{bcd\sqrt{d\cos(a+bx)}}$$

```
[Out] 2*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2563}

$$\frac{2\sqrt{c\sin(a+bx)}}{bcd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(b*c*d*Sqrt[d*Cos[a + b*x]])
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx=\frac{2\sqrt{c\sin(a+bx)}}{bcd\sqrt{d\cos(a+bx)}}$$

Mathematica [A] time = 0.06, size = 36, normalized size = 1.03

$$\frac{\sin(2(a+bx))}{b\sqrt{c\sin(a+bx)}\,(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] Sin[2*(a + b*x)]/(b*(d*Cos[a + b*x])^(3/2)*Sqrt[c*Sin[a + b*x]])
```

fricas [A] time = 0.51, size = 39, normalized size = 1.11

$$\frac{2\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{bcd^2\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2*sqrt(d*cos(b*x + a))*sqrt(c*sin(b*x + a))/(b*c*d^2*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\cos(bx+a))^{\frac{3}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a))), x)
```

maple [A] time = 0.10, size = 38, normalized size = 1.09

$$\frac{2\sin(bx+a)\cos(bx+a)}{b(d\cos(bx+a))^{\frac{3}{2}}\sqrt{c\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] 2/b*sin(b*x+a)*cos(b*x+a)/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\cos(bx+a))^{\frac{3}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(3/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*cos(b*x + a))^(3/2)*sqrt(c*sin(b*x + a))), x)
```

mupad [B] time = 0.81, size = 31, normalized size = 0.89

$$\frac{2\sqrt{c\,\sin\left(a+b\,x\right)}}{b\,c\,d\,\sqrt{d\,\cos\left(a+b\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^(1/2)),x)
[Out] (2*(c*sin(a + b*x))^(1/2))/(b*c*d*(d*cos(a + b*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\sin\left(a+bx\right)}\,\left(d\cos\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))**(3/2)/(c*sin(b*x+a))**(1/2),x)
[Out] Integral(1/(sqrt(c*sin(a + b*x))*(d*cos(a + b*x))**(3/2)), x)
```

3.298 $\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=75

$$\frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}$$

```
[Out] 2/5*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(5/2)+8/5*(c*sin(b*x+a))^(1/2
)/b/c/d^3/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2571, 2563}

$$\frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}+\frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Cos[a + b*x])^(7/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(5*b*c*d*(d*Cos[a + b*x])^(5/2)) + (8*Sqrt[c*Sin[a
 + b*x]])/(5*b*c*d^3*Sqrt[d*Cos[a + b*x]])
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{4\int \frac{1}{(d\cos(a+bx))^{3/2}\sqrt{c\sin(a+bx)}}\,dx}{5d^2}$$
$$= \frac{2\sqrt{c\sin(a+bx)}}{5bcd(d\cos(a+bx))^{5/2}} + \frac{8\sqrt{c\sin(a+bx)}}{5bcd^3\sqrt{d\cos(a+bx)}}$$

Mathematica [A] time = 0.15, size = 52, normalized size = 0.69

$$\frac{2(2\cos(2(a+bx))+3)\tan(a+bx)}{5bd^2\sqrt{c\sin(a+bx)}\,(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Cos[a + b*x])^(7/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*(3 + 2*Cos[2*(a + b*x)])*Tan[a + b*x])/(5*b*d^2*(d*Cos[a + b*x])^(3/2)*S
qrt[c*Sin[a + b*x]])
```

fricas [A] time = 0.50, size = 51, normalized size = 0.68

$$\frac{2\sqrt{d\cos(bx+a)}\left(4\cos(bx+a)^2+1\right)\sqrt{c\sin(bx+a)}}{5\,bcd^4\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas"
)
```

```
[Out] 2/5*sqrt(d*cos(b*x + a))*(4*cos(b*x + a)^2 + 1)*sqrt(c*sin(b*x + a))/(b*c*d
^4*cos(b*x + a)^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\cos(bx+a))^{\frac{7}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*cos(b*x + a))^(7/2)*sqrt(c*sin(b*x + a))), x)
```

maple [A] time = 0.11, size = 50, normalized size = 0.67

$$\frac{2\left(4\left(\cos^{2}(bx+a)\right)+1\right)\sin(bx+a)\cos(bx+a)}{5b\,(d\cos(bx+a))^{\frac{7}{2}}\sqrt{c\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] 2/5/b*(4*cos(b*x+a)^2+1)*sin(b*x+a)*cos(b*x+a)/(d*cos(b*x+a))^(7/2)/(c*sin(
b*x+a))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(d\cos(bx+a))^{\frac{7}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(7/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((d*cos(b*x + a))^(7/2)*sqrt(c*sin(b*x + a))), x)
```

mupad [B] time = 1.49, size = 77, normalized size = 1.03

$$\frac{8\sqrt{c\,\sin(a+b\,x)}\,\left(5\,\cos(2\,a+2\,b\,x)+\cos(4\,a+4\,b\,x)+4\right)}{5\,b\,c\,d^{3}\sqrt{d\,\cos(a+b\,x)}\,\left(4\,\cos(2\,a+2\,b\,x)+\cos(4\,a+4\,b\,x)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*cos(a + b*x))^(7/2)*(c*sin(a + b*x))^(1/2)),x)
```

```
[Out] (8*(c*sin(a + b*x))^(1/2)*(5*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 4))/(5*b
*c*d^3*(d*cos(a + b*x))^(1/2)*(4*cos(2*a + 2*b*x) + cos(4*a + 4*b*x) + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))**(7/2)/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

3.299 $\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=112

$$\frac{64\sqrt{c\sin(a+bx)}}{45bcd^5\sqrt{d\cos(a+bx)}}+\frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}$$

```
[Out] 2/9*(c*sin(b*x+a))^(1/2)/b/c/d/(d*cos(b*x+a))^(9/2)+16/45*(c*sin(b*x+a))^(1
/2)/b/c/d^3/(d*cos(b*x+a))^(5/2)+64/45*(c*sin(b*x+a))^(1/2)/b/c/d^5/(d*cos(
b*x+a))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2571, 2563}

$$\frac{64\sqrt{c\sin(a+bx)}}{45bcd^5\sqrt{d\cos(a+bx)}}+\frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}}+\frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*Cos[a + b*x])^(11/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[c*Sin[a + b*x]])/(9*b*c*d*(d*Cos[a + b*x])^(9/2)) + (16*Sqrt[c*Sin[
a + b*x]])/(45*b*c*d^3*(d*Cos[a + b*x])^(5/2)) + (64*Sqrt[c*Sin[a + b*x]])/
(45*b*c*d^5*Sqrt[d*Cos[a + b*x]])
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{1}{(d\cos(a+bx))^{11/2}\sqrt{c\sin(a+bx)}}\,dx = \frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}} + \frac{8\int \frac{1}{(d\cos(a+bx))^{7/2}\sqrt{c\sin(a+bx)}}\,dx}{9d^2}$$

$$= \frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}} + \frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}} + \frac{32\int \frac{1}{(d\cos(a+bx))^{3/2}}}{45d^4}$$

$$= \frac{2\sqrt{c\sin(a+bx)}}{9bcd(d\cos(a+bx))^{9/2}} + \frac{16\sqrt{c\sin(a+bx)}}{45bcd^3(d\cos(a+bx))^{5/2}} + \frac{64\sqrt{c\sin(a+b}}{45bcd^5\sqrt{d\cos(a-}}$$

Mathematica [A] time = 0.21, size = 67, normalized size = 0.60

$$\frac{2(20\cos(2(a+bx))+4\cos(4(a+bx))+21)\sec^5(a+bx)\sqrt{c\sin(a+bx)}\sqrt{d\cos(a+bx)}}{45bcd^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*Cos[a + b*x])^(11/2)*Sqrt[c*Sin[a + b*x]]),x]
```

```
[Out] (2*Sqrt[d*Cos[a + b*x]]*(21 + 20*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Sec
[a + b*x]^5*Sqrt[c*Sin[a + b*x]])/(45*b*c*d^6)
```

fricas [A] time = 0.60, size = 61, normalized size = 0.54

$$\frac{2\left(32\cos(bx+a)^4+8\cos(bx+a)^2+5\right)\sqrt{d\cos(bx+a)}\sqrt{c\sin(bx+a)}}{45\,bcd^6\cos(bx+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x, algorithm="fricas
")
```

```
[Out] 2/45*(32*cos(b*x + a)^4 + 8*cos(b*x + a)^2 + 5)*sqrt(d*cos(b*x + a))*sqrt(c
*sin(b*x + a))/(b*c*d^6*cos(b*x + a)^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d\cos(bx+a))^{\frac{11}{2}}\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

[Out] integrate(1/((d*cos(b*x + a))^(11/2)*sqrt(c*sin(b*x + a))), x)

maple [A] time = 0.13, size = 60, normalized size = 0.54

$$\frac{2\left(32\left(\cos^4\left(bx+a\right)\right)+8\left(\cos^2\left(bx+a\right)\right)+5\right)\sin\left(bx+a\right)\cos\left(bx+a\right)}{45b\left(d\cos\left(bx+a\right)\right)^{\frac{11}{2}}\sqrt{c\sin\left(bx+a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x)

[Out] 2/45/b*(32*cos(b*x+a)^4+8*cos(b*x+a)^2+5)*sin(b*x+a)*cos(b*x+a)/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(d\cos\left(bx+a\right)\right)^{\frac{11}{2}}\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*cos(b*x+a))^(11/2)/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")

[Out] integrate(1/((d*cos(b*x + a))^(11/2)*sqrt(c*sin(b*x + a))), x)

mupad [B] time = 3.83, size = 123, normalized size = 1.10

$$\frac{32\sqrt{c\ \sin\left(a+b\,x\right)}\ \left(162\ \cos\left(2\,a+2\,b\,x\right)+73\ \cos\left(4\,a+4\,b\,x\right)+18\ \cos\left(6\,a+6\,b\,x\right)+2\ \cos\left(8\,a+8\,b\,x\right)+105\right)}{45\,b\,c\,d^5\,\sqrt{d\ \cos\left(a+b\,x\right)}\ \left(56\ \cos\left(2\,a+2\,b\,x\right)+28\ \cos\left(4\,a+4\,b\,x\right)+8\ \cos\left(6\,a+6\,b\,x\right)+\cos\left(8\,a+8\,b\,x\right)+35\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d*cos(a + b*x))^(11/2)*(c*sin(a + b*x))^(1/2)),x)

[Out] (32*(c*sin(a + b*x))^(1/2)*(162*cos(2*a + 2*b*x) + 73*cos(4*a + 4*b*x) + 18*cos(6*a + 6*b*x) + 2*cos(8*a + 8*b*x) + 105))/(45*b*c*d^5*(d*cos(a + b*x))^(1/2)*(56*cos(2*a + 2*b*x) + 28*cos(4*a + 4*b*x) + 8*cos(6*a + 6*b*x) + cos(8*a + 8*b*x) + 35))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*cos(b*x+a))**(11/2)/(c*sin(b*x+a))**(1/2),x)

[Out] Timed out

3.300 $\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx$

Optimal. Leaf size=174

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}b}-\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}+\frac{\log\left(\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}$$

```
[Out] -1/2*arctan(-1+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))/b*2^(1/2)-1/2*arc
tan(1+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))/b*2^(1/2)-1/4*ln(1+cot(b*x
+a)-2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))/b*2^(1/2)+1/4*ln(1+cot(b*x+a
)+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2))/b*2^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2575, 297, 1162, 617, 204, 1165, 628}

$$\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}b}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}b}-\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}+\frac{\log\left(\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[a + b*x]]/Sqrt[Sin[a + b*x]],x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - A
rcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) - Lo
g[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(2*Sq
rt[2]*b) + Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a +
 b*x]]]/(2*Sqrt[2]*b)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}\,b} + \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}\,b} - \frac{\text{Subst}\left(\right.}{}$$
$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}\,b} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}\,b} - \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right.}{2\sqrt{2}\,b}$$

Mathematica [C] time = 0.02, size = 55, normalized size = 0.32

$$\frac{2\sqrt{\sin(a+bx)}\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\sin^2(a+bx)\right)}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[a + b*x]]/Sqrt[Sin[a + b*x]],x]
```

```
[Out] (2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4, Sin[a + b*x]^2]*
Sqrt[Sin[a + b*x]])/(b*Sqrt[Cos[a + b*x]])
```

fricas [B] time = 25.91, size = 1185, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*(b^(-4))^(1/4)*arctan(1/2*((sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x
 + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(4*b^2*sqrt(b^(-4))*cos(
b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2
)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)
*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + (sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*
x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin
```

```
(b*x + a)) + 2*cos(b*x + a)*sin(b*x + a) - 4*(b^2*cos(b*x + a)^4 - b^2*cos(
b*x + a)^2)*sqrt(b^(-4)))/((2*cos(b*x + a)^3 - cos(b*x + a))*sin(b*x + a)))
 - 1/4*sqrt(2)*(b^(-4))^(1/4)*arctan(1/2*((sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b
*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(4*b^2*sqrt(b^(-4))*co
s(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt
(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) +
1)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + (sqrt(2)*b^3*(b^(-4))^(3/4)*sin(
b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(s
in(b*x + a)) - 2*cos(b*x + a)*sin(b*x + a) + 4*(b^2*cos(b*x + a)^4 - b^2*co
s(b*x + a)^2)*sqrt(b^(-4)))/((2*cos(b*x + a)^3 - cos(b*x + a))*sin(b*x + a)
)) - 1/4*sqrt(2)*(b^(-4))^(1/4)*arctan(-1/2*((sqrt(2)*b^3*(b^(-4))^(3/4)*si
n(b*x + a) - sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt
(sin(b*x + a)) - sqrt(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) - 2*(sqr
t(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a
))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)*((sqrt(2)*b^3*(b^(-4))^(3/4)*
sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sq
rt(sin(b*x + a)) + 2*cos(b*x + a)*sin(b*x + a)))/(cos(b*x + a)*sin(b*x + a)
)) - 1/4*sqrt(2)*(b^(-4))^(1/4)*arctan(-1/2*((sqrt(2)*b^3*(b^(-4))^(3/4)*si
n(b*x + a) - sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt
(sin(b*x + a)) - sqrt(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) + 2*(sqr
t(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a
))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)*((sqrt(2)*b^3*(b^(-4))^(3/4)*
sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sq
rt(sin(b*x + a)) - 2*cos(b*x + a)*sin(b*x + a)))/(cos(b*x + a)*sin(b*x + a)
)) + 1/8*sqrt(2)*(b^(-4))^(1/4)*log(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x
 + a) + 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/
4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1) - 1/8*sqrt(2)*(
b^(-4))^(1/4)*log(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)
*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*s
qrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(bx+a)}}{\sqrt{\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(b*x + a))/sqrt(sin(b*x + a)), x)
```

maple [C] time = 0.12, size = 292, normalized size = 1.68

$$-\frac{\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}}\,\sqrt{\frac{-1+\cos(bx+a)}{\sin(bx+a)}}\,\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(bx+a)+\sin(bx+a)}{\sin(bx+a)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x)
```

```
[Out] -1/2/b/cos(b*x+a)^(1/2)*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
(I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^
(1/2))-I*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,
1/2*2^(1/2))+EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/
2*I,1/2*2^(1/2))+EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/
2+1/2*I,1/2*2^(1/2))-2*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2),1/2*2^(1/2)))*sin(b*x+a)^(3/2)/(-1+cos(b*x+a))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(bx+a)}}{\sqrt{\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2)/sin(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(b*x + a))/sqrt(sin(b*x + a)), x)
```

mupad [B] time = 1.60, size = 44, normalized size = 0.25

$$-\frac{2\cos(a+b\,x)^{3/2}\,\sqrt{\sin(a+b\,x)}\;{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{3}{4};\ \frac{7}{4};\ \cos(a+b\,x)^2\right)}{3\,b\left(\sin(a+b\,x)^2\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(1/2)/sin(a + b*x)^(1/2),x)
```

```
[Out] -(2*cos(a + b*x)^(3/2)*sin(a + b*x)^(1/2)*hypergeom([3/4, 3/4], 7/4, cos(a
+ b*x)^2))/(3*b*(sin(a + b*x)^2)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(1/2)/sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(cos(a + b*x))/sqrt(sin(a + b*x)), x)
```

3.301 $\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=199

$$-\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}\,b}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}\,b}-\frac{\log\left(\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{2\sqrt{2}\,b}+\text{lo}$$

```
[Out] 1/2*arctan(1-2^(1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2))/b*2^(1/2)-1/2*arcta
n(1+2^(1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2))/b*2^(1/2)-1/4*ln(1-2^(1/2)*s
in(b*x+a)^(1/2)/cos(b*x+a)^(1/2)+tan(b*x+a))/b*2^(1/2)+1/4*ln(1+2^(1/2)*sin
(b*x+a)^(1/2)/cos(b*x+a)^(1/2)+tan(b*x+a))/b*2^(1/2)-2*cos(b*x+a)^(1/2)/b/s
in(b*x+a)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2567, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}+\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}\,b}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}\,b}-\frac{\log\left(\tan(a+bx)-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{2\sqrt{2}\,b}+\text{lo}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(3/2)/Sin[a + b*x]^(3/2),x]
```

```
[Out] ArcTan[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b) - A
rcTan[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b) - Lo
g[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a + b*x]]/(2*Sq
rt[2]*b) + Log[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a
+ b*x]]/(2*Sqrt[2]*b) - (2*Sqrt[Cos[a + b*x]])/(b*Sqrt[Sin[a + b*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
```

```
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \int \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\,dx$$

$$= -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt{\sin(a+}}{\sqrt{\cos(a-}}\right.}{2b}$$

$$= -\frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}\,b} + \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}\,b} - \frac{2\sqrt{\cos(a}}{b\sqrt{\sin(a}}$$

$$= \frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}\,b} - \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}\,b} - \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right.}{2\sqrt{2}\,b}$$

Mathematica [**C**] time = 0.03, size = 55, normalized size = 0.28

$$-\frac{2\cos^2(a+bx)^{3/4}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};\sin^2(a+bx)\right)}{b\sqrt{\sin(a+bx)}\,\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(3/2)/Sin[a + b*x]^(3/2),x]
```

```
[Out] (-2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, Sin[a + b*x]^
2])/(b*Cos[a + b*x]^(3/2)*Sqrt[Sin[a + b*x]])
```

fricas [**B**] time = 50.21, size = 1545, normalized size = 7.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

[Out] 1/16*(4*(sqrt(2)*b*cos(b*x + a)^2 - sqrt(2)*b)*(b^(-4))^(1/4)*arctan(-((sqr
t(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a
))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - sqrt(4*b^2*sqrt(b^(-4))*cos(b*x
+ a)*sin(b*x + a) - 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*
(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)*(b^
2*sqrt(b^(-4)) + (sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-
4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 2*cos(b*x +
a)*sin(b*x + a)))/(2*cos(b*x + a)^2 - 1)) + 4*(sqrt(2)*b*cos(b*x + a)^2 -
sqrt(2)*b)*(b^(-4))^(1/4)*arctan(-((sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a)
+ sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x +
a)) + sqrt(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*(
b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(co
s(b*x + a))*sqrt(sin(b*x + a)) + 1)*(b^2*sqrt(b^(-4)) - (sqrt(2)*b^3*(b^(-4
))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x
+ a))*sqrt(sin(b*x + a)) + 2*cos(b*x + a)*sin(b*x + a)))/(2*cos(b*x + a)^2
- 1)) - 4*(sqrt(2)*b*cos(b*x + a)^2 - sqrt(2)*b)*(b^(-4))^(1/4)*arctan(1/2
*((sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(b
*x + a))*sqrt(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3
*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(
cos(b*x + a))*sqrt(sin(b*x + a)) + 1)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a))
- (sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(
b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 2*cos(b*x + a)*sin(b*x +
a) + 4*(b^2*cos(b*x + a)^4 - b^2*cos(b*x + a)^2)*sqrt(b^(-4)))/((2*cos(b*x
+ a)^3 - cos(b*x + a))*sin(b*x + a))) - 4*(sqrt(2)*b*cos(b*x + a)^2 - sqrt(
2)*b)*(b^(-4))^(1/4)*arctan(1/2*((sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) +
sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(4*b^2*sqrt(b^(-4))*cos(b*x + a
)*sin(b*x + a) - 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^
(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)*sqrt(c
os(b*x + a))*sqrt(sin(b*x + a)) - (sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a)
+ sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x +
a)) + 2*cos(b*x + a)*sin(b*x + a) - 4*(b^2*cos(b*x + a)^4 - b^2*cos(b*x + a
)^2)*sqrt(b^(-4)))/((2*cos(b*x + a)^3 - cos(b*x + a))*sin(b*x + a))) + (sqr
t(2)*b*cos(b*x + a)^2 - sqrt(2)*b)*(b^(-4))^(1/4)*log(4*b^2*sqrt(b^(-4))*co
s(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt
(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) +
1) - (sqrt(2)*b*cos(b*x + a)^2 - sqrt(2)*b)*(b^(-4))^(1/4)*log(4*b^2*sqrt(b
^(-4))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x +
a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x
+ a)) + 1) + (sqrt(2)*b*cos(b*x + a)^2 - sqrt(2)*b)*(b^(-4))^(1/4)*log(1/4
*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) + 1/8*(sqrt(2)*b^3*(b^(-4))^(3/
4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))
*sqrt(sin(b*x + a)) + 1/16) - (sqrt(2)*b*cos(b*x + a)^2 - sqrt(2)*b)*(b^(-4
))^(1/4)*log(1/4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) - 1/8*(sqrt(2)*
b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sq
rt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1/16) + 32*sqrt(cos(b*x + a))*sin(b*x

```
+ a)^(3/2))/(b*cos(b*x + a)^2 - b)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 0.11, size = 937, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x)
[Out] -1/2/b*(I*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/si
n(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2
-1/2*I,1/2*2^(1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)
-I*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a
))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,
1/2*2^(1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*cos(b*x+a)+I*((-1
+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2
)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(
1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)-I*((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a
))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,
1/2*2^(1/2))*cos(b*x+a)-((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*
EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/
2))*cos(b*x+a)-((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticP
i(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin
(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*cos(b*x+a)*2^(1/2))/sin
(b*x+a)^(1/2)/cos(b*x+a)^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)^{\frac{3}{2}}}{\sin\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2)/sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(3/2)/sin(b*x + a)^(3/2), x)
```

mupad [B] time = 1.62, size = 44, normalized size = 0.22

$$-\frac{2\cos\left(a+b\,x\right)^{5/2}\,{\left({\sin\left(a+b\,x\right)}^2\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{5}{4},\frac{5}{4};\ \frac{9}{4};\ {\cos\left(a+b\,x\right)}^2\right)}{5\,b\,\sqrt{\sin\left(a+b\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(3/2)/sin(a + b*x)^(3/2),x)
```

```
[Out] -(2*cos(a + b*x)^(5/2)*(sin(a + b*x)^2)^(1/4)*hypergeom([5/4, 5/4], 9/4, co
s(a + b*x)^2))/(5*b*sin(a + b*x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{3}{2}}\left(a+bx\right)}{\sin^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(3/2)/sin(b*x+a)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**(3/2)/sin(a + b*x)**(3/2), x)
```

3.302 $\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=201

$$-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}\,b}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}\,b}+\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}\,b}-\frac{\log}{}$$

```
[Out] -2/3*cos(b*x+a)^(3/2)/b/sin(b*x+a)^(3/2)+1/2*arctan(-1+2^(1/2)*cos(b*x+a)^(
1/2)/sin(b*x+a)^(1/2))/b*2^(1/2)+1/2*arctan(1+2^(1/2)*cos(b*x+a)^(1/2)/sin(
b*x+a)^(1/2))/b*2^(1/2)+1/4*ln(1+cot(b*x+a)-2^(1/2)*cos(b*x+a)^(1/2)/sin(b*
x+a)^(1/2))/b*2^(1/2)-1/4*ln(1+cot(b*x+a)+2^(1/2)*cos(b*x+a)^(1/2)/sin(b*x+
a)^(1/2))/b*2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2567, 2575, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}\,b}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{\sqrt{2}\,b}+\frac{\log\left(\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}+1\right)}{2\sqrt{2}\,b}-\frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(5/2)/Sin[a + b*x]^(5/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b))
+ ArcTan[1 + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(Sqrt[2]*b) +
 Log[1 + Cot[a + b*x] - (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[a + b*x]]]/(2
*Sqrt[2]*b) - Log[1 + Cot[a + b*x] + (Sqrt[2]*Sqrt[Cos[a + b*x]])/Sqrt[Sin[
a + b*x]]]/(2*Sqrt[2]*b) - (2*Cos[a + b*x]^(3/2))/(3*b*Sin[a + b*x]^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
```

```
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} - \int \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\,dx$$

$$= -\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}$$

$$= -\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{b}$$

$$= -\frac{2\cos^{\frac{3}{2}}(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \frac{\sqrt{\cos(a}{\sqrt{\sin(a}}\right.}{2b}$$

$$= \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}\,b} - \frac{\log\left(1+\cot(a+bx)+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{2\sqrt{2}\,b} - \frac{2\cos^{\frac{3}{2}}(a+}{3b\sin^{\frac{3}{2}}(a+}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}\,b} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\cos(a+bx)}}{\sqrt{\sin(a+bx)}}\right)}{\sqrt{2}\,b} + \frac{\log\left(1+\cot(a+bx)-\frac{\sqrt{2}\sqrt{\cos(a+b}}{\sqrt{\sin(a+bx}}\right.}{2\sqrt{2}\,b}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.28

$$-\frac{2\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(-\frac{3}{4},-\frac{3}{4};\frac{1}{4};\sin^2(a+bx)\right)}{3b\sin^{\frac{3}{2}}(a+bx)\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(5/2)/Sin[a + b*x]^(5/2),x]
```

```
[Out] (-2*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[-3/4, -3/4, 1/4, Sin[a + b*x]^
2])/(3*b*Sqrt[Cos[a + b*x]]*Sin[a + b*x]^(3/2))
```

fricas [B] time = 25.85, size = 1321, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*(sqrt(2)*b*cos(b*x + a)^2 - sqrt(2)*b)*(b^(-4))^(1/4)*arctan(1/2*((
sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x
+ a))*sqrt(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*(b
^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(cos
(b*x + a))*sqrt(sin(b*x + a)) + 1)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) +
(sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x
 + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 2*cos(b*x + a)*sin(b*x + a)
- 4*(b^2*cos(b*x + a)^4 - b^2*cos(b*x + a)^2)*sqrt(b^(-4)))/((2*cos(b*x + a
)^3 - cos(b*x + a))*sin(b*x + a))) + 6*(sqrt(2)*b*cos(b*x + a)^2 - sqrt(2)*
b)*(b^(-4))^(1/4)*arctan(1/2*((sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sq
rt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(4*b^2*sqrt(b^(-4))*cos(b*x + a)*s
in(b*x + a) - 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4
))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)*sqrt(cos(
b*x + a))*sqrt(sin(b*x + a)) + (sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + s
qrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a))
 - 2*cos(b*x + a)*sin(b*x + a) + 4*(b^2*cos(b*x + a)^4 - b^2*cos(b*x + a)^2
)*sqrt(b^(-4)))/((2*cos(b*x + a)^3 - cos(b*x + a))*sin(b*x + a))) - 6*(sqrt
(2)*b*cos(b*x + a)^2 - sqrt(2)*b)*(b^(-4))^(1/4)*arctan(1/2*((sqrt(2)*b^3*(
b^(-4))^(3/4)*sin(b*x + a) - sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(co
s(b*x + a))*sqrt(sin(b*x + a)) + sqrt(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b
*x + a) - 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(
1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)*((sqrt(2)*b^3
*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(
cos(b*x + a))*sqrt(sin(b*x + a)) + 2*cos(b*x + a)*sin(b*x + a)))/(cos(b*x +
 a)*sin(b*x + a))) - 6*(sqrt(2)*b*cos(b*x + a)^2 - sqrt(2)*b)*(b^(-4))^(1/4
)*arctan(1/2*((sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) - sqrt(2)*b*(b^(-4))
^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + sqrt(4*b^2*sqr
t(b^(-4))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x
 + a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(
b*x + a)) + 1)*((sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4
))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 2*cos(b*x +
a)*sin(b*x + a)))/(cos(b*x + a)*sin(b*x + a))) - 3*(sqrt(2)*b*cos(b*x + a)^
2 - sqrt(2)*b)*(b^(-4))^(1/4)*log(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x +
 a) + 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)
*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1) + 3*(sqrt(2)*b*co
s(b*x + a)^2 - sqrt(2)*b)*(b^(-4))^(1/4)*log(4*b^2*sqrt(b^(-4))*cos(b*x + a
)*sin(b*x + a) - 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^
(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1) + 16*c
os(b*x + a)^(3/2)*sqrt(sin(b*x + a)))/(b*cos(b*x + a)^2 - b)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [C] time = 0.12, size = 1261, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x)
[Out] -4/3/b*cos(b*x+a)^(5/2)*(-1+cos(b*x+a))^3*(3*I*((1-cos(b*x+a)+sin(b*x+a))/s
in(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x
+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)*cos(b*x+a)-3*I*((1-cos(b*x+a)+sin(b*x
+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+c
os(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+
a))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)*cos(b*x+a)+3*I*((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*
((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)-3*I*sin(b*x+a)*((1-cos(b*
x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*((1-cos(b*x+a)+sin(b*x+a))/s
in(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x
+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2),1/2-1/2*I,1/2*2^(1/2))*sin(b*x+a)*cos(b*x+a)+3*((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos
(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(b*x+a)*cos(b*x+a)-6*((1-cos(b*x+a)+sin(b
*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1
+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x
+a))^(1/2),1/2*2^(1/2))*sin(b*x+a)*cos(b*x+a)+3*sin(b*x+a)*((1-cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*
((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))+3*sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+
a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+co
s(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*sin(b*x+a)*((1-cos(b*x+a)+sin(b*x+a))/sin
```

```
(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a
))/sin(b*x+a))^(1/2)*EllipticF(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2*2^(1/2))+2*cos(b*x+a)^2*2^(1/2))/sin(b*x+a)^(3/2)/(-1+cos(b*x+a)+sin(b
*x+a))^3/(-1+cos(b*x+a)-sin(b*x+a))^3*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)^{\frac{5}{2}}}{\sin\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/2)/sin(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(5/2)/sin(b*x + a)^(5/2), x)
```

mupad [B] time = 1.90, size = 44, normalized size = 0.22

$$-\frac{2\cos\left(a+b\,x\right)^{7/2}\,{\left({\sin\left(a+b\,x\right)}^2\right)}^{3/4}\,{}_2\mathrm{F}_1\left(\frac{7}{4},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(a+b\,x\right)}^2\right)}{7\,b\,{\sin\left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(5/2)/sin(a + b*x)^(5/2),x)
```

```
[Out] -(2*cos(a + b*x)^(7/2)*(sin(a + b*x)^2)^(3/4)*hypergeom([7/4, 7/4], 11/4, c
os(a + b*x)^2))/(7*b*sin(a + b*x)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(5/2)/sin(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

3.303 $\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=226

$$-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}\,b}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}\,b}+\frac{\log\left(\tan(a+bx)-\frac{\sqrt{2}}{\sqrt{c}}\right.}{2\sqrt{2}\,b}$$

```
[Out] -2/5*cos(b*x+a)^(5/2)/b/sin(b*x+a)^(5/2)-1/2*arctan(1-2^(1/2)*sin(b*x+a)^(1
/2)/cos(b*x+a)^(1/2))/b*2^(1/2)+1/2*arctan(1+2^(1/2)*sin(b*x+a)^(1/2)/cos(b
*x+a)^(1/2))/b*2^(1/2)+1/4*ln(1-2^(1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2)+t
an(b*x+a))/b*2^(1/2)-1/4*ln(1+2^(1/2)*sin(b*x+a)^(1/2)/cos(b*x+a)^(1/2)+tan
(b*x+a))/b*2^(1/2)+2*cos(b*x+a)^(1/2)/b/sin(b*x+a)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2567, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}+\frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}\,b}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+1\right)}{\sqrt{2}\,b}+\frac{\log\left(\tan(a+bx)-\frac{\sqrt{2}}{\sqrt{c}}\right.}{2\sqrt{2}\,b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(7/2)/Sin[a + b*x]^(7/2),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b))
+ ArcTan[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]]]/(Sqrt[2]*b) +
 Log[1 - (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan[a + b*x]]/(2
*Sqrt[2]*b) - Log[1 + (Sqrt[2]*Sqrt[Sin[a + b*x]])/Sqrt[Cos[a + b*x]] + Tan
[a + b*x]]/(2*Sqrt[2]*b) - (2*Cos[a + b*x]^(5/2))/(5*b*Sin[a + b*x]^(5/2))
+ (2*Sqrt[Cos[a + b*x]])/(b*Sqrt[Sin[a + b*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
```

```
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
```

```
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} - \int \frac{\cos^{\frac{3}{2}}(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$$

$$= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \int \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\,dx$$

$$= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \frac{2\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} - \frac{\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{b} + \frac{\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,\right.}{b}$$

$$= -\frac{2\cos^{\frac{5}{2}}(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\sqrt{\cos(a+bx)}}{b\sqrt{\sin(a+bx)}} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{2b} + \frac{\mathrm{Subst}\left(\int \frac{}{1+}\right.}{}$$

$$= \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}\,b} - \frac{\log\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+bx)\right)}{2\sqrt{2}\,b} - \frac{2\cos^{\frac{5}{2}}(a+}{5b\sin^{\frac{5}{2}}(a+}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}\,b} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}\right)}{\sqrt{2}\,b} + \frac{\log\left(1-\frac{\sqrt{2}\sqrt{\sin(a+bx)}}{\sqrt{\cos(a+bx)}}+\tan(a+b\right.}{2\sqrt{2}\,b}$$

Mathematica [C] time = 0.04, size = 57, normalized size = 0.25

$$-\frac{2\cos^2(a+bx)^{3/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{5}{4};-\frac{1}{4};\sin^2(a+bx)\right)}{5b\sin^{\frac{5}{2}}(a+bx)\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(7/2)/Sin[a + b*x]^(7/2),x]
```

```
[Out] (-2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, Sin[a + b*x]
^2])/(5*b*Cos[a + b*x]^(3/2)*Sin[a + b*x]^(5/2))
```

fricas [B] time = 48.48, size = 1670, normalized size = 7.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/80*(32*(6*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a))*sin(b*x + a)^(3/2) - 20
*(sqrt(2)*b*cos(b*x + a)^4 - 2*sqrt(2)*b*cos(b*x + a)^2 + sqrt(2)*b)*(b^(-4
))^(1/4)*arctan(((sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-
4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + sqrt(4*b^2*
sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*sin(
b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(s
in(b*x + a)) + 1)*(b^2*sqrt(b^(-4)) + (sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x +
 a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*
x + a)) + 2*cos(b*x + a)*sin(b*x + a)))/(2*cos(b*x + a)^2 - 1)) - 20*(sqrt(
2)*b*cos(b*x + a)^4 - 2*sqrt(2)*b*cos(b*x + a)^2 + sqrt(2)*b)*(b^(-4))^(1/4
)*arctan(((sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/
4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - sqrt(4*b^2*sqrt(b^
(-4))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a
) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x
+ a)) + 1)*(b^2*sqrt(b^(-4)) - (sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + s
qrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a))
 + 2*cos(b*x + a)*sin(b*x + a)))/(2*cos(b*x + a)^2 - 1)) - 20*(sqrt(2)*b*co
s(b*x + a)^4 - 2*sqrt(2)*b*cos(b*x + a)^2 + sqrt(2)*b)*(b^(-4))^(1/4)*arcta
n(1/2*((sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*
sin(b*x + a))*sqrt(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) + 2*(sqrt(2
)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*
sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1)*sqrt(cos(b*x + a))*sqrt(sin(b*x
+ a)) - (sqrt(2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)
*sin(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - 2*cos(b*x + a)*sin(b
*x + a) + 4*(b^2*cos(b*x + a)^4 - b^2*cos(b*x + a)^2)*sqrt(b^(-4)))/((2*cos
(b*x + a)^3 - cos(b*x + a))*sin(b*x + a))) - 20*(sqrt(2)*b*cos(b*x + a)^4 -
 2*sqrt(2)*b*cos(b*x + a)^2 + sqrt(2)*b)*(b^(-4))^(1/4)*arctan(1/2*((sqrt(2
)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))*
sqrt(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*(b^(-4))
^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x +
 a))*sqrt(sin(b*x + a)) + 1)*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) - (sqrt(
2)*b^3*(b^(-4))^(3/4)*cos(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*sin(b*x + a))
*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 2*cos(b*x + a)*sin(b*x + a) - 4*(b
^2*cos(b*x + a)^4 - b^2*cos(b*x + a)^2)*sqrt(b^(-4)))/((2*cos(b*x + a)^3 -
cos(b*x + a))*sin(b*x + a))) + 5*(sqrt(2)*b*cos(b*x + a)^4 - 2*sqrt(2)*b*co
s(b*x + a)^2 + sqrt(2)*b)*(b^(-4))^(1/4)*log(4*b^2*sqrt(b^(-4))*cos(b*x + a
)*sin(b*x + a) + 2*(sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^
```

```
(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1) - 5*(s
qrt(2)*b*cos(b*x + a)^4 - 2*sqrt(2)*b*cos(b*x + a)^2 + sqrt(2)*b)*(b^(-4))^
(1/4)*log(4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) - 2*(sqrt(2)*b^3*(b^
(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(
b*x + a))*sqrt(sin(b*x + a)) + 1) + 5*(sqrt(2)*b*cos(b*x + a)^4 - 2*sqrt(2)
*b*cos(b*x + a)^2 + sqrt(2)*b)*(b^(-4))^(1/4)*log(1/4*b^2*sqrt(b^(-4))*cos(
b*x + a)*sin(b*x + a) + 1/8*(sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt
(2)*b*(b^(-4))^(1/4)*cos(b*x + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) +
1/16) - 5*(sqrt(2)*b*cos(b*x + a)^4 - 2*sqrt(2)*b*cos(b*x + a)^2 + sqrt(2)*
b)*(b^(-4))^(1/4)*log(1/4*b^2*sqrt(b^(-4))*cos(b*x + a)*sin(b*x + a) - 1/8*
(sqrt(2)*b^3*(b^(-4))^(3/4)*sin(b*x + a) + sqrt(2)*b*(b^(-4))^(1/4)*cos(b*x
 + a))*sqrt(cos(b*x + a))*sqrt(sin(b*x + a)) + 1/16))/(b*cos(b*x + a)^4 - 2
*b*cos(b*x + a)^2 + b)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 0.14, size = 1934, normalized size = 8.56

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x)
```

```
[Out] -8/5/b*cos(b*x+a)^(7/2)*(-1+cos(b*x+a))^4*(5*I*((1-cos(b*x+a)+sin(b*x+a))/s
in(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x
+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1
/2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)^3+5*I*((1-cos(b*x+a)+sin(b*x+a))/sin(
b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)
)/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)^2-5*I*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+si
n(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2)*cos(b*x+a)+5*I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^
(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*
x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2
*I,1/2*2^(1/2))*cos(b*x+a)-5*cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*
x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/
```

```
sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1
/2-1/2*I,1/2*2^(1/2))-5*cos(b*x+a)^3*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))
^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b
*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/
2*I,1/2*2^(1/2))-5*I*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*Ell
ipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))
*cos(b*x+a)^3-5*I*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*
x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(
1/2),1/2-1/2*I,1/2*2^(1/2))*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)-5*
cos(b*x+a)^2*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+s
in(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(
((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*cos(b
*x+a)^2*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-c
os(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+5*I*((1-cos(
b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*
x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))-5*I*((1-cos(b*x+a)+sin(b*x+a
))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos
(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(b*x+a)^2+5*((1-cos(b*x+a)+sin(b*x+a))/si
n(b*x+a))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+
a))/sin(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/
2),1/2-1/2*I,1/2*2^(1/2))*cos(b*x+a)+5*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a
))^(1/2)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin
(b*x+a))^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+
1/2*I,1/2*2^(1/2))*cos(b*x+a)+5*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2
)*((-1+cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a)
)^(1/2)*EllipticPi(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2-1/2*I,1
/2*2^(1/2))+5*((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a)+
sin(b*x+a))/sin(b*x+a))^(1/2)*((-1+cos(b*x+a))/sin(b*x+a))^(1/2)*EllipticPi
(((1-cos(b*x+a)+sin(b*x+a))/sin(b*x+a))^(1/2),1/2+1/2*I,1/2*2^(1/2))+12*cos
(b*x+a)^3*2^(1/2)-10*cos(b*x+a)*2^(1/2))/sin(b*x+a)^(5/2)/(-1+cos(b*x+a)+si
n(b*x+a))^4/(-1+cos(b*x+a)-sin(b*x+a))^4*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^{\frac{7}{2}}}{\sin(bx+a)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/2)/sin(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(7/2)/sin(b*x + a)^(7/2), x)
```

mupad [B] time = 1.88, size = 44, normalized size = 0.19

$$-\frac{2\cos\left(a+b\,x\right)^{9/2}\,{\left({\sin\left(a+b\,x\right)}^2\right)}^{5/4}\,{}_2\mathrm{F}_1\left(\frac{9}{4},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(a+b\,x\right)}^2\right)}{9\,b\,{\sin\left(a+b\,x\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(7/2)/sin(a + b*x)^(7/2),x)
```

```
[Out] -(2*cos(a + b*x)^(9/2)*(sin(a + b*x)^2)^(5/4)*hypergeom([9/4, 9/4], 13/4, c
os(a + b*x)^2))/(9*b*sin(a + b*x)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(7/2)/sin(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.304 $\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/4*cos(f*x+e)*hypergeom([-3/2, 2/3],[5/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(4/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\sqrt[3]{b\sin(e+fx)}\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, 2/3, 5/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\cos\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*cos(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^4, x)
```

maple [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \left(\cos^{4}\left(fx+e\right)\right)\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos \left(e + f\,x\right)^4\,{\left(b\,\sin \left(e + f\,x\right)\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(b*sin(e + f*x))^(1/3),x)
```

```
[Out] int(cos(e + f*x)^4*(b*sin(e + f*x))^(1/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

3.305 $\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/4*cos(f*x+e)*hypergeom([-1/2, 2/3],[5/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(4/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\sqrt[3]{b\sin(e+fx)}\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, 2/3, 5/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*cos(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^2, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(b*sin(e + f*x))^(1/3),x)
[Out] int(cos(e + f*x)^2*(b*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b \sin{\left(e + f x \right)}} \cos^{2}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(b*sin(f*x+e))**(1/3),x)
[Out] Integral((b*sin(e + f*x))**(1/3)*cos(e + f*x)**2, x)
```

3.306 $\int \sqrt[3]{b\sin(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/4*cos(f*x+e)*hypergeom([1/2, 2/3],[5/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(4/3
)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b*Sin[e +
 f*x])^(4/3))/(4*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{b\sin(e+fx)}\,dx = \frac{3\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{4/3}}{4bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\sqrt[3]{b\sin(e+fx)}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sin^2(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(1/3),x)
```

```
[Out] int((b*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\,\sin\left(e+f\,x\right)\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(1/3),x)
[Out] int((b*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(1/3),x)
[Out] Integral((b*sin(e + f*x))**(1/3), x)
```

3.307 $\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};\sin^2(e+fx)\right)}{4bf}$$

```
[Out] 3/4*hypergeom([2/3, 3/2],[5/3],sin(f*x+e)^2)*sec(f*x+e)*(b*sin(f*x+e))^(4/3
)*(cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};\sin^2(e+fx)\right)}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 3/2, 5/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(4/3))/(4*b*f)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^2(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{4/3}}{4bf}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\sqrt[3]{b\sin(e+fx)}\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};\sin^2(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 3/2, 5/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sec\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*sec(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^2, x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\left(\sec^{2}\left(fx+e\right)\right)\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,\sin\left(e+f\,x\right)\right)^{1/3}}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(1/3)/cos(e + f*x)^2,x)
[Out] int((b*sin(e + f*x))^(1/3)/cos(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\sin\left(e+fx\right)}\,\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(b*sin(f*x+e))**(1/3),x)
[Out] Integral((b*sin(e + f*x))**(1/3)*sec(e + f*x)**2, x)
```

3.308 $\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};\sin^2(e+fx)\right)}{4bf}$$

```
[Out] 3/4*hypergeom([2/3, 5/2],[5/3],sin(f*x+e)^2)*sec(f*x+e)*(b*sin(f*x+e))^(4/3
)*(cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\sin(e+fx))^{4/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};\sin^2(e+fx)\right)}{4bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 5/2, 5/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(4/3))/(4*b*f)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^4(e+fx)\sqrt[3]{b\sin(e+fx)}\,dx=\frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{4/3}}{4bf}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\sqrt[3]{b\sin(e+fx)}\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};\sin^2(e+fx)\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[2/3, 5/2, 5/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(1/3)*Tan[e + f*x])/(4*f)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sec\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(1/3)*sec(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^4, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int\left(\sec^{4}\left(fx+e\right)\right)\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(1/3)*sec(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3}}{{\cos\left(e+f\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(1/3)/cos(e + f*x)^4,x)
[Out] int((b*sin(e + f*x))^(1/3)/cos(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(b*sin(f*x+e))**(1/3),x)
[Out] Timed out
```

3.309 $\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(-\frac{3}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/8*cos(f*x+e)*hypergeom([-3/2, 4/3],[7/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(8/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(-\frac{3}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^4(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)(b\sin(e+fx))^{5/3}\,{}_2F_1\left(-\frac{3}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, 4/3, 7/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}} b\cos\left(fx+e\right)^{4}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*b*cos(f*x + e)^4*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^4, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \left(\cos^{4}\left(fx+e\right)\right)\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(e+f\,x\right)^4\,{\left(b\,\sin\left(e+f\,x\right)\right)}^{5/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(b*sin(e + f*x))^(5/3),x)
[Out] int(cos(e + f*x)^4*(b*sin(e + f*x))^(5/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(b*sin(f*x+e))**(5/3),x)
[Out] Timed out
```

3.310 $\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/8*cos(f*x+e)*hypergeom([-1/2, 4/3],[7/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(8/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^2(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)(b\sin(e+fx))^{5/3}\,{}_2F_1\left(-\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, 4/3, 7/3, Sin[e + f*x]^2]*(
b*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}b\cos\left(fx+e\right)^{2}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*b*cos(f*x + e)^2*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^2, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(fx+e\right)\right)\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)\right)}^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(b*sin(e + f*x))^(5/3),x)
[Out] int(cos(e + f*x)^2*(b*sin(e + f*x))^(5/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(b*sin(f*x+e))**(5/3),x)
[Out] Timed out
```

3.311 $\int (b\sin(e+fx))^{5/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/8*cos(f*x+e)*hypergeom([1/2, 4/3],[7/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(8/3
)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b*Sin[e +
 f*x])^(8/3))/(8*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\sin(e+fx))^{5/3}\,dx=\frac{3\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{8/3}}{8bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)(b\sin(e+fx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\sin^2(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/2, 4/3, 7/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}b\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*b*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(f*x+e))^(5/3),x)
```

```
[Out] int((b*sin(f*x+e))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(b\,\sin\left(e+f\,x\right)\right)^{5/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(5/3),x)
[Out] int((b*sin(e + f*x))^(5/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(e+fx\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(f*x+e))**(5/3),x)
[Out] Integral((b*sin(e + f*x))**(5/3), x)
```

3.312 $\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{7}{3};\sin^2(e+fx)\right)}{8bf}$$

```
[Out] 3/8*hypergeom([4/3, 3/2],[7/3],sin(f*x+e)^2)*sec(f*x+e)*(b*sin(f*x+e))^(8/3
)*(cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{7}{3};\sin^2(e+fx)\right)}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 3/2, 7/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(8/3))/(8*b*f)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^2(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{7}{3};\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{8/3}}{8bf}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)(b\sin(e+fx))^{5/3}\,{}_2F_1\left(\frac{4}{3},\frac{3}{2};\frac{7}{3};\sin^2(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 3/2, 7/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}b\sec\left(fx+e\right)^{2}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*b*sec(f*x + e)^2*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^2, x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\left(\sec^{2}\left(fx+e\right)\right)\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,\sin\left(e+f\,x\right)\right)^{5/3}}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(5/3)/cos(e + f*x)^2,x)
[Out] int((b*sin(e + f*x))^(5/3)/cos(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(b*sin(f*x+e))**(5/3),x)
[Out] Timed out
```

3.313 $\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(\frac{4}{3},\frac{5}{2};\frac{7}{3};\sin^2(e+fx)\right)}{8bf}$$

```
[Out] 3/8*hypergeom([4/3, 5/2],[7/3],sin(f*x+e)^2)*sec(f*x+e)*(b*sin(f*x+e))^(8/3
)*(cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\sin(e+fx))^{8/3}\,{}_2F_1\left(\frac{4}{3},\frac{5}{2};\frac{7}{3};\sin^2(e+fx)\right)}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 5/2, 7/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(8/3))/(8*b*f)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^4(e+fx)(b\sin(e+fx))^{5/3}\,dx=\frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{4}{3},\frac{5}{2};\frac{7}{3};\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{8/3}}{8bf}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)(b\sin(e+fx))^{5/3}\,{}_2F_1\left(\frac{4}{3},\frac{5}{2};\frac{7}{3};\sin^2(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[4/3, 5/2, 7/3, Sin[e + f*x]^2]*(b
*Sin[e + f*x])^(5/3)*Tan[e + f*x])/(8*f)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}b\sec\left(fx+e\right)^{4}\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*b*sec(f*x + e)^4*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^4, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int\left(\sec^{4}\left(fx+e\right)\right)\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}\sec\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(5/3)*sec(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,\sin\left(e+f\,x\right)\right)^{5/3}}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(e + f*x))^(5/3)/cos(e + f*x)^4,x)
[Out] int((b*sin(e + f*x))^(5/3)/cos(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(b*sin(f*x+e))**(5/3),x)
[Out] Timed out
```

3.314 $\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/2*cos(f*x+e)*hypergeom([-3/2, 1/3],[4/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(2/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-3/2, 1/3, 4/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)}{2f\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, 1/3, 4/3, Sin[e + f*x]^2]*T
an[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}\cos\left(fx+e\right)^{4}}{b\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*cos(f*x + e)^4/(b*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int\frac{\cos^{4}\left(fx+e\right)}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^4}{{\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(b*sin(e + f*x))^(1/3),x)
[Out] int(cos(e + f*x)^4/(b*sin(e + f*x))^(1/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(b*sin(f*x+e))**(1/3),x)
[Out] Timed out
```

3.315 $\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/2*cos(f*x+e)*hypergeom([-1/2, 1/3],[4/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(2/
3)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[-1/2, 1/3, 4/3, Sin[e + f*x]^2]*(b*Sin[e
+ f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};\sin^2(e+fx)\right)}{2f\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, 1/3, 4/3, Sin[e + f*x]^2]*T
an[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}\cos\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*cos(f*x + e)^2/(b*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(fx+e\right)}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^2}{\left(b\,\sin\left(e+f\,x\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(b*sin(e + f*x))^(1/3),x)
[Out] int(cos(e + f*x)^2/(b*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2\left(e+fx\right)}{\sqrt[3]{b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(b*sin(f*x+e))**(1/3),x)
[Out] Integral(cos(e + f*x)**2/(b*sin(e + f*x))**(1/3), x)
```

3.316 $\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

```
[Out] 3/2*cos(f*x+e)*hypergeom([1/3, 1/2],[4/3],sin(f*x+e)^2)*(b*sin(f*x+e))^(2/3
)/b/f/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$\frac{3\cos(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(-1/3),x]
```

```
[Out] (3*Cos[e + f*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[e + f*x]^2]*(b*Sin[e +
 f*x])^(2/3))/(2*b*f*Sqrt[Cos[e + f*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\sin(e+fx)}}\,dx=\frac{3\cos(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(e+fx)\right)(b\sin(e+fx))^{2/3}}{2bf\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2f\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(-1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, Sin[e + f*x]^2]*Ta
n[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b \sin\left(fx + e\right)\right)^{\frac{2}{3}}}{b \sin\left(fx + e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)/(b*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sin\left(fx + e\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(-1/3), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sin\left(fx + e\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(1/(b*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sin\left(fx + e\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(-1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\sin\left(e+fx\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sin(e + f*x))^(1/3),x)
```

```
[Out] int(1/(b*sin(e + f*x))^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral((b*sin(e + f*x))**(-1/3), x)
```

3.317 $\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{3}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2bf}$$

```
[Out] 3/2*hypergeom([1/3, 3/2],[4/3],sin(f*x+e)^2)*sec(f*x+e)*(b*sin(f*x+e))^(2/3
)*(cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{3}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 3/2, 4/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(2/3))/(2*b*f)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx = \frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{3}{2};\frac{4}{3};\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{2/3}}{2bf}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{3}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2f\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 3/2, 4/3, Sin[e + f*x]^2]*Ta
n[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}\sec\left(fx+e\right)^{2}}{b\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*sec(f*x + e)^2/(b*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}\left(fx+e\right)}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(b*sin(e + f*x))^(1/3)),x)
[Out] int(1/(cos(e + f*x)^2*(b*sin(e + f*x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}{\left(e + f x \right)}}{\sqrt[3]{b \sin{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(b*sin(f*x+e))**(1/3),x)
[Out] Integral(sec(e + f*x)**2/(b*sin(e + f*x))**(1/3), x)
```

3.318 $\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{5}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2bf}$$

```
[Out] 3/2*hypergeom([1/3, 5/2],[4/3],sin(f*x+e)^2)*sec(f*x+e)*(b*sin(f*x+e))^(2/3
)*(cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)(b\sin(e+fx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{5}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 5/2, 4/3, Sin[e + f*x]^2]*Se
c[e + f*x]*(b*Sin[e + f*x])^(2/3))/(2*b*f)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\sqrt[3]{b\sin(e+fx)}}\,dx=\frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(\frac{1}{3},\frac{5}{2};\frac{4}{3};\sin^2(e+fx)\right)\sec(e+fx)(b\sin(e+fx))^{2/3}}{2bf}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\,{}_2F_1\left(\frac{1}{3},\frac{5}{2};\frac{4}{3};\sin^2(e+fx)\right)}{2f\sqrt[3]{b\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(b*Sin[e + f*x])^(1/3),x]
```

```
[Out] (3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[1/3, 5/2, 4/3, Sin[e + f*x]^2]*Ta
n[e + f*x])/(2*f*(b*Sin[e + f*x])^(1/3))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{2}{3}}\sec\left(fx+e\right)^{4}}{b\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^(2/3)*sec(f*x + e)^4/(b*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int\frac{\sec^{4}\left(fx+e\right)}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

```
[Out] int(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^4/(b*sin(f*x + e))^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(e+f\,x\right)^4\,{\left(b\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(b*sin(e + f*x))^(1/3)),x)
[Out] int(1/(cos(e + f*x)^4*(b*sin(e + f*x))^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(e+fx\right)}{\sqrt[3]{b\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(b*sin(f*x+e))**(1/3),x)
[Out] Integral(sec(e + f*x)**4/(b*sin(e + f*x))**(1/3), x)
```

3.319 $\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}\,(b\sin(e+fx))^{2/3}}$$

```
[Out] -3/2*cos(f*x+e)*hypergeom([-3/2, -1/3],[2/3],sin(f*x+e)^2)/b/f/(b*sin(f*x+e
))^(2/3)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}\,(b\sin(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Cos[e + f*x]*Hypergeometric2F1[-3/2, -1/3, 2/3, Sin[e + f*x]^2])/(2*b*f
*Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx=-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}\,(b\sin(e+fx))^{2/3}}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2f(b\sin(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-3/2, -1/3, 2/3, Sin[e + f*x]^2]
*Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\cos\left(fx+e\right)^{4}}{b^{2}\cos\left(fx+e\right)^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)*cos(f*x + e)^4/(b^2*cos(f*x + e)^2 - b^2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(5/3), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\cos^{4}\left(fx+e\right)}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e))^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^4}{\left(b\,\sin\left(e+f\,x\right)\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(b*sin(e + f*x))^(5/3),x)
```

```
[Out] int(cos(e + f*x)^4/(b*sin(e + f*x))^(5/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

3.320 $\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}\,(b\sin(e+fx))^{2/3}}$$

```
[Out] -3/2*cos(f*x+e)*hypergeom([-1/2, -1/3],[2/3],sin(f*x+e)^2)/b/f/(b*sin(f*x+e
))^(2/3)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}\,(b\sin(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Cos[e + f*x]*Hypergeometric2F1[-1/2, -1/3, 2/3, Sin[e + f*x]^2])/(2*b*f
*Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}\,(b\sin(e+fx))^{2/3}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};\sin^2(e+fx)\right)}{2f(b\sin(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/2, -1/3, 2/3, Sin[e + f*x]^2]
*Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\cos\left(fx+e\right)^{2}}{b^{2}\cos\left(fx+e\right)^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)*cos(f*x + e)^2/(b^2*cos(f*x + e)^2 - b^2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(fx+e\right)}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(e+f\,x\right)^2}{\left(b\,\sin\left(e+f\,x\right)\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(b*sin(e + f*x))^(5/3),x)
```

```
[Out] int(cos(e + f*x)^2/(b*sin(e + f*x))^(5/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

3.321 $\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}\,(b\sin(e+fx))^{2/3}}$$

```
[Out] -3/2*cos(f*x+e)*hypergeom([-1/3, 1/2],[2/3],sin(f*x+e)^2)/b/f/(b*sin(f*x+e)
)^(2/3)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2643}

$$-\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}\,(b\sin(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[e + f*x])^(-5/3),x]
```

```
[Out] (-3*Cos[e + f*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sin[e + f*x]^2])/(2*b*f*
Sqrt[Cos[e + f*x]^2]*(b*Sin[e + f*x])^(2/3))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\cos(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf\sqrt{\cos^2(e+fx)}\,(b\sin(e+fx))^{2/3}}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2f(b\sin(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[e + f*x])^(-5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sin[e + f*x]^2]*
Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}}{b^2\cos\left(fx+e\right)^2-b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)/(b^2*cos(f*x + e)^2 - b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^(-5/3), x)
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(1/(b*sin(f*x+e))^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^(-5/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\sin\left(e+fx\right)\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sin(e + f*x))^(5/3),x)
```

```
[Out] int(1/(b*sin(e + f*x))^(5/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(e+fx\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Integral((b*sin(e + f*x))**(-5/3), x)
```

3.322 $\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf(b\sin(e+fx))^{2/3}}$$

```
[Out] -3/2*hypergeom([-1/3, 3/2],[2/3],sin(f*x+e)^2)*sec(f*x+e)*(cos(f*x+e)^2)^(1
/2)/b/f/(b*sin(f*x+e))^(2/3)
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$-\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf(b\sin(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 3/2, 2/3, Sin[e + f*x]^2]*
Sec[e + f*x])/(2*b*f*(b*Sin[e + f*x])^(2/3))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx=-\frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};\sin^2(e+fx)\right)\sec(e+fx)}{2bf(b\sin(e+fx))^{2/3}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2f(b\sin(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 3/2, 2/3, Sin[e + f*x]^2]*
Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sec\left(fx+e\right)^{2}}{b^{2}\cos\left(fx+e\right)^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)*sec(f*x + e)^2/(b^2*cos(f*x + e)^2 - b^2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e))^(5/3), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int\frac{\sec^{2}\left(fx+e\right)}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)\right)}^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(b*sin(e + f*x))^(5/3)),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(b*sin(e + f*x))^(5/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

3.323 $\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf(b\sin(e+fx))^{2/3}}$$

```
[Out] -3/2*hypergeom([-1/3, 5/2],[2/3],sin(f*x+e)^2)*sec(f*x+e)*(cos(f*x+e)^2)^(1
/2)/b/f/(b*sin(f*x+e))^(2/3)
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2577}

$$-\frac{3\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2bf(b\sin(e+fx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 5/2, 2/3, Sin[e + f*x]^2]*
Sec[e + f*x])/(2*b*f*(b*Sin[e + f*x])^(2/3))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{(b\sin(e+fx))^{5/3}}\,dx = -\frac{3\sqrt{\cos^2(e+fx)}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{2}{3};\sin^2(e+fx)\right)\sec(e+fx)}{2bf(b\sin(e+fx))^{2/3}}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.95

$$-\frac{3\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\,{}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{2}{3};\sin^2(e+fx)\right)}{2f(b\sin(e+fx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/(b*Sin[e + f*x])^(5/3),x]
```

```
[Out] (-3*Sqrt[Cos[e + f*x]^2]*Hypergeometric2F1[-1/3, 5/2, 2/3, Sin[e + f*x]^2]*
Tan[e + f*x])/(2*f*(b*Sin[e + f*x])^(5/3))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\sin\left(fx+e\right)\right)^{\frac{1}{3}}\sec\left(fx+e\right)^{4}}{b^{2}\cos\left(fx+e\right)^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*sin(f*x + e))^(1/3)*sec(f*x + e)^4/(b^2*cos(f*x + e)^2 - b^2),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^4/(b*sin(f*x + e))^(5/3), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(fx+e\right)}{\left(b\sin\left(fx+e\right)\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

```
[Out] int(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(b*sin(f*x+e))^(5/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(e+f\,x\right)^4\,\left(b\,\sin\left(e+f\,x\right)\right)^{5/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^4*(b*sin(e + f*x))^(5/3)),x)
```

```
[Out] int(1/(cos(e + f*x)^4*(b*sin(e + f*x))^(5/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4/(b*sin(f*x+e))**(5/3),x)
```

```
[Out] Timed out
```

3.324 $\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}+\frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

```
[Out] -1/2*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))/b+1/4*ln(1-sin(b*x+a)^(2/3)/co
s(b*x+a)^(2/3)+sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3))/b-1/2*arctan(1/3*(1-2*sin
(b*x+a)^(2/3)/cos(b*x+a)^(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] time = 0.15, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2574, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}+\frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(1/3)/Cos[a + b*x]^(1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]])/
(2*b) - Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/(2*b) + Log[1 - Sin[
a + b*x]^(2/3)/Cos[a + b*x]^(2/3) + Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3)]/
(4*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx = \frac{3\,\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$= \frac{3\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}}{\cos^{\frac{2}{3}}}\right.}{4b}$$

$$= -\frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\right.}{2b}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b}$$

Mathematica [C] time = 0.04, size = 57, normalized size = 0.45

$$\frac{3\sin^{\frac{4}{3}}(a+bx)\cos^2(a+bx)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\sin^2(a+bx)\right)}{4b\cos^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(1/3)/Cos[a + b*x]^(1/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, Sin[a + b*x]^2]*
Sin[a + b*x]^(4/3))/(4*b*Cos[a + b*x]^(4/3))
```

fricas [A] time = 0.47, size = 144, normalized size = 1.12

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cos(bx+a)-2\sqrt{3}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}}{3\cos(bx+a)}\right) - 2\log\left(\frac{\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)}{\cos(bx+a)}\right) + \log\left(\frac{\cos(bx+a)^2-\cos(bx\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*cos(b*x + a) - 2*sqrt(3)*cos(b*x + a)^(
1/3)*sin(b*x + a)^(2/3))/cos(b*x + a)) - 2*log((cos(b*x + a)^(1/3)*sin(b*x
+ a)^(2/3) + cos(b*x + a))/cos(b*x + a)) + log((cos(b*x + a)^2 - cos(b*x +
a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3))/cos(b*
x + a)^2))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\sin^{\frac{1}{3}}\left(bx+a\right)}{\cos\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x)
```

```
[Out] int(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)^{\frac{1}{3}}}{\cos\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(1/3)/cos(b*x + a)^(1/3), x)
```

mupad [B] time = 1.24, size = 44, normalized size = 0.34

$$-\frac{3\cos\left(a+b\,x\right)^{2/3}\sin\left(a+b\,x\right)^{4/3}{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ \cos\left(a+b\,x\right)^2\right)}{2\,b\,{\left({\sin\left(a+b\,x\right)}^2\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^(1/3)/cos(a + b*x)^(1/3),x)
```

```
[Out] -(3*cos(a + b*x)^(2/3)*sin(a + b*x)^(4/3)*hypergeom([1/3, 1/3], 4/3, cos(a
+ b*x)^2))/(2*b*(sin(a + b*x)^2)^(2/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\sin(a + bx)}}{\sqrt[3]{\cos(a + bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(1/3)/cos(b*x+a)**(1/3),x)
```

```
[Out] Integral(sin(a + b*x)**(1/3)/cos(a + b*x)**(1/3), x)
```

3.325 $\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx$

Optimal. Leaf size=224

$$\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}-\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b}+\frac{\tan^{-1}}{}$$

```
[Out] arctan(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3))/b+1/2*arctan(2*sin(b*x+a)^(1/3)/c
os(b*x+a)^(1/3)-3^(1/2))/b+1/2*arctan(2*sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3)+3
^(1/2))/b+1/4*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3)-sin(b*x+a)^(1/3)*3^(1/
2)/cos(b*x+a)^(1/3))*3^(1/2)/b-1/4*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3)+s
in(b*x+a)^(1/3)*3^(1/2)/cos(b*x+a)^(1/3))*3^(1/2)/b
```

Rubi [A] time = 0.33, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2574, 295, 634, 618, 204, 628, 203}

$$\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}-\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b}+\frac{\tan^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3),x]
```

```
[Out] -ArcTan[Sqrt[3] - (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) + ArcTan
[Sqrt[3] + (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) + ArcTan[Sin[a
+ b*x]^(1/3)/Cos[a + b*x]^(1/3)]/b + (Sqrt[3]*Log[1 - (Sqrt[3]*Sin[a + b*x]
^(1/3))/Cos[a + b*x]^(1/3) + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b)
- (Sqrt[3]*Log[1 + (Sqrt[3]*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3) + Sin[a
+ b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx = \frac{3\,\mathrm{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$= \frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}-\frac{\sqrt{3}}{2}}{1+\sqrt{3}x+}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b} + \frac{\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin}}{\sqrt[3]{\cos}}\right.}{4b}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\right.}{4b}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} + \frac{\sqrt{3}\log\left(1\right.}{}$$

Mathematica [C] time = 0.04, size = 57, normalized size = 0.25

$$\frac{3\sin^{\frac{5}{3}}(a+bx)\cos^2(a+bx)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{5}{6};\frac{11}{6};\sin^2(a+bx)\right)}{5b\cos^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6, Sin[a + b*x]^2]
*Sin[a + b*x]^(5/3))/(5*b*Cos[a + b*x]^(5/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sin^{\frac{2}{3}}(bx+a)}{\cos(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x)
```

```
[Out] int(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^{\frac{2}{3}}}{\cos(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(2/3)/cos(b*x + a)^(2/3), x)
```

mupad [B] time = 1.05, size = 44, normalized size = 0.20

$$-\frac{3\cos(a+b\,x)^{1/3}\sin(a+b\,x)^{5/3}\,{}_2F_1\left(\frac{1}{6},\frac{1}{6};\ \frac{7}{6};\ \cos(a+b\,x)^2\right)}{b\left(\sin(a+b\,x)^2\right)^{5/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^(2/3)/cos(a + b*x)^(2/3),x)
```

```
[Out] -(3*cos(a + b*x)^(1/3)*sin(a + b*x)^(5/3)*hypergeom([1/6, 1/6], 7/6, cos(a
+ b*x)^2))/(b*(sin(a + b*x)^2)^(5/6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(2/3)/cos(b*x+a)**(2/3),x)
```

```
[Out] Integral(sin(a + b*x)**(2/3)/cos(a + b*x)**(2/3), x)
```

3.326 $\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)} dx$

Optimal. Leaf size=249

$$\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}+\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}-\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}-\frac{\tan^{-1}\left(\sqrt{3}-\right.}{2}$$

```
[Out] arctan(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3))/b+1/2*arctan(2*cos(b*x+a)^(1/3)/s
in(b*x+a)^(1/3)-3^(1/2))/b+1/2*arctan(2*cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3)+3
^(1/2))/b+3*sin(b*x+a)^(1/3)/b/cos(b*x+a)^(1/3)+1/4*ln(1+cos(b*x+a)^(2/3)/s
in(b*x+a)^(2/3)-cos(b*x+a)^(1/3)*3^(1/2)/sin(b*x+a)^(1/3))*3^(1/2)/b-1/4*ln
(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3)+cos(b*x+a)^(1/3)*3^(1/2)/sin(b*x+a)^(1
/3))*3^(1/2)/b
```

Rubi [A] time = 0.35, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2566, 2575, 295, 634, 618, 204, 628, 203}

$$\frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}+\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}-\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}-\frac{\tan^{-1}\left(\sqrt{3}-\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3),x]
```

```
[Out] -ArcTan[Sqrt[3] - (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) + ArcTan
[Sqrt[3] + (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) + ArcTan[Cos[a
+ b*x]^(1/3)/Sin[a + b*x]^(1/3)]/b + (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Si
n[a + b*x]^(2/3) - (Sqrt[3]*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b)
- (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3) + (Sqrt[3]*Cos[a +
 b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b) + (3*Sin[a + b*x]^(1/3))/(b*Cos[a +
 b*x]^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} - \int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx$$

$$= \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} + \frac{3\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{3\sqrt[3]{\sin(a+bx)}}{b\sqrt[3]{\cos(a+bx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} - \frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} - \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\right.}{4b}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} + \frac{\sqrt{3}\log\left(\right.}{}$$

Mathematica [C] time = 0.05, size = 57, normalized size = 0.23

$$\frac{3\sin^{\frac{7}{3}}(a+bx)\sqrt[6]{\cos^2(a+bx)}\ {}_2F_1\left(\frac{7}{6},\frac{7}{6};\frac{13}{6};\sin^2(a+bx)\right)}{7b\sqrt[3]{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(1/6)*Hypergeometric2F1[7/6, 7/6, 13/6, Sin[a + b*x]^2]
*Sin[a + b*x]^(7/3))/(7*b*Cos[a + b*x]^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sin^{\frac{4}{3}}\left(bx+a\right)}{\cos\left(bx+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x)
[Out] int(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)^{\frac{4}{3}}}{\cos\left(bx+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^(4/3)/cos(b*x + a)^(4/3), x)
```

mupad [B] time = 1.64, size = 44, normalized size = 0.18

$$\frac{3\sin\left(a+b\,x\right)^{7/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{6},-\frac{1}{6};\ \frac{5}{6};\ \cos\left(a+b\,x\right)^2\right)}{b\cos\left(a+b\,x\right)^{1/3}\,{\left(\sin\left(a+b\,x\right)^2\right)}^{7/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^(4/3)/cos(a + b*x)^(4/3),x)
```

```
[Out] (3*sin(a + b*x)^(7/3)*hypergeom([-1/6, -1/6], 5/6, cos(a + b*x)^2))/(b*cos(
a + b*x)^(1/3)*(sin(a + b*x)^2)^(7/6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(4/3)/cos(b*x+a)**(4/3),x)
```

```
[Out] Timed out
```

3.327 $\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}+\frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

```
[Out] 1/4*ln(1+cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3)-cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3
))/b-1/2*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))/b+3/2*sin(b*x+a)^(2/3)/b/c
os(b*x+a)^(2/3)-1/2*arctan(1/3*(1-2*cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))*3^(1
/2))*3^(1/2)/b
```

Rubi [A] time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2566, 2575, 275, 292, 31, 634, 618, 204, 628}

$$\frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)}+\frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(5/3)/Cos[a + b*x]^(5/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]])/
(2*b) + Log[1 + Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3) - Cos[a + b*x]^(2/3)/
Sin[a + b*x]^(2/3)]/(4*b) - Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/
(2*b) + (3*Sin[a + b*x]^(2/3))/(2*b*Cos[a + b*x]^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sin^{\frac{5}{3}}(a+bx)}{\cos^{\frac{5}{3}}(a+bx)}\,dx = \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} - \int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx$$

$$= \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\sin^{\frac{2}{3}}(a+bx)}{2b\cos^{\frac{2}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-}\right.}{}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\sin}{2b\cos}$$

Mathematica [C] time = 0.06, size = 57, normalized size = 0.37

$$\frac{3\sin^{\frac{8}{3}}(a+bx)\sqrt[3]{\cos^2(a+bx)}\ {}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\sin^2(a+bx)\right)}{8b\cos^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(5/3)/Cos[a + b*x]^(5/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, Sin[a + b*x]^2]*
Sin[a + b*x]^(8/3))/(8*b*Cos[a + b*x]^(2/3))
```

fricas [A] time = 0.87, size = 197, normalized size = 1.27

$$\frac{2\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}-\sqrt{3}\sin(bx+a)}{3\sin(bx+a)}\right)\cos(bx+a)+\cos(bx+a)\log\left(\frac{4\left(\cos(bx+a)^2-\cos(bx+a)^{\frac{4}{3}}\sin(bx+a)^{\frac{2}{3}}\right.}{\cos(bx+a)^2}\right.}{4b\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3)
- sqrt(3)*sin(b*x + a))/sin(b*x + a))*cos(b*x + a) + cos(b*x + a)*log(4*(co
s(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*s
in(b*x + a)^(4/3) - 1)/(cos(b*x + a)^2 - 1)) - 2*cos(b*x + a)*log(-2*(cos(b
*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin(b*x + a))/sin(b*x + a)) + 6*cos(b*x
+ a)^(1/3)*sin(b*x + a)^(2/3))/(b*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin(bx+a)^{\frac{5}{3}}}{\cos(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^(5/3)/cos(b*x + a)^(5/3), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\sin^{\frac{5}{3}}(bx+a)}{\cos(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x)
```

```
[Out] int(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^{\frac{5}{3}}}{\cos(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(5/3)/cos(b*x+a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^(5/3)/cos(b*x + a)^(5/3), x)
```

mupad [B] time = 1.14, size = 44, normalized size = 0.28

$$\frac{3\sin(a+b\,x)^{8/3}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\ \frac{2}{3};\ \cos(a+b\,x)^2\right)}{2\,b\cos(a+b\,x)^{2/3}\left(\sin(a+b\,x)^2\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^(5/3)/cos(a + b*x)^(5/3),x)
```

```
[Out] (3*sin(a + b*x)^(8/3)*hypergeom([-1/3, -1/3], 2/3, cos(a + b*x)^2))/(2*b*co
s(a + b*x)^(2/3)*(sin(a + b*x)^2)^(4/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(5/3)/cos(b*x+a)**(5/3),x)
```

```
[Out] Timed out
```

3.328 $\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}+\frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

```
[Out] 1/2*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))/b-1/4*ln(1-sin(b*x+a)^(2/3)/cos
(b*x+a)^(2/3)+sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3))/b+3/4*sin(b*x+a)^(4/3)/b/c
os(b*x+a)^(4/3)+1/2*arctan(1/3*(1-2*sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))*3^(1
/2))*3^(1/2)/b
```

Rubi [A] time = 0.11, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2566, 2574, 275, 292, 31, 634, 618, 204, 628}

$$\frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)}+\frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^(7/3)/Cos[a + b*x]^(7/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]])/(
2*b) + Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/(2*b) - Log[1 - Sin[a
 + b*x]^(2/3)/Cos[a + b*x]^(2/3) + Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3)]/(
4*b) + (3*Sin[a + b*x]^(4/3))/(4*b*Cos[a + b*x]^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sin^{\frac{7}{3}}(a+bx)}{\cos^{\frac{7}{3}}(a+bx)}\,dx = \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} - \int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx$$

$$= \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$= \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} + \frac{3\sin^{\frac{4}{3}}(a+bx)}{4b\cos^{\frac{4}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} + \frac{3\sin}{4b\cos}$$

Mathematica [C] time = 0.05, size = 57, normalized size = 0.37

$$\frac{3\sin^{\frac{10}{3}}(a+bx)\cos^2(a+bx)^{2/3}\,{}_2F_1\left(\frac{5}{3},\frac{5}{3};\frac{8}{3};\sin^2(a+bx)\right)}{10b\cos^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^(7/3)/Cos[a + b*x]^(7/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(2/3)*Hypergeometric2F1[5/3, 5/3, 8/3, Sin[a + b*x]^2]*
Sin[a + b*x]^(10/3))/(10*b*Cos[a + b*x]^(4/3))
```

fricas [A] time = 1.12, size = 195, normalized size = 1.26

$$-\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cos(bx+a)-2\sqrt{3}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}}{3\cos(bx+a)}\right)\cos(bx+a)^2-2\cos(bx+a)^2\log\left(\frac{\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(b}{\cos(bx+a)}\right.}{4\,b\cos(bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*cos(b*x + a) - 2*sqrt(3)*cos(b*x + a)^
(1/3)*sin(b*x + a)^(2/3))/cos(b*x + a))*cos(b*x + a)^2 - 2*cos(b*x + a)^2*l
og((cos(b*x + a)^(1/3)*sin(b*x + a)^(2/3) + cos(b*x + a))/cos(b*x + a)) + c
os(b*x + a)^2*log((cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3) +
 cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3))/cos(b*x + a)^2) - 3*cos(b*x + a)^(2
/3)*sin(b*x + a)^(4/3))/(b*cos(b*x + a)^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sin^{\frac{7}{3}}(bx+a)}{\cos(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x)
[Out] int(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(bx+a)^{\frac{7}{3}}}{\cos(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^(7/3)/cos(b*x+a)^(7/3),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^(7/3)/cos(b*x + a)^(7/3), x)
```

mupad [B] time = 1.64, size = 44, normalized size = 0.28

$$\frac{3\sin(a+b\,x)^{10/3}\,{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\ \frac{1}{3};\ \cos(a+b\,x)^2\right)}{4\,b\cos(a+b\,x)^{4/3}\left(\sin(a+b\,x)^2\right)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^(7/3)/cos(a + b*x)^(7/3),x)
[Out] (3*sin(a + b*x)^(10/3)*hypergeom([-2/3, -2/3], 1/3, cos(a + b*x)^2))/(4*b*c
os(a + b*x)^(4/3)*(sin(a + b*x)^2)^(5/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**(7/3)/cos(b*x+a)**(7/3),x)
[Out] Timed out
```

3.329 $\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx$

Optimal. Leaf size=128

$$-\frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b}+\frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

```
[Out] -1/4*ln(1+cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3)-cos(b*x+a)^(2/3)/sin(b*x+a)^(2/
3))/b+1/2*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))/b+1/2*arctan(1/3*(1-2*cos
(b*x+a)^(2/3)/sin(b*x+a)^(2/3))*3^(1/2))*3^(1/2)/b
```

Rubi [A] time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2575, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b}+\frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(1/3)/Sin[a + b*x]^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]])/(
2*b) - Log[1 + Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3) - Cos[a + b*x]^(2/3)/S
in[a + b*x]^(2/3)]/(4*b) + Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/(
2*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx = -\frac{3\,\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= -\frac{3\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+\cdots}{\sin^{\frac{2}{3}}(a+\cdots}\right.}{4b}$$

$$= -\frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\frac{2\,\text{c}\cdots}{\text{si}\cdots}\right.}{2b}$$

$$= \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} - \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.45

$$\frac{3\sin^{\frac{2}{3}}(a+bx)\sqrt[3]{\cos^2(a+bx)}\;{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\sin^2(a+bx)\right)}{2b\cos^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(1/3)/Sin[a + b*x]^(1/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, Sin[a + b*x]^2]*
Sin[a + b*x]^(2/3))/(2*b*Cos[a + b*x]^(2/3))
```

fricas [A] time = 0.97, size = 152, normalized size = 1.19

$$-\frac{2\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}-\sqrt{3}\sin(bx+a)}{3\sin(bx+a)}\right)+\log\left(\frac{4\left(\cos(bx+a)^2-\cos(bx+a)^{\frac{4}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{4}{3}}-1\right)}{\cos(bx+a)^2-1}\right)\cdots}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3)
 - sqrt(3)*sin(b*x + a))/sin(b*x + a)) + log(4*(cos(b*x + a)^2 - cos(b*x +
a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3) - 1)/(c
os(b*x + a)^2 - 1)) - 2*log(-2*(cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin
(b*x + a))/sin(b*x + a)))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{1}{3}}\left(bx+a\right)}{\sin\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x)
```

```
[Out] int(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)^{\frac{1}{3}}}{\sin\left(bx+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(1/3)/sin(b*x + a)^(1/3), x)
```

mupad [B] time = 1.48, size = 44, normalized size = 0.34

$$-\frac{3\cos\left(a+b\,x\right)^{4/3}\sin\left(a+b\,x\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{2}{3},\frac{2}{3};\ \frac{5}{3};\ \cos\left(a+b\,x\right)^2\right)}{4\,b\,{\left({\sin\left(a+b\,x\right)}^2\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(1/3)/sin(a + b*x)^(1/3),x)
```

```
[Out] -(3*cos(a + b*x)^(4/3)*sin(a + b*x)^(2/3)*hypergeom([2/3, 2/3], 5/3, cos(a
+ b*x)^2))/(4*b*(sin(a + b*x)^2)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{\cos (a+bx)}}{\sqrt[3]{\sin (a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(1/3)/sin(b*x+a)**(1/3),x)
```

```
[Out] Integral(cos(a + b*x)**(1/3)/sin(a + b*x)**(1/3), x)
```

3.330 $\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx$

Optimal. Leaf size=225

$$-\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}+\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b}-$$

```
[Out] -arctan(cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3))/b-1/2*arctan(2*cos(b*x+a)^(1/3)/
sin(b*x+a)^(1/3)-3^(1/2))/b-1/2*arctan(2*cos(b*x+a)^(1/3)/sin(b*x+a)^(1/3)+
3^(1/2))/b-1/4*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3)-cos(b*x+a)^(1/3)*3^(1
/2)/sin(b*x+a)^(1/3))*3^(1/2)/b+1/4*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3)+
cos(b*x+a)^(1/3)*3^(1/2)/sin(b*x+a)^(1/3))*3^(1/2)/b
```

Rubi [A] time = 0.30, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2575, 295, 634, 618, 204, 628, 203}

$$-\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}+1\right)}{4b}+\frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3),x]
```

```
[Out] ArcTan[Sqrt[3] - (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) - ArcTan[
Sqrt[3] + (2*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)]/(2*b) - ArcTan[Cos[a +
 b*x]^(1/3)/Sin[a + b*x]^(1/3)]/b - (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin
[a + b*x]^(2/3) - (Sqrt[3]*Cos[a + b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b) +
 (Sqrt[3]*Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3) + (Sqrt[3]*Cos[a +
b*x]^(1/3))/Sin[a + b*x]^(1/3)])/(4*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\,\text{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}\cdots}{1+\sqrt{\cdots}}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \cdots\right.}{4b}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)} - \frac{\sqrt{3}\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{4b} + \frac{\sqrt{3}\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+\cdots\right.}{4b}$$

$$= \frac{\tan^{-1}\left(\sqrt{3} - \frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} - \frac{\tan^{-1}\left(\sqrt{3} + \frac{2\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{2b} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1\cdots\right.}{}$$

Mathematica [C] time = 0.03, size = 55, normalized size = 0.24

$$\frac{3\sqrt[3]{\sin(a+bx)}\,\sqrt[6]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{6};\frac{7}{6};\sin^2(a+bx)\right)}{b\sqrt[3]{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3),x]
```

```
[Out] (3*(Cos[a + b*x]^2)^(1/6)*Hypergeometric2F1[1/6, 1/6, 7/6, Sin[a + b*x]^2]*
Sin[a + b*x]^(1/3))/(b*Cos[a + b*x]^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{2}{3}}(bx+a)}{\sin(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x)
```

```
[Out] int(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^{\frac{2}{3}}}{\sin(bx+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(2/3)/sin(b*x + a)^(2/3), x)
```

mupad [B] time = 1.12, size = 44, normalized size = 0.20

$$-\frac{3\cos(a+b\,x)^{5/3}\sin(a+b\,x)^{1/3}\,{}_2F_1\left(\frac{5}{6},\frac{5}{6};\ \frac{11}{6};\ \cos(a+b\,x)^2\right)}{5\,b\left(\sin(a+b\,x)^2\right)^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(2/3)/sin(a + b*x)^(2/3),x)
```

```
[Out] -(3*cos(a + b*x)^(5/3)*sin(a + b*x)^(1/3)*hypergeom([5/6, 5/6], 11/6, cos(a
 + b*x)^2))/(5*b*(sin(a + b*x)^2)^(1/6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(2/3)/sin(b*x+a)**(2/3),x)
```

```
[Out] Integral(cos(a + b*x)**(2/3)/sin(a + b*x)**(2/3), x)
```

3.331 $\displaystyle\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx$

Optimal. Leaf size=250

$$-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}-\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}+\frac{\tan^{-1}\left(\sqrt{3}-\right.}{2}$$

```
[Out] -arctan(sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3))/b-1/2*arctan(2*sin(b*x+a)^(1/3)/
cos(b*x+a)^(1/3)-3^(1/2))/b-1/2*arctan(2*sin(b*x+a)^(1/3)/cos(b*x+a)^(1/3)+
3^(1/2))/b-3*cos(b*x+a)^(1/3)/b/sin(b*x+a)^(1/3)-1/4*ln(1+sin(b*x+a)^(2/3)/
cos(b*x+a)^(2/3)-sin(b*x+a)^(1/3)*3^(1/2)/cos(b*x+a)^(1/3))*3^(1/2)/b+1/4*l
n(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3)+sin(b*x+a)^(1/3)*3^(1/2)/cos(b*x+a)^(
1/3))*3^(1/2)/b
```

Rubi [A] time = 0.33, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2567, 2574, 295, 634, 618, 204, 628, 203}

$$-\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}}-\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}+\frac{\sqrt{3}\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+1\right)}{4b}+\frac{\tan^{-1}\left(\sqrt{3}-\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3),x]
```

```
[Out] ArcTan[Sqrt[3] - (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) - ArcTan[
Sqrt[3] + (2*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3)]/(2*b) - ArcTan[Sin[a +
 b*x]^(1/3)/Cos[a + b*x]^(1/3)]/b - (Sqrt[3]*Log[1 - (Sqrt[3]*Sin[a + b*x]^
(1/3))/Cos[a + b*x]^(1/3) + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b) +
 (Sqrt[3]*Log[1 + (Sqrt[3]*Sin[a + b*x]^(1/3))/Cos[a + b*x]^(1/3) + Sin[a +
 b*x]^(2/3)/Cos[a + b*x]^(2/3)])/(4*b) - (3*Cos[a + b*x]^(1/3))/(b*Sin[a +
b*x]^(1/3))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}} - \int \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\,dx$$

$$= -\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}} - \frac{3\,\mathrm{Subst}\left(\int \frac{x^4}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}} - \frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \frac{\mathrm{Subst}\left(\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \frac{3\sqrt[3]{\cos(a+bx)}}{b\sqrt[3]{\sin(a+bx)}} - \frac{\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{4b} - \frac{\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1-\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{\sqrt{3}\log\left(1+\frac{\sqrt{3}\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right.}{4b}$$

$$= \frac{\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} - \frac{\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{2b} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b} - \frac{\sqrt{3}\log\left(1-\right.}{}$$

Mathematica [C] time = 0.03, size = 55, normalized size = 0.22

$$-\frac{3\cos^2(a+bx)^{5/6}\,{}_2F_1\left(-\frac{1}{6},-\frac{1}{6};\frac{5}{6};\sin^2(a+bx)\right)}{b\sqrt[3]{\sin(a+bx)}\,\cos^{\frac{5}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3),x]
```

```
[Out] (-3*(Cos[a + b*x]^2)^(5/6)*Hypergeometric2F1[-1/6, -1/6, 5/6, Sin[a + b*x]^
2])/(b*Cos[a + b*x]^(5/3)*Sin[a + b*x]^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{4}{3}}(bx+a)}{\sin(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x)
[Out] int(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^{\frac{4}{3}}}{\sin(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^(4/3)/sin(b*x + a)^(4/3), x)
```

mupad [B] time = 1.70, size = 44, normalized size = 0.18

$$-\frac{3\cos(a+b\,x)^{7/3}\left(\sin(a+b\,x)^2\right)^{1/6}{}_2\mathrm{F}_1\left(\frac{7}{6},\frac{7}{6};\ \frac{13}{6};\ \cos(a+b\,x)^2\right)}{7\,b\sin(a+b\,x)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(4/3)/sin(a + b*x)^(4/3),x)
```

```
[Out] -(3*cos(a + b*x)^(7/3)*(sin(a + b*x)^2)^(1/6)*hypergeom([7/6, 7/6], 13/6, c
os(a + b*x)^2))/(7*b*sin(a + b*x)^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(4/3)/sin(b*x+a)**(4/3),x)
```

```
[Out] Timed out
```

3.332 $\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}+\frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

```
[Out] 1/2*ln(1+sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))/b-1/4*ln(1-sin(b*x+a)^(2/3)/cos
(b*x+a)^(2/3)+sin(b*x+a)^(4/3)/cos(b*x+a)^(4/3))/b-3/2*cos(b*x+a)^(2/3)/b/s
in(b*x+a)^(2/3)+1/2*arctan(1/3*(1-2*sin(b*x+a)^(2/3)/cos(b*x+a)^(2/3))*3^(1
/2))*3^(1/2)/b
```

Rubi [A] time = 0.11, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2567, 2574, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)}+\frac{\log\left(\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\log\left(\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+1\right)}{4b}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(5/3)/Sin[a + b*x]^(5/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*Sin[a + b*x]^(2/3))/Cos[a + b*x]^(2/3))/Sqrt[3]])/(
2*b) + Log[1 + Sin[a + b*x]^(2/3)/Cos[a + b*x]^(2/3)]/(2*b) - Log[1 - Sin[a
 + b*x]^(2/3)/Cos[a + b*x]^(2/3) + Sin[a + b*x]^(4/3)/Cos[a + b*x]^(4/3)]/(
4*b) - (3*Cos[a + b*x]^(2/3))/(2*b*Sin[a + b*x]^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{3}}(a+bx)}{\sin^{\frac{5}{3}}(a+bx)}\,dx = -\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} - \int \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\,dx$$

$$= -\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\sin(a+bx)}}{\sqrt[3]{\cos(a+bx)}}\right)}{b}$$

$$= -\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{3\,\text{Subst}\left(\int \right.}{}$$

$$= \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cos^{\frac{2}{3}}(a+bx)}{2b\sin^{\frac{2}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{}{-3} \right.}{}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{\log\left(1-\frac{\sin^{\frac{2}{3}}(a+bx)}{\cos^{\frac{2}{3}}(a+bx)}+\frac{\sin^{\frac{4}{3}}(a+bx)}{\cos^{\frac{4}{3}}(a+bx)}\right)}{4b} - \frac{3\cos}{2b\sin}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.37

$$-\frac{3\cos^2(a+bx)^{2/3}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};\sin^2(a+bx)\right)}{2b\sin^{\frac{2}{3}}(a+bx)\cos^{\frac{4}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(5/3)/Sin[a + b*x]^(5/3),x]
```

```
[Out] (-3*(Cos[a + b*x]^2)^(2/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, Sin[a + b*x]^
2])/(2*b*Cos[a + b*x]^(4/3)*Sin[a + b*x]^(2/3))
```

fricas [A] time = 0.97, size = 189, normalized size = 1.22

$$-\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cos(bx+a)-2\sqrt{3}\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}}{3\cos(bx+a)}\right)\sin(bx+a)-2\log\left(\frac{\cos(bx+a)^{\frac{1}{3}}\sin(bx+a)^{\frac{2}{3}}+\cos(bx+a)}{\cos(bx+a)}\right)\sin(bx+}{4b\sin(bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*cos(b*x + a) - 2*sqrt(3)*cos(b*x + a)^
(1/3)*sin(b*x + a)^(2/3))/cos(b*x + a))*sin(b*x + a) - 2*log((cos(b*x + a)^
(1/3)*sin(b*x + a)^(2/3) + cos(b*x + a))/cos(b*x + a))*sin(b*x + a) + log((
cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3) + cos(b*x + a)^(2/3)
*sin(b*x + a)^(4/3))/cos(b*x + a)^2)*sin(b*x + a) + 6*cos(b*x + a)^(2/3)*si
n(b*x + a)^(1/3))/(b*sin(b*x + a))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{5}{3}}(bx+a)}{\sin(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x)
```

```
[Out] int(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^{\frac{5}{3}}}{\sin(bx+a)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/3)/sin(b*x+a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(5/3)/sin(b*x + a)^(5/3), x)
```

mupad [B] time = 1.20, size = 44, normalized size = 0.28

$$-\frac{3\cos(a+b\,x)^{8/3}\left(\sin(a+b\,x)^2\right)^{1/3}{}_2\mathrm{F}_1\left(\frac{4}{3},\frac{4}{3};\ \frac{7}{3};\ \cos(a+b\,x)^2\right)}{8\,b\sin(a+b\,x)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(5/3)/sin(a + b*x)^(5/3),x)
```

```
[Out] -(3*cos(a + b*x)^(8/3)*(sin(a + b*x)^2)^(1/3)*hypergeom([4/3, 4/3], 7/3, co
s(a + b*x)^2))/(8*b*sin(a + b*x)^(2/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(5/3)/sin(b*x+a)**(5/3),x)
```

```
[Out] Timed out
```

3.333 $\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx$

Optimal. Leaf size=155

$$-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}+\frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

```
[Out] 1/4*ln(1+cos(b*x+a)^(4/3)/sin(b*x+a)^(4/3)-cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3
))/b-1/2*ln(1+cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))/b-3/4*cos(b*x+a)^(4/3)/b/s
in(b*x+a)^(4/3)-1/2*arctan(1/3*(1-2*cos(b*x+a)^(2/3)/sin(b*x+a)^(2/3))*3^(1
/2))*3^(1/2)/b
```

Rubi [A] time = 0.13, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2567, 2575, 275, 292, 31, 634, 618, 204, 628}

$$-\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)}+\frac{\log\left(\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{4b}-\frac{\log\left(\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}+1\right)}{2b}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(7/3)/Sin[a + b*x]^(7/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - (2*Cos[a + b*x]^(2/3))/Sin[a + b*x]^(2/3))/Sqrt[3]])/
(2*b) + Log[1 + Cos[a + b*x]^(4/3)/Sin[a + b*x]^(4/3) - Cos[a + b*x]^(2/3)/
Sin[a + b*x]^(2/3)]/(4*b) - Log[1 + Cos[a + b*x]^(2/3)/Sin[a + b*x]^(2/3)]/
(2*b) - (3*Cos[a + b*x]^(4/3))/(4*b*Sin[a + b*x]^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{3}}(a+bx)}{\sin^{\frac{7}{3}}(a+bx)}\,dx = -\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} - \int \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\,dx$$

$$= -\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{x^3}{1+x^6}\,dx, x, \frac{\sqrt[3]{\cos(a+bx)}}{\sqrt[3]{\sin(a+bx)}}\right)}{b}$$

$$= -\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} + \frac{3\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} - \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} + \frac{\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b}$$

$$= -\frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} + \frac{\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} + \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cos^{\frac{4}{3}}(a+bx)}{4b\sin^{\frac{4}{3}}(a+bx)} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-\right.}{}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}}{\sqrt{3}}\right)}{2b} + \frac{\log\left(1+\frac{\cos^{\frac{4}{3}}(a+bx)}{\sin^{\frac{4}{3}}(a+bx)}-\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{4b} - \frac{\log\left(1+\frac{\cos^{\frac{2}{3}}(a+bx)}{\sin^{\frac{2}{3}}(a+bx)}\right)}{2b} - \frac{3\cos}{4b\sin}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.37

$$-\frac{3\sqrt[3]{\cos^2(a+bx)}\ {}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};\sin^2(a+bx)\right)}{4b\sin^{\frac{4}{3}}(a+bx)\cos^{\frac{2}{3}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(7/3)/Sin[a + b*x]^(7/3),x]
```

```
[Out] (-3*(Cos[a + b*x]^2)^(1/3)*Hypergeometric2F1[-2/3, -2/3, 1/3, Sin[a + b*x]^
2])/(4*b*Cos[a + b*x]^(2/3)*Sin[a + b*x]^(4/3))
```

fricas [A] time = 0.95, size = 219, normalized size = 1.41

$$2\left(\sqrt{3}\cos(bx+a)^2-\sqrt{3}\right)\arctan\left(\frac{2\sqrt{3}\cos(bx+a)^{\frac{2}{3}}\sin(bx+a)^{\frac{1}{3}}-\sqrt{3}\sin(bx+a)}{3\sin(bx+a)}\right)+\left(\cos(bx+a)^2-1\right)\log\left(\frac{4\left(\cos(bx+a)^2-\cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(sqrt(3)*cos(b*x + a)^2 - sqrt(3))*arctan(1/3*(2*sqrt(3)*cos(b*x + a
)^(2/3)*sin(b*x + a)^(1/3) - sqrt(3)*sin(b*x + a))/sin(b*x + a)) + (cos(b*x
 + a)^2 - 1)*log(4*(cos(b*x + a)^2 - cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3)
+ cos(b*x + a)^(2/3)*sin(b*x + a)^(4/3) - 1)/(cos(b*x + a)^2 - 1)) - 2*(cos
(b*x + a)^2 - 1)*log(-2*(cos(b*x + a)^(2/3)*sin(b*x + a)^(1/3) + sin(b*x +
a))/sin(b*x + a)) + 3*cos(b*x + a)^(4/3)*sin(b*x + a)^(2/3))/(b*cos(b*x + a
)^2 - b)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int\frac{\cos^{\frac{7}{3}}(bx+a)}{\sin(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x)
```

```
[Out] int(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^{\frac{7}{3}}}{\sin(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/3)/sin(b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(7/3)/sin(b*x + a)^(7/3), x)
```

mupad [B] time = 1.91, size = 44, normalized size = 0.28

$$-\frac{3\cos(a+b\,x)^{10/3}\left(\sin(a+b\,x)^2\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{5}{3},\frac{5}{3};\ \frac{8}{3};\ \cos(a+b\,x)^2\right)}{10\,b\sin(a+b\,x)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(7/3)/sin(a + b*x)^(7/3),x)
```

```
[Out] -(3*cos(a + b*x)^(10/3)*(sin(a + b*x)^2)^(2/3)*hypergeom([5/3, 5/3], 8/3, c
os(a + b*x)^2))/(10*b*sin(a + b*x)^(4/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(7/3)/sin(b*x+a)**(7/3),x)
```

```
[Out] Timed out
```

3.334 $\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx$

Optimal. Leaf size=16

$$-\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

```
[Out] -3/5*cos(x)^(5/3)/sin(x)^(5/3)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2563}

$$-\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^(2/3)/Sin[x]^(8/3),x]
```

```
[Out] (-3*Cos[x]^(5/3))/(5*Sin[x]^(5/3))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin^{\frac{8}{3}}(x)}\,dx = -\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{3\cos^{\frac{5}{3}}(x)}{5\sin^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^(2/3)/Sin[x]^(8/3),x]
[Out] (-3*Cos[x]^(5/3))/(5*Sin[x]^(5/3))
```

fricas [A] time = 0.86, size = 18, normalized size = 1.12

$$\frac{3 \cos(x)^{\frac{5}{3}} \sin(x)^{\frac{1}{3}}}{5 \left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(2/3)/sin(x)^(8/3),x, algorithm="fricas")
[Out] 3/5*cos(x)^(5/3)*sin(x)^(1/3)/(cos(x)^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)^{\frac{2}{3}}}{\sin(x)^{\frac{8}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(2/3)/sin(x)^(8/3),x, algorithm="giac")
[Out] integrate(cos(x)^(2/3)/sin(x)^(8/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\cos^{\frac{2}{3}}(x)}{\sin(x)^{\frac{8}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(2/3)/sin(x)^(8/3),x)
[Out] int(cos(x)^(2/3)/sin(x)^(8/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)^{\frac{2}{3}}}{\sin(x)^{\frac{8}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(2/3)/sin(x)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)^(2/3)/sin(x)^(8/3), x)
```

mupad [B] time = 0.83, size = 10, normalized size = 0.62

$$-\frac{3\cos(x)^{5/3}}{5\sin(x)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(2/3)/sin(x)^(8/3),x)
```

```
[Out] -(3*cos(x)^(5/3))/(5*sin(x)^(5/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**(2/3)/sin(x)**(8/3),x)
```

```
[Out] Timed out
```

3.335 $\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx$

Optimal. Leaf size=16

$$\frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

```
[Out] 3/5*sin(x)^(5/3)/cos(x)^(5/3)
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2563}

$$\frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^(2/3)/Cos[x]^(8/3),x]
```

```
[Out] (3*Sin[x]^(5/3))/(5*Cos[x]^(5/3))
```

Rule 2563

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Cos[e + f*x])^(n + 1))
/(a*b*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos^{\frac{8}{3}}(x)}\,dx = \frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$\frac{3\sin^{\frac{5}{3}}(x)}{5\cos^{\frac{5}{3}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^(2/3)/Cos[x]^(8/3),x]
```

```
[Out] (3*Sin[x]^(5/3))/(5*Cos[x]^(5/3))
```

fricas [A] time = 0.82, size = 10, normalized size = 0.62

$$\frac{3\,\sin(x)^{\frac{5}{3}}}{5\,\cos(x)^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(2/3)/cos(x)^(8/3),x, algorithm="fricas")
```

```
[Out] 3/5*sin(x)^(5/3)/cos(x)^(5/3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)^{\frac{2}{3}}}{\cos(x)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(2/3)/cos(x)^(8/3),x, algorithm="giac")
```

```
[Out] integrate(sin(x)^(2/3)/cos(x)^(8/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sin^{\frac{2}{3}}(x)}{\cos(x)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^(2/3)/cos(x)^(8/3),x)
```

```
[Out] int(sin(x)^(2/3)/cos(x)^(8/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)^{\frac{2}{3}}}{\cos(x)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^(2/3)/cos(x)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^(2/3)/cos(x)^(8/3), x)
```

mupad [B] time = 0.82, size = 94, normalized size = 5.88

$$\frac{6\,2^{2/3}\tan\left(\frac{x}{2}\right)^{5/3}\left(1-\tan\left(\frac{x}{2}\right)^2\right)^{1/3}+6\,2^{2/3}\tan\left(\frac{x}{2}\right)^{11/3}\left(1-\tan\left(\frac{x}{2}\right)^2\right)^{1/3}}{5\tan\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)^2\left(10\tan\left(\frac{x}{2}\right)^2-5\tan\left(\frac{x}{2}\right)^2\left(\tan\left(\frac{x}{2}\right)^2+1\right)+10\right)+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^(2/3)/cos(x)^(8/3),x)
```

```
[Out] (6*2^(2/3)*tan(x/2)^(5/3)*(1 - tan(x/2)^2)^(1/3) + 6*2^(2/3)*tan(x/2)^(11/3
)*(1 - tan(x/2)^2)^(1/3))/(5*tan(x/2)^2 - tan(x/2)^2*(10*tan(x/2)^2 - 5*tan
(x/2)^2*(tan(x/2)^2 + 1) + 10) + 5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**(2/3)/cos(x)**(8/3),x)
```

```
[Out] Timed out
```

3.336 $\int \cos^n(e+fx)\sin^m(e+fx)\,dx$

Optimal. Leaf size=80

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}\cos^{n+1}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(n+1)}$$

```
[Out] -cos(f*x+e)^(1+n)*hypergeom([1/2-1/2*m, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)^2
)*sin(f*x+e)^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2576}

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}\cos^{n+1}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^n*Sin[e + f*x]^m,x]
```

```
[Out] -((Cos[e + f*x]^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)/2,
Cos[e + f*x]^2]*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1 +
 n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \cos^n(e+fx)\sin^m(e+fx)\,dx = -\frac{\cos^{1+n}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\sin^{-1+m}(e+fx)\sin^2(e+f}{f(1+n)}$$

Mathematica [A] time = 0.11, size = 79, normalized size = 0.99

$$\frac{\sin^{m+1}(e+fx)\cos^{n-1}(e+fx)\cos^2(e+fx)^{\frac{1-n}{2}}\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^n*Sin[e + f*x]^m,x]
```

```
[Out] (Cos[e + f*x]^(-1 + n)*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1[(1 +
m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x]^(1 + m))/(f*(1 + m
))
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\cos\left(fx+e\right)^{n}\sin\left(fx+e\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^n*sin(f*x+e)^m,x, algorithm="fricas")
```

```
[Out] integral(cos(f*x + e)^n*sin(f*x + e)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)^{n}\sin\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^n*sin(f*x+e)^m,x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^n*sin(f*x + e)^m, x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \left(\cos^{n}\left(fx+e\right)\right)\left(\sin^{m}\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^n*sin(f*x+e)^m,x)
```

```
[Out] int(cos(f*x+e)^n*sin(f*x+e)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(fx+e\right)^{n}\sin\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^n*sin(f*x+e)^m,x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^n*sin(f*x + e)^m, x)
```

mupad [B] time = 2.35, size = 71, normalized size = 0.89

$$-\frac{\cos\left(e+f\,x\right)^{n+1}\,\sin\left(e+f\,x\right)^{m+1}\,{}_2F_1\left(\frac{1}{2}-\frac{m}{2},\frac{n}{2}+\frac{1}{2};\ \frac{n}{2}+\frac{3}{2};\ \cos\left(e+f\,x\right)^2\right)}{f\,\left(n+1\right)\,\left(\sin\left(e+f\,x\right)^2\right)^{\frac{m}{2}+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^n*sin(e + f*x)^m,x)
```

```
[Out] -(cos(e + f*x)^(n + 1)*sin(e + f*x)^(m + 1)*hypergeom([1/2 - m/2, n/2 + 1/2
], n/2 + 3/2, cos(e + f*x)^2))/(f*(n + 1)*(sin(e + f*x)^2)^(m/2 + 1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{m}\left(e+fx\right)\cos^{n}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**n*sin(f*x+e)**m,x)
```

```
[Out] Integral(sin(e + f*x)**m*cos(e + f*x)**n, x)
```

3.337 $\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx$

Optimal. Leaf size=85

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)}$$

```
[Out] -(d*cos(f*x+e))^(1+n)*hypergeom([1/2-1/2*m, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+
e)^2)*sin(f*x+e)^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/d/f/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2576}

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*Sin[e + f*x]^m,x]
```

```
[Out] -(((d*Cos[e + f*x])^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)
/2, Cos[e + f*x]^2]*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(d*
f*(1 + n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(e+fx))^n \sin^m(e+fx)\,dx = -\frac{(d\cos(e+fx))^{1+n}\,{}_2F_1\left(\frac{1-m}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)\sin^{-1+m}(e+fx)\sin^2(}{df(1+n)}$$

Mathematica [A] time = 0.12, size = 82, normalized size = 0.96

$$\frac{d\sin^{m+1}(e+fx)\cos^2(e+fx)^{\frac{1-n}{2}}(d\cos(e+fx))^{n-1}\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*Sin[e + f*x]^m,x]
```

```
[Out] (d*(d*Cos[e + f*x])^(-1 + n)*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1
[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x]^(1 + m))/(f*
(1 + m))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\cos\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*sin(f*x+e)^m,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^n*sin(f*x + e)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*sin(f*x+e)^m,x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n*sin(f*x + e)^m, x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(fx+e\right)\right)^{n}\left(\sin^{m}\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*sin(f*x+e)^m,x)
```

```
[Out] int((d*cos(f*x+e))^n*sin(f*x+e)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*sin(f*x+e)^m,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^n*sin(f*x + e)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^m\,\left(d\,\cos\left(e+f\,x\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^m*(d*cos(e + f*x))^n,x)
[Out] int(sin(e + f*x)^m*(d*cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(e+fx\right)\right)^n \sin^m\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*sin(f*x+e)**m,x)
[Out] Integral((d*cos(e + f*x))**n*sin(e + f*x)**m, x)
```

3.338 $\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx$

Optimal. Leaf size=83

$$-\frac{b\sin^2(e+fx)^{\frac{1-m}{2}}\cos^{n+1}(e+fx)(b\sin(e+fx))^{m-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(n+1)}$$

```
[Out] -b*cos(f*x+e)^(1+n)*hypergeom([1/2-1/2*m, 1/2+1/2*n],[3/2+1/2*n],cos(f*x+e)
^2)*(b*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2576}

$$-\frac{b\sin^2(e+fx)^{\frac{1-m}{2}}\cos^{n+1}(e+fx)(b\sin(e+fx))^{m-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^n*(b*Sin[e + f*x])^m,x]
```

```
[Out] -((b*Cos[e + f*x]^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 + n)/2
, Cos[e + f*x]^2]*(b*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(
f*(1 + n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \cos^n(e+fx)(b\sin(e+fx))^m\,dx = -\frac{b\cos^{1+n}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)(b\sin(e+fx))^{-1+m}\sin}{f(1+n)}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 1.02

$$\frac{\sin(e+fx)\cos^{n-1}(e+fx)\cos^2(e+fx)^{\frac{1-n}{2}}(b\sin(e+fx))^m\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^n*(b*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]^(-1 + n)*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1[(1 +
m)/2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(b*Sin[e + f*x])^m
)/(f*(1 + m))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)\right)^{m}\cos\left(fx+e\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^n*(b*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e))^m*cos(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{m}\cos\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^n*(b*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e))^m*cos(f*x + e)^n, x)
```

maple [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \left(\cos^{n}\left(fx+e\right)\right)\left(b\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^n*(b*sin(f*x+e))^m,x)
```

```
[Out] int(cos(f*x+e)^n*(b*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)\right)^{m}\cos\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^n*(b*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e))^m*cos(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^n\,\left(b\,\sin\left(e+f\,x\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^n*(b*sin(e + f*x))^m,x)
[Out] int(cos(e + f*x)^n*(b*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(e+fx\right)\right)^m \cos^n\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**n*(b*sin(f*x+e))**m,x)
[Out] Integral((b*sin(e + f*x))**m*cos(e + f*x)**n, x)
```

3.339 $\int (d\cos(e+fx))^n (b\sin(e+fx))^m\,dx$

Optimal. Leaf size=88

$$-\frac{b\sin^2(e+fx)^{\frac{1-m}{2}}(b\sin(e+fx))^{m-1}(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)}$$

```
[Out] -b*(d*cos(f*x+e))^(1+n)*hypergeom([1/2-1/2*m, 1/2+1/2*n],[3/2+1/2*n],cos(f*
x+e)^2)*(b*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/d/f/(1+n)
```

Rubi [A] time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2576}

$$-\frac{b\sin^2(e+fx)^{\frac{1-m}{2}}(b\sin(e+fx))^{m-1}(d\cos(e+fx))^{n+1}\,{}_2F_1\left(\frac{1-m}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(e+fx)\right)}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^n*(b*Sin[e + f*x])^m,x]
```

```
[Out] -((b*(d*Cos[e + f*x])^(1 + n)*Hypergeometric2F1[(1 - m)/2, (1 + n)/2, (3 +
n)/2, Cos[e + f*x]^2]*(b*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2
))/(d*f*(1 + n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(e+fx))^n(b\sin(e+fx))^m\,dx = -\frac{b(d\cos(e+fx))^{1+n}\,{}_2F_1\left(\frac{1-m}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(e+fx)\right)(b\sin(e+fx))^{-1+}}{df(1+n)}$$

Mathematica [A] time = 0.09, size = 85, normalized size = 0.97

$$\frac{\tan(e+fx)\cos^2(e+fx)^{\frac{1-n}{2}}(b\sin(e+fx))^m(d\cos(e+fx))^n\,{}_2F_1\left(\frac{m+1}{2},\frac{1-n}{2};\frac{m+3}{2};\sin^2(e+fx)\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[e + f*x])^n*(b*Sin[e + f*x])^m,x]
```

```
[Out] ((d*Cos[e + f*x])^n*(Cos[e + f*x]^2)^((1 - n)/2)*Hypergeometric2F1[(1 + m)/
2, (1 - n)/2, (3 + m)/2, Sin[e + f*x]^2]*(b*Sin[e + f*x])^m*Tan[e + f*x])/(
f*(1 + m))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\cos\left(fx+e\right)\right)^{n}\left(b\sin\left(fx+e\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e))^n*(b*sin(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(fx+e\right)\right)^{n}\left(b\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((d*cos(f*x + e))^n*(b*sin(f*x + e))^m, x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(fx+e\right)\right)^{n}\left(b\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x)
```

```
[Out] int((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(fx+e\right)\right)^{n}\left(b\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^n*(b*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(f*x + e))^n*(b*sin(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\,\cos\left(e+f\,x\right)\right)^n\,\left(b\,\sin\left(e+f\,x\right)\right)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^n*(b*sin(e + f*x))^m,x)
[Out] int((d*cos(e + f*x))^n*(b*sin(e + f*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(e+fx\right)\right)^{m}\left(d\cos\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**n*(b*sin(f*x+e))**m,x)
[Out] Integral((b*sin(e + f*x))**m*(d*cos(e + f*x))**n, x)
```

3.340 $\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=74

$$\frac{(c\sin(a+bx))^{m+5}}{bc^5(m+5)}-\frac{2(c\sin(a+bx))^{m+3}}{bc^3(m+3)}+\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}$$

```
[Out] (c*sin(b*x+a))^(1+m)/b/c/(1+m)-2*(c*sin(b*x+a))^(3+m)/b/c^3/(3+m)+(c*sin(b*
x+a))^(5+m)/b/c^5/(5+m)
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2564, 270}

$$-\frac{2(c\sin(a+bx))^{m+3}}{bc^3(m+3)}+\frac{(c\sin(a+bx))^{m+5}}{bc^5(m+5)}+\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5*(c*Sin[a + b*x])^m,x]
```

```
[Out] (c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m)) - (2*(c*Sin[a + b*x])^(3 + m))/(b*c^
3*(3 + m)) + (c*Sin[a + b*x])^(5 + m)/(b*c^5*(5 + m))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \cos^5(a+bx)(c\sin(a+bx))^m\,dx = \frac{\text{Subst}\left(\int x^m\left(1-\frac{x^2}{c^2}\right)^2\,dx,x,c\sin(a+bx)\right)}{bc}$$
$$= \frac{\text{Subst}\left(\int \left(x^m-\frac{2x^{2+m}}{c^2}+\frac{x^{4+m}}{c^4}\right)dx,x,c\sin(a+bx)\right)}{bc}$$
$$= \frac{(c\sin(a+bx))^{1+m}}{bc(1+m)} - \frac{2(c\sin(a+bx))^{3+m}}{bc^3(3+m)} + \frac{(c\sin(a+bx))^{5+m}}{bc^5(5+m)}$$

Mathematica [A] time = 0.31, size = 55, normalized size = 0.74

$$\frac{\sin(a+bx)\left(\frac{\sin^4(a+bx)}{m+5}-\frac{2\sin^2(a+bx)}{m+3}+\frac{1}{m+1}\right)(c\sin(a+bx))^m}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5*(c*Sin[a + b*x])^m,x]
[Out] (Sin[a + b*x]*(c*Sin[a + b*x])^m*((1 + m)^(-1) - (2*Sin[a + b*x]^2)/(3 + m)
 + Sin[a + b*x]^4/(5 + m)))/b
```

fricas [A] time = 0.63, size = 70, normalized size = 0.95

$$\frac{\left(\left(m^2+4\,m+3\right)\cos\left(bx+a\right)^4+4\left(m+1\right)\cos\left(bx+a\right)^2+8\right)\left(c\sin\left(bx+a\right)\right)^m\sin\left(bx+a\right)}{bm^3+9\,bm^2+23\,bm+15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="fricas")
[Out] ((m^2 + 4*m + 3)*cos(b*x + a)^4 + 4*(m + 1)*cos(b*x + a)^2 + 8)*(c*sin(b*x
+ a))^m*sin(b*x + a)/(b*m^3 + 9*b*m^2 + 23*b*m + 15*b)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 1.78, size = 0, normalized size = 0.00

$$\int \left(\cos^5 (bx + a)\right) (c \sin (bx + a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5*(c*sin(b*x+a))^m,x)
```

```
[Out] int(cos(b*x+a)^5*(c*sin(b*x+a))^m,x)
```

maxima [A] time = 0.56, size = 77, normalized size = 1.04

$$\frac{\frac{c^m \sin(bx+a)^m \sin(bx+a)^5}{m+5} - \frac{2\,c^m \sin(bx+a)^m \sin(bx+a)^3}{m+3} + \frac{(c\sin(bx+a))^{m+1}}{c(m+1)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] (c^m*sin(b*x + a)^m*sin(b*x + a)^5/(m + 5) - 2*c^m*sin(b*x + a)^m*sin(b*x +
 a)^3/(m + 3) + (c*sin(b*x + a))^(m + 1)/(c*(m + 1)))/b
```

mupad [B] time = 1.61, size = 132, normalized size = 1.78

$$\frac{(c\ \sin(a+b\,x))^m\ \left(150\ \sin(a+b\,x)+25\ \sin(3\,a+3\,b\,x)+3\ \sin(5\,a+5\,b\,x)+24\,m\ \sin(a+b\,x)+28\,m\ \sin(3\right.}{16\,b\ \left(m^3+9\,m^2+23\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5*(c*sin(a + b*x))^m,x)
```

```
[Out] ((c*sin(a + b*x))^m*(150*sin(a + b*x) + 25*sin(3*a + 3*b*x) + 3*sin(5*a + 5
*b*x) + 24*m*sin(a + b*x) + 28*m*sin(3*a + 3*b*x) + 4*m*sin(5*a + 5*b*x) +
2*m^2*sin(a + b*x) + 3*m^2*sin(3*a + 3*b*x) + m^2*sin(5*a + 5*b*x)))/(16*b*
(23*m + 9*m^2 + m^3 + 15))
```

sympy [A] time = 66.31, size = 2050, normalized size = 27.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5*(c*sin(b*x+a))**m,x)
```

```
[Out] Piecewise((x*(c*sin(a))**m*cos(a)**5, Eq(b, 0)), ((log(sin(a + b*x))/b + co
s(a + b*x)**2/(2*b*sin(a + b*x)**2) - cos(a + b*x)**4/(4*b*sin(a + b*x)**4)
```

)/c**5, Eq(m, -5)), ((16*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(
8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**
2) + 32*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x
/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) + 16*log(tan(a
/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan
(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16*log(tan(a/2 + b*x/2))*tan(
a/2 + b*x/2)**6/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*t
an(a/2 + b*x/2)**2) - 32*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(8*b*tan
(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - 16
*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(8*b*tan(a/2 + b*x/2)**6 + 16*b*
tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) - tan(a/2 + b*x/2)**8/(8*b*t
an(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4 + 8*b*tan(a/2 + b*x/2)**2) +
18*tan(a/2 + b*x/2)**4/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*x/2)**4
+ 8*b*tan(a/2 + b*x/2)**2) - 1/(8*b*tan(a/2 + b*x/2)**6 + 16*b*tan(a/2 + b*
x/2)**4 + 8*b*tan(a/2 + b*x/2)**2))/c**3, Eq(m, -3)), ((-log(tan(a/2 + b*x/
2)**2 + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2
)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a
/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/
2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 6*
log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4
*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2
+ b) - 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/
2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b
*x/2)**2 + b) - log(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**8 + 4*b*t
an(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b)
 + log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*t
an(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b)
 + 4*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b
*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 +
b) + 6*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4
*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2
+ b) + 4*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 +
 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**
2 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2
)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*tan(a/2 +
 b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 +
 b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*tan(a/2 + b*x/2)**4/(b*tan(a/
2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan
(a/2 + b*x/2)**2 + b) - 4*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*
tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b
))/c, Eq(m, -1)), (c**m*m**2*sin(a + b*x)*sin(a + b*x)**m*cos(a + b*x)**4/(
b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 4*c**m*m*sin(a + b*x)**3*sin(a + b*x)*
*m*cos(a + b*x)**2/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 8*c**m*m*sin(a + b
*x)*sin(a + b*x)**m*cos(a + b*x)**4/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) + 8
*c**m*sin(a + b*x)**5*sin(a + b*x)**m/(b*m**3 + 9*b*m**2 + 23*b*m + 15*b) +

```
 20*c**m*sin(a + b*x)**3*sin(a + b*x)**m*cos(a + b*x)**2/(b*m**3 + 9*b*m**2
 + 23*b*m + 15*b) + 15*c**m*sin(a + b*x)*sin(a + b*x)**m*cos(a + b*x)**4/(b
*m**3 + 9*b*m**2 + 23*b*m + 15*b), True))
```

3.341 $\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=50

$$\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}-\frac{(c\sin(a+bx))^{m+3}}{bc^3(m+3)}$$

```
[Out] (c*sin(b*x+a))^(1+m)/b/c/(1+m)-(c*sin(b*x+a))^(3+m)/b/c^3/(3+m)
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2564, 14}

$$\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}-\frac{(c\sin(a+bx))^{m+3}}{bc^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

```
[Out] (c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m)) - (c*Sin[a + b*x])^(3 + m)/(b*c^3*(3
 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \cos^3(a+bx)(c\sin(a+bx))^m\,dx = \frac{\text{Subst}\left(\int x^m\left(1-\frac{x^2}{c^2}\right)dx, x, c\sin(a+bx)\right)}{bc}$$
$$= \frac{\text{Subst}\left(\int \left(x^m-\frac{x^{2+m}}{c^2}\right)dx, x, c\sin(a+bx)\right)}{bc}$$
$$= \frac{(c\sin(a+bx))^{1+m}}{bc(1+m)} - \frac{(c\sin(a+bx))^{3+m}}{bc^3(3+m)}$$

Mathematica [A] time = 0.09, size = 48, normalized size = 0.96

$$\frac{\sin(a+bx)((m+1)\cos(2(a+bx))+m+5)(c\sin(a+bx))^m}{2b(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

```
[Out] ((5 + m + (1 + m)*Cos[2*(a + b*x)])*Sin[a + b*x]*(c*Sin[a + b*x])^m)/(2*b*(
1 + m)*(3 + m))
```

fricas [A] time = 0.66, size = 46, normalized size = 0.92

$$\frac{\left((m+1)\cos(bx+a)^2+2\right)(c\sin(bx+a))^m\sin(bx+a)}{bm^2+4\,bm+3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] ((m + 1)*cos(b*x + a)^2 + 2)*(c*sin(b*x + a))^m*sin(b*x + a)/(b*m^2 + 4*b*m
 + 3*b)
```

giac [A] time = 3.05, size = 92, normalized size = 1.84

$$-\frac{(c\sin(bx+a))^m\,m\sin(bx+a)^3+(c\sin(bx+a))^m\sin(bx+a)^3-(c\sin(bx+a))^m\,m\sin(bx+a)-3\,(c\sin(b}{\left(m^2+4\,m+3\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] -((c*sin(b*x + a))^m*m*sin(b*x + a)^3 + (c*sin(b*x + a))^m*sin(b*x + a)^3 -
 (c*sin(b*x + a))^m*m*sin(b*x + a) - 3*(c*sin(b*x + a))^m*sin(b*x + a))/((m
^2 + 4*m + 3)*b)
```

maple [F] time = 1.17, size = 0, normalized size = 0.00

$$\int \left(\cos^3(bx+a)\right)(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

```
[Out] int(cos(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

maxima [A] time = 0.43, size = 53, normalized size = 1.06

$$-\frac{\frac{c^m \sin(bx+a)^m \sin(bx+a)^3}{m+3} - \frac{(c\sin(bx+a))^{m+1}}{c(m+1)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] -(c^m*sin(b*x + a)^m*sin(b*x + a)^3/(m + 3) - (c*sin(b*x + a))^(m + 1)/(c*(
m + 1)))/b
```

mupad [B] time = 0.88, size = 62, normalized size = 1.24

$$\frac{(c\,\sin(a+b\,x))^m\,\left(9\,\sin(a+b\,x)+\sin(3\,a+3\,b\,x)+m\,\sin(a+b\,x)+m\,\sin(3\,a+3\,b\,x)\right)}{4\,b\,\left(m^2+4\,m+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*(c*sin(a + b*x))^m,x)
```

```
[Out] ((c*sin(a + b*x))^m*(9*sin(a + b*x) + sin(3*a + 3*b*x) + m*sin(a + b*x) + m
*sin(3*a + 3*b*x)))/(4*b*(4*m + m^2 + 3))
```

sympy [A] time = 13.28, size = 530, normalized size = 10.60

$$\begin{cases} x\left(c\sin\left(a\right)\right)^{m}\cos^{3}\left(a\right) \\ \frac{-\frac{\log\left(\sin\left(a+bx\right)\right)}{b}-\frac{\cos^{2}\left(a+bx\right)}{2b\sin^{2}\left(a+bx\right)}}{c^{3}} \\ \frac{-\frac{\log\left(\tan^{2}\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^{2}\left(\frac{a}{2}+\frac{bx}{2}\right)+b}-\frac{2\log\left(\tan^{2}\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)\tan^{2}\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^{2}\left(\frac{a}{2}+\frac{bx}{2}\right)+b}-\frac{\log\left(\tan^{2}\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{b\tan^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^{2}\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)\tan^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)}{b\tan^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)+2b\tan^{2}\left(\frac{a}{2}+\frac{bx}{2}\right)+b}+\frac{2\log\left(\tan\left(\frac{a}{2}+\cdots\right.\right.}{b\tan^{4}\left(\frac{a}{2}+\frac{bx}{2}\right)\cdots}}{c} \\ \frac{c^{m}m\sin\left(a+bx\right)\sin^{m}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{bm^{2}+4bm+3b}+\frac{2c^{m}\sin^{3}\left(a+bx\right)\sin^{m}\left(a+bx\right)}{bm^{2}+4bm+3b}+\frac{3c^{m}\sin\left(a+bx\right)\sin^{m}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{bm^{2}+4bm+3b} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*(c*sin(b*x+a))**m,x)
```

```
[Out] Piecewise((x*(c*sin(a))**m*cos(a)**3, Eq(b, 0)), ((-log(sin(a + b*x))/b - c
os(a + b*x)**2/(2*b*sin(a + b*x)**2))/c**3, Eq(m, -3)), ((-log(tan(a/2 + b*
x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x
/2)**2 + b) - 2*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2
 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2)**2 + 1)/
(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2
))*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b
) + 2*log(tan(a/2 + b*x/2))*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*
b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2))/(b*tan(a/2 + b*x/2)**4 +
 2*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**
4 + 2*b*tan(a/2 + b*x/2)**2 + b))/c, Eq(m, -1)), (c**m*m*sin(a + b*x)*sin(a
 + b*x)**m*cos(a + b*x)**2/(b*m**2 + 4*b*m + 3*b) + 2*c**m*sin(a + b*x)**3*
sin(a + b*x)**m/(b*m**2 + 4*b*m + 3*b) + 3*c**m*sin(a + b*x)*sin(a + b*x)**
m*cos(a + b*x)**2/(b*m**2 + 4*b*m + 3*b), True))
```

3.342 $\int \cos(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=24

$$\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}$$

```
[Out] (c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 30}

$$\frac{(c\sin(a+bx))^{m+1}}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*(c*Sin[a + b*x])^m,x]
```

```
[Out] (c*Sin[a + b*x])^(1 + m)/(b*c*(1 + m))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)(c\sin(a+bx))^m\,dx &= \frac{\text{Subst}\left(\int x^m\,dx, x, c\sin(a+bx)\right)}{bc}\\ &= \frac{(c\sin(a+bx))^{1+m}}{bc(1+m)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.04

$$\frac{\sin(a+bx)(c\sin(a+bx))^m}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sin[a + b*x]*(c*Sin[a + b*x])^m)/(b*(1 + m))
```

fricas [A] time = 0.62, size = 24, normalized size = 1.00

$$\frac{(c\sin(bx+a))^m\sin(bx+a)}{bm+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] (c*sin(b*x + a))^m*sin(b*x + a)/(b*m + b)
```

giac [A] time = 1.93, size = 24, normalized size = 1.00

$$\frac{(c\sin(bx+a))^{m+1}}{bc(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] (c*sin(b*x + a))^(m + 1)/(b*c*(m + 1))
```

maple [A] time = 0.00, size = 25, normalized size = 1.04

$$\frac{(c\sin(bx+a))^{1+m}}{bc(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*(c*sin(b*x+a))^m,x)
```

```
[Out] (c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

maxima [A] time = 0.47, size = 24, normalized size = 1.00

$$\frac{(c\sin(bx+a))^{m+1}}{bc(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] (c*sin(b*x + a))^(m + 1)/(b*c*(m + 1))
```

mupad [B] time = 0.61, size = 25, normalized size = 1.04

$$\frac{\sin(a+b\,x)\,\left(c\,\sin(a+b\,x)\right)^m}{b\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(c*sin(a + b*x))^m,x)
```

```
[Out] (sin(a + b*x)*(c*sin(a + b*x))^m)/(b*(m + 1))
```

sympy [A] time = 2.04, size = 58, normalized size = 2.42

$$\begin{cases} \frac{x\cos(a)}{c\sin(a)} & \text{for } b = 0 \wedge m = -1 \\ x\,(c\sin(a))^m\cos(a) & \text{for } b = 0 \\ \frac{\log(\sin(a+bx))}{bc} & \text{for } m = -1 \\ \frac{c^m\sin(a+bx)\sin^m(a+bx)}{bm+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*(c*sin(b*x+a))**m,x)
```

```
[Out] Piecewise((x*cos(a)/(c*sin(a)), Eq(b, 0) & Eq(m, -1)), (x*(c*sin(a))**m*cos
(a), Eq(b, 0)), (log(sin(a + b*x))/(b*c), Eq(m, -1)), (c**m*sin(a + b*x)*si
n(a + b*x)**m/(b*m + b), True))
```

3.343 $\int \sec(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=48

$$\frac{(c\sin(a+bx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[Out] hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+a))^(1+m)/b/c
/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 364}

$$\frac{(c\sin(a+bx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x]
)^(1 + m))/(b*c*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \sec(a+bx)(c\sin(a+bx))^m\,dx = \frac{\text{Subst}\left(\int \frac{x^m}{1-\frac{x^2}{c^2}}\,dx, x, c\sin(a+bx)\right)}{bc}$$
$$= \frac{{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.06

$$\frac{\sin(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+1}{2}+1;\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, Sin[a + b*x]^2]*Sin[a + b*x
]*(c*Sin[a + b*x])^m)/(b*(1 + m))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left((c\sin(bx+a))^m\sec(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m*sec(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \sec(bx+a)(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*(c*sin(b*x+a))^m,x)
```

```
[Out] int(sec(b*x+a)*(c*sin(b*x+a))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\ \sin(a+b\,x))^m}{\cos(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m/cos(a + b*x),x)
```

```
[Out] int((c*sin(a + b*x))^m/cos(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(a+bx))^m \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*(c*sin(b*x+a))**m,x)
```

```
[Out] Integral((c*sin(a + b*x))**m*sec(a + b*x), x)
```

3.344 $\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=48

$$\frac{(c\sin(a+bx))^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[Out] hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+a))^(1+m)/b/c
/(1+m)
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2564, 364}

$$\frac{(c\sin(a+bx))^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x]
)^(1 + m))/(b*c*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \sec^3(a+bx)(c\sin(a+bx))^m\,dx = \frac{\text{Subst}\left(\int \frac{x^m}{\left(1-\frac{x^2}{c^2}\right)^2}\,dx, x, c\sin(a+bx)\right)}{bc}$$
$$= \frac{{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.06

$$\frac{\sin(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+1}{2}+1;\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^3*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, Sin[a + b*x]^2]*Sin[a + b*x
]*(c*Sin[a + b*x])^m)/(b*(1 + m))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left((c\sin(bx+a))^m\sec(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m*sec(b*x + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m\sec(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^3, x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \left(\sec^3(bx+a)\right)(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

```
[Out] int(sec(b*x+a)^3*(c*sin(b*x+a))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m \sec(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,\sin(a+b\,x))^m}{\cos(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m/cos(a + b*x)^3,x)
```

```
[Out] int((c*sin(a + b*x))^m/cos(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(a+bx))^m \sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3*(c*sin(b*x+a))**m,x)
```

```
[Out] Integral((c*sin(a + b*x))**m*sec(a + b*x)**3, x)
```

3.345 $\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=68

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([-3/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x
+a))^(1+m)/b/c/(1+m)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2577}

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)\,{}_2F_1\left(-\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.93

$$\frac{\sqrt{\cos^2(a+bx)}\,\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m)/2, Sin[a +
 b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c\sin\left(bx+a\right)\right)^{m}\cos\left(bx+a\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m*cos(b*x + a)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sin\left(bx+a\right)\right)^{m}\cos\left(bx+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m*cos(b*x + a)^4, x)
```

maple [F] time = 1.26, size = 0, normalized size = 0.00

$$\int \left(\cos^{4}\left(bx+a\right)\right)\left(c\sin\left(bx+a\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

```
[Out] int(cos(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sin\left(bx+a\right)\right)^{m}\cos\left(bx+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m*cos(b*x + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+bx)^4 (c\sin(a+bx))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4*(c*sin(a + b*x))^m,x)
[Out] int(cos(a + b*x)^4*(c*sin(a + b*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*(c*sin(b*x+a))**m,x)
[Out] Timed out
```

3.346 $\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=68

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([-1/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x
+a))^(1+m)/b/c/(1+m)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2577}

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]
*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^2(a+bx)(c\sin(a+bx))^m\,dx = \frac{\cos(a+bx)\,{}_2F_1\left(-\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.93

$$\frac{\sqrt{\cos^2(a+bx)}\,\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, Sin[a +
 b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left((c\sin(bx+a))^m\cos(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m*cos(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m\cos(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m*cos(b*x + a)^2, x)
```

maple [F] time = 1.11, size = 0, normalized size = 0.00

$$\int \left(\cos^2(bx+a)\right)(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

```
[Out] int(cos(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m\cos(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m*cos(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)^2\,{\left(c\,\sin(a+b\,x)\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*(c*sin(a + b*x))^m,x)
[Out] int(cos(a + b*x)^2*(c*sin(a + b*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*(c*sin(b*x+a))**m,x)
[Out] Timed out
```

3.347 $\int (c\sin(a+bx))^m\,dx$

Optimal. Leaf size=68

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+
a))^(1+m)/b/c/(1+m)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2643}

$$\frac{\cos(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^m,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*
(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (c\sin(a+bx))^m\,dx=\frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.93

$$\frac{\sqrt{\cos^2(a+bx)}\,\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left((c\sin(bx+a))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m, x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^m,x)
```

```
[Out] int((c*sin(b*x+a))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\ \sin (a+b\,x))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m,x)
```

```
[Out] int((c*sin(a + b*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin (a+bx))^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**m,x)
```

```
[Out] Integral((c*sin(a + b*x))**m, x)
```

3.348 $\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{\cos^2(a+bx)}\,\sec(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[Out] hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*sec(b*x+a)*(c*sin(b*x+
a))^(1+m)*(cos(b*x+a)^2)^(1/2)/b/c/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2577}

$$\frac{\sqrt{\cos^2(a+bx)}\,\sec(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*Sec[a + b*x]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^2(a+bx)(c\sin(a+bx))^m\,dx=\frac{\sqrt{\cos^2(a+bx)}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)\sec(a+bx)(c\sin(a+bx))^{1+m}}{bc(1+m)}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.93

$$\frac{\sqrt{\cos^2(a+bx)}\,\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left((c\sin(bx+a))^m\sec(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m*sec(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m\sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^2, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(\sec^2(bx+a)\right)(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

```
[Out] int(sec(b*x+a)^2*(c*sin(b*x+a))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m\sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\ \sin(a+b\,x))^m}{\cos(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m/cos(a + b*x)^2,x)
[Out] int((c*sin(a + b*x))^m/cos(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(a+bx))^m \sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*(c*sin(b*x+a))**m,x)
[Out] Integral((c*sin(a + b*x))**m*sec(a + b*x)**2, x)
```

3.349 $\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{\cos^2(a+bx)}\,\sec(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[Out] hypergeom([5/2, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*sec(b*x+a)*(c*sin(b*x+
a))^(1+m)*(cos(b*x+a)^2)^(1/2)/b/c/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2577}

$$\frac{\sqrt{\cos^2(a+bx)}\,\sec(a+bx)(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*Sec[a + b*x]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sec^4(a+bx)(c\sin(a+bx))^m\,dx = \frac{\sqrt{\cos^2(a+bx)}\,{}_2F_1\left(\frac{5}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)\sec(a+bx)(c\sin(a+bx))^{1+}}{bc(1+m)}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.93

$$\frac{\sqrt{\cos^2(a+bx)}\,\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[5/2, (1 + m)/2, (3 + m)/2, Sin[a +
b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left((c\sin(bx+a))^m\sec(bx+a)^4\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral((c*sin(b*x + a))^m*sec(b*x + a)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m\sec(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^4, x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \left(\sec^4(bx+a)\right)(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

```
[Out] int(sec(b*x+a)^4*(c*sin(b*x+a))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^m\sec(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m*sec(b*x + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,\sin(a+b\,x))^m}{\cos(a+b\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m/cos(a + b*x)^4,x)
```

```
[Out] int((c*sin(a + b*x))^m/cos(a + b*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4*(c*sin(b*x+a))**m,x)
```

```
[Out] Timed out
```

3.350 $\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=75

$$\frac{d\sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt[4]{\cos^2(a+bx)}}$$

```
[Out] d*hypergeom([-1/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+a))^(1+m
)*(d*cos(b*x+a))^(1/2)/b/c/(1+m)/(cos(b*x+a)^2)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2577}

$$\frac{d\sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt[4]{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d*Sqrt[d*Cos[a + b*x]]*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, Sin[a
 + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*(Cos[a + b*x]^2)^(1/4))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx=\frac{d\sqrt{d\cos(a+bx)}\,{}_2F_1\left(-\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt[4]{\cos^2(a+bx)}}$$

Mathematica [A] time = 0.13, size = 78, normalized size = 1.04

$$\frac{d^2\cos^2(a+bx)^{3/4}\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d^2*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, S
in[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m)*Sqrt[d*Cos[a + b
*x]])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\cos\left(bx+a\right)}\,\left(c\sin\left(bx+a\right)\right)^{m}d\cos\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m*d*cos(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}\left(c\sin\left(bx+a\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}\left(c\sin\left(bx+a\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

```
[Out] int((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}\left(c\sin\left(bx+a\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cos(a+bx))^{3/2} (c\sin(a+bx))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^m,x)
[Out] int((d*cos(a + b*x))^(3/2)*(c*sin(a + b*x))^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*(c*sin(b*x+a))**m,x)
[Out] Timed out
```

3.351 $\int \sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=75

$$\frac{d\sqrt[4]{\cos^2(a+bx)}\,(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{d\cos(a+bx)}}$$

```
[Out] d*(cos(b*x+a)^2)^(1/4)*hypergeom([1/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)
*(c*sin(b*x+a))^(1+m)/b/c/(1+m)/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2577}

$$\frac{d\sqrt[4]{\cos^2(a+bx)}\,(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^m\,dx = \frac{d\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)\sqrt{d\cos(a+bx)}}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 1.00

$$\frac{\sqrt[4]{\cos^2(a+bx)}\,\tan(a+bx)\sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^m\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

```
[Out] (Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)
/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\cos\left(bx+a\right)}\,\left(c\sin\left(bx+a\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\cos\left(bx+a\right)}\,\left(c\sin\left(bx+a\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int\sqrt{d\cos\left(bx+a\right)}\,\left(c\sin\left(bx+a\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

```
[Out] int((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\cos\left(bx+a\right)}\,\left(c\sin\left(bx+a\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,\cos\left(a+b\,x\right)}\,{\left(c\,\sin\left(a+b\,x\right)\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^m,x)
[Out] int((d*cos(a + b*x))^(1/2)*(c*sin(a + b*x))^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \sin{\left(a + b x \right)}\right)^{m} \sqrt{d \cos{\left(a + b x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*(c*sin(b*x+a))**m,x)
[Out] Integral((c*sin(a + b*x))**m*sqrt(d*cos(a + b*x)), x)
```

3.352 $\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=75

$$\frac{d\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)(d\cos(a+bx))^{3/2}}$$

```
[Out] d*(cos(b*x+a)^2)^(3/4)*hypergeom([3/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)
*(c*sin(b*x+a))^(1+m)/b/c/(1+m)/(d*cos(b*x+a))^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2577}

$$\frac{d\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^m/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m)*(d*Cos[a + b*x])^(3/2))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx = \frac{d\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bc(1+m)(d\cos(a+bx))^{3/2}}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 1.00

$$\frac{\cos^2(a+bx)^{3/4}\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{b(m+1)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^m/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\,\left(c\sin\left(bx+a\right)\right)^{m}}{d\cos\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m/(d*cos(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{m}}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m/sqrt(d*cos(b*x + a)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{m}}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x)
```

```
[Out] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{m}}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^m/sqrt(d*cos(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,\sin (a+b\,x))^m}{\sqrt{d\,\cos (a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(1/2),x)
```

```
[Out] int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin (a+bx))^m}{\sqrt{d\cos (a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**m/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral((c*sin(a + b*x))**m/sqrt(d*cos(a + b*x)), x)
```

3.353 $\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt[4]{\cos^2(a+bx)}\,(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)\sqrt{d\cos(a+bx)}}$$

```
[Out] (cos(b*x+a)^2)^(1/4)*hypergeom([5/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(
c*sin(b*x+a))^(1+m)/b/c/d/(1+m)/(d*cos(b*x+a))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2577}

$$\frac{\sqrt[4]{\cos^2(a+bx)}\,(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m)*Sqrt[d*Cos[a + b*x]])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx = \frac{\sqrt[4]{\cos^2(a+bx)}\,{}_2F_1\left(\frac{5}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)\sqrt{d\cos(a+bx)}}$$

Mathematica [A] time = 0.09, size = 78, normalized size = 1.01

$$\frac{\sqrt[4]{\cos^2(a+bx)}\,\tan(a+bx)\sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^m\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bd^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] (Sqrt[d*Cos[a + b*x]]*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)
/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*d^2*(1 +
 m))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\,\left(c\sin\left(bx+a\right)\right)^{m}}{d^{2}\cos\left(bx+a\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m/(d^2*cos(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{m}}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(3/2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{m}}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x)
```

```
[Out] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{m}}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,\sin(a+b\,x))^m}{(d\,\cos(a+b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(3/2),x)
[Out] int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**m/(d*cos(b*x+a))**(3/2),x)
[Out] Integral((c*sin(a + b*x))**m/(d*cos(a + b*x))**(3/2), x)
```

3.354 $\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)(d\cos(a+bx))^{3/2}}$$

```
[Out] (cos(b*x+a)^2)^(3/4)*hypergeom([7/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(
c*sin(b*x+a))^(1+m)/b/c/d/(1+m)/(d*cos(b*x+a))^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2577}

$$\frac{\cos^2(a+bx)^{3/4}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m)*(d*Cos[a + b*x])^(3/2))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx=\frac{\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{7}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)(d\cos(a+bx))^{3/2}}$$

Mathematica [A] time = 0.08, size = 78, normalized size = 1.01

$$\frac{\cos^2(a+bx)^{3/4}\tan(a+bx)(c\sin(a+bx))^m\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bd^2(m+1)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^m/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(c*Sin[a + b*x])^m*Tan[a + b*x])/(b*d^2*(1 + m)*Sqrt[d*Cos[a + b*
x]])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\cos\left(bx+a\right)}\,\left(c\sin\left(bx+a\right)\right)^{m}}{d^{3}\cos\left(bx+a\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*(c*sin(b*x + a))^m/(d^3*cos(b*x + a)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{m}}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(5/2), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{m}}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] int((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^{m}}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m/(d*cos(b*x + a))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,\sin\left(a+b\,x\right))^m}{(d\,\cos\left(a+b\,x\right))^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(5/2),x)
[Out] int((c*sin(a + b*x))^m/(d*cos(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**m/(d*cos(b*x+a))**(5/2),x)
[Out] Timed out
```

3.355 $\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx$

Optimal. Leaf size=76

$$-\frac{(d\cos(a+bx))^{n+5}}{bd^5(n+5)}+\frac{2(d\cos(a+bx))^{n+3}}{bd^3(n+3)}-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)/b/d/(1+n)+2*(d*cos(b*x+a))^(3+n)/b/d^3/(3+n)-(d*cos(b
*x+a))^(5+n)/b/d^5/(5+n)
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 270}

$$\frac{2(d\cos(a+bx))^{n+3}}{bd^3(n+3)}-\frac{(d\cos(a+bx))^{n+5}}{bd^5(n+5)}-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^5,x]
```

```
[Out] -((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n))) + (2*(d*Cos[a + b*x])^(3 + n))/(b
*d^3*(3 + n)) - (d*Cos[a + b*x])^(5 + n)/(b*d^5*(5 + n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^n \sin^5(a+bx)\,dx = -\frac{\text{Subst}\left(\int x^n\left(1-\frac{x^2}{d^2}\right)^2\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \left(x^n-\frac{2x^{2+n}}{d^2}+\frac{x^{4+n}}{d^4}\right)dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)}+\frac{2(d\cos(a+bx))^{3+n}}{bd^3(3+n)}-\frac{(d\cos(a+bx))^{5+n}}{bd^5(5+n)}$$

Mathematica [A] time = 0.31, size = 83, normalized size = 1.09

$$-\frac{\cos(a+bx)\left(-4\left(n^2+8n+7\right)\cos(2(a+bx))+\left(n^2+4n+3\right)\cos(4(a+bx))+3n^2+28n+89\right)(d\cos(a+bx))}{8b(n+1)(n+3)(n+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^5,x]
```

```
[Out] -1/8*(Cos[a + b*x]*(d*Cos[a + b*x])^n*(89 + 28*n + 3*n^2 - 4*(7 + 8*n + n^2
)*Cos[2*(a + b*x)] + (3 + 4*n + n^2)*Cos[4*(a + b*x)]))/(b*(1 + n)*(3 + n)*
(5 + n))
```

fricas [A] time = 0.63, size = 84, normalized size = 1.11

$$-\frac{\left(\left(n^2+4\,n+3\right)\cos\left(bx+a\right)^5-2\left(n^2+6\,n+5\right)\cos\left(bx+a\right)^3+\left(n^2+8\,n+15\right)\cos\left(bx+a\right)\right)\left(d\cos\left(bx+a\right)\right)^n}{bn^3+9\,bn^2+23\,bn+15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -((n^2 + 4*n + 3)*cos(b*x + a)^5 - 2*(n^2 + 6*n + 5)*cos(b*x + a)^3 + (n^2
+ 8*n + 15)*cos(b*x + a))*(d*cos(b*x + a))^n/(b*n^3 + 9*b*n^2 + 23*b*n + 15
*b)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 1.29, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n \left(\sin^5(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a)^5,x)
```

```
[Out] int((d*cos(b*x+a))^n*sin(b*x+a)^5,x)
```

maxima [A] time = 0.38, size = 78, normalized size = 1.03

$$-\frac{\frac{d^n\cos(bx+a)^n\cos(bx+a)^5}{n+5} - \frac{2\,d^n\cos(bx+a)^n\cos(bx+a)^3}{n+3} + \frac{(d\cos(bx+a))^{n+1}}{d(n+1)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -(d^n*cos(b*x + a)^n*cos(b*x + a)^5/(n + 5) - 2*d^n*cos(b*x + a)^n*cos(b*x
+ a)^3/(n + 3) + (d*cos(b*x + a))^(n + 1)/(d*(n + 1)))/b
```

mupad [B] time = 1.54, size = 132, normalized size = 1.74

$$-\frac{(d\,\cos(a+b\,x))^n\,\left(150\,\cos(a+b\,x)-25\,\cos(3\,a+3\,b\,x)+3\,\cos(5\,a+5\,b\,x)+24\,n\,\cos(a+b\,x)-28\,n\,\cos\right.}{16\,b\,\left(n^3+9\,n^2+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^5*(d*cos(a + b*x))^n,x)
```

```
[Out] -((d*cos(a + b*x))^n*(150*cos(a + b*x) - 25*cos(3*a + 3*b*x) + 3*cos(5*a +
5*b*x) + 24*n*cos(a + b*x) - 28*n*cos(3*a + 3*b*x) + 4*n*cos(5*a + 5*b*x) +
 2*n^2*cos(a + b*x) - 3*n^2*cos(3*a + 3*b*x) + n^2*cos(5*a + 5*b*x)))/(16*b
*(23*n + 9*n^2 + n^3 + 15))
```

sympy [A] time = 75.79, size = 2462, normalized size = 32.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a)**5,x)
```

```
[Out] Piecewise((x*(d*cos(a))**n*sin(a)**5, Eq(b, 0)), ((-log(cos(a + b*x))/b + s
in(a + b*x)**4/(4*b*cos(a + b*x)**4) - sin(a + b*x)**2/(2*b*cos(a + b*x)**2
))/d**5, Eq(n, -5)), ((2*log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2)**8/(b*t
an(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) - 4*log(tan(a/2 + b*x/2)
- 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 +
 b) + 2*log(tan(a/2 + b*x/2) - 1)/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*
x/2)**4 + b) + 2*log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 +
 b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) - 4*log(tan(a/2 + b*x/2) + 1)*tan
(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) + 2*
log(tan(a/2 + b*x/2) + 1)/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4
+ b) - 2*log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/
2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) + 4*log(tan(a/2 + b*x/2)**2 + 1)*tan(a
/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4 + b) - 2*lo
g(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)**4
 + b) + 4*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x/2)
**4 + b) + 4*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 - 2*b*tan(a/2 + b*x
/2)**4 + b))/d**3, Eq(n, -3)), ((-log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2
)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2
)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2) - 1)*tan(a/2 +
 b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 +
 b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 6*log(tan(a/2 + b*x/2) - 1)*tan
(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan
(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2) -
1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6
*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2
) - 1)/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x
/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) + 1)*tan(a/2 +
 b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 +
 b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x/2) + 1)*tan
(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan
(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 6*log(tan(a/2 + b*x/2) +
1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6
*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 4*log(tan(a/2 + b*x
/2) + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)*
*6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 +
 b*x/2) + 1)/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2
 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2)**2 + 1)*
tan(a/2 + b*x/2)**8/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*
tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 4*log(tan(a/2 + b*x/2)
**2 + 1)*tan(a/2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)*
*6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 6*log(tan(a/2
 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2
+ b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) + 4*lo
g(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4*b
*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 +
```

```
b) + log(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*
x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/
2 + b*x/2)**6/(b*tan(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/
2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2 + b) - 8*tan(a/2 + b*x/2)**4/(b*tan
(a/2 + b*x/2)**8 + 4*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*
tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**8 + 4
*b*tan(a/2 + b*x/2)**6 + 6*b*tan(a/2 + b*x/2)**4 + 4*b*tan(a/2 + b*x/2)**2
+ b))/d, Eq(n, -1)), (-d**n*n**2*sin(a + b*x)**4*cos(a + b*x)*cos(a + b*x)*
*n/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) - 8*d**n*n*sin(a + b*x)**4*cos(a + b
*x)*cos(a + b*x)**n/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) - 4*d**n*n*sin(a +
b*x)**2*cos(a + b*x)**3*cos(a + b*x)**n/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b)
 - 15*d**n*sin(a + b*x)**4*cos(a + b*x)*cos(a + b*x)**n/(b*n**3 + 9*b*n**2
+ 23*b*n + 15*b) - 20*d**n*sin(a + b*x)**2*cos(a + b*x)**3*cos(a + b*x)**n/
(b*n**3 + 9*b*n**2 + 23*b*n + 15*b) - 8*d**n*cos(a + b*x)**5*cos(a + b*x)**
n/(b*n**3 + 9*b*n**2 + 23*b*n + 15*b), True))
```

3.356 $\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx$

Optimal. Leaf size=50

$$\frac{(d\cos(a+bx))^{n+3}}{bd^3(n+3)}-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)/b/d/(1+n)+(d*cos(b*x+a))^(3+n)/b/d^3/(3+n)
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 14}

$$\frac{(d\cos(a+bx))^{n+3}}{bd^3(n+3)}-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^3,x]
```

```
[Out] -((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n))) + (d*Cos[a + b*x])^(3 + n)/(b*d^3
*(3 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^n \sin^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int x^n\left(1-\frac{x^2}{d^2}\right)dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{\text{Subst}\left(\int \left(x^n-\frac{x^{2+n}}{d^2}\right)dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)} + \frac{(d\cos(a+bx))^{3+n}}{bd^3(3+n)}$$

Mathematica [A] time = 0.12, size = 50, normalized size = 1.00

$$\frac{\cos(a+bx)((n+1)\cos(2(a+bx))-n-5)(d\cos(a+bx))^n}{2b(n+1)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^3,x]
```

```
[Out] (Cos[a + b*x]*(d*Cos[a + b*x])^n*(-5 - n + (1 + n)*Cos[2*(a + b*x)]))/(2*b*
(1 + n)*(3 + n))
```

fricas [A] time = 0.76, size = 50, normalized size = 1.00

$$\frac{\left((n+1)\cos(bx+a)^3-(n+3)\cos(bx+a)\right)(d\cos(bx+a))^n}{bn^2+4\,bn+3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((n + 1)*cos(b*x + a)^3 - (n + 3)*cos(b*x + a))*(d*cos(b*x + a))^n/(b*n^2 +
 4*b*n + 3*b)
```

giac [A] time = 1.73, size = 100, normalized size = 2.00

$$\frac{\frac{(d\cos(bx+a))^n n\cos(bx+a)^3}{b}+\frac{(d\cos(bx+a))^n\cos(bx+a)^3}{b}-\frac{(d\cos(bx+a))^n n\cos(bx+a)}{b}-\frac{3\,(d\cos(bx+a))^n\cos(bx+a)}{b}}{n^2+4\,n+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] ((d*cos(b*x + a))^n*n*cos(b*x + a)^3/b + (d*cos(b*x + a))^n*cos(b*x + a)^3/
b - (d*cos(b*x + a))^n*n*cos(b*x + a)/b - 3*(d*cos(b*x + a))^n*cos(b*x + a)
/b)/(n^2 + 4*n + 3)
```

maple [C] time = 2.14, size = 1076, normalized size = 21.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a)^3,x)
```

```
[Out] -1/8*exp(I*(b*x+a))^(-n)*d^n*(1/2)^n*(exp(2*I*(b*x+a))+1)^n/(3+n)/(1+n)/b*(
n+9)*exp(-1/2*I*(Pi*n*csgn(I*cos(b*x+a))^3-Pi*n*csgn(I*(exp(2*I*(b*x+a))+1)
)*csgn(I*cos(b*x+a))^2-Pi*n*csgn(I*cos(b*x+a))^2*csgn(I*exp(-I*(b*x+a)))+Pi
*n*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))*csgn(I*exp(-I*(b*x+a)))-
Pi*n*csgn(I*cos(b*x+a))*csgn(I*d*cos(b*x+a))^2+Pi*n*csgn(I*d)*csgn(I*cos(b*
x+a))*csgn(I*d*cos(b*x+a))+Pi*n*csgn(I*d*cos(b*x+a))^3-Pi*n*csgn(I*d)*csgn(
I*d*cos(b*x+a))^2+2*b*x+2*a))-1/8*exp(I*(b*x+a))^(-n)*d^n*(1/2)^n*(exp(2*I*
(b*x+a))+1)^n/(3+n)/(1+n)/b*(n+9)*exp(1/2*I*(-Pi*n*csgn(I*cos(b*x+a))^3+Pi*
n*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2+Pi*n*csgn(I*cos(b*x+a))
^2*csgn(I*exp(-I*(b*x+a)))-Pi*n*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x
+a))*csgn(I*exp(-I*(b*x+a)))+Pi*n*csgn(I*cos(b*x+a))*csgn(I*d*cos(b*x+a))^2
-Pi*n*csgn(I*d)*csgn(I*cos(b*x+a))*csgn(I*d*cos(b*x+a))-Pi*n*csgn(I*d*cos(b
*x+a))^3+Pi*n*csgn(I*d)*csgn(I*d*cos(b*x+a))^2+2*b*x+2*a))+1/8/(b*n+3*b)*(e
xp(2*I*(b*x+a))+1)^n*(1/2)^n*d^n*exp(I*(b*x+a))^(-n)*exp(-1/2*I*(Pi*n*csgn(
I*cos(b*x+a))^3-Pi*n*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))^2-Pi*n
*csgn(I*cos(b*x+a))^2*csgn(I*exp(-I*(b*x+a)))-Pi*n*csgn(I*cos(b*x+a))*csgn(
I*d*cos(b*x+a))^2+Pi*n*csgn(I*d)*csgn(I*cos(b*x+a))*csgn(I*d*cos(b*x+a))+Pi
*n*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))*csgn(I*exp(-I*(b*x+a)))+
Pi*n*csgn(I*d*cos(b*x+a))^3-Pi*n*csgn(I*d)*csgn(I*d*cos(b*x+a))^2+6*b*x+6*a
))+1/8*exp(I*(b*x+a))^(-n)*d^n*(1/2)^n*(exp(2*I*(b*x+a))+1)^n/(b*n+3*b)*exp
(1/2*I*(-Pi*n*csgn(I*cos(b*x+a))^3+Pi*n*csgn(I*(exp(2*I*(b*x+a))+1))*csgn(I
*cos(b*x+a))^2+Pi*n*csgn(I*cos(b*x+a))^2*csgn(I*exp(-I*(b*x+a)))-Pi*n*csgn(
I*(exp(2*I*(b*x+a))+1))*csgn(I*cos(b*x+a))*csgn(I*exp(-I*(b*x+a)))+Pi*n*csg
n(I*cos(b*x+a))*csgn(I*d*cos(b*x+a))^2-Pi*n*csgn(I*d)*csgn(I*cos(b*x+a))*cs
gn(I*d*cos(b*x+a))-Pi*n*csgn(I*d*cos(b*x+a))^3+Pi*n*csgn(I*d)*csgn(I*d*cos(
b*x+a))^2+6*b*x+6*a))
```

maxima [A] time = 0.37, size = 52, normalized size = 1.04

$$\frac{\frac{d^n \cos(bx+a)^n \cos(bx+a)^3}{n+3} - \frac{(d\cos(bx+a))^{n+1}}{d(n+1)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (d^n*cos(b*x + a)^n*cos(b*x + a)^3/(n + 3) - (d*cos(b*x + a))^(n + 1)/(d*(n
 + 1)))/b
```

mupad [B] time = 0.91, size = 65, normalized size = 1.30

$$-\frac{(d\cos(a+b\,x))^n\,(9\cos(a+b\,x)-\cos(3\,a+3\,b\,x)+n\cos(a+b\,x)-n\cos(3\,a+3\,b\,x))}{4\,b\,\left(n^2+4\,n+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*(d*cos(a + b*x))^n,x)
```

```
[Out] -((d*cos(a + b*x))^n*(9*cos(a + b*x) - cos(3*a + 3*b*x) + n*cos(a + b*x) -
n*cos(3*a + 3*b*x)))/(4*b*(4*n + n^2 + 3))
```

sympy [A] time = 12.78, size = 694, normalized size = 13.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((x*(d*cos(a))**n*sin(a)**3, Eq(b, 0)), ((log(cos(a + b*x))/b + si
n(a + b*x)**2/(2*b*cos(a + b*x)**2))/d**3, Eq(n, -3)), ((-log(tan(a/2 + b*x
/2) - 1)*tan(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)*
*2 + b) - 2*log(tan(a/2 + b*x/2) - 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/
2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) - 1)/(b*tan(a/2
 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) + 1)*tan
(a/2 + b*x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) - 2*
log(tan(a/2 + b*x/2) + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*b*
tan(a/2 + b*x/2)**2 + b) - log(tan(a/2 + b*x/2) + 1)/(b*tan(a/2 + b*x/2)**4
 + 2*b*tan(a/2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2)**2 + 1)*tan(a/2 + b*
x/2)**4/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/2 + b*x/2)**2 + b) + 2*log(tan(a
/2 + b*x/2)**2 + 1)*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4 + 2*b*tan(a/
2 + b*x/2)**2 + b) + log(tan(a/2 + b*x/2)**2 + 1)/(b*tan(a/2 + b*x/2)**4 +
2*b*tan(a/2 + b*x/2)**2 + b) - 2*tan(a/2 + b*x/2)**2/(b*tan(a/2 + b*x/2)**4
 + 2*b*tan(a/2 + b*x/2)**2 + b))/d, Eq(n, -1)), (-d**n*n*sin(a + b*x)**2*co
s(a + b*x)*cos(a + b*x)**n/(b*n**2 + 4*b*n + 3*b) - 3*d**n*sin(a + b*x)**2*
cos(a + b*x)*cos(a + b*x)**n/(b*n**2 + 4*b*n + 3*b) - 2*d**n*cos(a + b*x)**
3*cos(a + b*x)**n/(b*n**2 + 4*b*n + 3*b), True))
```

3.357 $\int (d\cos(a+bx))^n \sin(a+bx)\,dx$

Optimal. Leaf size=25

$$-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)/b/d/(1+n)
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 30}

$$-\frac{(d\cos(a+bx))^{n+1}}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x],x]
```

```
[Out] -((d*Cos[a + b*x])^(1 + n)/(b*d*(1 + n)))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^n \sin(a+bx)\,dx &= -\frac{\text{Subst}\left(\int x^n\,dx, x, d\cos(a+bx)\right)}{bd}\\ &= -\frac{(d\cos(a+bx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.04

$$-\frac{\cos(a+bx)(d\cos(a+bx))^n}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x],x]
[Out] -((Cos[a + b*x]*(d*Cos[a + b*x])^n)/(b*(1 + n)))
```

fricas [A] time = 0.69, size = 25, normalized size = 1.00

$$-\frac{(d\cos(bx+a))^n\cos(bx+a)}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a),x, algorithm="fricas")
[Out] -(d*cos(b*x + a))^n*cos(b*x + a)/(b*n + b)
```

giac [A] time = 1.92, size = 25, normalized size = 1.00

$$-\frac{(d\cos(bx+a))^{n+1}}{bd(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a),x, algorithm="giac")
[Out] -(d*cos(b*x + a))^(n + 1)/(b*d*(n + 1))
```

maple [A] time = 0.00, size = 26, normalized size = 1.04

$$-\frac{(d\cos(bx+a))^{1+n}}{bd(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a),x)
[Out] -(d*cos(b*x+a))^(1+n)/b/d/(1+n)
```

maxima [A] time = 0.33, size = 25, normalized size = 1.00

$$-\frac{(d\cos(bx+a))^{n+1}}{bd(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -(d*cos(b*x + a))^(n + 1)/(b*d*(n + 1))
```

mupad [B] time = 0.18, size = 26, normalized size = 1.04

$$-\frac{\cos(a+b\,x)\,\left(d\,\cos(a+b\,x)\right)^n}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*(d*cos(a + b*x))^n,x)
```

```
[Out] -(cos(a + b*x)*(d*cos(a + b*x))^n)/(b*(n + 1))
```

sympy [A] time = 1.92, size = 61, normalized size = 2.44

$$\begin{cases} \frac{x\sin(a)}{d\cos(a)} & \text{for}\ b=0\wedge n=-1 \\ x\left(d\cos(a)\right)^n\sin(a) & \text{for}\ b=0 \\ -\frac{\log(\cos(a+bx))}{bd} & \text{for}\ n=-1 \\ -\frac{d^n\cos(a+bx)\cos^n(a+bx)}{bn+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a)/(d*cos(a)), Eq(b, 0) & Eq(n, -1)), (x*(d*cos(a))**n*sin
(a), Eq(b, 0)), (-log(cos(a + b*x))/(b*d), Eq(n, -1)), (-d**n*cos(a + b*x)*
cos(a + b*x)**n/(b*n + b), True))
```

3.358 $\int (d\cos(a+bx))^n \csc(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{(d\cos(a+bx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)/b/
d/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2565, 364}

$$-\frac{(d\cos(a+bx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x],x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2])/(b*d*(1 + n)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^n \csc(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^n}{1-\frac{x^2}{d^2}}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)}{bd(1+n)}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 1.06

$$-\frac{\cos(a+bx)(d\cos(a+bx))^n\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+1}{2}+1;\cos^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x],x]
```

```
[Out] -((Cos[a + b*x]*(d*Cos[a + b*x])^n*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 +
 n)/2, Cos[a + b*x]^2])/(b*(1 + n)))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left((d\cos(bx+a))^n \csc(bx+a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n \csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n \csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a),x)
[Out] int((d*cos(b*x+a))^n*csc(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d \cos (bx + a))^n \csc (bx + a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\ \cos (a + b\,x))^n}{\sin (a + b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^n/sin(a + b*x),x)
[Out] int((d*cos(a + b*x))^n/sin(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d \cos (a + bx))^n \csc (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a),x)
[Out] Integral((d*cos(a + b*x))**n*csc(a + b*x), x)
```

3.359 $\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{(d\cos(a+bx))^{n+1}\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)/b/
d/(1+n)
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 364}

$$-\frac{(d\cos(a+bx))^{n+1}\,{}_2F_1\left(2,\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^3,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[2, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2])/(b*d*(1 + n)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^n}{\left(1-\frac{x^2}{d^2}\right)^2}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(2,\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)}{bd(1+n)}$$

Mathematica [B] time = 2.70, size = 154, normalized size = 3.14

$$-\frac{2^{-n-3}\cos(a+bx)(d\cos(a+bx))^n\left(2^{n+1}\,{}_2F_1(1,n+1;n+2;\cos(a+bx))+2^{n+1}\,{}_2F_1(2,n+1;n+2;\cos(a+bx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^3,x]
```

```
[Out] -((2^(-3 - n)*Cos[a + b*x]*(d*Cos[a + b*x])^n*(2^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, Cos[a + b*x]] + 2^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 +
n, Cos[a + b*x]] + (Hypergeometric2F1[n, 1 + n, 2 + n, (Cos[a + b*x]*Sec[(a
 + b*x)/2]^2)/2] + Hypergeometric2F1[1 + n, 1 + n, 2 + n, (Cos[a + b*x]*Sec
[(a + b*x)/2]^2)/2])*(Sec[(a + b*x)/2]^2)^(1 + n)))/(b*(1 + n)))
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left((d\cos(bx+a))^n\csc(bx+a)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n\csc(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^3, x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n \left(\csc^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a)^3,x)
[Out] int((d*cos(b*x+a))^n*csc(b*x+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n \csc(bx+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,\cos(a+b\,x))^n}{\sin(a+b\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^n/sin(a + b*x)^3,x)
[Out] int((d*cos(a + b*x))^n/sin(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(a+bx))^n \csc^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a)**3,x)
[Out] Integral((d*cos(a + b*x))**n*csc(a + b*x)**3, x)
```

3.360 $\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{(d\cos(a+bx))^{n+1}\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([3, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)/b/
d/(1+n)
```

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2565, 364}

$$-\frac{(d\cos(a+bx))^{n+1}\,{}_2F_1\left(3,\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^5,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[3, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2])/(b*d*(1 + n)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$\int (d\cos(a+bx))^n \csc^5(a+bx)\,dx = -\frac{\text{Subst}\left(\int \frac{x^n}{\left(1-\frac{x^2}{d^2}\right)^3}\,dx, x, d\cos(a+bx)\right)}{bd}$$
$$= -\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(3, \frac{1+n}{2}; \frac{3+n}{2}; \cos^2(a+bx)\right)}{bd(1+n)}$$

Mathematica [B] time = 4.02, size = 244, normalized size = 4.98

$$-\frac{2^{-n-5}\cos(a+bx)(d\cos(a+bx))^n \left(3\, 2^{n+1}\,{}_2F_1(1,n+1;n+2;\cos(a+bx)) + 3\, 2^{n+1}\,{}_2F_1(2,n+1;n+2;\cos(a+b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^5,x]
```

```
[Out] -((2^(-5 - n)*Cos[a + b*x]*(d*Cos[a + b*x])^n*(3*2^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, Cos[a + b*x]] + 3*2^(1 + n)*Hypergeometric2F1[2, 1 + n,
2 + n, Cos[a + b*x]] + 2^(2 + n)*Hypergeometric2F1[3, 1 + n, 2 + n, Cos[a +
 b*x]] + 2*Hypergeometric2F1[-1 + n, 1 + n, 2 + n, (Cos[a + b*x]*Sec[(a + b
*x)/2]^2)/2]*(Sec[(a + b*x)/2]^2)^(1 + n) + 3*Hypergeometric2F1[n, 1 + n, 2
 + n, (Cos[a + b*x]*Sec[(a + b*x)/2]^2)/2]*(Sec[(a + b*x)/2]^2)^(1 + n) + 3
*Hypergeometric2F1[1 + n, 1 + n, 2 + n, (Cos[a + b*x]*Sec[(a + b*x)/2]^2)/2
]*(Sec[(a + b*x)/2]^2)^(1 + n)))/(b*(1 + n)))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left((d\cos(bx+a))^n \csc(bx+a)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n \csc(bx+a)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^5,x, algorithm="giac")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^5, x)
```

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n \left(\csc^5(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a)^5,x)
[Out] int((d*cos(b*x+a))^n*csc(b*x+a)^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n \csc(bx+a)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^5,x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\ \cos(a+b\,x))^n}{\sin(a+b\,x)^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^n/sin(a + b*x)^5,x)
[Out] int((d*cos(a + b*x))^n/sin(a + b*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a)**5,x)
[Out] Timed out
```

3.361 $\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([-3/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)
*sin(b*x+a)/b/d/(1+n)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2576}

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^4,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, C
os[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n \sin^4(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(-\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.11, size = 68, normalized size = 0.99

$$-\frac{\sin(2(a+bx))(d\cos(a+bx))^n\,{}_2F_1\left(-\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{2b(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^4,x]
```

```
[Out] -1/2*((d*Cos[a + b*x])^n*Hypergeometric2F1[-3/2, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2]*Sin[2*(a + b*x)])/(b*(1 + n)*Sqrt[Sin[a + b*x]^2])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(\cos\left(bx+a\right)^{4}-2\,\cos\left(bx+a\right)^{2}+1\right)\left(d\cos\left(bx+a\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*(d*cos(b*x + a))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(bx+a\right)\right)^{n}\sin\left(bx+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*sin(b*x + a)^4, x)
```

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(bx+a\right)\right)^{n}\left(\sin^{4}\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a)^4,x)
```

```
[Out] int((d*cos(b*x+a))^n*sin(b*x+a)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(bx+a\right)\right)^{n}\sin\left(bx+a\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n*sin(b*x + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin (a + b\,x)^4\,(d\,\cos (a + b\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^4*(d*cos(a + b*x))^n,x)
```

```
[Out] int(sin(a + b*x)^4*(d*cos(a + b*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a)**4,x)
```

```
[Out] Timed out
```

3.362 $\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([-1/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)
*sin(b*x+a)/b/d/(1+n)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2576}

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Sin[a + b*x]^2,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2, C
os[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(-\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.08, size = 68, normalized size = 0.99

$$-\frac{\sin(2(a+bx))(d\cos(a+bx))^n\,{}_2F_1\left(-\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{2b(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Sin[a + b*x]^2,x]
```

```
[Out] -1/2*((d*Cos[a + b*x])^n*Hypergeometric2F1[-1/2, (1 + n)/2, (3 + n)/2, Cos[
a + b*x]^2]*Sin[2*(a + b*x)])/(b*(1 + n)*Sqrt[Sin[a + b*x]^2])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(bx+a\right)^2-1\right)\left(d\cos\left(bx+a\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*(d*cos(b*x + a))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(bx+a\right)\right)^n \sin\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*sin(b*x + a)^2, x)
```

maple [F] time = 1.28, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(bx+a\right)\right)^n \left(\sin^2\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*sin(b*x+a)^2,x)
```

```
[Out] int((d*cos(b*x+a))^n*sin(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(bx+a\right)\right)^n \sin\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(a+b\,x)^2\,(d\,\cos(a+b\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*(d*cos(a + b*x))^n,x)
[Out] int(sin(a + b*x)^2*(d*cos(a + b*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(a+bx))^n \sin^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*sin(b*x+a)**2,x)
[Out] Integral((d*cos(a + b*x))**n*sin(a + b*x)**2, x)
```

3.363 $\int (d\cos(a+bx))^n\,dx$

Optimal. Leaf size=69

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)*
sin(b*x+a)/b/d/(1+n)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2643}

$$-\frac{\sin(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Co
s[a + b*x]^2]*Sin[a + b*x])/(b*d*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (d\cos(a+bx))^n\,dx=-\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sin(a+bx)}{bd(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.93

$$-\frac{\sqrt{\sin^2(a+bx)}\cot(a+bx)(d\cos(a+bx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n,x]
```

```
[Out] -(((d*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(1 + n)))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left((d\cos(bx+a))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n,x)
```

```
[Out] int((d*cos(b*x+a))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cos(a+b\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^n,x)
```

```
[Out] int((d*cos(a + b*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(a+bx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n,x)
```

```
[Out] Integral((d*cos(a + b*x))**n, x)
```

3.364 $\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{\sin^2(a+bx)}\,\csc(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*csc(b*x+a)*hypergeom([3/2, 1/2+1/2*n],[3/2+1/2*n],cos
(b*x+a)^2)*(sin(b*x+a)^2)^(1/2)/b/d/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2576}

$$-\frac{\sqrt{\sin^2(a+bx)}\,\csc(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{3}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^2,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[3/2, (1 + n)/2,
(3 + n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*d*(1 + n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\csc(a+bx)\,{}_2F_1\left(\frac{3}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{bd(1+n)}$$

Mathematica [A] time = 0.19, size = 80, normalized size = 1.16

$$\frac{d\csc(a+bx)\left(-\cot^2(a+bx)\right)^{\frac{1-n}{2}}(d\cos(a+bx))^{n-1}\,{}_2F_1\left(\frac{1-n}{2},1-\frac{n}{2};2-\frac{n}{2};\csc^2(a+bx)\right)}{b(n-2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^2,x]
```

```
[Out] (d*(d*Cos[a + b*x])^(-1 + n)*(-Cot[a + b*x]^2)^((1 - n)/2)*Csc[a + b*x]*Hyp
ergeometric2F1[(1 - n)/2, 1 - n/2, 2 - n/2, Csc[a + b*x]^2])/(b*(-2 + n))
```

fricas **[F]** time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left((d\cos(bx+a))^n\csc(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a)^2, x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^2, x)
```

maple **[F]** time = 0.41, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n\left(\csc^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a)^2,x)
```

```
[Out] int((d*cos(b*x+a))^n*csc(b*x+a)^2,x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n\csc(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\ \cos(a+b\,x))^n}{\sin(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^n/sin(a + b*x)^2,x)
[Out] int((d*cos(a + b*x))^n/sin(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(a+bx))^n \csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a)**2,x)
[Out] Integral((d*cos(a + b*x))**n*csc(a + b*x)**2, x)
```

3.365 $\int (d\cos(a+bx))^n \csc^4(a+bx)\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{\sin^2(a+bx)}\csc(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*csc(b*x+a)*hypergeom([5/2, 1/2+1/2*n],[3/2+1/2*n],cos
(b*x+a)^2)*(sin(b*x+a)^2)^(1/2)/b/d/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2576}

$$-\frac{\sqrt{\sin^2(a+bx)}\csc(a+bx)(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{5}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Csc[a + b*x]^4,x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[5/2, (1 + n)/2,
(3 + n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*d*(1 + n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n\csc^4(a+bx)\,dx = -\frac{(d\cos(a+bx))^{1+n}\csc(a+bx)\,{}_2F_1\left(\frac{5}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{bd(1+n)}$$

Mathematica [A] time = 0.21, size = 82, normalized size = 1.19

$$\frac{d\csc^3(a+bx)\left(-\cot^2(a+bx)\right)^{\frac{1-n}{2}}(d\cos(a+bx))^{n-1}\,{}_2F_1\left(\frac{1-n}{2},2-\frac{n}{2};3-\frac{n}{2};\csc^2(a+bx)\right)}{b(n-4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Csc[a + b*x]^4,x]
```

```
[Out] (d*(d*Cos[a + b*x])^(-1 + n)*(-Cot[a + b*x]^2)^((1 - n)/2)*Csc[a + b*x]^3*H
ypergeometric2F1[(1 - n)/2, 2 - n/2, 3 - n/2, Csc[a + b*x]^2])/(b*(-4 + n))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left((d\cos(bx+a))^n\csc(bx+a)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] integral((d*cos(b*x + a))^n*csc(b*x + a)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n\csc(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^4, x)
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n\left(\csc^4(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*csc(b*x+a)^4,x)
```

```
[Out] int((d*cos(b*x+a))^n*csc(b*x+a)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\cos(bx+a))^n\csc(bx+a)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*csc(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n*csc(b*x + a)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d \cos (a + b x))^n}{\sin (a + b x)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^n/sin(a + b*x)^4,x)
[Out] int((d*cos(a + b*x))^n/sin(a + b*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*csc(b*x+a)**4,x)
[Out] Timed out
```

3.366 $\int (d\cos(a+bx))^n (c\sin(a+bx))^{5/2}\,dx$

Optimal. Leaf size=76

$$-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{3}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sin^2(a+bx)^{3/4}}$$

```
[Out] -c*(d*cos(b*x+a))^(1+n)*hypergeom([-3/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^
2)*(c*sin(b*x+a))^(3/2)/b/d/(1+n)/(sin(b*x+a)^2)^(3/4)
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2576}

$$-\frac{c(c\sin(a+bx))^{3/2}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{3}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sin^2(a+bx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-3/4, (1 + n)/2, (3 + n)/2,
 Cos[a + b*x]^2]*(c*Sin[a + b*x])^(3/2))/(b*d*(1 + n)*(Sin[a + b*x]^2)^(3/4
)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{5/2}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\,{}_2F_1\left(-\frac{3}{4},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)(c\sin(a+bx))^{3/2}}{bd(1+n)\sin^2(a+bx)^{3/4}}$$

Mathematica [B] time = 0.42, size = 158, normalized size = 2.08

$$\frac{\cot(a+bx)(c\sin(a+bx))^{5/2}(d\cos(a+bx))^n\left((n+1)\cos^2(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{n+3}{2};\frac{n+5}{2};\cos^2(a+bx)\right)-(n+3)\,{}_2F_1\left(\right.\right.}{2b(n+1)(n+3)\sin^2(a+bx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(5/2),x]
```

```
[Out] ((d*Cos[a + b*x])^n*Cot[a + b*x]*(-((3 + n)*Hypergeometric2F1[-3/4, (1 + n)
/2, (3 + n)/2, Cos[a + b*x]^2]) - (3 + n)*Hypergeometric2F1[1/4, (1 + n)/2,
 (3 + n)/2, Cos[a + b*x]^2] + (1 + n)*Cos[a + b*x]^2*Hypergeometric2F1[1/4,
 (3 + n)/2, (5 + n)/2, Cos[a + b*x]^2])*(c*Sin[a + b*x])^(5/2))/(2*b*(1 + n
)*(3 + n)*(Sin[a + b*x]^2)^(3/4))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2\cos\left(bx+a\right)^2-c^2\right)\sqrt{c\sin\left(bx+a\right)}\,\left(d\cos\left(bx+a\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*cos(b*x + a)^2 - c^2)*sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^
n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}\left(d\cos\left(bx+a\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(5/2)*(d*cos(b*x + a))^n, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(bx+a\right)\right)^n\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x)
```

```
[Out] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin\left(bx+a\right)\right)^{\frac{5}{2}}\left(d\cos\left(bx+a\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^(5/2)*(d*cos(b*x + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cos(a+b\,x))^n\,(c\sin(a+b\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(5/2),x)
[Out] int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*(c*sin(b*x+a))**(5/2),x)
[Out] Timed out
```

3.367 $\int (d\cos(a+bx))^n(c\sin(a+bx))^{3/2}\,dx$

Optimal. Leaf size=76

$$-\frac{c\sqrt{c\sin(a+bx)}\,(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{1}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt[4]{\sin^2(a+bx)}}$$

```
[Out] -c*(d*cos(b*x+a))^(1+n)*hypergeom([-1/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^
2)*(c*sin(b*x+a))^(1/2)/b/d/(1+n)/(sin(b*x+a)^2)^(1/4)
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2576}

$$-\frac{c\sqrt{c\sin(a+bx)}\,(d\cos(a+bx))^{n+1}\,{}_2F_1\left(-\frac{1}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt[4]{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[-1/4, (1 + n)/2, (3 + n)/2,
 Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]])/(b*d*(1 + n)*(Sin[a + b*x]^2)^(1/4))
)
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n(c\sin(a+bx))^{3/2}\,dx=-\frac{c(d\cos(a+bx))^{1+n}\,{}_2F_1\left(-\frac{1}{4},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sqrt{c\sin(a+bx)}}{bd(1+n)\sqrt[4]{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.16, size = 76, normalized size = 1.00

$$-\frac{\cot(a+bx)(c\sin(a+bx))^{3/2}(d\cos(a+bx))^n\,{}_2F_1\left(-\frac{1}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)\sqrt[4]{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -(((d*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[-1/4, (1 + n)/2, (3 +
n)/2, Cos[a + b*x]^2]*(c*Sin[a + b*x])^(3/2))/(b*(1 + n)*(Sin[a + b*x]^2)^(
1/4)))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c\sin\left(bx+a\right)}\left(d\cos\left(bx+a\right)\right)^{n}c\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n*c*sin(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}\left(d\cos\left(bx+a\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)*(d*cos(b*x + a))^n, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(bx+a\right)\right)^{n}\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

```
[Out] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}\left(d\cos\left(bx+a\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)*(d*cos(b*x + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\cos(a+b\,x))^n\,(c\sin(a+b\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(3/2),x)
```

```
[Out] int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*(c*sin(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

3.368 $\int (d\cos(a+bx))^n \sqrt{c\sin(a+bx)}\,dx$

Optimal. Leaf size=76

$$-\frac{c\sqrt[4]{\sin^2(a+bx)}\,(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{c\sin(a+bx)}}$$

```
[Out] -c*(d*cos(b*x+a))^(1+n)*hypergeom([1/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2
)*(sin(b*x+a)^2)^(1/4)/b/d/(1+n)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2576}

$$-\frac{c\sqrt[4]{\sin^2(a+bx)}\,(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/4, (1 + n)/2, (3 + n)/2,
Cos[a + b*x]^2]*(Sin[a + b*x]^2)^(1/4))/(b*d*(1 + n)*Sqrt[c*Sin[a + b*x]]))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int (d\cos(a+bx))^n \sqrt{c\sin(a+bx)}\,dx = -\frac{c(d\cos(a+bx))^{1+n}\,{}_2F_1\left(\frac{1}{4},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sqrt[4]{\sin^2(a+bx)}}{bd(1+n)\sqrt{c\sin(a+bx)}}$$

Mathematica [A] time = 0.10, size = 82, normalized size = 1.08

$$-\frac{\sin(a+bx)\cos(a+bx)\sqrt{c\sin(a+bx)}\,(d\cos(a+bx))^n\,{}_2F_1\left(\frac{1}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)\sin^2(a+bx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((Cos[a + b*x]*(d*Cos[a + b*x])^n*Hypergeometric2F1[1/4, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sin[a + b*x]*Sqrt[c*Sin[a + b*x]])/(b*(1 + n)*(Sin[a +
 b*x]^2)^(3/4)))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c\sin\left(bx+a\right)}\,\left(d\cos\left(bx+a\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c\sin\left(bx+a\right)}\,\left(d\cos\left(bx+a\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n, x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(bx+a\right)\right)^{n}\sqrt{c\sin\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

```
[Out] int((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{c\sin\left(bx+a\right)}\,\left(d\cos\left(bx+a\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d\ \cos (a + b\,x))^n\ \sqrt{c\ \sin (a + b\,x)}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(1/2),x)
[Out] int((d*cos(a + b*x))^n*(c*sin(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c \sin (a + bx)}\ (d \cos (a + bx))^n\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n*(c*sin(b*x+a))**(1/2),x)
[Out] Integral(sqrt(c*sin(a + b*x))*(d*cos(a + b*x))**n, x)
```

3.369 $\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=76

$$-\frac{c\sin^2(a+bx)^{3/4}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{3}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)(c\sin(a+bx))^{3/2}}$$

```
[Out] -c*(d*cos(b*x+a))^(1+n)*hypergeom([3/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2
)*(sin(b*x+a)^2)^(3/4)/b/d/(1+n)/(c*sin(b*x+a))^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2576}

$$-\frac{c\sin^2(a+bx)^{3/4}(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{3}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bd(n+1)(c\sin(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((c*(d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[3/4, (1 + n)/2, (3 + n)/2,
Cos[a + b*x]^2]*(Sin[a + b*x]^2)^(3/4))/(b*d*(1 + n)*(c*Sin[a + b*x])^(3/2)
))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \frac{(d\cos(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx=-\frac{c(d\cos(a+bx))^{1+n}\,{}_2F_1\left(\frac{3}{4},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sin^2(a+bx)^{3/4}}{bd(1+n)(c\sin(a+bx))^{3/2}}$$

Mathematica [A] time = 0.11, size = 82, normalized size = 1.08

$$-\frac{\sin(a+bx)\cos(a+bx)(d\cos(a+bx))^n\,{}_2F_1\left(\frac{3}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)\sqrt[4]{\sin^2(a+bx)}\,\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((Cos[a + b*x]*(d*Cos[a + b*x])^n*Hypergeometric2F1[3/4, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(1 + n)*Sqrt[c*Sin[a + b*x]]*(Sin[a +
 b*x]^2)^(1/4)))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c\sin\left(bx+a\right)}\,\left(d\cos\left(bx+a\right)\right)^{n}}{c\sin\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n/(c*sin(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(bx+a\right)\right)^{n}}{\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(bx+a\right)\right)^{n}}{\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(bx+a\right)\right)^{n}}{\sqrt{c\sin\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,\cos\left(a+b\,x\right))^n}{\sqrt{c\,\sin\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^n/(c*sin(a + b*x))^(1/2),x)
```

```
[Out] int((d*cos(a + b*x))^n/(c*sin(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\cos\left(a+bx\right))^n}{\sqrt{c\sin\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral((d*cos(a + b*x))**n/sqrt(c*sin(a + b*x)), x)
```

3.370 $\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\sqrt[4]{\sin^2(a+bx)}\,(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{5}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bcd(n+1)\sqrt{c\sin(a+bx)}}$$

```
[Out] -(d*cos(b*x+a))^(1+n)*hypergeom([5/4, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)*
(sin(b*x+a)^2)^(1/4)/b/c/d/(1+n)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2576}

$$-\frac{\sqrt[4]{\sin^2(a+bx)}\,(d\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{5}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bcd(n+1)\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -(((d*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[5/4, (1 + n)/2, (3 + n)/2, Co
s[a + b*x]^2]*(Sin[a + b*x]^2)^(1/4))/(b*c*d*(1 + n)*Sqrt[c*Sin[a + b*x]]))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rubi steps

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = -\frac{(d\cos(a+bx))^{1+n}\,{}_2F_1\left(\frac{5}{4},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sqrt[4]{\sin^2(a+bx)}}{bcd(1+n)\sqrt{c\sin(a+bx)}}$$

Mathematica [A] time = 0.13, size = 79, normalized size = 1.01

$$-\frac{\sqrt[4]{\sin^2(a+bx)}\,\cot(a+bx)\sqrt{c\sin(a+bx)}\,(d\cos(a+bx))^n\,{}_2F_1\left(\frac{5}{4},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bc^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -((((d*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[5/4, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]]*(Sin[a + b*x]^2)^(1/4))/(b*c^2*(1
 + n)))
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{c\sin\left(bx+a\right)}\,\left(d\cos\left(bx+a\right)\right)^{n}}{c^{2}\cos\left(bx+a\right)^{2}-c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*sin(b*x + a))*(d*cos(b*x + a))^n/(c^2*cos(b*x + a)^2 - c^2
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(bx+a\right)\right)^{n}}{\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(bx+a\right)\right)^{n}}{\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

```
[Out] int((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d\cos\left(bx+a\right)\right)^{n}}{\left(c\sin\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((d*cos(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^n/(c*sin(a + b*x))^(3/2),x)
[Out] int((d*cos(a + b*x))^n/(c*sin(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d\cos(a+bx))^n}{(c\sin(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**n/(c*sin(b*x+a))**(3/2),x)
[Out] Integral((d*cos(a + b*x))**n/(c*sin(a + b*x))**(3/2), x)
```

3.371 $\int \sqrt{b\sec(e+fx)}\,\sin^7(e+fx)\,dx$

Optimal. Leaf size=85

$$\frac{2b^7}{13f(b\sec(e+fx))^{13/2}}-\frac{2b^5}{3f(b\sec(e+fx))^{9/2}}+\frac{6b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] 2/13*b^7/f/(b*sec(f*x+e))^(13/2)-2/3*b^5/f/(b*sec(f*x+e))^(9/2)+6/5*b^3/f/(
b*sec(f*x+e))^(5/2)-2*b/f/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$\frac{2b^7}{13f(b\sec(e+fx))^{13/2}}-\frac{2b^5}{3f(b\sec(e+fx))^{9/2}}+\frac{6b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^7,x]
```

```
[Out] (2*b^7)/(13*f*(b*Sec[e + f*x])^(13/2)) - (2*b^5)/(3*f*(b*Sec[e + f*x])^(9/2
)) + (6*b^3)/(5*f*(b*Sec[e + f*x])^(5/2)) - (2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\,\sin^7(e+fx)\,dx = \frac{b^7\,\mathrm{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{15/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^7\,\mathrm{Subst}\left(\int \left(-\frac{1}{x^{15/2}}+\frac{3}{b^2x^{11/2}}-\frac{3}{b^4x^{7/2}}+\frac{1}{b^6x^{3/2}}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b^7}{13f(b\sec(e+fx))^{13/2}}-\frac{2b^5}{3f(b\sec(e+fx))^{9/2}}+\frac{6b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{}{f\sqrt{}}$$

Mathematica [A] time = 0.34, size = 58, normalized size = 0.68

$$\frac{(-8939\cos(e+fx)+887\cos(3(e+fx))-155\cos(5(e+fx))+15\cos(7(e+fx)))\sqrt{b\sec(e+fx)}}{6240f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^7,x]
```

```
[Out] ((-8939*Cos[e + f*x] + 887*Cos[3*(e + f*x)] - 155*Cos[5*(e + f*x)] + 15*Cos
[7*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(6240*f)
```

fricas [A] time = 0.81, size = 56, normalized size = 0.66

$$\frac{2\left(15\cos\left(fx+e\right)^7-65\cos\left(fx+e\right)^5+117\cos\left(fx+e\right)^3-195\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{195\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/195*(15*cos(f*x + e)^7 - 65*cos(f*x + e)^5 + 117*cos(f*x + e)^3 - 195*cos
(f*x + e))*sqrt(b/cos(f*x + e))/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)-256/195*(390*b^2*sqrt(-b)*(-sqrt(-b
)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^9+702*b^3*(-s
qrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^8-1716*
b^3*sqrt(-b)*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)
))^4+b))^7-1716*b^4*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x
+exp(1)))^4+b))^6+1872*b^4*sqrt(-b)*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt
(-b*tan(1/2*(f*x+exp(1)))^4+b))^5+1040*b^5*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))
^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^4+26*b^6*sqrt(-b)*(-sqrt(-b)*tan(1/2
*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))-572*b^5*sqrt(-b)*(-sqr
t(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^3-156*b^6
*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^2+2
*b^7)*sign(cos(f*x+exp(1)))/(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(
1/2*(f*x+exp(1)))^4+b)-sqrt(-b))^13/f
```

maple [B] time = 0.38, size = 517, normalized size = 6.08

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(60\left(\cos^{7}\left(fx+e\right)\right)-260\left(\cos^{5}\left(fx+e\right)\right)+468\left(\cos^{3}\left(fx+e\right)\right)+195\cos\left(fx+e\right)\sqrt{-\frac{\cos\cdots}{\left(\cos\cdots\right.}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/390/f*(-1+cos(f*x+e))^2*(60*cos(f*x+e)^7-260*cos(f*x+e)^5+468*cos(f*x+e)^
3+195*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-195*cos(f*x+e)*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)+195*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-195*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e
)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-78
0*cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^4
```

maxima [A] time = 0.32, size = 63, normalized size = 0.74

$$\frac{2\left(15\,b^{6}-\frac{65\,b^{6}}{\cos\left(fx+e\right)^{2}}+\frac{117\,b^{6}}{\cos\left(fx+e\right)^{4}}-\frac{195\,b^{6}}{\cos\left(fx+e\right)^{6}}\right)b}{195\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/195*(15*b^6 - 65*b^6/cos(f*x + e)^2 + 117*b^6/cos(f*x + e)^4 - 195*b^6/co
s(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(13/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^7\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^7*(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^7*(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**7*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.372 $\int \sqrt{b\sec(e+fx)}\,\sin^5(e+fx)\,dx$

Optimal. Leaf size=63

$$-\frac{2b^5}{9f(b\sec(e+fx))^{9/2}}+\frac{4b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] -2/9*b^5/f/(b*sec(f*x+e))^(9/2)+4/5*b^3/f/(b*sec(f*x+e))^(5/2)-2*b/f/(b*sec
(f*x+e))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$-\frac{2b^5}{9f(b\sec(e+fx))^{9/2}}+\frac{4b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^5,x]
```

```
[Out] (-2*b^5)/(9*f*(b*Sec[e + f*x])^(9/2)) + (4*b^3)/(5*f*(b*Sec[e + f*x])^(5/2)
) - (2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\,\sin^5(e+fx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{11/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^5\,\text{Subst}\left(\int \left(\frac{1}{x^{11/2}} - \frac{2}{b^2x^{7/2}} + \frac{1}{b^4x^{3/2}}\right) dx, x, b\sec(e+fx)\right)}{f}$$
$$= -\frac{2b^5}{9f(b\sec(e+fx))^{9/2}} + \frac{4b^3}{5f(b\sec(e+fx))^{5/2}} - \frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Mathematica [A] time = 0.21, size = 48, normalized size = 0.76

$$-\frac{(554\cos(e+fx) - 47\cos(3(e+fx)) + 5\cos(5(e+fx)))\sqrt{b\sec(e+fx)}}{360f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^5,x]
```

```
[Out] -1/360*((554*Cos[e + f*x] - 47*Cos[3*(e + f*x)] + 5*Cos[5*(e + f*x)])*Sqrt[
b*Sec[e + f*x]])/f
```

fricas [A] time = 0.70, size = 46, normalized size = 0.73

$$-\frac{2\left(5\cos\left(fx+e\right)^5 - 18\cos\left(fx+e\right)^3 + 45\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{45\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(5*cos(f*x + e)^5 - 18*cos(f*x + e)^3 + 45*cos(f*x + e))*sqrt(b/cos(f
*x + e))/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)-64/45*(-60*b^2*(-sqrt(-b)*tan(1/2*(
f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^6+90*b^2*sqrt(-b)*(-sqrt
(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^5+162*b^3*
(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^4+18
*b^4*sqrt(-b)*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1
)))^4+b))-108*b^3*sqrt(-b)*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1
/2*(f*x+exp(1)))^4+b))^3-72*b^4*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*
tan(1/2*(f*x+exp(1)))^4+b))^2+2*b^5)*sign(cos(f*x+exp(1)))/(-sqrt(-b)*tan(1
/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b)-sqrt(-b))^9/f
```

maple [B] time = 0.19, size = 507, normalized size = 8.05

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(20\left(\cos^{5}\left(fx+e\right)\right)-72\left(\cos^{3}\left(fx+e\right)\right)-45\cos\left(fx+e\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}\,\ln\left(-\frac{2\left(2\left(\cos^{2}\left(f\right.\right.\right.}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/90/f*(-1+cos(f*x+e))^2*(20*cos(f*x+e)^5-72*cos(f*x+e)^3-45*cos(f*x+e)*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)+45*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*c
os(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-45*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-1)/sin(f*x+e)^2)+45*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+180*cos(f*x+e))*(cos(f*x
+e)+1)^2*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^4
```

maxima [A] time = 0.32, size = 50, normalized size = 0.79

$$-\frac{2\left(5\,b^{4}-\frac{18\,b^{4}}{\cos\left(fx+e\right)^{2}}+\frac{45\,b^{4}}{\cos\left(fx+e\right)^{4}}\right)b}{45\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/45*(5*b^4 - 18*b^4/cos(f*x + e)^2 + 45*b^4/cos(f*x + e)^4)*b/(f*(b/cos(f
*x + e))^(9/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(e+f\,x\right)^5\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^5*(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.373 $\int \sqrt{b \sec(e+fx)} \sin^3(e+fx)\,dx$

Optimal. Leaf size=41

$$\frac{2b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] 2/5*b^3/f/(b*sec(f*x+e))^(5/2)-2*b/f/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2b^3}{5f(b\sec(e+fx))^{5/2}}-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^3,x]
```

```
[Out] (2*b^3)/(5*f*(b*Sec[e + f*x])^(5/2)) - (2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\sin^3(e+fx)\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{7/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^3\,\text{Subst}\left(\int \left(-\frac{1}{x^{7/2}}+\frac{1}{b^2x^{3/2}}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b^3}{5f(b\sec(e+fx))^{5/2}} - \frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Mathematica [A] time = 0.16, size = 36, normalized size = 0.88

$$\frac{(\cos(3(e+fx))-17\cos(e+fx))\sqrt{b\sec(e+fx)}}{10f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^3,x]
[Out] ((-17*Cos[e + f*x] + Cos[3*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(10*f)
```

fricas [A] time = 0.69, size = 34, normalized size = 0.83

$$\frac{2\left(\cos\left(fx+e\right)^3-5\cos\left(fx+e\right)\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{5f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 2/5*(cos(f*x + e)^3 - 5*cos(f*x + e))*sqrt(b/cos(f*x + e))/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)-16/5*(5*b^2*sqrt(-b)*(-sqrt(-b)*tan
```

```
(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))-5*b*sqrt(-b)*(-sqr
t(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^3-5*b^2*(
-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^2+b^3
)*sign(cos(f*x+exp(1)))/(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*
(f*x+exp(1)))^4+b)-sqrt(-b))^5/f
```

maple [B] time = 0.21, size = 497, normalized size = 12.12

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(4\left(\cos^{3}\left(fx+e\right)\right)-5\cos\left(fx+e\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}\ \ln\left(-\frac{2\left(\cos^{2}\left(fx+e\right)\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}-\left(\cos^{2}\left(fx\right.\right.}{\sin\left(fx\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/10/f*(-1+cos(f*x+e))^2*(4*cos(f*x+e)^3-5*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-co
s(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e
)^2)+5*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-5*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+5
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)-20*cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))
^(1/2)/sin(f*x+e)^4
```

maxima [A] time = 0.39, size = 35, normalized size = 0.85

$$\frac{2\left(b^{2}-\frac{5\,b^{2}}{\cos\left(fx+e\right)^{2}}\right)b}{5\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b^2 - 5*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(e+f\,x\right)^3\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^3*(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

3.374 $\int \sqrt{b \sec(e+fx)} \sin(e+fx)\,dx$

Optimal. Leaf size=18

$$-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] -2*b/f/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x],x]
```

```
[Out] (-2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{b\sec(e+fx)}\sin(e+fx)\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\
&= -\frac{2b}{f\sqrt{b\sec(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 1.00

$$-\frac{2b}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x],x]
```

```
[Out] (-2*b)/(f*Sqrt[b*Sec[e + f*x]])
```

fricas [A] time = 0.57, size = 23, normalized size = 1.28

$$-\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b/cos(f*x + e))*cos(f*x + e)/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)-2*sqrt(b*cos(f*x+exp(1)))*sign(cos(
f*x+exp(1)))/f
```

maple [A] time = 0.03, size = 17, normalized size = 0.94

$$-\frac{2b}{f\sqrt{b\sec\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2*b/f/(b*sec(f*x+e))^(1/2)
```

maxima [A] time = 0.55, size = 23, normalized size = 1.28

$$-\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(b/cos(f*x + e))*cos(f*x + e)/f
```

mupad [B] time = 0.20, size = 23, normalized size = 1.28

$$-\frac{2\cos\left(e+fx\right)\sqrt{\frac{b}{\cos\left(e+fx\right)}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(b/cos(e + f*x))^(1/2),x)
```

```
[Out] -(2*cos(e + f*x)*(b/cos(e + f*x))^(1/2))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(e+fx\right)}\,\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*sin(e + f*x), x)
```

3.375 $\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}$$

```
[Out] arctan((b*sec(f*x+e))^(1/2)/b^(1/2))*b^(1/2)/f-arctanh((b*sec(f*x+e))^(1/2)
/b^(1/2))*b^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2622, 329, 298, 203, 206}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[b]*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/f - (Sqrt[b]*ArcTanh[Sqrt[b*
Sec[e + f*x]]/Sqrt[b]])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}
\int \csc(e+fx)\sqrt{b\sec(e+fx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{bf}\\
&= \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{bf}\\
&= -\frac{b\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} + \frac{b\,\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+f
\end{aligned}$$

$$= \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}$$

Mathematica [A] time = 0.35, size = 73, normalized size = 1.26

$$\frac{\sqrt{b\sec(e+fx)}\left(\log\left(1-\sqrt{\sec(e+fx)}\right)-\log\left(\sqrt{\sec(e+fx)}+1\right)+2\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{2f\sqrt{\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((2*ArcTan[Sqrt[Sec[e + f*x]]] + Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt
[Sec[e + f*x]]])*Sqrt[b*Sec[e + f*x]])/(2*f*Sqrt[Sec[e + f*x]])
```

fricas [B] time = 0.71, size = 247, normalized size = 4.26

$$\left[\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)+\sqrt{-b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}-6b\cos(fx+e}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{4f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1
)/b) + sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*s
qrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*co
s(f*x + e) + 1)))/f, -1/4*(2*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f
*x + e) - 1)/sqrt(b)) - sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 +
 cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*
x + e)^2 - 2*cos(f*x + e) + 1)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)-2*(-1/4*sqrt(-b)*ln(ab
s(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b)))-1/
2*b*atan((-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4
+b))/sqrt(-b))/sqrt(-b))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 0.18, size = 169, normalized size = 2.91

$$\frac{\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)\left(\ln\left(-\frac{2\left(2\left(\cos^2(fx+e)\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\left(\cos^2(fx+e)\right)+2\cos(fx+e)-2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-1\right)}{\sin(fx+e)^2}\right)-\arctan\left(\right.\right.}{2f\sin(fx+e)^2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/2/f*(b/cos(f*x+e))^(1/2)*cos(f*x+e)*(ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-1)/sin(f*x+e)^2)-arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)))*(-1+cos(f*x+e))/sin(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

maxima [A] time = 0.54, size = 72, normalized size = 1.24

$$\frac{b\left(\frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{\sqrt{b}}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(2*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/sqrt(b) + log(-(sqrt(b) - sqr
t(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/sqrt(b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{b}{\cos(e+fx)}}}{\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/sin(e + f*x),x)
```

```
[Out] int((b/cos(e + f*x))^(1/2)/sin(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(e+fx\right)}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*csc(e + f*x), x)
```

3.376 $\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal. Leaf size=93

$$-\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf}+\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}-\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}$$

```
[Out] -1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(3/2)/b/f+3/4*arctan((b*sec(f*x+e))^(1/2)/
b^(1/2))*b^(1/2)/f-3/4*arctanh((b*sec(f*x+e))^(1/2)/b^(1/2))*b^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2622, 288, 329, 298, 203, 206}

$$-\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf}+\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}-\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (3*Sqrt[b]*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (3*Sqrt[b]*ArcTanh
[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (Cot[e + f*x]^2*(b*Sec[e + f*x])^(3
/2))/(2*b*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^3(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^{5/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4bf}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2bf}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2bf} - \frac{(3b)\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4f}$$

$$= \frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{\cot^2(e+fx)(b\sec}{2bf}$$

Mathematica [A] time = 0.62, size = 95, normalized size = 1.02

$$-\frac{\sqrt{b\sec(e+fx)}\left(-3\log\left(1-\sqrt{\sec(e+fx)}\right)+3\log\left(\sqrt{\sec(e+fx)}+1\right)+\frac{4\csc^2(e+fx)}{\sqrt{\sec(e+fx)}}-6\tan^{-1}\left(\sqrt{\sec(e+fx)}\right.\right.}{8f\sqrt{\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/8*((-6*ArcTan[Sqrt[Sec[e + f*x]]] - 3*Log[1 - Sqrt[Sec[e + f*x]]] + 3*Lo
g[1 + Sqrt[Sec[e + f*x]]] + (4*Csc[e + f*x]^2)/Sqrt[Sec[e + f*x]])*Sqrt[b*S
ec[e + f*x]])/(f*Sqrt[Sec[e + f*x]])
```

fricas [B] time = 0.71, size = 354, normalized size = 3.81

$$\left[\frac{6\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)+3\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\,\log\left(\frac{b\cos(fx+e)^2-4\left(\cos\right.}{}\right.}{16\left(f\cos\left(fx+e\right)^2-f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(6*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x +
 e))*(cos(f*x + e) + 1)/b) + 3*(cos(f*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x
 + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) -
 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*sqrt(b/co
s(f*x + e))*cos(f*x + e))/(f*cos(f*x + e)^2 - f), -1/16*(6*(cos(f*x + e)^2
- 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) -
3*(cos(f*x + e)^2 - 1)*sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 +
cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x
 + e)^2 - 2*cos(f*x + e) + 1)) - 8*sqrt(b/cos(f*x + e))*cos(f*x + e))/(f*co
s(f*x + e)^2 - f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*(-1/16*sqrt(-b*tan(1/
2*(f*x+exp(1)))^4+b)+2*(3/32*sqrt(-b)*ln(abs(-sqrt(-b)*tan(1/2*(f*x+exp(1))
)^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b)))+1/16*b*sqrt(-b)/((-sqrt(-b)*tan(1/
2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^2-b)+3/16*b*atan((-sq
rt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))/sqrt(-b)
)/sqrt(-b)))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 0.22, size = 603, normalized size = 6.48

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\left(8\left(\cos^{2}\left(fx+e\right)\right)\left(-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)^{\frac{3}{2}}+16\cos\left(fx+e\right)\left(-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)^{\frac{3}{2}}+4\left(\cos^{2}\left(fx+e\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))*(8*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
+16*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+4*cos(f*x+e)^2*ln(-2*(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*cos(f*x+e)^2*ar
ctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-cos(f*x+e)^2*ln(-(2*cos(f*x+
e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+8*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(3/2)+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-4*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-4*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-1)/sin(f*x+e)^2)+3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+ln(-
(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2))*cos(f*x+e)*(b/
cos(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/sin(f*x+e)^4
```

maxima [A] time = 0.43, size = 106, normalized size = 1.14

$$\frac{b\left(\frac{4\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}{b^2-\frac{b^2}{\cos\left(fx+e\right)^2}}+\frac{6\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{3\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos\left(fx+e\right)}}}\right)}{\sqrt{b}}\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*b*(4*(b/cos(f*x + e))^(3/2)/(b^2 - b^2/cos(f*x + e)^2) + 6*arctan(sqrt(
b/cos(f*x + e))/sqrt(b))/sqrt(b) + 3*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/
(sqrt(b) + sqrt(b/cos(f*x + e))))/sqrt(b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^3,x)
```

```
[Out] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(e + f x\right)} \csc^{3}\left(e + f x\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**3, x)
```

3.377 $\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal. Leaf size=123

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3f}-\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf}+\frac{21\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}-\frac{21\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}}{}\right.}{32f}$$

```
[Out] -7/16*cot(f*x+e)^2*(b*sec(f*x+e))^(3/2)/b/f-1/4*cot(f*x+e)^4*(b*sec(f*x+e))
^(7/2)/b^3/f+21/32*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))*b^(1/2)/f-21/32*arc
tanh((b*sec(f*x+e))^(1/2)/b^(1/2))*b^(1/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2622, 288, 329, 298, 203, 206}

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3f}-\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf}+\frac{21\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}-\frac{21\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}}{}\right.}{32f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (21*Sqrt[b]*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) - (21*Sqrt[b]*ArcT
anh[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) - (7*Cot[e + f*x]^2*(b*Sec[e + f*
x])^(3/2))/(16*b*f) - (Cot[e + f*x]^4*(b*Sec[e + f*x])^(7/2))/(4*b^3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^5(e+fx)\sqrt{b\sec(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^{9/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3 f} + \frac{7\,\text{Subst}\left(\int \frac{x^{5/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3 f}$$

$$= -\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3 f} + \frac{21\,\text{Sul}}{}$$

$$= -\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3 f} + \frac{21\,\text{Sul}}{}$$

$$= -\frac{7\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{7/2}}{4b^3 f} - \frac{(21b)\,\text{S}}{}$$

$$= \frac{21\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{21\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{7\cot^2(e+fx)(b}{16b}$$

Mathematica [A] time = 1.01, size = 107, normalized size = 0.87

$$\frac{b\left(-16\csc^4(e+fx) - 28\csc^2(e+fx) + 21\sqrt{\sec(e+fx)}\left(\log\left(1-\sqrt{\sec(e+fx)}\right) - \log\left(\sqrt{\sec(e+fx)}+1\right)\right) + 4}{64f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (b*(-28*Csc[e + f*x]^2 - 16*Csc[e + f*x]^4 + 42*ArcTan[Sqrt[Sec[e + f*x]]]*
Sqrt[Sec[e + f*x]] + 21*(Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt[Sec[e +
 f*x]]])*Sqrt[Sec[e + f*x]]))/(64*f*Sqrt[b*Sec[e + f*x]])
```

fricas [B] time = 0.93, size = 438, normalized size = 3.56

$$\left[\frac{42\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)+21\left(\cos\left(fx+e\right)^4-2\,\cos\left(f\right.\right.}{128\left(f\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/128*(42*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(1/2*sqrt
(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 21*(cos(f*x + e)^4 - 2*co
s(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(
f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x +
e)^2 + 2*cos(f*x + e) + 1)) + 8*(7*cos(f*x + e)^3 - 11*cos(f*x + e))*sqrt(b
/cos(f*x + e)))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f), -1/128*(42*(co
s(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e)
)*(cos(f*x + e) - 1)/sqrt(b)) - 21*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*
sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*s
qrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e
) + 1)) - 8*(7*cos(f*x + e)^3 - 11*cos(f*x + e))*sqrt(b/cos(f*x + e)))/(f*c
os(f*x + e)^4 - 2*f*cos(f*x + e)^2 + f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*(2*sqrt(-b*tan(1/2*(f
*x+exp(1)))^4+b)*(-5/128-1/256*tan(1/2*(f*x+exp(1)))^2)+2*(21/256*sqrt(-b)*
ln(abs(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b)
))-1/128*(-b^2*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(
1)))^4+b))-b*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)
))^4+b))^3-10*b*sqrt(-b)*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2
*(f*x+exp(1)))^4+b))^2+10*b^2*sqrt(-b))/((-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2
+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^2-b)^2+21/128*b*atan((-sqrt(-b)*tan(1/
2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))/sqrt(-b))/sqrt(-b)))*
sign(cos(f*x+exp(1)))/f
```

maple [B] time = 0.23, size = 1089, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/64/f*(72*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+56*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-11*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+32*cos(f*x+e)^3*ln(-2*(2*cos(f
*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-21*cos(f*x+e)^3*arctan(
1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-104*cos(f*x+e)*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)+44*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+11*
cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)-32*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)+21*cos(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-8
8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-88*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)+11*cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)-32*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)+21*cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2))+44*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-11*ln(-(2*cos(f*x+e)^2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+32*ln(-2*(2*cos(f*x+e)^2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-21*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)))*cos(f*x+e)*(b/cos(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)/sin(f*x+e)^4
```

maxima [A] time = 0.42, size = 138, normalized size = 1.12

$$\frac{b\left(\frac{42\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{21\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos\left(fx+e\right)}}}\right)}{\sqrt{b}}+\frac{4\left(7\,b^2\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}-11\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}\right)}{b^4-\frac{2\,b^4}{\cos\left(fx+e\right)^2}+\frac{b^4}{\cos\left(fx+e\right)^4}}\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/64*b*(42*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/sqrt(b) + 21*log(-(sqrt(b)
- sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/sqrt(b) + 4*(7*b^
2*(b/cos(f*x + e))^(3/2) - 11*(b/cos(f*x + e))^(7/2))/(b^4 - 2*b^4/cos(f*x
+ e)^2 + b^4/cos(f*x + e)^4))/f
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{\sin\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^5,x)
```

```
[Out] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^5, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.378 $\int \sqrt{b \sec(e+fx)} \sin^6(e+fx)\,dx$

Optimal. Leaf size=123

$$-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}-\frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}}+\frac{80\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+f}}{77f}$$

```
[Out] -40/77*b*sin(f*x+e)/f/(b*sec(f*x+e))^(1/2)-20/77*b*sin(f*x+e)^3/f/(b*sec(f*
x+e))^(1/2)-2/11*b*sin(f*x+e)^5/f/(b*sec(f*x+e))^(1/2)+80/77*(cos(1/2*f*x+1
/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos
(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2627, 3771, 2641}

$$-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}-\frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}}+\frac{80\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+f}}{77f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^6,x]
```

```
[Out] (80*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(77*
f) - (40*b*Sin[e + f*x])/(77*f*Sqrt[b*Sec[e + f*x]]) - (20*b*Sin[e + f*x]^3
)/(77*f*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]^5)/(11*f*Sqrt[b*Sec[e + f
*x]])
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}\,\sin^6(e+fx)\,dx &= -\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}+\frac{10}{11}\int \sqrt{b\sec(e+fx)}\,\sin^4(e+fx)\,dx\\ &= -\frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}+\frac{60}{77}\int \sqrt{b\sec(e+fx)}\,\sin^2(e+\\ &= -\frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}+\frac{40}{77}\int \sqrt{}\\ &= -\frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{20b\sin^3(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^5(e+fx)}{11f\sqrt{b\sec(e+fx)}}+\frac{1}{77}\big(40\\ &= \frac{80\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{77f}-\frac{40b\sin(e+fx)}{77f\sqrt{b\sec(e+fx)}}-\frac{2}{77}\end{aligned}$$

Mathematica [A] time = 0.15, size = 73, normalized size = 0.59

$$\frac{\sqrt{b\sec(e+fx)}\left(-435\sin(2(e+fx))+68\sin(4(e+fx))-7\sin(6(e+fx))+1280\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\right.}{1232f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^6,x]
```

```
[Out] (Sqrt[b*Sec[e + f*x]]*(1280*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] -
435*Sin[2*(e + f*x)] + 68*Sin[4*(e + f*x)] - 7*Sin[6*(e + f*x)]))/(1232*f)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^6-3\,\cos\left(fx+e\right)^4+3\,\cos\left(fx+e\right)^2-1\right)\sqrt{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(b
*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^6, x)
```

maple [C] time = 0.34, size = 165, normalized size = 1.34

$$-\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(7\left(\cos^6\left(fx+e\right)\right)-7\left(\cos^5\left(fx+e\right)\right)+40i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i\left(-1+c\right.}{\sin}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/77/f*(-1+cos(f*x+e))*(7*cos(f*x+e)^6-7*cos(f*x+e)^5+40*I*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin
(f*x+e),I)*sin(f*x+e)-24*cos(f*x+e)^4+24*cos(f*x+e)^3+37*cos(f*x+e)^2-37*co
s(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^6\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^6*(b/cos(e + f*x))^(1/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.379 $\int \sqrt{b \sec(e+fx)} \sin^4(e+fx) \, dx$

Optimal. Leaf size=95

$$-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}+\frac{8\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{7f}$$

```
[Out] -4/7*b*sin(f*x+e)/f/(b*sec(f*x+e))^(1/2)-2/7*b*sin(f*x+e)^3/f/(b*sec(f*x+e)
)^(1/2)+8/7*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1
/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2627, 3771, 2641}

$$-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}+\frac{8\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{7f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^4,x]
```

```
[Out] (8*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(7*f)
 - (4*b*Sin[e + f*x])/(7*f*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]^3)/(7*
f*Sqrt[b*Sec[e + f*x]])
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}\sin^4(e+fx)\,dx &= -\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}+\frac{6}{7}\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx\\ &= -\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}+\frac{4}{7}\int \sqrt{b\sec(e+fx)}\,dx\\ &= -\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{7f\sqrt{b\sec(e+fx)}}+\frac{1}{7}\left(4\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right.\\ &= \frac{8\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{7f}-\frac{4b\sin(e+fx)}{7f\sqrt{b\sec(e+fx)}}-\frac{2b}{7f\sqrt{}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 61, normalized size = 0.64

$$\frac{\sqrt{b\sec(e+fx)}\left(-10\sin(2(e+fx))+\sin(4(e+fx))+32\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{28f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^4,x]
```

```
[Out] (Sqrt[b*Sec[e + f*x]]*(32*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] - 10
*Sin[2*(e + f*x)] + Sin[4*(e + f*x)]))/(28*f)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^4, x)
```

maple [C] time = 0.23, size = 143, normalized size = 1.51

$$\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(-4i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\operatorname{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)+\cos^4\left(fx+e\right)\cdots}{7f\sin\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/7/f*(-1+cos(f*x+e))*(-4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+cos(f*x+e)
^4-cos(f*x+e)^3-3*cos(f*x+e)^2+3*cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e)
)^(1/2)/sin(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(e+f\,x\right)^4\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^4*(b/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(e+fx\right)}\,\sin^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*sin(e + f*x)**4, x)
```

3.380 $\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx$

Optimal. Leaf size=67

$$\frac{4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

```
[Out] -2/3*b*sin(f*x+e)/f/(b*sec(f*x+e))^(1/2)+4/3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/c
os(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b
*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2627, 3771, 2641}

$$\frac{4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^2,x]
```

```
[Out] (4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f)
 - (2*b*Sin[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]])
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}\sin^2(e+fx)\,dx &= -\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{2}{3}\int \sqrt{b\sec(e+fx)}\,dx\\ &= -\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{1}{3}\left(2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\\ &= \frac{4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}-\frac{2b\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 51, normalized size = 0.76

$$-\frac{\sqrt{b\sec(e+fx)}\left(\sin(2(e+fx))-4\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^2,x]
```

```
[Out] -1/3*(Sqrt[b*Sec[e + f*x]]*(-4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]
 + Sin[2*(e + f*x)]))/f
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\sqrt{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(b*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^2, x)
```

maple [C] time = 0.19, size = 123, normalized size = 1.84

$$-\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(2i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\operatorname{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)+\cos^{2}\left(fx+e\right)-}{3f\sin\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/3/f*(-1+cos(f*x+e))*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+cos(f*x+e)
^2-cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(e+f\,x\right)^{2}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^2*(b/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(e+fx\right)}\,\sin^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*sin(e + f*x)**2, x)
```

3.381 $\int \sqrt{b \sec(e+fx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] 2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2
*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2641}

$$\frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}\,dx &= \left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx\\ &= \frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 1.00

$$\frac{2\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]],x]
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e)), x)
```

maple [C] time = 0.17, size = 98, normalized size = 2.58

$$-\frac{2i\sqrt{\frac{b}{\cos\left(fx+e\right)}}\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\left(\cos\left(fx+e\right)+1\right)^{2}}{f\sin\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2),x)
[Out] -2*I/f*(b/cos(f*x+e))^(1/2)*(-1+cos(f*x+e))*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(cos(
f*x+e)+1)^2/sin(f*x+e)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e)), x)
```

mupad [B] time = 0.56, size = 35, normalized size = 0.92

$$\frac{2\sqrt{\cos\left(e + f\,x\right)}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\;\mathrm{F}\left(\frac{e}{2} + \frac{f\,x}{2}\middle|2\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2),x)
[Out] (2*cos(e + f*x)^(1/2)*(b/cos(e + f*x))^(1/2)*ellipticF(e/2 + (f*x)/2, 2))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(e + fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*sec(e + f*x)), x)
```

3.382 $\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] -b*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)+(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f
*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x
+e))^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2625, 3771, 2641}

$$\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}-\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -((b*Csc[e + f*x])/(f*Sqrt[b*Sec[e + f*x]])) + (Sqrt[Cos[e + f*x]]*Elliptic
F[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/f
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}+\frac{1}{2}\int\sqrt{b\sec(e+fx)}\,dx$$
$$= -\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}+\frac{1}{2}\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx$$
$$= -\frac{b\csc(e+fx)}{f\sqrt{b\sec(e+fx)}}+\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{f}$$

Mathematica [A] time = 0.12, size = 47, normalized size = 0.76

$$\frac{\sqrt{b\sec(e+fx)}\left(\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)-\cot(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((-Cot[e + f*x] + Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[b*Sec[
e + f*x]])/f
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^2, x)
```

maple [C] time = 0.20, size = 184, normalized size = 2.97

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)+i\sqrt{\frac{}{\cos}}\right.}{f\sin\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(-1+cos(f*x+e))^2*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+I*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+
cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*
x+e))^(1/2)/sin(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{{\sin\left(e+f\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^2,x)
```

```
[Out] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(e+fx\right)}\,\csc^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**2, x)
```

3.383 $\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal. Leaf size=95

$$-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}-\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}}+\frac{5\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{6f}$$

```
[Out] -5/6*b*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)-1/3*b*csc(f*x+e)^3/f/(b*sec(f*x+e)
)^(1/2)+5/6*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1
/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2625, 3771, 2641}

$$-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}-\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}}+\frac{5\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-5*b*Csc[e + f*x])/(6*f*Sqrt[b*Sec[e + f*x]]) - (b*Csc[e + f*x]^3)/(3*f*Sq
rt[b*Sec[e + f*x]]) + (5*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[
b*Sec[e + f*x]])/(6*f)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx &= -\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{5}{6}\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx\\ &= -\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}}-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{5}{12}\int \sqrt{b\sec(e+fx)}\,dx\\ &= -\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}}-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{1}{12}\left(5\sqrt{\cos(e+fx)}\,\sqrt{b\sec(e+\right.\\ &= -\frac{5b\csc(e+fx)}{6f\sqrt{b\sec(e+fx)}}-\frac{b\csc^3(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{5\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right.}{6f}\end{aligned}$$

Mathematica [A] time = 0.24, size = 63, normalized size = 0.66

$$\frac{\sqrt{b\sec(e+fx)}\left(5\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)-\cot(e+fx)\left(2\csc^2(e+fx)+5\right)\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[b*Sec[e + f*x]],x]
[Out] ((-(Cot[e + f*x]*(5 + 2*Csc[e + f*x]^2)) + 5*Sqrt[Cos[e + f*x]]*EllipticF[(
e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]])/(6*f)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^4, x)
```

maple [C] time = 0.23, size = 335, normalized size = 3.53

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(5i\left(\cos^{3}\left(fx+e\right)\right)\sin\left(fx+e\right)\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)+5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/6/f*(-1+cos(f*x+e))^2*(5*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)+5*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)-5*I*(1/(cos(f*x+
e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e
)-5*cos(f*x+e)^3+7*cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(1/2)/sin(f*
x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{\sin\left(e+f\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^4,x)
```

```
[Out] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(e+fx\right)} \csc^4\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*sec(e + f*x))*csc(e + f*x)**4, x)
```

3.384 $\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx$

Optimal. Leaf size=123

$$-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}+\frac{3\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{4f}$$

```
[Out] -3/4*b*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)-3/10*b*csc(f*x+e)^3/f/(b*sec(f*x+e
))^(1/2)-1/5*b*csc(f*x+e)^5/f/(b*sec(f*x+e))^(1/2)+3/4*(cos(1/2*f*x+1/2*e)^
2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e
)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2625, 3771, 2641}

$$-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}+\frac{3\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6*Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-3*b*Csc[e + f*x])/(4*f*Sqrt[b*Sec[e + f*x]]) - (3*b*Csc[e + f*x]^3)/(10*f
*Sqrt[b*Sec[e + f*x]]) - (b*Csc[e + f*x]^5)/(5*f*Sqrt[b*Sec[e + f*x]]) + (3
*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(4*f)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \csc^6(e+fx)\sqrt{b\sec(e+fx)}\,dx = -\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}+\frac{9}{10}\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx$$
$$= -\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}+\frac{3}{4}\int \csc^2(e+fx)\sqrt{b\sec(e+f$$
$$= -\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}+\frac{3}{8}\int \sqrt{b$$
$$= -\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}+\frac{1}{8}\left(3\sqrt{\mathrm{co}}\right.$$
$$= -\frac{3b\csc(e+fx)}{4f\sqrt{b\sec(e+fx)}}-\frac{3b\csc^3(e+fx)}{10f\sqrt{b\sec(e+fx)}}-\frac{b\csc^5(e+fx)}{5f\sqrt{b\sec(e+fx)}}+\frac{3\sqrt{\cos(e}}{}$$

Mathematica [A] time = 0.48, size = 73, normalized size = 0.59

$$\frac{\sqrt{b\sec(e+fx)}\left(15\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)-\cot(e+fx)\left(4\csc^4(e+fx)+6\csc^2(e+fx)+15\right)\right)}{20f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6*Sqrt[b*Sec[e + f*x]],x]
[Out] ((-(Cot[e + f*x]*(15 + 6*Csc[e + f*x]^2 + 4*Csc[e + f*x]^4)) + 15*Sqrt[Cos[
e + f*x]]*EllipticF[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]])/(20*f)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^6, x)
```

maple [C] time = 0.25, size = 485, normalized size = 3.94

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(15i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\left(\cos^{5}\left(fx+e\right)\right)\sin\left(fx+e\right)\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)+1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/20/f*(-1+cos(f*x+e))^2*(15*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*cos(f*x+e)^5*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)+15*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f
*x+e)^4*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-30*I*(1/(cos(f
*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*sin(f*x+e)*E
llipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-30*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)*EllipticF(I*(-1+cos(f
*x+e))/sin(f*x+e),I)+15*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+1
5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-15*cos(f*x+e)^5+36*cos(f*x+e)^3-25
*cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^9
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*csc(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{\sin\left(e+f\,x\right)^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^6,x)
```

```
[Out] int((b/cos(e + f*x))^(1/2)/sin(e + f*x)^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.385 $\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx$

Optimal. Leaf size=83

$$\frac{2b^7}{11f(b\sec(e+fx))^{11/2}}-\frac{6b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{2b^3}{f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] 2/11*b^7/f/(b*sec(f*x+e))^(11/2)-6/7*b^5/f/(b*sec(f*x+e))^(7/2)+2*b^3/f/(b*
sec(f*x+e))^(3/2)+2*b*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$\frac{2b^7}{11f(b\sec(e+fx))^{11/2}}-\frac{6b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{2b^3}{f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^7,x]
```

```
[Out] (2*b^7)/(11*f*(b*Sec[e + f*x])^(11/2)) - (6*b^5)/(7*f*(b*Sec[e + f*x])^(7/2
)) + (2*b^3)/(f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}\sin^7(e+fx)\,dx = \frac{b^7\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{13/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^7\,\text{Subst}\left(\int \left(-\frac{1}{x^{13/2}}+\frac{3}{b^2x^{9/2}}-\frac{3}{b^4x^{5/2}}+\frac{1}{b^6\sqrt{x}}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b^7}{11f(b\sec(e+fx))^{11/2}} - \frac{6b^5}{7f(b\sec(e+fx))^{7/2}} + \frac{2b^3}{f(b\sec(e+fx))^{3/2}} + \frac{2b}{}$$

Mathematica [A] time = 0.17, size = 52, normalized size = 0.63

$$\frac{b(809\cos(2(e+fx))-90\cos(4(e+fx))+7\cos(6(e+fx))+3370)\sqrt{b\sec(e+fx)}}{1232f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^7,x]
```

```
[Out] (b*(3370 + 809*Cos[2*(e + f*x)] - 90*Cos[4*(e + f*x)] + 7*Cos[6*(e + f*x)])
*Sqrt[b*Sec[e + f*x]])/(1232*f)
```

fricas [A] time = 0.75, size = 54, normalized size = 0.65

$$\frac{2\left(7\,b\cos\left(fx+e\right)^6-33\,b\cos\left(fx+e\right)^4+77\,b\cos\left(fx+e\right)^2+77\,b\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{77\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="fricas")
```

```
[Out] 2/77*(7*b*cos(f*x + e)^6 - 33*b*cos(f*x + e)^4 + 77*b*cos(f*x + e)^2 + 77*b
)*sqrt(b/cos(f*x + e))/f
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^7, x)
```

maple [B] time = 0.32, size = 969, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x)
```

```
[Out] 1/154/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(28*cos(f*x+e)^7+77*cos(f*x+e)^3
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)-77*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+231*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-231*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)-132*cos(f*x+e)^5+231*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-231*co
s(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+77*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-77*ln(-(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)+308*cos(f*x+e)^3+308*cos(f*x+e))*(b/cos(f*x+e))^(3/2)/s
in(f*x+e)^4
```

maxima [A] time = 0.31, size = 72, normalized size = 0.87

$$\frac{2\,b\left(\frac{7\,b^{6}}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{11}{2}}}-\frac{33\,b^{4}}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}}+\frac{77\,b^{2}}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}+77\,\sqrt{\frac{b}{\cos\left(fx+e\right)}}\right)}{77\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^7,x, algorithm="maxima")
```

```
[Out] 2/77*b*(7*b^6/(b/cos(f*x + e))^(11/2) - 33*b^4/(b/cos(f*x + e))^(7/2) + 77*
b^2/(b/cos(f*x + e))^(3/2) + 77*sqrt(b/cos(f*x + e)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^7\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^7*(b/cos(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)^7*(b/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**7,x)
```

```
[Out] Timed out
```

3.386 $\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx$

Optimal. Leaf size=63

$$-\frac{2b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{4b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] -2/7*b^5/f/(b*sec(f*x+e))^(7/2)+4/3*b^3/f/(b*sec(f*x+e))^(3/2)+2*b*(b*sec(f
*x+e))^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$-\frac{2b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{4b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] (-2*b^5)/(7*f*(b*Sec[e + f*x])^(7/2)) + (4*b^3)/(3*f*(b*Sec[e + f*x])^(3/2)
) + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}\sin^5(e+fx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{9/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^5\,\text{Subst}\left(\int \left(\frac{1}{x^{9/2}}-\frac{2}{b^2x^{5/2}}+\frac{1}{b^4\sqrt{x}}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= -\frac{2b^5}{7f(b\sec(e+fx))^{7/2}}+\frac{4b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Mathematica [A] time = 0.11, size = 42, normalized size = 0.67

$$\frac{b(44\cos(2(e+fx))-3\cos(4(e+fx))+215)\sqrt{b\sec(e+fx)}}{84f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] (b*(215 + 44*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(
84*f)
```

fricas [A] time = 0.66, size = 43, normalized size = 0.68

$$-\frac{2\left(3\,b\cos\left(fx+e\right)^4-14\,b\cos\left(fx+e\right)^2-21\,b\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{21\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -2/21*(3*b*cos(f*x + e)^4 - 14*b*cos(f*x + e)^2 - 21*b)*sqrt(b/cos(f*x + e)
)/f
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^5, x)
```

maple [B] time = 0.23, size = 959, normalized size = 15.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x)
```

```
[Out] -1/42/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(-21*cos(f*x+e)^3*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/
sin(f*x+e)^2)+21*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*c
os(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-63*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-1)/sin(f*x+e)^2)+63*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+12*cos(f
*x+e)^5-63*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x
+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+63*cos(f*x+e)*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)
/sin(f*x+e)^2)-21*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+21*ln(-(2*cos(f*x+e)^2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-
56*cos(f*x+e)^3-84*cos(f*x+e))*(b/cos(f*x+e))^(3/2)/sin(f*x+e)^4
```

maxima [A] time = 0.42, size = 55, normalized size = 0.87

$$-\frac{2\,b\left(\frac{3\,b^{4}}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}}-\frac{14\,b^{2}}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}-21\,\sqrt{\frac{b}{\cos\left(fx+e\right)}}\right)}{21\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -2/21*b*(3*b^4/(b/cos(f*x + e))^(7/2) - 14*b^2/(b/cos(f*x + e))^(3/2) - 21*
sqrt(b/cos(f*x + e)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(e+f\,x\right)^5\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(b/cos(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)^5*(b/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

3.387 $\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx$

Optimal. Leaf size=41

$$\frac{2b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] 2/3*b^3/f/(b*sec(f*x+e))^(3/2)+2*b*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] (2*b^3)/(3*f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}\sin^3(e+fx)\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{5/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^3\,\text{Subst}\left(\int \left(-\frac{1}{x^{5/2}}+\frac{1}{b^2\sqrt{x}}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b^3}{3f(b\sec(e+fx))^{3/2}}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Mathematica [A] time = 0.07, size = 30, normalized size = 0.73

$$\frac{b(\cos(2(e+fx))+7)\sqrt{b\sec(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
[Out] (b*(7 + Cos[2*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(3*f)
```

fricas [A] time = 0.57, size = 31, normalized size = 0.76

$$\frac{2\left(b\cos\left(fx+e\right)^2+3\,b\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
[Out] 2/3*(b*cos(f*x + e)^2 + 3*b)*sqrt(b/cos(f*x + e))/f
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^3, x)
```

maple [B] time = 0.19, size = 949, normalized size = 23.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x)
```

```
[Out] 1/6/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(3*cos(f*x+e)^3*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)-3*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+9*cos(f*x+e)^2*(-cos(f*x
+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-1)/sin(f*x+e)^2)-9*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-
(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+9*cos(f*x+e)*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)-9*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*ln(-2*(2
*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e
)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(3/2)-3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin
(f*x+e)^2)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+4*cos(f*x+e)^3+12*cos(f*x+e
))*(b/cos(f*x+e))^(3/2)/sin(f*x+e)^4
```

maxima [A] time = 0.36, size = 37, normalized size = 0.90

$$\frac{2\,b\left(\frac{b^2}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}+3\,\sqrt{\frac{b}{\cos\left(fx+e\right)}}\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 2/3*b*(b^2/(b/cos(f*x + e))^(3/2) + 3*sqrt(b/cos(f*x + e)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(e+f\,x\right)^3\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^3*(b/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**3,x)
[Out] Timed out
```

3.388 $\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx$

Optimal. Leaf size=18

$$\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

[Out] 2*b*(b*sec(f*x+e))^(1/2)/f

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x],x]

[Out] (2*b*Sqrt[b*Sec[e + f*x]])/f

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{3/2}\sin(e+fx)\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{2b\sqrt{b\sec(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.03, size = 18, normalized size = 1.00

$$\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x],x]
```

```
[Out] (2*b*Sqrt[b*Sec[e + f*x]])/f
```

fricas [A] time = 0.69, size = 18, normalized size = 1.00

$$\frac{2\,b\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] 2*b*sqrt(b/cos(f*x + e))/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)b*4*b*sign(cos(f*x+exp(1)))/f/(-sqrt
(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b)+sqrt(-b))
```

maple [A] time = 0.02, size = 17, normalized size = 0.94

$$\frac{2b\sqrt{b\sec\left(fx+e\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e),x)
```

```
[Out] 2*b*(b*sec(f*x+e))^(1/2)/f
```

maxima [A] time = 0.46, size = 23, normalized size = 1.28

$$\frac{2\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\cos\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] 2*(b/cos(f*x + e))^(3/2)*cos(f*x + e)/f
```

mupad [B] time = 0.48, size = 18, normalized size = 1.00

$$\frac{2\,b\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(b/cos(e + f*x))^(3/2),x)
```

```
[Out] (2*b*(b/cos(e + f*x))^(1/2))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e),x)
```

```
[Out] Timed out
```

3.389 $\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=77

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] -b^(3/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-b^(3/2)*arctanh((b*sec(f*x+
e))^(1/2)/b^(1/2))/f+2*b*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2622, 321, 329, 212, 206, 203}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}+\frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -((b^(3/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/f) - (b^(3/2)*ArcTanh[Sqrt
[b*Sec[e + f*x]]/Sqrt[b]])/f + (2*b*Sqrt[b*Sec[e + f*x]])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^{3/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{bf}$$

$$= \frac{2b\sqrt{b\sec(e+fx)}}{f} + \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{2b\sqrt{b\sec(e+fx)}}{f} + \frac{(2b)\operatorname{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f}$$

$$= \frac{2b\sqrt{b\sec(e+fx)}}{f} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} - \frac{b^2\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} + \frac{2b\sqrt{b\sec(e+fx)}}{f}$$

Mathematica [A] time = 0.96, size = 85, normalized size = 1.10

$$\frac{(b\sec(e+fx))^{3/2}\left(4\sqrt{\sec(e+fx)}+\log\left(1-\sqrt{\sec(e+fx)}\right)-\log\left(\sqrt{\sec(e+fx)}+1\right)-2\tan^{-1}\left(\sqrt{\sec(e+fx)}\right.\right.}{2f\sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] ((-2*ArcTan[Sqrt[Sec[e + f*x]]] + Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqr
t[Sec[e + f*x]]] + 4*Sqrt[Sec[e + f*x]])*(b*Sec[e + f*x])^(3/2))/(2*f*Sec[e
 + f*x]^(3/2))
```

fricas [B] time = 0.80, size = 278, normalized size = 3.61

$$\left[\frac{2\sqrt{-b}\,b\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)+\sqrt{-b}\,b\log\left(\frac{b\cos(fx+e)^2+4\left(\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}-6b\cos(fx\right.}{\cos(fx+e)^2+2\cos(fx+e)+1}}{4f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b)*b*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) +
 1)/b) + sqrt(-b)*b*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f*x + e
))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 +
2*cos(f*x + e) + 1)) + 8*b*sqrt(b/cos(f*x + e)))/f, 1/4*(2*b^(3/2)*arctan(1
/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) + b^(3/2)*log((b*cos(f*
x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) +
 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) + 8*b*sqrt(b/
cos(f*x + e)))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)b*2*(-2*b/(sqrt(-b)*tan
(1/2*(f*x+exp(1)))^2-sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b)-sqrt(-b))-1/4*sqrt(
-b)*ln(abs(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^
4+b)))+1/2*b*atan((-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+e
xp(1)))^4+b))/sqrt(-b))/sqrt(-b))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 0.21, size = 235, normalized size = 3.05

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{3}\left(4\cos\left(fx+e\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}+\cos\left(fx+e\right)\ln\left(-\frac{2\left(2\left(\cos^{2}\left(fx+e\right)\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}-\left(\cos^{2}\left(fx+e\right)\right)\right.}{\sin\left(fx+e\right)^{2}}\right.\right.}{2f\sin\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(-1+cos(f*x+e))^3*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+
cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f
*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2
)+cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+4*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2))*(b/cos(f*x+e))^(3/2)*cos(f*x+e)^2/sin(f*x+e)^6/(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
```

maxima [A] time = 0.51, size = 87, normalized size = 1.13

$$-\frac{\left(2\sqrt{b}\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}}\right)-\sqrt{b}\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos\left(fx+e\right)}}}\right)-4\sqrt{\frac{b}{\cos\left(fx+e\right)}}\right)b}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(b)*arctan(sqrt(b/cos(f*x + e))/sqrt(b)) - sqrt(b)*log(-(sqrt(b
) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) - 4*sqrt(b/cos(
f*x + e)))*b/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(3/2)/sin(e + f*x),x)
```

```
[Out] int((b/cos(e + f*x))^(3/2)/sin(e + f*x), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.390 $\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=113

$$-\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}-\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}+\frac{5b\sqrt{b\sec(e+fx)}}{2f}-\frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf}$$

```
[Out] -5/4*b^(3/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-5/4*b^(3/2)*arctanh((b*
sec(f*x+e))^(1/2)/b^(1/2))/f-1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(5/2)/b/f+5/2*
b*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2622, 288, 321, 329, 212, 206, 203}

$$-\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}-\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}+\frac{5b\sqrt{b\sec(e+fx)}}{2f}-\frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-5*b^(3/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (5*b^(3/2)*ArcTan
h[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) + (5*b*Sqrt[b*Sec[e + f*x]])/(2*f) -
 (Cot[e + f*x]^2*(b*Sec[e + f*x])^(5/2))/(2*b*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^3(e+fx)(b\sec(e+fx))^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^{7/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf} + \frac{5\,\text{Subst}\left(\int \frac{x^{3/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4bf}$$

$$= \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\right.}\right.}{}$$

$$= \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\right.}{2}$$

$$= \frac{5b\sqrt{b\sec(e+fx)}}{2f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{5/2}}{2bf} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{b-x^2}\right.}{4}$$

$$= -\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} + \frac{5b\sqrt{b\sec(e+f}}{2f}$$

Mathematica [A] time = 2.33, size = 97, normalized size = 0.86

$$-\frac{(b\sec(e+fx))^{3/2}\left(-5\log\left(1-\sqrt{\sec(e+fx)}\right)+5\log\left(\sqrt{\sec(e+fx)}+1\right)+4\left(\csc^2(e+fx)-5\right)\sqrt{\sec(e+fx)}\right.}{8f\sec^{\frac{3}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/8*((10*ArcTan[Sqrt[Sec[e + f*x]]] - 5*Log[1 - Sqrt[Sec[e + f*x]]] + 5*Lo
g[1 + Sqrt[Sec[e + f*x]]] + 4*(-5 + Csc[e + f*x]^2)*Sqrt[Sec[e + f*x]])*(b*
Sec[e + f*x])^(3/2))/(f*Sec[e + f*x]^(3/2))
```

fricas [B] time = 0.56, size = 388, normalized size = 3.43

$$\left[\frac{10\left(b\cos\left(fx+e\right)^{2}-b\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\left(\cos\left(fx+e\right)+1\right)}{2b}\right)+5\left(b\cos\left(fx+e\right)^{2}-b\right)\sqrt{-b}\log\left(\frac{b\cos\left(fx+e\right)^{2}+}{}\right.}{16\left(f\cos\left(fx+e\right)^{2}-f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(10*(b*cos(f*x + e)^2 - b)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*
x + e))*(cos(f*x + e) + 1)/b) + 5*(b*cos(f*x + e)^2 - b)*sqrt(-b)*log((b*co
s(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x +
e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*(5*b
*cos(f*x + e)^2 - 4*b)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^2 - f), 1/16*(
10*(b*cos(f*x + e)^2 - b)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x
+ e) - 1)/sqrt(b)) + 5*(b*cos(f*x + e)^2 - b)*sqrt(b)*log((b*cos(f*x + e)^2
 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos
(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) + 8*(5*b*cos(f*x + e)
^2 - 4*b)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^2 - f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)b*2*(1/16*sqrt(-b*tan(1
/2*(f*x+exp(1)))^4+b)+2*(-5/32*sqrt(-b)*ln(abs(-sqrt(-b)*tan(1/2*(f*x+exp(1
)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b)))+1/16*(b*sqrt(-b)*(-sqrt(-b)*tan(
1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))+16*b*(-sqrt(-b)*tan
(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^2-17*b^2)/(-b*(-sq
rt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))+(-sqrt(-
b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^3+sqrt(-b)*(
-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^2-b*s
qrt(-b))+5/16*b*atan((-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*
x+exp(1)))^4+b))/sqrt(-b))/sqrt(-b)))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 0.19, size = 644, normalized size = 5.70

$$\frac{\left(\cos\left(fx+e\right)+1\right)\left(-1+\cos\left(fx+e\right)\right)^{3}\left(4\left(\cos^{3}\left(fx+e\right)\right)\left(-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)^{\frac{3}{2}}+8\left(\cos^{2}\left(fx+e\right)\right)\left(-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/8/f*(cos(f*x+e)+1)*(-1+cos(f*x+e))^3*(4*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(3/2)+8*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+4*cos(
f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-16*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-4*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-1)/sin(f*x+e)^2)-5*cos(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2))-cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-1)/sin(f*x+e)^2)+4*cos(f*x+e)*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^
(1/2)-1)/sin(f*x+e)^2)+5*cos(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2))+cos(f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/
2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(
f*x+e)^2)+16*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(b/cos(f*x+e))^(3/2)*cos
(f*x+e)^2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)/sin(f*x+e)^8
```

maxima [A] time = 0.68, size = 123, normalized size = 1.09

$$\frac{\left(\frac{4\,b^{2}\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{b^{2}-\frac{b^{2}}{\cos\left(fx+e\right)^{2}}}-10\,\sqrt{b}\,\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}}\right)+5\,\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos\left(fx+e\right)}}}\right)+16\,\sqrt{\frac{b}{\cos\left(fx+e\right)}}\right)b}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(4*b^2*sqrt(b/cos(f*x + e))/(b^2 - b^2/cos(f*x + e)^2) - 10*sqrt(b)*arc
tan(sqrt(b/cos(f*x + e))/sqrt(b)) + 5*sqrt(b)*log(-(sqrt(b) - sqrt(b/cos(f*
x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) + 16*sqrt(b/cos(f*x + e)))*b/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^3,x)
[Out] int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

3.391 $\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx$

Optimal. Leaf size=128

$$\frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}+\frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{16b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{3f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] 8/3*b^3*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)+20/9*b^3*sin(f*x+e)^3/f/(b*sec(f*
x+e))^(3/2)-16/3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*Ellipt
icE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)+2*b
*sin(f*x+e)^5*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.15, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2624, 2627, 3771, 2639}

$$\frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}+\frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{16b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{3f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sin^5(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^6,x]
```

```
[Out] (-16*b^2*EllipticE[(e + f*x)/2, 2])/(3*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
f*x]]) + (8*b^3*Sin[e + f*x])/(3*f*(b*Sec[e + f*x])^(3/2)) + (20*b^3*Sin[e
+ f*x]^3)/(9*f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e +
f*x]^5)/f
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}\sin^6(e+fx)\,dx = \frac{2b\sqrt{b\sec(e+fx)}\,\sin^5(e+fx)}{f} - \left(10b^2\right)\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\,\sin^5(e+fx)}{f} - \frac{1}{3}\left(20b^2\right)\int \frac{\sin}{\sqrt{b\,s}}$$
$$= \frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\,\sin^5(e+}{f}$$
$$= \frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{20b^3\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\,\sin^5(e+}{f}$$
$$= -\frac{16b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{3f\sqrt{\cos(e+fx)}\,\sqrt{b\sec(e+fx)}} + \frac{8b^3\sin(e+fx)}{3f(b\sec(e+fx))^{3/2}} + \frac{20b^3\sin^3(e}{9f(b\sec(e+}$$

Mathematica [A] time = 0.15, size = 70, normalized size = 0.55

$$-\frac{b\sqrt{b\sec(e+fx)}\left(-158\sin(e+fx)-13\sin(3(e+fx))+\sin(5(e+fx))+384\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{72f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^6,x]
```

```
[Out] -1/72*(b*Sqrt[b*Sec[e + f*x]]*(384*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2
, 2] - 158*Sin[e + f*x] - 13*Sin[3*(e + f*x)] + Sin[5*(e + f*x)]))/f
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^6-3\,b\cos\left(fx+e\right)^4+3\,b\cos\left(fx+e\right)^2-b\right)\sqrt{b\sec\left(fx+e\right)}\,\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^6 - 3*b*cos(f*x + e)^4 + 3*b*cos(f*x + e)^2 - b)*
sqrt(b*sec(f*x + e))*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^6, x)
```

maple [C] time = 0.29, size = 330, normalized size = 2.58

$$\frac{2\left(\cos^{6}\left(fx+e\right)-24i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)+24i\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x)
```

```
[Out] 2/9/f*(cos(f*x+e)^6-24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+24
*I*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-24*I*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*sin(f*x+e)+24*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-5*cos(f*x+e)^4+
19*cos(f*x+e)^2-24*cos(f*x+e)+9)*cos(f*x+e)*(b/cos(f*x+e))^(3/2)/sin(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^6\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(b/cos(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)^6*(b/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

3.392 $\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx$

Optimal. Leaf size=98

$$\frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{24b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sin^3(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] 12/5*b^3*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)-24/5*b^2*(cos(1/2*f*x+1/2*e)^2)^
(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)
^(1/2)/(b*sec(f*x+e))^(1/2)+2*b*sin(f*x+e)^3*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2624, 2627, 3771, 2639}

$$\frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{24b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2b\sin^3(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (-24*b^2*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
f*x]]) + (12*b^3*Sin[e + f*x])/(5*f*(b*Sec[e + f*x])^(3/2)) + (2*b*Sqrt[b*S
ec[e + f*x]]*Sin[e + f*x]^3)/f
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}\sin^4(e+fx)\,dx = \frac{2b\sqrt{b\sec(e+fx)}\,\sin^3(e+fx)}{f} - \left(6b^2\right)\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

$$= \frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\,\sin^3(e+fx)}{f} - \frac{1}{5}\left(12b^2\right)\int \frac{}{\sqrt{b\,\mathrm{s}}}$$

$$= \frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+fx)}\,\sin^3(e+fx)}{f} - \frac{\left(12b^2\right)\int\sqrt{\cos}}{5\sqrt{\cos(e+fx)}}$$

$$= -\frac{24b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{12b^3\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{2b\sqrt{b\sec(e+}}{}$$

Mathematica [A] time = 0.13, size = 60, normalized size = 0.61

$$\frac{b\sqrt{b\sec(e+fx)}\left(21\sin(e+fx)+\sin(3(e+fx))-48\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{10f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (b*Sqrt[b*Sec[e + f*x]]*(-48*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2] +
 21*Sin[e + f*x] + Sin[3*(e + f*x)]))/(10*f)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\cos\left(fx+e\right)^4 - 2\,b\cos\left(fx+e\right)^2 + b\right)\sqrt{b\sec\left(fx+e\right)}\,\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + b)*sqrt(b*sec(f*x + e))*s
ec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

maple [C] time = 0.23, size = 320, normalized size = 3.27

$$-\frac{2\left(12i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)-12i\cos\left(fx+e\right)\sin\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x)
```

```
[Out] -2/5/f*(12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-12*I*cos(f*x+e
)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ell
ipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f
*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f
*x+e)-12*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^
(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+cos(f*x+e)^4-8*cos(f*x+e)^2
+12*cos(f*x+e)-5)*cos(f*x+e)*(b/cos(f*x+e))^(3/2)/sin(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^4\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^4*(b/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**4,x)
[Out] Timed out
```

3.393 $\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx$

Optimal. Leaf size=66

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -4*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*
x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)+2*b*sin(f*x+e)*(b
*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2624, 3771, 2639}

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (-4*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]]) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/f
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}\sin^2(e+fx)\,dx = \frac{2b\sqrt{b\sec(e+fx)}\,\sin(e+fx)}{f} - \left(2b^2\right)\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= \frac{2b\sqrt{b\sec(e+fx)}\,\sin(e+fx)}{f} - \frac{\left(2b^2\right)\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\,\sqrt{b\sec(e+fx)}}$$
$$= -\frac{4b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\,\sqrt{b\sec(e+fx)}} + \frac{2b\sqrt{b\sec(e+fx)}\,\sin(e+fx)}{f}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 0.73

$$\frac{2b\sqrt{b\sec(e+fx)}\left(\sin(e+fx)-2\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (2*b*Sqrt[b*Sec[e + f*x]]*(-2*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2]
+ Sin[e + f*x]))/f
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-b\right)\sqrt{b\sec\left(fx+e\right)}\,\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - b)*sqrt(b*sec(f*x + e))*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
```

[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^2, x)

maple [C] time = 0.20, size = 310, normalized size = 4.70

$$2\left(-2i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\;\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)+2i\cos\left(fx+e\right)\sin\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x)

[Out] 2/f*(-2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+2*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+cos(f*x+e)^2-2*cos(f*x+e)+1)*cos(f*x+e)*(b/cos(f*x+e))^(3/2)/sin(f*x+e)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e))^(3/2)*sin(f*x + e)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(e+f\,x\right)^{2}\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f*x)^2*(b/cos(e + f*x))^(3/2),x)

[Out] int(sin(e + f*x)^2*(b/cos(e + f*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2)*sin(f*x+e)**2,x)
```

```
[Out] Timed out
```

3.394 $\int (b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=66

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -2*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*
x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)+2*b*sin(f*x+e)*(b
*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3771, 2639}

$$\frac{2b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}-\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]]) + (2*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/f
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\sec(e+fx))^{3/2}\,dx = \frac{2b\sqrt{b\sec(e+fx)}\,\sin(e+fx)}{f} - b^2\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= \frac{2b\sqrt{b\sec(e+fx)}\,\sin(e+fx)}{f} - \frac{b^2\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\,\sqrt{b\sec(e+fx)}}$$
$$= -\frac{2b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\,\sqrt{b\sec(e+fx)}} + \frac{2b\sqrt{b\sec(e+fx)}\,\sin(e+fx)}{f}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 0.73

$$\frac{2b\sqrt{b\sec(e+fx)}\left(\sin(e+fx)-\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b*Sqrt[b*Sec[e + f*x]]*(-(Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2])
+ Sin[e + f*x]))/f
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)}\,b\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*b*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

[Out] integrate((b*sec(f*x + e))^(3/2), x)

maple [C] time = 0.26, size = 322, normalized size = 4.88

$$\frac{2\left(\cos\left(fx+e\right)+1\right)^{2}\left(-1+\cos\left(fx+e\right)\right)^{2}\left(i\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\ \mathrm{EllipticE}\left(\frac{i(-1\cdots}{s\cdots}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(f*x+e))^(3/2),x)

[Out] 2/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-cos(f*x+e)+1)*cos(f*x+e)*(b/cos(f*x+e))^(3/2)/sin(f*x+e)^5

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b/cos(e + f*x))^(3/2),x)

[Out] int((b/cos(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(3/2), x)
```

3.395 $\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=90

$$-\frac{3b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}+\frac{3b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

```
[Out] -3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*
x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)-b*csc(f*x+e)*(b*s
ec(f*x+e))^(1/2)/f+3*b*sin(f*x+e)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2625, 3768, 3771, 2639}

$$-\frac{3b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f}+\frac{3b\sin(e+fx)\sqrt{b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-3*b^2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]]) - (b*Csc[e + f*x]*Sqrt[b*Sec[e + f*x]])/f + (3*b*Sqrt[b*Sec[e + f*x]]*S
in[e + f*x])/f
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx = -\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f} + \frac{3}{2}\int (b\sec(e+fx))^{3/2}\,dx$$

$$= -\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f} + \frac{3b\sqrt{b\sec(e+fx)}\,\sin(e+fx)}{f} - \frac{1}{2}\left(3b^2\right)$$

$$= -\frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f} + \frac{3b\sqrt{b\sec(e+fx)}\,\sin(e+fx)}{f} - \frac{\left(3b^2\right.}{2\sqrt{\cos(}}$$

$$= -\frac{3b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\csc(e+fx)\sqrt{b\sec(e+fx)}}{f} + \frac{3b\sqrt{b}}{}$$

Mathematica [A] time = 0.15, size = 57, normalized size = 0.63

$$\frac{b\sqrt{b\sec(e+fx)}\left(3\sin(e+fx)-\csc(e+fx)-3\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (b*Sqrt[b*Sec[e + f*x]]*(-Csc[e + f*x] - 3*Sqrt[Cos[e + f*x]]*EllipticE[(e
+ f*x)/2, 2] + 3*Sin[e + f*x]))/f
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)}\,b\csc\left(fx+e\right)^2\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*b*csc(f*x + e)^2*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

maple [C] time = 0.20, size = 322, normalized size = 3.58

$$-\frac{\left(\cos\left(fx+e\right)+1\right)^{2}\left(-1+\cos\left(fx+e\right)\right)^{2}\left(3i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(
f*x+e)*cos(f*x+e)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*
sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*(
1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+c
os(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)+3*cos(f*x+e)-2)*cos(f*x+e)*(b/cos(f*x+e))^(3/2)/sin(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}}{\sin\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^2,x)
```

```
[Out] int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.396 $\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{7b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}-\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}+\frac{7b\sin(e+fx)}{2}$$

```
[Out] -7/2*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*
f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)-7/6*b*csc(f*x+e
)*(b*sec(f*x+e))^(1/2)/f-1/3*b*csc(f*x+e)^3*(b*sec(f*x+e))^(1/2)/f+7/2*b*si
n(f*x+e)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.13, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2625, 3768, 3771, 2639}

$$-\frac{7b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}-\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}+\frac{7b\sin(e+fx)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-7*b^2*EllipticE[(e + f*x)/2, 2])/(2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) - (7*b*Csc[e + f*x]*Sqrt[b*Sec[e + f*x]])/(6*f) - (b*Csc[e + f*x]^3*S
qrt[b*Sec[e + f*x]])/(3*f) + (7*b*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/(2*f)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \csc^4(e+fx)(b\sec(e+fx))^{3/2}\,dx &= -\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}+\frac{7}{6}\int \csc^2(e+fx)(b\sec(e+fx))^{3/2}\,dx\\ &= -\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}+\frac{7}{4}\int (b\\ &= -\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}+\frac{7b\sqrt{b}}{}\\ &= -\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}-\frac{b\csc^3(e+fx)\sqrt{b\sec(e+fx)}}{3f}+\frac{7b\sqrt{b}}{}\\ &= -\frac{7b^2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{7b\csc(e+fx)\sqrt{b\sec(e+fx)}}{6f}-\frac{b\,\mathrm{cs}}{}\end{aligned}$$

Mathematica [A] time = 0.21, size = 77, normalized size = 0.62

$$-\frac{b\sin(e+fx)\sqrt{b\sec(e+fx)}\left(2\csc^4(e+fx)+7\csc^2(e+fx)+21\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)-\right.}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/6*(b*(-21 + 7*Csc[e + f*x]^2 + 2*Csc[e + f*x]^4 + 21*Sqrt[Cos[e + f*x]]*
Csc[e + f*x]*EllipticE[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/
f
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)}\,b\csc\left(fx+e\right)^4\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sec(f*x + e))*b*csc(f*x + e)^4*sec(f*x + e), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)\right)^{\frac{3}{2}} \csc \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^4, x)

maple [C] time = 0.22, size = 622, normalized size = 5.02

$$\frac{\left(\cos \left(fx+e\right)+1\right)^{2} \left(-1+\cos \left(fx+e\right)\right)^{2} \left(21 i \left(\cos^{3}\left(fx+e\right)\right) \sin \left(fx+e\right) \sqrt{\frac{1}{\cos \left(fx+e\right)+1}}\, \sqrt{\frac{\cos \left(fx+e\right)}{\cos \left(fx+e\right)+1}}\, \mathrm{EllipticF}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x)

[Out] 1/6/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(21*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)+21*I*cos(f*x+e)^2*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*cos(f*x+e)^2*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+21*cos(f*x+e)^3-14*cos(f*x+e)^2-21*cos(f*x+e)+12)*cos(f*x+e)*(b/cos(f*x+e))^(3/2)/sin(f*x+e)^7

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)\right)^{\frac{3}{2}} \csc \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^(3/2)*csc(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^4,x)
[Out] int((b/cos(e + f*x))^(3/2)/sin(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

3.397 $\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx$

Optimal. Leaf size=85

$$\frac{2b^7}{9f(b\sec(e+fx))^{9/2}}-\frac{6b^5}{5f(b\sec(e+fx))^{5/2}}+\frac{6b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] 2/9*b^7/f/(b*sec(f*x+e))^(9/2)-6/5*b^5/f/(b*sec(f*x+e))^(5/2)+2/3*b*(b*sec(
f*x+e))^(3/2)/f+6*b^3/f/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$\frac{2b^7}{9f(b\sec(e+fx))^{9/2}}-\frac{6b^5}{5f(b\sec(e+fx))^{5/2}}+\frac{6b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^7,x]
```

```
[Out] (2*b^7)/(9*f*(b*Sec[e + f*x])^(9/2)) - (6*b^5)/(5*f*(b*Sec[e + f*x])^(5/2))
 + (6*b^3)/(f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int (b\sec(e+fx))^{5/2}\sin^7(e+fx)\,dx = \frac{b^7\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{11/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^7\,\text{Subst}\left(\int \left(-\frac{1}{x^{11/2}}+\frac{3}{b^2x^{7/2}}-\frac{3}{b^4x^{3/2}}+\frac{\sqrt{x}}{b^6}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b^7}{9f(b\sec(e+fx))^{9/2}} - \frac{6b^5}{5f(b\sec(e+fx))^{5/2}} + \frac{6b^3}{f\sqrt{b\sec(e+fx)}} + \frac{2b(b\text{se}}{}$$

Mathematica [A] time = 0.42, size = 52, normalized size = 0.61

$$\frac{b(1803\cos(2(e+fx))-78\cos(4(e+fx))+5\cos(6(e+fx))+2366)(b\sec(e+fx))^{3/2}}{720f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^7,x]
```

```
[Out] (b*(2366 + 1803*Cos[2*(e + f*x)] - 78*Cos[4*(e + f*x)] + 5*Cos[6*(e + f*x)]
)*(b*Sec[e + f*x])^(3/2))/(720*f)
```

fricas [A] time = 0.56, size = 70, normalized size = 0.82

$$\frac{2\left(5\,b^2\cos\left(fx+e\right)^6-27\,b^2\cos\left(fx+e\right)^4+135\,b^2\cos\left(fx+e\right)^2+15\,b^2\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{45\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^2*cos(f*x + e)^6 - 27*b^2*cos(f*x + e)^4 + 135*b^2*cos(f*x + e)^2
 + 15*b^2)*sqrt(b/cos(f*x + e))/(f*cos(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^7, x)
```

maple [B] time = 0.21, size = 532, normalized size = 6.26

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(20\left(\cos^{6}\left(fx+e\right)\right)-108\left(\cos^{4}\left(fx+e\right)\right)-135\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}\left(\cos^{2}\left(fx+e\right)\right)\ln\left(-\frac{2\left(2\left(\cos\right.\right.}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x)
```

```
[Out] 1/90/f*(-1+cos(f*x+e))^2*(20*cos(f*x+e)^6-108*cos(f*x+e)^4-135*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)+135*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2
)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-135
*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*(2*cos(f*x+e)^2*(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+135*cos(f*x+e)*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-c
os(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+
e)^2)+540*cos(f*x+e)^2+60)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(5/2)
/sin(f*x+e)^4
```

maxima [A] time = 0.54, size = 66, normalized size = 0.78

$$\frac{2\left(15\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}+\frac{5\,b^{6}-\frac{27\,b^{6}}{\cos\left(fx+e\right)^{2}}+\frac{135\,b^{6}}{\cos\left(fx+e\right)^{4}}}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{9}{2}}}\right)b}{45\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^7,x, algorithm="maxima")
```

```
[Out] 2/45*(15*(b/cos(f*x + e))^(3/2) + (5*b^6 - 27*b^6/cos(f*x + e)^2 + 135*b^6/
cos(f*x + e)^4)/(b/cos(f*x + e))^(9/2))*b/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^7\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^7*(b/cos(e + f*x))^(5/2),x)
[Out] int(sin(e + f*x)^7*(b/cos(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**7,x)
[Out] Timed out
```

3.398 $\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx$

Optimal. Leaf size=63

$$-\frac{2b^5}{5f(b\sec(e+fx))^{5/2}}+\frac{4b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] -2/5*b^5/f/(b*sec(f*x+e))^(5/2)+2/3*b*(b*sec(f*x+e))^(3/2)/f+4*b^3/f/(b*sec
(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$-\frac{2b^5}{5f(b\sec(e+fx))^{5/2}}+\frac{4b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^5,x]
```

```
[Out] (-2*b^5)/(5*f*(b*Sec[e + f*x])^(5/2)) + (4*b^3)/(f*Sqrt[b*Sec[e + f*x]]) +
(2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int (b\sec(e+fx))^{5/2}\sin^5(e+fx)\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{7/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^5\,\text{Subst}\left(\int \left(\frac{1}{x^{7/2}} - \frac{2}{b^2x^{3/2}} + \frac{\sqrt{x}}{b^4}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= -\frac{2b^5}{5f(b\sec(e+fx))^{5/2}} + \frac{4b^3}{f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Mathematica [A] time = 0.33, size = 42, normalized size = 0.67

$$\frac{b(108\cos(2(e+fx)) - 3\cos(4(e+fx)) + 151)(b\sec(e+fx))^{3/2}}{60f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^5,x]
[Out] (b*(151 + 108*Cos[2*(e + f*x)] - 3*Cos[4*(e + f*x)])*(b*Sec[e + f*x])^(3/2)
)/(60*f)
```

fricas [A] time = 0.69, size = 57, normalized size = 0.90

$$-\frac{2\left(3\,b^2\cos\left(fx+e\right)^4 - 30\,b^2\cos\left(fx+e\right)^2 - 5\,b^2\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{15\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="fricas")
[Out] -2/15*(3*b^2*cos(f*x + e)^4 - 30*b^2*cos(f*x + e)^2 - 5*b^2)*sqrt(b/cos(f*x
 + e))/(f*cos(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^5, x)
```

maple [B] time = 0.20, size = 522, normalized size = 8.29

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(6\left(\cos^{4}\left(fx+e\right)\right)-15\left(\cos^{2}\left(fx+e\right)\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}\ \ln\left(-\frac{2\left(\cos^{2}\left(fx+e\right)\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}-(\cdots}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x)
```

```
[Out] -1/15/f*(-1+cos(f*x+e))^2*(6*cos(f*x+e)^4-15*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)+15*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*cos(f*x+e)^2*ln(-2*(2*cos(
f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-15*cos(f*x+e)*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1
)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
-1)/sin(f*x+e)^2)+15*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*ln(-2*
(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-60*cos(f*x+e)^2
-10)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(5/2)/sin(f*x+e)^4
```

maxima [A] time = 0.40, size = 52, normalized size = 0.83

$$\frac{2\left(5\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}-\frac{3\left(b^{4}-\frac{10\,b^{4}}{\cos\left(fx+e\right)^{2}}\right)}{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}\right)b}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] 2/15*(5*(b/cos(f*x + e))^(3/2) - 3*(b^4 - 10*b^4/cos(f*x + e)^2)/(b/cos(f*x
 + e))^(5/2))*b/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(e+f\,x\right)^5\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(b/cos(e + f*x))^(5/2),x)
[Out] int(sin(e + f*x)^5*(b/cos(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**5,x)
[Out] Timed out
```

3.399 $\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx$

Optimal. Leaf size=41

$$\frac{2b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)/f+2*b^3/f/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2b^3}{f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^3,x]
```

```
[Out] (2*b^3)/(f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int (b\sec(e+fx))^{5/2}\sin^3(e+fx)\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{3/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^3\,\text{Subst}\left(\int \left(-\frac{1}{x^{3/2}}+\frac{\sqrt{x}}{b^2}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b^3}{f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Mathematica [A] time = 0.20, size = 32, normalized size = 0.78

$$\frac{b(3\cos(2(e+fx))+5)(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^3,x]
[Out] (b*(5 + 3*Cos[2*(e + f*x)])*(b*Sec[e + f*x])^(3/2))/(3*f)
```

fricas [A] time = 0.58, size = 42, normalized size = 1.02

$$\frac{2\left(3\,b^2\cos\left(fx+e\right)^2+b^2\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="fricas")
[Out] 2/3*(3*b^2*cos(f*x + e)^2 + b^2)*sqrt(b/cos(f*x + e))/(f*cos(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^3, x)
```

maple [B] time = 0.18, size = 357, normalized size = 8.71

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\left(12\left(\cos^{3}\left(fx+e\right)\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}+12\left(\cos^{2}\left(fx+e\right)\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}-3\left(\cos^{2}\left(fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x)
```

```
[Out] -1/6/f*(-1+cos(f*x+e))*(12*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)+12*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3*cos(f*x+e)^2*ln(-(
2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+
e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*cos(f*x+e)^2*l
n(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*co
s(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+4*cos(f*x+
e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2))*cos(f*x+e)*(b/cos(f*x+e))^(5/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/s
in(f*x+e)^2
```

maxima [A] time = 0.33, size = 36, normalized size = 0.88

$$\frac{2\left(\frac{3b^{2}}{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}+\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\right)b}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 2/3*(3*b^2/sqrt(b/cos(f*x + e)) + (b/cos(f*x + e))^(3/2))*b/f
```

mupad [B] time = 0.91, size = 50, normalized size = 1.22

$$\frac{b^{2}\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,\left(\frac{13\,\cos\left(e+f\,x\right)}{3}+\cos\left(3\,e+3\,f\,x\right)\right)}{f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(b/cos(e + f*x))^(5/2),x)
```

```
[Out] (b^2*(b/cos(e + f*x))^(1/2)*((13*cos(e + f*x))/3 + cos(3*e + 3*f*x)))/(f*(c
os(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

3.400 $\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx$

Optimal. Leaf size=20

$$\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

[Out] 2/3*b*(b*sec(f*x+e))^(3/2)/f

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x],x]

[Out] (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{5/2}\sin(e+fx)\,dx &= \frac{b\,\text{Subst}\left(\int \sqrt{x}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= \frac{2b(b\sec(e+fx))^{3/2}}{3f}\end{aligned}$$

Mathematica [A] time = 0.04, size = 20, normalized size = 1.00

$$\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x],x]
```

```
[Out] (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

fricas [A] time = 0.63, size = 28, normalized size = 1.40

$$\frac{2\, b^{2} \sqrt{\frac{b}{\cos\left(f x + e\right)}}}{3\, f \cos\left(f x + e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] 2/3*b^2*sqrt(b/cos(f*x + e))/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)-4/3*b^2*(3*b*(-sqrt(-b)*tan(1/2*(f*
x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^2-b^2)*sign(cos(f*x+exp(1)
))/(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b)+sq
rt(-b))^3/f
```

maple [A] time = 0.02, size = 17, normalized size = 0.85

$$\frac{2b \left(b \sec\left(f x + e\right)\right)^{\frac{3}{2}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e),x)
```

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)/f
```

maxima [A] time = 0.56, size = 23, normalized size = 1.15

$$\frac{2 \left(\frac{b}{\cos\left(f x + e\right)}\right)^{\frac{5}{2}} \cos\left(f x + e\right)}{3\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] 2/3*(b/cos(f*x + e))^(5/2)*cos(f*x + e)/f
```

mupad [B] time = 0.62, size = 39, normalized size = 1.95

$$\frac{4\,b^2\,\cos\left(e+f\,x\right)\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{3\,f\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(b/cos(e + f*x))^(5/2),x)
```

```
[Out] (4*b^2*cos(e + f*x)*(b/cos(e + f*x))^(1/2))/(3*f*(cos(2*e + 2*f*x) + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e),x)
```

```
[Out] Timed out
```

3.401 $\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=78

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] b^(5/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-b^(5/2)*arctanh((b*sec(f*x+e
))^(1/2)/b^(1/2))/f+2/3*b*(b*sec(f*x+e))^(3/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2622, 321, 329, 298, 203, 206}

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f}+\frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b^(5/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/f - (b^(5/2)*ArcTanh[Sqrt[b*
Sec[e + f*x]]/Sqrt[b]])/f + (2*b*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\int \csc(e+fx)(b\sec(e+fx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int \frac{x^{5/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{bf} \\
&= \frac{2b(b\sec(e+fx))^{3/2}}{3f} + \frac{b\,\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{f} \\
&= \frac{2b(b\sec(e+fx))^{3/2}}{3f} + \frac{(2b)\,\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} \\
&= \frac{2b(b\sec(e+fx))^{3/2}}{3f} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} + \frac{b^3\,\text{Subst}\left(\right.}{} \\
&= \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} - \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{f} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 87, normalized size = 1.12

$$\frac{(b\sec(e+fx))^{5/2}\left(4\sec^{\frac{3}{2}}(e+fx)+3\log\left(1-\sqrt{\sec(e+fx)}\right)-3\log\left(\sqrt{\sec(e+fx)}+1\right)+6\tan^{-1}\left(\sqrt{\sec(e+f\right.\right.}{6f\sec^{\frac{5}{2}}(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((b*Sec[e + f*x])^(5/2)*(6*ArcTan[Sqrt[Sec[e + f*x]]] + 3*Log[1 - Sqrt[Sec[
e + f*x]]] - 3*Log[1 + Sqrt[Sec[e + f*x]]] + 4*Sec[e + f*x]^(3/2)))/(6*f*Se
c[e + f*x]^(5/2))
```

fricas [B] time = 0.67, size = 328, normalized size = 4.21

$$\left[\frac{6\sqrt{-b}\,b^2\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)\cos\left(fx+e\right)+3\sqrt{-b}\,b^2\cos\left(fx+e\right)\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2-c\right.}{\cos(fx\right.}}{12f\cos\left(fx+e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(-b)*b^2*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e
) + 1)/b)*cos(f*x + e) + 3*sqrt(-b)*b^2*cos(f*x + e)*log((b*cos(f*x + e)^2
- 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos
(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) + 8*b^2*sqrt(b/cos(f*
x + e)))/(f*cos(f*x + e)), -1/12*(6*b^(5/2)*arctan(1/2*sqrt(b/cos(f*x + e))
*(cos(f*x + e) - 1)/sqrt(b))*cos(f*x + e) - 3*b^(5/2)*cos(f*x + e)*log((b*c
os(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x +
e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) - 8*b^2*
sqrt(b/cos(f*x + e)))/(f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
```

```
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*b^2*(1/4*sqrt(-b)*ln(
abs(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b)))+
1/3*(-6*b*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^
4+b))^2+2*b^2)/(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(
1)))^4+b)+sqrt(-b))^3+1/2*b*atan((-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b
*tan(1/2*(f*x+exp(1)))^4+b))/sqrt(-b))/sqrt(-b))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 0.17, size = 237, normalized size = 3.04

$$\frac{\left(-1+\cos\left(fx+e\right)\right)\left(3\left(\cos^2\left(fx+e\right)\right)\ln\left(-\frac{2\left(2\left(\cos^2\left(fx+e\right)\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^2}}-\left(\cos^2\left(fx+e\right)\right)+2\cos\left(fx+e\right)-2\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^2}}-1\right)}{\sin\left(fx+e\right)^2}\right)\right.}{6f\sqrt{-\frac{\cos}{\left(\cos\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/6/f*(-1+cos(f*x+e))*(3*cos(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*cos(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2))-4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-4*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cos(f*x+e)*(b/cos(f*x+e))^(5/2)/(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)/sin(f*x+e)^2
```

maxima [A] time = 0.43, size = 87, normalized size = 1.12

$$\frac{\left(6\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}}\right)+3\,b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos\left(fx+e\right)}}}\right)+4\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\right)b}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(6*b^(3/2)*arctan(sqrt(b/cos(f*x + e))/sqrt(b)) + 3*b^(3/2)*log(-(sqrt(
b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) + 4*(b/cos(f*x
 + e))^(3/2))*b/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(5/2)/sin(e + f*x),x)
```

```
[Out] int((b/cos(e + f*x))^(5/2)/sin(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.402 $\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=113

$$\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}-\frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}+\frac{7b(b\sec(e+fx))^{3/2}}{6f}-\frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf}$$

```
[Out] 7/4*b^(5/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-7/4*b^(5/2)*arctanh((b*s
ec(f*x+e))^(1/2)/b^(1/2))/f+7/6*b*(b*sec(f*x+e))^(3/2)/f-1/2*cot(f*x+e)^2*(
b*sec(f*x+e))^(7/2)/b/f
```

Rubi [A] time = 0.08, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2622, 288, 321, 329, 298, 203, 206}

$$\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}-\frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f}+\frac{7b(b\sec(e+fx))^{3/2}}{6f}-\frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (7*b^(5/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) - (7*b^(5/2)*ArcTanh
[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*f) + (7*b*(b*Sec[e + f*x])^(3/2))/(6*f)
- (Cot[e + f*x]^2*(b*Sec[e + f*x])^(7/2))/(2*b*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^3(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^{9/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf} + \frac{7\,\text{Subst}\left(\int \frac{x^{5/2}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4bf}$$

$$= \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf} + \frac{(7b)\,\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\right.}{}$$

$$= \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf} + \frac{(7b)\,\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\right.}{}$$

$$= \frac{7b(b\sec(e+fx))^{3/2}}{6f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{2bf} - \frac{\left(7b^3\right)\text{Subst}\left(\int \frac{1}{b-x^2}\right.}{}$$

$$= \frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} - \frac{7b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4f} + \frac{7b(b\sec(e+fx))^{3}}{6f}$$

Mathematica [A] time = 1.90, size = 109, normalized size = 0.96

$$\frac{b^3\left(-12\csc^2(e+fx)+16\sec^2(e+fx)+21\sqrt{\sec(e+fx)}\left(\log\left(1-\sqrt{\sec(e+fx)}\right)-\log\left(\sqrt{\sec(e+fx)}+1\right)\right)+4\right.}{24f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b^3*(-12*Csc[e + f*x]^2 + 42*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[Sec[e + f*x]]
 + 21*(Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt[Sec[e + f*x]]])*Sqrt[Sec[
e + f*x]] + 16*Sec[e + f*x]^2))/(24*f*Sqrt[b*Sec[e + f*x]])
```

fricas [B] time = 0.74, size = 448, normalized size = 3.96

$$\left[\frac{42\left(b^2\cos\left(fx+e\right)^3-b^2\cos\left(fx+e\right)\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)+21\left(b^2\cos\left(fx+e\right)^3-b^2\cos\ldots}{48\left(f\cos\left(fx\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(42*(b^2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(-b)*arctan(1/2*sqrt(
-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 21*(b^2*cos(f*x + e)^3 - b
^2*cos(f*x + e))*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f
*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e
)^2 + 2*cos(f*x + e) + 1)) + 8*(7*b^2*cos(f*x + e)^2 - 4*b^2)*sqrt(b/cos(f*
x + e)))/(f*cos(f*x + e)^3 - f*cos(f*x + e)), -1/48*(42*(b^2*cos(f*x + e)^3
 - b^2*cos(f*x + e))*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e)
- 1)/sqrt(b)) - 21*(b^2*cos(f*x + e)^3 - b^2*cos(f*x + e))*sqrt(b)*log((b*c
os(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x +
e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) - 8*(7*b
^2*cos(f*x + e)^2 - 4*b^2)*sqrt(b/cos(f*x + e)))/(f*cos(f*x + e)^3 - f*cos(
f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*b^2*(-1/16*sqrt(-b*ta
n(1/2*(f*x+exp(1)))^4+b)+2*(7/32*sqrt(-b)*ln(abs(-sqrt(-b)*tan(1/2*(f*x+exp
(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b)))+1/3*(-3*b*(-sqrt(-b)*tan(1/2*(
f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^2+b^2)/(-sqrt(-b)*tan(1/
2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b)+sqrt(-b))^3+1/16*b*sqr
t(-b)/((-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b
))^2-b)+7/16*b*atan((-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x
+exp(1)))^4+b))/sqrt(-b))/sqrt(-b)))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 0.22, size = 699, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/24/f*(-1+cos(f*x+e))*(24*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/
2)+48*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-3*cos(f*x+e)^4*ln(-
(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x
+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+24*cos(f*x+e)^4
*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*
cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-21*cos(f
*x+e)^4*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+24*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-4*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-28*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(f*x+
e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+
2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-24*cos
(f*x+e)^2*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*
x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)
+21*cos(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+16*cos(f*
x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+16*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2))*cos(f*x+e)*(b/cos(f*x+e))^(5/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)/sin(f*x+e)^4
```

maxima [A] time = 0.66, size = 123, normalized size = 1.09

$$\frac{\left(\frac{12\, b^{2} \left(\frac{b}{\cos\left(f x+e\right)}\right)^{\frac{3}{2}}}{b^{2}-\frac{b^{2}}{\cos\left(f x+e\right)^{2}}} + 42\, b^{\frac{3}{2}} \arctan\left(\frac{\sqrt{\frac{b}{\cos\left(f x+e\right)}}}{\sqrt{b}}\right) + 21\, b^{\frac{3}{2}} \log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos\left(f x+e\right)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos\left(f x+e\right)}}}\right) + 16\, \left(\frac{b}{\cos\left(f x+e\right)}\right)^{\frac{3}{2}}\right) b}{24\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(12*b^2*(b/cos(f*x + e))^(3/2)/(b^2 - b^2/cos(f*x + e)^2) + 42*b^(3/2)
*arctan(sqrt(b/cos(f*x + e))/sqrt(b)) + 21*b^(3/2)*log(-(sqrt(b) - sqrt(b/c
os(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) + 16*(b/cos(f*x + e))^(3/2)
)*b/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^3,x)
[Out] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

3.403 $\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=143

$$\frac{77b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}-\frac{77b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}-\frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3f}+\frac{77b(b\sec(e+fx))^{3/2}}{48f}-$$

```
[Out] 77/32*b^(5/2)*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f-77/32*b^(5/2)*arctanh(
(b*sec(f*x+e))^(1/2)/b^(1/2))/f+77/48*b*(b*sec(f*x+e))^(3/2)/f-11/16*cot(f*
x+e)^2*(b*sec(f*x+e))^(7/2)/b/f-1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(11/2)/b^3/
f
```

Rubi [A] time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2622, 288, 321, 329, 298, 203, 206}

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3f}+\frac{77b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}-\frac{77b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f}+\frac{77b(b\sec(e+fx))^{3/2}}{48f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (77*b^(5/2)*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) - (77*b^(5/2)*ArcT
anh[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*f) + (77*b*(b*Sec[e + f*x])^(3/2))/(
48*f) - (11*Cot[e + f*x]^2*(b*Sec[e + f*x])^(7/2))/(16*b*f) - (Cot[e + f*x]
^4*(b*Sec[e + f*x])^(11/2))/(4*b^3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
```

```
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^5(e+fx)(b\sec(e+fx))^{5/2}\,dx = \frac{\text{Subst}\left(\int \frac{x^{13/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3 f} + \frac{11\,\text{Subst}\left(\int \frac{x^{9/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx\right.}{8b^3 f}$$

$$= -\frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{11/2}}{4b^3 f} + \frac{77\,\text{S}}{}$$

$$= \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\,\text{s}}{4b}$$

$$= \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\,\text{s}}{4b}$$

$$= \frac{77b(b\sec(e+fx))^{3/2}}{48f} - \frac{11\cot^2(e+fx)(b\sec(e+fx))^{7/2}}{16bf} - \frac{\cot^4(e+fx)(b\,\text{s}}{4b}$$

$$= \frac{77b^{5/2}\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} - \frac{77b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32f} + \frac{77b(b\sec(e+f}{48f}$$

Mathematica [A] time = 1.32, size = 119, normalized size = 0.83

$$\frac{b^3\left(-48\csc^4(e+fx) - 180\csc^2(e+fx) + 128\sec^2(e+fx) + 231\sqrt{\sec(e+fx)}\left(\log\left(1-\sqrt{\sec(e+fx)}\right) - \log\left(\sqrt{}\right.\right.\right.}{192f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b^3*(-180*Csc[e + f*x]^2 - 48*Csc[e + f*x]^4 + 462*ArcTan[Sqrt[Sec[e + f*x
]]]*Sqrt[Sec[e + f*x]] + 231*(Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqrt[Se
```

```
c[e + f*x]]])*Sqrt[Sec[e + f*x]] + 128*Sec[e + f*x]^2))/(192*f*Sqrt[b*Sec[e
 + f*x]])
```

fricas [B] time = 0.94, size = 542, normalized size = 3.79

$$\left[\frac{462\left(b^2\cos\left(fx+e\right)^5-2\,b^2\cos\left(fx+e\right)^3+b^2\cos\left(fx+e\right)\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{\frac{b}{\cos\left(fx+e\right)}}\left(\cos\left(fx+e\right)+1\right)}{2\,b}\right)+231\left(b^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/384*(462*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))*
sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 2
31*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))*sqrt(-b)*
log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/c
os(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1))
 + 8*(77*b^2*cos(f*x + e)^4 - 121*b^2*cos(f*x + e)^2 + 32*b^2)*sqrt(b/cos(f
*x + e)))/(f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e)), -1/384*
(462*(b^2*cos(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))*sqrt(b)
*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) - 231*(b^2*cos
(f*x + e)^5 - 2*b^2*cos(f*x + e)^3 + b^2*cos(f*x + e))*sqrt(b)*log((b*cos(f
*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e))
+ 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)) - 8*(77*b^2*
cos(f*x + e)^4 - 121*b^2*cos(f*x + e)^2 + 32*b^2)*sqrt(b/cos(f*x + e)))/(f*
cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*b^2*(2*sqrt(-b*tan(1/
2*(f*x+exp(1)))^4+b)*(-9/128-1/256*tan(1/2*(f*x+exp(1)))^2)+2*(77/256*sqrt(
-b)*ln(abs(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^
4+b)))-1/128*(-b^2*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+
exp(1)))^4+b))-b*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+ex
```

```
p(1)))^4+b))^3-18*b*sqrt(-b)*(-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan
(1/2*(f*x+exp(1)))^4+b))^2+18*b^2*sqrt(-b))/((-sqrt(-b)*tan(1/2*(f*x+exp(1)
))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^2-b)^2+1/3*(-3*b*(-sqrt(-b)*tan(1/
2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b))^2+b^2)/(-sqrt(-b)*tan
(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b)+sqrt(-b))^3+77/128*
b*atan((-sqrt(-b)*tan(1/2*(f*x+exp(1)))^2+sqrt(-b*tan(1/2*(f*x+exp(1)))^4+b
))/sqrt(-b))/sqrt(-b)))*sign(cos(f*x+exp(1)))/f
```

maple [B] time = 0.18, size = 1161, normalized size = 8.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/192/f*(408*cos(f*x+e)^5*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+360*cos(f*x
+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+288*cos(f*x+e)^5*ln(-2*(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-231*cos(f*x+e)^5*arctan(
1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-57*cos(f*x+e)^5*ln(-(2*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-504*cos(f*x+e)^3*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(3/2)-288*cos(f*x+e)^4*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+231*cos(f*x+e)^4*arctan(1/2/(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2))+100*cos(f*x+e)^4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)+57*cos(f*x+e)^4*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f
*x+e)^2)-456*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-288*cos(f*x+
e)^3*ln(-2*(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+231*
cos(f*x+e)^3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-456*cos(f*x+e
)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+57*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+288*cos(f*x+e)^2*ln(-2*(2*cos
(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-231*cos(f*x+e)^2*arct
an(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+484*cos(f*x+e)^2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-57*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)-128*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*cos(
f*x+e)*(b/cos(f*x+e))^(5/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/sin(f*x+e)
^4
```

maxima [A] time = 0.43, size = 155, normalized size = 1.08

$$\frac{462\, b^{\frac{3}{2}} \arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right) + 231\, b^{\frac{3}{2}} \log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right) + 128 \left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}} + \frac{12\left(15\, b^4\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}} - 19\, b^2\left(\frac{b}{\cos(fx+e)}\right)^{\frac{7}{2}}\right.}{b^4-\frac{2\,b^4}{\cos(fx+e)^2}+\frac{b^4}{\cos(fx+e)^4}}}{192\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/192*(462*b^(3/2)*arctan(sqrt(b/cos(f*x + e))/sqrt(b)) + 231*b^(3/2)*log(-
(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e)))) + 128*(b
/cos(f*x + e))^(3/2) + 12*(15*b^4*(b/cos(f*x + e))^(3/2) - 19*b^2*(b/cos(f*
x + e))^(7/2))/(b^4 - 2*b^4/cos(f*x + e)^2 + b^4/cos(f*x + e)^4))*b/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos(e+f\,x)}\right)^{5/2}}{\sin\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^5,x)
```

```
[Out] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.404 $\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx$

Optimal. Leaf size=130

$$\frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}}+\frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}}-\frac{80b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{21f}+\frac{2b\sin^5(e+fx)(b\,}{3f}$$

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)*sin(f*x+e)^5/f+40/21*b^3*sin(f*x+e)/f/(b*sec(f*x
+e))^(1/2)+20/21*b^3*sin(f*x+e)^3/f/(b*sec(f*x+e))^(1/2)-80/21*b^2*(cos(1/2
*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2
))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.15, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2624, 2627, 3771, 2641}

$$\frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}}+\frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}}-\frac{80b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{21f}+\frac{2b\sin^5(e+fx)(b\,}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^6,x]
```

```
[Out] (-80*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])
/(21*f) + (40*b^3*Sin[e + f*x])/(21*f*Sqrt[b*Sec[e + f*x]]) + (20*b^3*Sin[e
 + f*x]^3)/(21*f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e
+ f*x]^5)/(3*f)
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^{5/2}\sin^6(e+fx)\,dx &= \frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f}-\frac{1}{3}\left(10b^2\right)\int\sqrt{b\sec(e+fx)}\,\sin^4(e+f\\ &= \frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+fx)}{3f}-\frac{1}{7}\left(20b^2\right)\int\sqrt{b\\ &= \frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}}+\frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+}{3f}\\ &= \frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx)}}+\frac{20b^3\sin^3(e+fx)}{21f\sqrt{b\sec(e+fx)}}+\frac{2b(b\sec(e+fx))^{3/2}\sin^5(e+}{3f}\\ &= -\frac{80b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{21f}+\frac{40b^3\sin(e+fx)}{21f\sqrt{b\sec(e+fx}}\end{aligned}$$

Mathematica [A] time = 0.19, size = 74, normalized size = 0.57

$$-\frac{b^2\sqrt{b\sec(e+fx)}\left(-58\sin(2(e+fx))+3\sin(4(e+fx))-56\tan(e+fx)+320\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\right.}{84f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^6,x]
```

```
[Out] -1/84*(b^2*Sqrt[b*Sec[e + f*x]]*(320*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)
/2, 2] - 58*Sin[2*(e + f*x)] + 3*Sin[4*(e + f*x)] - 56*Tan[e + f*x]))/f
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b^2\cos\left(fx+e\right)^6-3\,b^2\cos\left(fx+e\right)^4+3\,b^2\cos\left(fx+e\right)^2-b^2\right)\sqrt{b\sec\left(fx+e\right)}\,\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^6 - 3*b^2*cos(f*x + e)^4 + 3*b^2*cos(f*x + e)^2
 - b^2)*sqrt(b*sec(f*x + e))*sec(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^6, x)
```

maple [C] time = 0.24, size = 168, normalized size = 1.29

$$-\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(-40i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x)
```

```
[Out] -2/21/f*(-1+cos(f*x+e))*(-40*I*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e)
,I)+3*cos(f*x+e)^5-3*cos(f*x+e)^4-16*cos(f*x+e)^3+16*cos(f*x+e)^2-7*cos(f*x
+e)+7)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(5/2)/sin(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^{6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^6\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(b/cos(e + f*x))^(5/2),x)
```

```
[Out] int(sin(e + f*x)^6*(b/cos(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

3.405 $\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx$

Optimal. Leaf size=100

$$\frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}-\frac{8b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\sin^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)*sin(f*x+e)^3/f+4/3*b^3*sin(f*x+e)/f/(b*sec(f*x+e
))^(1/2)-8/3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(
sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2624, 2627, 3771, 2641}

$$\frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}-\frac{8b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\sin^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^4,x]
```

```
[Out] (-8*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/
(3*f) + (4*b^3*Sin[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]]) + (2*b*(b*Sec[e + f
*x])^(3/2)*Sin[e + f*x]^3)/(3*f)
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\sec(e+fx))^{5/2}\sin^4(e+fx)\,dx = \frac{2b(b\sec(e+fx))^{3/2}\sin^3(e+fx)}{3f} - \left(2b^2\right)\int \sqrt{b\sec(e+fx)}\,\sin^2(e+fx)\,d$$

$$= \frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^3(e+fx)}{3f} - \frac{1}{3}\left(4b^2\right)\int \sqrt{b\,se}$$

$$= \frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}} + \frac{2b(b\sec(e+fx))^{3/2}\sin^3(e+fx)}{3f} - \frac{1}{3}\left(4b^2\sqrt{\cos(e+}\right.$$

$$= -\frac{8b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{4b^3\sin(e+fx)}{3f\sqrt{b\sec(e+fx)}}$$

Mathematica [A] time = 0.13, size = 64, normalized size = 0.64

$$-\frac{b^2\sqrt{b\sec(e+fx)}\left(-\sin(2(e+fx)) - 2\tan(e+fx) + 8\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^4,x]
```

```
[Out] -1/3*(b^2*Sqrt[b*Sec[e + f*x]]*(8*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2,
 2] - Sin[2*(e + f*x)] - 2*Tan[e + f*x]))/f
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos\left(fx+e\right)^4 - 2\,b^2\cos\left(fx+e\right)^2 + b^2\right)\sqrt{b\sec\left(fx+e\right)}\,\sec\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(f*x + e)^4 - 2*b^2*cos(f*x + e)^2 + b^2)*sqrt(b*sec(f*x +
 e))*sec(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}} \sin\left(fx+e\right)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^4, x)
```

maple [C] time = 0.20, size = 144, normalized size = 1.44

$$\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(4i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\operatorname{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos^3\right.}{3f\sin\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x)
```

```
[Out] 2/3/f*(-1+cos(f*x+e))*(4*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+
cos(f*x+e)^3-cos(f*x+e)^2+cos(f*x+e)-1)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(
f*x+e))^(5/2)/sin(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}} \sin\left(fx+e\right)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^4\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(b/cos(e + f*x))^(5/2),x)
```

```
[Out] int(sin(e + f*x)^4*(b/cos(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**4,x)
```

```
[Out] Timed out
```

3.406 $\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx$

Optimal. Leaf size=70

$$\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{4b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}$$

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)*sin(f*x+e)/f-4/3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)
*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2624, 3771, 2641}

$$\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}-\frac{4b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^2,x]
```

```
[Out] (-4*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/
(3*f) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Rule 2624

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(f*a*(n - 1)), x] + Dist[(b^2*(m + 1))/(a^2*(n - 1)), Int[(a*Csc[e + f*x])
^(m + 2)*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
n, 1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\sec(e+fx))^{5/2}\sin^2(e+fx)\,dx = \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f} - \frac{1}{3}\left(2b^2\right)\int \sqrt{b\sec(e+fx)}\,dx$$
$$= \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f} - \frac{1}{3}\left(2b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int$$
$$= -\frac{4b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b(b\sec(e+fx))^{3/2}}{3f}$$

Mathematica [A] time = 0.12, size = 52, normalized size = 0.74

$$\frac{2b^2\sqrt{b\sec(e+fx)}\left(\tan(e+fx)-2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2)*Sin[e + f*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[b*Sec[e + f*x]]*(-2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2
] + Tan[e + f*x]))/(3*f)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(b^2\cos\left(fx+e\right)^2-b^2\right)\sqrt{b\sec\left(fx+e\right)}\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - b^2)*sqrt(b*sec(f*x + e))*sec(f*x + e)^2, x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="giac")
```

[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^2, x)

maple [C] time = 0.18, size = 126, normalized size = 1.80

$$\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(2i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\operatorname{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)+\cos\left(\right.}{3f\sin\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x)

[Out] 2/3/f*(-1+cos(f*x+e))*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)+cos(f*x+e)-1)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(5/2)/sin(f*x+e)^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(f*x+e))^(5/2)*sin(f*x+e)^2,x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e))^(5/2)*sin(f*x + e)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(e+f\,x\right)^{2}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f*x)^2*(b/cos(e + f*x))^(5/2),x)

[Out] int(sin(e + f*x)^2*(b/cos(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(f*x+e))**(5/2)*sin(f*x+e)**2,x)

[Out] Timed out

3.407 $\int (b\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{2b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] 2/3*b*(b*sec(f*x+e))^(3/2)*sin(f*x+e)/f+2/3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2
)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)
*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3771, 2641}

$$\frac{2b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\sin(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(
3*f) + (2*b*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (b\sec(e+fx))^{5/2}\,dx = \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f} + \frac{1}{3}b^2\int \sqrt{b\sec(e+fx)}\,dx$$
$$= \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f} + \frac{1}{3}\left(b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}$$
$$= \frac{2b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f} + \frac{2b(b\sec(e+fx))^{3/2}\sin(e+fx)}{3f}$$

Mathematica [A] time = 0.08, size = 51, normalized size = 0.73

$$\frac{2b^2\sqrt{b\sec(e+fx)}\left(\tan(e+fx)+\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*b^2*Sqrt[b*Sec[e + f*x]]*(Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] +
 Tan[e + f*x]))/(3*f)
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}\,b^2\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*b^2*sec(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2), x)
```

maple [C] time = 0.17, size = 128, normalized size = 1.83

$$-\frac{2\left(-1+\cos\left(fx+e\right)\right)\left(i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\;\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)-\cos}{3f\sin\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(5/2),x)
```

```
[Out] -2/3/f*(-1+cos(f*x+e))*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-c
os(f*x+e)+1)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(5/2)/sin(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(5/2),x)
```

```
[Out] int((b/cos(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(e+fx\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(5/2), x)
```

3.408 $\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=98

$$-\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{5b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

```
[Out] 2/3*b*csc(f*x+e)*(b*sec(f*x+e))^(3/2)/f-5/3*b^3*csc(f*x+e)/f/(b*sec(f*x+e))
^(1/2)+5/3*b^2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(si
n(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2626, 2625, 3771, 2641}

$$-\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{5b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-5*b^3*Csc[e + f*x])/(3*f*Sqrt[b*Sec[e + f*x]]) + (5*b^2*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*f) + (2*b*Csc[e + f*x]
*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx &= \frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}+\frac{1}{3}\left(5b^2\right)\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\\ &= -\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}+\frac{1}{6}\left(5b^2\right)\int \sqrt{b\,\mathrm{se}}\\ &= -\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{2b\csc(e+fx)(b\sec(e+fx))^{3/2}}{3f}+\frac{1}{6}\left(5b^2\sqrt{\cos(e+}\right.\\ &= -\frac{5b^3\csc(e+fx)}{3f\sqrt{b\sec(e+fx)}}+\frac{5b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3f}\end{aligned}$$

Mathematica [A] time = 0.17, size = 67, normalized size = 0.68

$$\frac{b\sin(e+fx)(b\sec(e+fx))^{3/2}\left(-3\cot^2(e+fx)+5\cos^{\frac{3}{2}}(e+fx)\csc(e+fx)F\left(\frac{1}{2}(e+fx)\middle|2\right)+2\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b*(2 - 3*Cot[e + f*x]^2 + 5*Cos[e + f*x]^(3/2)*Csc[e + f*x]*EllipticF[(e +
 f*x)/2, 2])*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(3*f)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)}\,b^2\csc\left(fx+e\right)^2\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

[Out] integral(sqrt(b*sec(f*x + e))*b^2*csc(f*x + e)^2*sec(f*x + e)^2, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^2, x)

maple [C] time = 0.19, size = 202, normalized size = 2.06

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(5i\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+5i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x)

[Out] 1/3/f*(-1+cos(f*x+e))^2*(5*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)+5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-5*cos(f*x+e)^2+2)*cos(f*x+e)*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(5/2)/sin(f*x+e)^5

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}{{\sin\left(e+f\,x\right)}^{2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^2,x)
```

```
[Out] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.409 $\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx$

Optimal. Leaf size=123

$$-\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}}+\frac{5b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{2f}-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}+\frac{b\csc(e$$

```
[Out] b*csc(f*x+e)*(b*sec(f*x+e))^(3/2)/f-1/3*b*csc(f*x+e)^3*(b*sec(f*x+e))^(3/2)
/f-5/2*b^3*csc(f*x+e)/f/(b*sec(f*x+e))^(1/2)+5/2*b^2*(cos(1/2*f*x+1/2*e)^2)
^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^
(1/2)*(b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2625, 2626, 3771, 2641}

$$-\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}}+\frac{5b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{2f}-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}+\frac{b\csc(e$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-5*b^3*Csc[e + f*x])/(2*f*Sqrt[b*Sec[e + f*x]]) + (5*b^2*Sqrt[Cos[e + f*x]
]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(2*f) + (b*Csc[e + f*x]*(
b*Sec[e + f*x])^(3/2))/f - (b*Csc[e + f*x]^3*(b*Sec[e + f*x])^(3/2))/(3*f)
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2626

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(n - 1)), x] + Dist[(b^2*(m + n - 2))/(n - 1), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \csc^4(e+fx)(b\sec(e+fx))^{5/2}\,dx &= -\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}+\frac{3}{2}\int \csc^2(e+fx)(b\sec(e+fx))^{5/2}\,dx\\
&= \frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f}-\frac{b\csc^3(e+fx)(b\sec(e+fx))^{3/2}}{3f}+\frac{1}{2}\Big(5b\\
&= -\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}}+\frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f}-\frac{b\csc^3(e+fx)(b\\
&\qquad 3f\\
&= -\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}}+\frac{b\csc(e+fx)(b\sec(e+fx))^{3/2}}{f}-\frac{b\csc^3(e+fx)(b\\
&\qquad 3f\\
&= -\frac{5b^3\csc(e+fx)}{2f\sqrt{b\sec(e+fx)}}+\frac{5b^2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{2f}\end{aligned}$$

Mathematica [A] time = 0.39, size = 79, normalized size = 0.64

$$\frac{b\sin(e+fx)(b\sec(e+fx))^{3/2}\left(-\left(\cot^2(e+fx)\left(2\csc^2(e+fx)+11\right)\right)+15\cos^{\frac{3}{2}}(e+fx)\csc(e+fx)F\left(\frac{1}{2}(e+fx\right.\right.}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (b*(4 - Cot[e + f*x]^2*(11 + 2*Csc[e + f*x]^2) + 15*Cos[e + f*x]^(3/2)*Csc[
e + f*x]*EllipticF[(e + f*x)/2, 2])*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(6
*f)
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}\,b^2\csc\left(fx+e\right)^4\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*b^2*csc(f*x + e)^4*sec(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)\right)^{\frac{5}{2}} \csc \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^4, x)
```

maple [C] time = 0.23, size = 352, normalized size = 2.86

$$-\frac{\left(-1+\cos \left(fx+e\right)\right)^{2} \left(15 i \sqrt{\frac{1}{\cos \left(fx+e\right)+1}}\, \sqrt{\frac{\cos \left(fx+e\right)}{\cos \left(fx+e\right)+1}}\, \left(\cos^{4}\left(fx+e\right)\right) \sin \left(fx+e\right) \mathrm{EllipticF}\left(\frac{i\left(-1+\cos \left(fx+e\right)\right)}{\sin \left(fx+e\right)}, i\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/6/f*(-1+cos(f*x+e))^2*(15*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*cos(f*x+e)^4*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+
e),I)+15*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*sin(f*x+e)-15*I*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(
f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)-15*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin
(f*x+e)*cos(f*x+e)-15*cos(f*x+e)^4+21*cos(f*x+e)^2-4)*cos(f*x+e)*(cos(f*x+e
)+1)^2*(b/cos(f*x+e))^(5/2)/sin(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)\right)^{\frac{5}{2}} \csc \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(5/2)*csc(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{5/2}}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^4,x)
[Out] int((b/cos(e + f*x))^(5/2)/sin(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

3.410 $\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=87

$$\frac{2b^7}{15f(b\sec(e+fx))^{15/2}}-\frac{6b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{6b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] 2/15*b^7/f/(b*sec(f*x+e))^(15/2)-6/11*b^5/f/(b*sec(f*x+e))^(11/2)+6/7*b^3/f
/(b*sec(f*x+e))^(7/2)-2/3*b/f/(b*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$\frac{2b^7}{15f(b\sec(e+fx))^{15/2}}-\frac{6b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{6b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^7/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (2*b^7)/(15*f*(b*Sec[e + f*x])^(15/2)) - (6*b^5)/(11*f*(b*Sec[e + f*x])^(11
/2)) + (6*b^3)/(7*f*(b*Sec[e + f*x])^(7/2)) - (2*b)/(3*f*(b*Sec[e + f*x])^(
3/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\sin^7(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b^7\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{17/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^7\,\text{Subst}\left(\int \left(-\frac{1}{x^{17/2}}+\frac{3}{b^2x^{13/2}}-\frac{3}{b^4x^{9/2}}+\frac{1}{b^6x^{5/2}}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b^7}{15f(b\sec(e+fx))^{15/2}}-\frac{6b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{6b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+}$$

Mathematica [A] time = 0.21, size = 52, normalized size = 0.60

$$\frac{b(4035\cos(2(e+fx))-798\cos(4(e+fx))+77\cos(6(e+fx))-7410)}{18480f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^7/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (b*(-7410 + 4035*Cos[2*(e + f*x)] - 798*Cos[4*(e + f*x)] + 77*Cos[6*(e + f*
x)]))/(18480*f*(b*Sec[e + f*x])^(3/2))
```

fricas [A] time = 0.65, size = 61, normalized size = 0.70

$$\frac{2\left(77\cos\left(fx+e\right)^8-315\cos\left(fx+e\right)^6+495\cos\left(fx+e\right)^4-385\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{1155\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*cos(f*x + e)^8 - 315*cos(f*x + e)^6 + 495*cos(f*x + e)^4 - 385*c
os(f*x + e)^2)*sqrt(b/cos(f*x + e))/(b*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2/1155*(-147840*b*sqrt(-b)*(-sqr
t(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^11-88704*b^2*
(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^10+54208
0*b^2*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)
^4+b))^9+190080*b^3*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1
))/2)^4+b))^8-633600*b^3*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*
tan((f*x+exp(1))/2)^4+b))^7-98560*b^4*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt
(-b*tan((f*x+exp(1))/2)^4+b))^6+295680*b^4*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp
(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^5+26880*b^5*(-sqrt(-b)*tan((f*x
+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^4+1920*b^6*sqrt(-b)*(-sqrt(
-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))-58240*b^5*sqrt(
-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^3-13
440*b^6*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^
2+128*b^7)/(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b
)-sqrt(-b))^15/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [A] time = 0.22, size = 56, normalized size = 0.64

$$\frac{2\left(77\left(\cos^{6}\left(fx+e\right)\right)-315\left(\cos^{4}\left(fx+e\right)\right)+495\left(\cos^{2}\left(fx+e\right)\right)-385\right)\cos\left(fx+e\right)}{1155f\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/1155/f*(77*cos(f*x+e)^6-315*cos(f*x+e)^4+495*cos(f*x+e)^2-385)*cos(f*x+e)
/(b/cos(f*x+e))^(1/2)
```

maxima [A] time = 1.13, size = 63, normalized size = 0.72

$$\frac{2\left(77\,b^{6}-\frac{315\,b^{6}}{\cos\left(fx+e\right)^{2}}+\frac{495\,b^{6}}{\cos\left(fx+e\right)^{4}}-\frac{385\,b^{6}}{\cos\left(fx+e\right)^{6}}\right)b}{1155\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(77*b^6 - 315*b^6/cos(f*x + e)^2 + 495*b^6/cos(f*x + e)^4 - 385*b^6/
cos(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(15/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^7}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^7/(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^7/(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**7/(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

3.411 $\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=65

$$-\frac{2b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{4b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] -2/11*b^5/f/(b*sec(f*x+e))^(11/2)+4/7*b^3/f/(b*sec(f*x+e))^(7/2)-2/3*b/f/(b
*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$-\frac{2b^5}{11f(b\sec(e+fx))^{11/2}}+\frac{4b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-2*b^5)/(11*f*(b*Sec[e + f*x])^(11/2)) + (4*b^3)/(7*f*(b*Sec[e + f*x])^(7/
2)) - (2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{13/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{b^5\,\text{Subst}\left(\int \left(\frac{1}{x^{13/2}} - \frac{2}{b^2x^{9/2}} + \frac{1}{b^4x^{5/2}}\right)dx, x, b\sec(e+fx)\right)}{f}$$

$$= -\frac{2b^5}{11f(b\sec(e+fx))^{11/2}} + \frac{4b^3}{7f(b\sec(e+fx))^{7/2}} - \frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.17, size = 42, normalized size = 0.65

$$\frac{b(180\cos(2(e+fx)) - 21\cos(4(e+fx)) - 415)}{924f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (b*(-415 + 180*Cos[2*(e + f*x)] - 21*Cos[4*(e + f*x)]))/(924*f*(b*Sec[e + f
*x])^(3/2))
```

fricas [A] time = 0.78, size = 51, normalized size = 0.78

$$-\frac{2\left(21\cos\left(fx+e\right)^6 - 66\cos\left(fx+e\right)^4 + 77\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{231\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/231*(21*cos(f*x + e)^6 - 66*cos(f*x + e)^4 + 77*cos(f*x + e)^2)*sqrt(b/c
os(f*x + e))/(b*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2/231*(4928*b*(-sqrt(-b)*tan((f*
x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^8-2464*b*sqrt(-b)*(-sqrt(-
b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^7-12320*b^2*(-sq
rt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^6+2464*b^2*s
qrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^
5+8096*b^3*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b
))^4+352*b^4*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp
(1))/2)^4+b))-352*b^3*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan
((f*x+exp(1))/2)^4+b))^3-1760*b^4*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*
tan((f*x+exp(1))/2)^4+b))^2+32*b^5)/(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-
b*tan((f*x+exp(1))/2)^4+b)-sqrt(-b))^11/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [A] time = 0.17, size = 46, normalized size = 0.71

$$-\frac{2\left(21\left(\cos^{4}\left(fx+e\right)\right)-66\left(\cos^{2}\left(fx+e\right)\right)+77\right)\cos\left(fx+e\right)}{231f\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/231/f*(21*cos(f*x+e)^4-66*cos(f*x+e)^2+77)*cos(f*x+e)/(b/cos(f*x+e))^(1/
2)
```

maxima [A] time = 0.42, size = 50, normalized size = 0.77

$$-\frac{2\left(21\,b^{4}-\frac{66\,b^{4}}{\cos\left(fx+e\right)^{2}}+\frac{77\,b^{4}}{\cos\left(fx+e\right)^{4}}\right)b}{231\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -2/231*(21*b^4 - 66*b^4/cos(f*x + e)^2 + 77*b^4/cos(f*x + e)^4)*b/(f*(b/cos
(f*x + e))^(11/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(e+f\,x\right)^{5}}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^5/(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.412 $\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=43

$$\frac{2b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] 2/7*b^3/f/(b*sec(f*x+e))^(7/2)-2/3*b/f/(b*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2b^3}{7f(b\sec(e+fx))^{7/2}}-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (2*b^3)/(7*f*(b*Sec[e + f*x])^(7/2)) - (2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{b^3\operatorname{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{9/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^3\operatorname{Subst}\left(\int \left(-\frac{1}{x^{9/2}}+\frac{1}{b^2x^{5/2}}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b^3}{7f(b\sec(e+fx))^{7/2}} - \frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.10, size = 32, normalized size = 0.74

$$\frac{b(3\cos(2(e+fx))-11)}{21f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
[Out] (b*(-11 + 3*Cos[2*(e + f*x)]))/(21*f*(b*Sec[e + f*x])^(3/2))
```

fricas [A] time = 0.61, size = 41, normalized size = 0.95

$$\frac{2\left(3\cos\left(fx+e\right)^4-7\cos\left(fx+e\right)^2\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{21\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 2/21*(3*cos(f*x + e)^4 - 7*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))/(b*f)
```

giac [A] time = 1.69, size = 50, normalized size = 1.16

$$\frac{2\left(3\,b^4-\frac{7\,b^4}{\cos\left(fx+e\right)^2}\right)\cos\left(fx+e\right)^3}{21\,b^4f\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*(3*b^4 - 7*b^4/cos(f*x + e)^2)*cos(f*x + e)^3/(b^4*f*sqrt(b/cos(f*x +
e)))
```

maple [A] time = 0.16, size = 36, normalized size = 0.84

$$\frac{2\left(3\left(\cos^2\left(fx+e\right)\right)-7\right)\cos\left(fx+e\right)}{21f\sqrt{\frac{b}{\cos(fx+e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/21/f*(3*cos(f*x+e)^2-7)*cos(f*x+e)/(b/cos(f*x+e))^(1/2)
```

maxima [A] time = 0.42, size = 37, normalized size = 0.86

$$\frac{2\left(3\,b^2-\frac{7\,b^2}{\cos(fx+e)^2}\right)b}{21\,f\left(\frac{b}{\cos(fx+e)}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*(3*b^2 - 7*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(7/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(e+f\,x\right)^3}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3/(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.413 $\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=20

$$-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] -2/3*b/f/(b*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{2b}{3f(b\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 20, normalized size = 1.00

$$-\frac{2b}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2))
```

fricas [A] time = 0.47, size = 28, normalized size = 1.40

$$-\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}}{3\,bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] -2/3*sqrt(b/cos(f*x + e))*cos(f*x + e)^2/(b*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)-2/3*cos(f*x+exp(1))/f*sqrt(b*f*cos(
f*x+exp(1))/f)*abs(f)*sign(cos(f*x+exp(1)))*sign(f)/b/f
```

maple [A] time = 0.03, size = 17, normalized size = 0.85

$$-\frac{2b}{3f\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x)
[Out] -2/3*b/f/(b*sec(f*x+e))^(3/2)
```

maxima [A] time = 0.47, size = 23, normalized size = 1.15

$$-\frac{2\cos\left(fx+e\right)}{3f\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] -2/3*cos(f*x + e)/(f*sqrt(b/cos(f*x + e)))
```

mupad [B] time = 0.54, size = 28, normalized size = 1.40

$$-\frac{2\cos\left(e+fx\right)^2\sqrt{\frac{b}{\cos\left(e+fx\right)}}}{3bf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(b/cos(e + f*x))^(1/2),x)
[Out] -(2*cos(e + f*x)^2*(b/cos(e + f*x))^(1/2))/(3*b*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sin(e + f*x)/sqrt(b*sec(e + f*x)), x)
```

3.414 $\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=59

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}\,f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}\,f}$$

```
[Out] -arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f/b^(1/2)-arctanh((b*sec(f*x+e))^(1/2
)/b^(1/2))/f/b^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2622, 329, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}\,f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -(ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(Sqrt[b]*f)) - ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(Sqrt[b]*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{bf}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{bf}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}\,f} - \frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}\,f}$$

Mathematica [A] time = 0.10, size = 73, normalized size = 1.24

$$-\frac{\sqrt{\sec(e+fx)}\left(-\log\left(1-\sqrt{\sec(e+fx)}\right)+\log\left(\sqrt{\sec(e+fx)}+1\right)+2\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{2f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[b*Sec[e + f*x]],x]
```

[Out] -1/2*((2*ArcTan[Sqrt[Sec[e + f*x]]] - Log[1 - Sqrt[Sec[e + f*x]]] + Log[1 + Sqrt[Sec[e + f*x]]])*Sqrt[Sec[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])

fricas [B] time = 0.91, size = 253, normalized size = 4.29

$$\left[\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}(\cos(fx+e)+1)}{2b}\right)-\sqrt{-b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2-\cos(fx+e)\right)\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}-6b\cos(fx+e)}{\cos(fx+e)^2+2\cos(fx+e)+1}\right.}{4bf}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/4*(2*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) - sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/(b*f), 1/4*(2*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) + sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/(b*f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/2*(-atan((-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))/sqrt(-b))/sqrt(-b)-1/2*ln(abs(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b)))/sqrt(-b))/sign(tan((f*x+exp(1))/2)^2-1)

maple [B] time = 0.18, size = 161, normalized size = 2.73

$$-\frac{\left(\ln\left(-\frac{2\left(\cos^2(fx+e)\right)\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-\left(\cos^2(fx+e)\right)+2\cos(fx+e)-2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}-1}{\sin(fx+e)^2}\right)+\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}}\right)\right)(-1+\mathrm{c}}{2f\sin(fx+e)^2\sqrt{-\frac{\cos(fx+e)}{(\cos(fx+e)+1)^2}}\sqrt{\frac{b}{\cos(fx+e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)
^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+arc
tan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))*(-1+cos(f*x+e))/sin(f*x+e)^2
/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/(b/cos(f*x+e))^(1/2)
```

maxima [A] time = 0.45, size = 73, normalized size = 1.24

$$-\frac{b\left(\frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}-\frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos\left(fx+e\right)}}}\right)}{b^{\frac{3}{2}}}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*(2*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/b^(3/2) - log(-(sqrt(b) - sq
rt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/b^(3/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(b/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(b/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(b*sec(e + f*x)), x)
```

3.415 $\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=93

$$-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf}-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,f}$$

```
[Out] -1/4*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/f/b^(1/2)-1/4*arctanh((b*sec(f*x+
e))^(1/2)/b^(1/2))/f/b^(1/2)-1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(1/2)/b/f
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2622, 288, 329, 212, 206, 203}

$$-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf}-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(4*Sqrt[b]*f) - ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(4*Sqrt[b]*f) - (Cot[e + f*x]^2*Sqrt[b*Sec[e + f*x]])/(2*b
*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^{3/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{4bf}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf} + \frac{\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2bf}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf} - \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4f} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,f} - \frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2bf}$$

Mathematica [A] time = 1.10, size = 93, normalized size = 1.00

$$\frac{\sqrt{\sec(e+fx)}\left(\log\left(1-\sqrt{\sec(e+fx)}\right)-\log\left(\sqrt{\sec(e+fx)}+1\right)-\frac{4\csc^2(e+fx)}{\sec^{\frac{3}{2}}(e+fx)}-2\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{8f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((-2*ArcTan[Sqrt[Sec[e + f*x]]] + Log[1 - Sqrt[Sec[e + f*x]]] - Log[1 + Sqr
t[Sec[e + f*x]]] - (4*Csc[e + f*x]^2)/Sec[e + f*x]^(3/2))*Sqrt[Sec[e + f*x]
])/(8*f*Sqrt[b*Sec[e + f*x]])
```

fricas [B] time = 0.60, size = 361, normalized size = 3.88

$$\left[\frac{2\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\left(\cos\left(fx+e\right)+1\right)}{2b}\right)+8\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)^2-\right.}{16\left(bf\cos\left(fx+e\right)^2-bf\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x +
 e))*(cos(f*x + e) + 1)/b) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 - (cos(f
*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x
 + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^
2 + 2*cos(f*x + e) + 1)))/(b*f*cos(f*x + e)^2 - b*f), 1/16*(2*(cos(f*x + e)
^2 - 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(b))
 + 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sqrt(b)*log
((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f
*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/(b
*f*cos(f*x + e)^2 - b*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/16*(-sqrt(-b*tan((f
*x+exp(1))/2)^4+b)/b+2*(sqrt(-b)/(-(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b
*tan((f*x+exp(1))/2)^4+b))^2+b)-atan((-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(
-b*tan((f*x+exp(1))/2)^4+b))/sqrt(-b))/sqrt(-b)-1/2*ln(abs(-sqrt(-b)*tan((f
*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b)))/sqrt(-b)))/sign(tan((f*x
+exp(1))/2)^2-1)
```

maple [B] time = 0.21, size = 425, normalized size = 4.57

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\left(8\left(\cos^{2}\left(fx+e\right)\right)\left(-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)^{\frac{3}{2}}+16\cos\left(fx+e\right)\left(-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)^{\frac{3}{2}}+8\left(-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))*(8*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
+16*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+8*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(3/2)-4*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x
+e)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-cos(f*x+e)^2*ln(-(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+4*cos(f*x+e)*(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2))+ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2
+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2))/(-co
s(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)/sin(f*x+e)^4/(b/cos(f*x+e))^(1/2)
```

maxima [A] time = 0.75, size = 105, normalized size = 1.13

$$\frac{b\left(\frac{4\sqrt{\frac{b}{\cos(fx+e)}}}{b^2-\frac{b^2}{\cos(fx+e)^2}}-\frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{3}{2}}}\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*b*(4*sqrt(b/cos(f*x + e))/(b^2 - b^2/cos(f*x + e)^2) - 2*arctan(sqrt(b/
cos(f*x + e))/sqrt(b))/b^(3/2) + log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqr
t(b) + sqrt(b/cos(f*x + e))))/b^(3/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+fx\right)^3\sqrt{\frac{b}{\cos(e+fx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/sqrt(b*sec(e + f*x)), x)
```

3.416 $\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3f}-\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf}-\frac{5\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f}$$

```
[Out] -1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(5/2)/b^3/f-5/32*arctan((b*sec(f*x+e))^(1/
2)/b^(1/2))/f/b^(1/2)-5/32*arctanh((b*sec(f*x+e))^(1/2)/b^(1/2))/f/b^(1/2)-
5/16*cot(f*x+e)^2*(b*sec(f*x+e))^(1/2)/b/f
```

Rubi [A] time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2622, 288, 329, 212, 206, 203}

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3f}-\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf}-\frac{5\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f}-\frac{5\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-5*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*Sqrt[b]*f) - (5*ArcTanh[Sqrt[
b*Sec[e + f*x]]/Sqrt[b]])/(32*Sqrt[b]*f) - (5*Cot[e + f*x]^2*Sqrt[b*Sec[e +
 f*x]])/(16*b*f) - (Cot[e + f*x]^4*(b*Sec[e + f*x])^(5/2))/(4*b^3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^{7/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3 f} + \frac{5\,\text{Subst}\left(\int \frac{x^{3/2}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3 f}$$

$$= -\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3 f} + \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\right.}{3}$$

$$= -\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3 f} + \frac{5\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx\right.}{16}$$

$$= -\frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{5/2}}{4b^3 f} - \frac{5\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx,\right.}{32}$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,f} - \frac{5\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,f} - \frac{5\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16bf} - \text{cot}$$

Mathematica [A] time = 1.92, size = 107, normalized size = 0.87

$$-\frac{\sqrt{\sec(e+fx)}\left(-5\log\left(1-\sqrt{\sec(e+fx)}\right)+5\log\left(\sqrt{\sec(e+fx)}+1\right)+4\left(4\csc^4(e+fx)+\csc^2(e+fx)-5\right)\right.}{64f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/64*((10*ArcTan[Sqrt[Sec[e + f*x]]] - 5*Log[1 - Sqrt[Sec[e + f*x]]] + 5*L
og[1 + Sqrt[Sec[e + f*x]]] + 4*(-5 + Csc[e + f*x]^2 + 4*Csc[e + f*x]^4)*Sqr
t[Sec[e + f*x]])*Sqrt[Sec[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])
```

fricas [B] time = 0.87, size = 450, normalized size = 3.66

$$\left[\frac{10\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)-5\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx\right.\right.}{128\left(bf\cos\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/128*(10*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(1/2*sqrt
(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) - 5*(cos(f*x + e)^4 - 2*cos
(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f
*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e
)^2 + 2*cos(f*x + e) + 1)) + 8*(5*cos(f*x + e)^4 - 9*cos(f*x + e)^2)*sqrt(b
/cos(f*x + e)))/(b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x + e)^2 + b*f), 1/128*(1
0*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x
 + e))*(cos(f*x + e) - 1)/sqrt(b)) + 5*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 +
 1)*sqrt(b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(
b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x
 + e) + 1)) + 8*(5*cos(f*x + e)^4 - 9*cos(f*x + e)^2)*sqrt(b/cos(f*x + e)))
/(b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x + e)^2 + b*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/64*(2*sqrt(-b*tan((
f*x+exp(1))/2)^4+b)*(-1/4*tan((f*x+exp(1))/2)^2/b-3/2/b)+2*(1/2*(-b*(-sqrt(
-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))-(-sqrt(-b)*tan(
(f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^3-6*sqrt(-b)*(-sqrt(-b)
*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^2+6*b*sqrt(-b))/(-
(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^2+b)^2-5
/2*atan((-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))/
sqrt(-b))/sqrt(-b)-5/4*ln(abs(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((
f*x+exp(1))/2)^4+b)))/sqrt(-b)))/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [B] time = 0.22, size = 729, normalized size = 5.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/64/f*(40*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+24*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-20*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)-5*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+
e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
1/2)-1)/sin(f*x+e)^2)-5*cos(f*x+e)^3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2))-72*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+40*cos(f*x+e)
^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+5*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+
e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+5*cos(f*x+e)^2*arctan(1/2/(-cos
(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-56*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-20
*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+5*cos(f*x+e)*ln(-(2*cos(f*
x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+5*cos(f*x+e)*arctan(1/2/
(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-5*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(c
os(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)-1)/sin(f*x+e)^2)-5*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)))/sin(f*x+e)^4/(b/cos(f*x+e))^(1/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)
```

maxima [A] time = 1.46, size = 138, normalized size = 1.12

$$\frac{b\left(\frac{4\left(5\,b^2\sqrt{\frac{b}{\cos\left(fx+e\right)}}-9\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\right)}{b^4-\frac{2\,b^4}{\cos\left(fx+e\right)^2}+\frac{b^4}{\cos\left(fx+e\right)^4}}-\frac{10\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}+\frac{5\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos\left(fx+e\right)}}}\right)}{b^{\frac{3}{2}}}\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/64*b*(4*(5*b^2*sqrt(b/cos(f*x + e)) - 9*(b/cos(f*x + e))^(5/2))/(b^4 - 2*
b^4/cos(f*x + e)^2 + b^4/cos(f*x + e)^4) - 10*arctan(sqrt(b/cos(f*x + e))/s
qrt(b))/b^(3/2) + 5*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b
/cos(f*x + e))))/b^(3/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin \left(e+f\,x\right)^5\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(b*sec(f*x+e))**(1/2),x)
[Out] Integral(csc(e + f*x)**5/sqrt(b*sec(e + f*x)), x)
```

3.417 $\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}-\frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}}-\frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}}+\frac{16E\left(\frac{1}{2}(e+fx)\middle|2\right)}{39f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -8/39*b*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)-20/117*b*sin(f*x+e)^3/f/(b*sec(f*
x+e))^(3/2)-2/13*b*sin(f*x+e)^5/f/(b*sec(f*x+e))^(3/2)+16/39*(cos(1/2*f*x+1
/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/c
os(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2627, 3771, 2639}

$$-\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}}-\frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}}-\frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}}+\frac{16E\left(\frac{1}{2}(e+fx)\middle|2\right)}{39f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (16*EllipticE[(e + f*x)/2, 2])/(39*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]
]) - (8*b*Sin[e + f*x])/(39*f*(b*Sec[e + f*x])^(3/2)) - (20*b*Sin[e + f*x]^
3)/(117*f*(b*Sec[e + f*x])^(3/2)) - (2*b*Sin[e + f*x]^5)/(13*f*(b*Sec[e + f
*x])^(3/2))
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}} + \frac{10}{13}\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= -\frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}} + \frac{20}{39}\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$
$$= -\frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}} - \frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}} + \frac{8}{39}\int \frac{}{\sqrt{b\sec}}$$
$$= -\frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}} - \frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}} - \frac{2b\sin^5(e+fx)}{13f(b\sec(e+fx))^{3/2}} + \frac{8\int\sqrt{c}}{39\sqrt{\cos(e+}}$$
$$= \frac{16E\left(\frac{1}{2}(e+fx)\middle|2\right)}{39f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{8b\sin(e+fx)}{39f(b\sec(e+fx))^{3/2}} - \frac{20b\sin^3(e+fx)}{117f(b\sec(e+fx))^{3/2}} - \frac{}{13}$$

Mathematica [A] time = 0.46, size = 73, normalized size = 0.59

$$\frac{-317\sin(2(e+fx))+76\sin(4(e+fx))-9\sin(6(e+fx))+\frac{768E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{1872f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] ((768*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] - 317*Sin[2*(e + f*x)]
+ 76*Sin[4*(e + f*x)] - 9*Sin[6*(e + f*x)])/(1872*f*Sqrt[b*Sec[e + f*x]])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^6-3\cos\left(fx+e\right)^4+3\cos\left(fx+e\right)^2-1\right)\sqrt{b\sec\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(b
*sec(f*x + e))/(b*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{6}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

maple [C] time = 0.24, size = 338, normalized size = 2.75

$$\frac{2\left(9\left(\cos^{8}\left(fx+e\right)\right)-37\left(\cos^{6}\left(fx+e\right)\right)+24i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\operatorname{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/117/f*(9*cos(f*x+e)^8-37*cos(f*x+e)^6+24*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(
f*x+e)*cos(f*x+e)-24*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)
*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+24*I
*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-24*I*EllipticE(I*(-1+cos(f*x+e))/sin(
f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)+59*cos(f*x+e)^4-55*cos(f*x+e)^2+24*cos(f*x+e))*(b/cos(f*x+e))^(1/2)/si
n(f*x+e)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{6}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^6}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^6/(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

3.418 $\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}-\frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}}+\frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -4/15*b*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)-2/9*b*sin(f*x+e)^3/f/(b*sec(f*x+e
))^(3/2)+8/15*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin
(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2627, 3771, 2639}

$$-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}-\frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}}+\frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (8*EllipticE[(e + f*x)/2, 2])/(15*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]
) - (4*b*Sin[e + f*x])/(15*f*(b*Sec[e + f*x])^(3/2)) - (2*b*Sin[e + f*x]^3)
/(9*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= -\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}+\frac{2}{3}\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}}-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}+\frac{4}{15}\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}}-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}+\frac{4\int \sqrt{\cos(e+fx)}\,dx}{15\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= \frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{4b\sin(e+fx)}{15f(b\sec(e+fx))^{3/2}}-\frac{2b\sin^3(e+fx)}{9f(b\sec(e+fx))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.33, size = 63, normalized size = 0.66

$$
\frac{-68\sin(2(e+fx))+10\sin(4(e+fx))+\frac{192E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{360f\sqrt{b\sec(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
[Out] ((192*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] - 68*Sin[2*(e + f*x)] +
 10*Sin[4*(e + f*x)])/(360*f*Sqrt[b*Sec[e + f*x]])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\sqrt{b\sec\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b*sec(f*x + e))/(b*se
c(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

maple [C] time = 0.20, size = 328, normalized size = 3.45

$$-\frac{2\left(5\left(\cos^{6}\left(fx+e\right)\right)+12i\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\ \mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/45/f*(5*cos(f*x+e)^6+12*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(
f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2
)-12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+12*I*EllipticE(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)-12*I*sin(f*x+e)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-16*cos(f*x+
e)^4+23*cos(f*x+e)^2-12*cos(f*x+e))*(b/cos(f*x+e))^(1/2)/sin(f*x+e)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^4}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^4/(b/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**4/sqrt(b*sec(e + f*x)), x)
```

3.419 $\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=67

$$\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

```
[Out] -2/5*b*sin(f*x+e)/f/(b*sec(f*x+e))^(3/2)+4/5*(cos(1/2*f*x+1/2*e)^2)^(1/2)/c
os(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/
(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2627, 3771, 2639}

$$\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (4*EllipticE[(e + f*x)/2, 2])/(5*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
 - (2*b*Sin[e + f*x])/(5*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= -\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{2}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\ &= -\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{2\int \sqrt{\cos(e+fx)}\,dx}{5\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= \frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{5f(b\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 60, normalized size = 0.90

$$-\frac{\sqrt{b\sec(e+fx)}\left(\sin(e+fx)+\sin(3(e+fx))-8\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{10bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/10*(Sqrt[b*Sec[e + f*x]]*(-8*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2
] + Sin[e + f*x] + Sin[3*(e + f*x)]))/(b*f)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^2-1\right)\sqrt{b\sec\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(b*sec(f*x + e))/(b*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

maple [C] time = 0.23, size = 318, normalized size = 4.75

$$-\frac{2\left(2i\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \text{EllipticE}\left(\frac{i(-1+\cos(fx+e))}{\sin(fx+e)}, i\right)-2i\sqrt{\frac{1}{\cos(fx+e)+1}}\sqrt{\frac{\cos\cdots}{\cos(\cdots}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/5/f*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(
f*x+e)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-2*I*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+2*I*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),
I)-2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-cos(f*x+e)^4+3*cos(f*x+e)^2-2*c
os(f*x+e))*(b/cos(f*x+e))^(1/2)/sin(f*x+e)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^{2}}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^2/(b/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(e + fx)}{\sqrt{b\sec(e + fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sin(e + f*x)**2/sqrt(b*sec(e + f*x)), x)
```

3.420 $\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] 2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2
*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2639}

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx &= \frac{\int \sqrt{\cos(e+fx)}\,dx}{\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= \frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 1.00

$$\frac{2E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sec[e + f*x]],x]
[Out] (2*EllipticE[(e + f*x)/2, 2])/(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))/(b*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sec(f*x + e)), x)
```

maple [C] time = 0.19, size = 306, normalized size = 8.05

$$\frac{2\left(i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\text{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)-i\cos\left(fx+e\right)\sin\left(fx+e\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 2/f*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF
(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-I*EllipticE(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)+I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-I*Elli
pticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-cos(f*x+e)^2+cos(f*x+e))*(b/cos(f*x+e))^(1
/2)/sin(f*x+e)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(1/(b/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*sec(e + f*x)), x)
```

3.421 $\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=63

$$-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -b*csc(f*x+e)/f/(b*sec(f*x+e))^(3/2)-(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f
*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f
*x+e))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2625, 3771, 2639}

$$-\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -((b*Csc[e + f*x])/(f*(b*Sec[e + f*x])^(3/2))) - EllipticE[(e + f*x)/2, 2]/
(f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= -\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{1}{2}\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\ &= -\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{\int \sqrt{\cos(e+fx)}\,dx}{2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= -\frac{b\csc(e+fx)}{f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 48, normalized size = 0.76

$$\frac{-\cot(e+fx)-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[b*Sec[e + f*x]],x]
[Out] (-Cot[e + f*x] - EllipticE[(e + f*x)/2, 2]/Sqrt[Cos[e + f*x]])/(f*Sqrt[b*Se
c[e + f*x]])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^2}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^2/(b*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

maple [C] time = 0.19, size = 316, normalized size = 5.02

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\operatorname{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)-i\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e))^2*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-I
*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*
x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+I*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)*sin(f*x+e)-I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e))*(cos(f*x+e)
+1)^2*(b/cos(f*x+e))^(1/2)/b/sin(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(e+fx\right)^{2}\sqrt{\frac{b}{\cos\left(e+fx\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(b*sec(e + f*x)), x)
```

3.422 $\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -1/2*b*csc(f*x+e)/f/(b*sec(f*x+e))^(3/2)-1/3*b*csc(f*x+e)^3/f/(b*sec(f*x+e)
)^(3/2)-1/2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1
/2*f*x+1/2*e),2^(1/2))/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2625, 3771, 2639}

$$-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -(b*Csc[e + f*x])/(2*f*(b*Sec[e + f*x])^(3/2)) - (b*Csc[e + f*x]^3)/(3*f*(b
*Sec[e + f*x])^(3/2)) - EllipticE[(e + f*x)/2, 2]/(2*f*Sqrt[Cos[e + f*x]]*S
qrt[b*Sec[e + f*x]])
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= -\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}+\frac{1}{2}\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx\\ &= -\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}}-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{1}{4}\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx\\ &= -\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}}-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{\int \sqrt{\cos(e+fx)}\,dx}{4\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= -\frac{b\csc(e+fx)}{2f(b\sec(e+fx))^{3/2}}-\frac{b\csc^3(e+fx)}{3f(b\sec(e+fx))^{3/2}}-\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.19, size = 74, normalized size = 0.78

$$-\frac{\tan(e+fx)\left(2\csc^4(e+fx)+\csc^2(e+fx)+3\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)-3\right)}{6f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/6*((-3 + Csc[e + f*x]^2 + 2*Csc[e + f*x]^4 + 3*Sqrt[Cos[e + f*x]]*Csc[e
+ f*x]*EllipticE[(e + f*x)/2, 2])*Tan[e + f*x])/(f*Sqrt[b*Sec[e + f*x]])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^4}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^4/(b*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

maple [C] time = 0.24, size = 618, normalized size = 6.51

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^2\left(3i\left(\cos^3\left(fx+e\right)\right)\sin\left(fx+e\right)\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/6/f*(-1+cos(f*x+e))^2*(3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*sin(f*
x+e)-3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*sin(f*x+e)+3*I*EllipticE(I*
(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x
+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)-3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f
*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x
+e)^2*sin(f*x+e)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*s
in(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-3*I*EllipticE(I*(-1+cos(f*x+e
))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e
)+1))^(1/2)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)-3*cos(f*x+e)^3+2*cos(f
*x+e)^2+3*cos(f*x+e))*(cos(f*x+e)+1)^2*(b/cos(f*x+e))^(1/2)/b/sin(f*x+e)^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^4/sqrt(b*sec(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^4\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4\left(e+fx\right)}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(b*sec(f*x+e))**(1/2),x)
[Out] Integral(csc(e + f*x)**4/sqrt(b*sec(e + f*x)), x)
```

3.423 $\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=123

$$-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}}-\frac{7E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -7/20*b*csc(f*x+e)/f/(b*sec(f*x+e))^(3/2)-7/30*b*csc(f*x+e)^3/f/(b*sec(f*x+
e))^(3/2)-1/5*b*csc(f*x+e)^5/f/(b*sec(f*x+e))^(3/2)-7/20*(cos(1/2*f*x+1/2*e
)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/f/cos(f
*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2625, 3771, 2639}

$$-\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}}-\frac{7E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-7*b*Csc[e + f*x])/(20*f*(b*Sec[e + f*x])^(3/2)) - (7*b*Csc[e + f*x]^3)/(3
0*f*(b*Sec[e + f*x])^(3/2)) - (b*Csc[e + f*x]^5)/(5*f*(b*Sec[e + f*x])^(3/2
)) - (7*EllipticE[(e + f*x)/2, 2])/(20*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e +
f*x]])
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{7}{10}\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

$$= -\frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} + \frac{7}{20}\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$$

$$= -\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}} - \frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} - \frac{7}{40}\int \frac{1}{\sqrt{b\sec(}}$$

$$= -\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}} - \frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} - \frac{7\int\sqrt{\mathrm{co}}}{40\sqrt{\cos(e+f}}$$

$$= -\frac{7b\csc(e+fx)}{20f(b\sec(e+fx))^{3/2}} - \frac{7b\csc^3(e+fx)}{30f(b\sec(e+fx))^{3/2}} - \frac{b\csc^5(e+fx)}{5f(b\sec(e+fx))^{3/2}} - \frac{7E\left(\frac{1}{2}\right.}{20f\sqrt{\cos(e+}}$$

Mathematica [A] time = 0.13, size = 86, normalized size = 0.70

$$-\frac{\tan(e+fx)\left(12\csc^6(e+fx)+2\csc^4(e+fx)+7\csc^2(e+fx)+21\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)\right.}{60f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/60*((-21 + 7*Csc[e + f*x]^2 + 2*Csc[e + f*x]^4 + 12*Csc[e + f*x]^6 + 21*
Sqrt[Cos[e + f*x]]*Csc[e + f*x]*EllipticE[(e + f*x)/2, 2])*Tan[e + f*x])/(f
*Sqrt[b*Sec[e + f*x]])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^6}{b\sec\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^6/(b*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{6}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

maple [C] time = 0.25, size = 918, normalized size = 7.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/60/f*(-1+cos(f*x+e))^2*(21*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f
*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*
x+e)+42*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2
)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)-21*I*EllipticE(
I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)+21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*s
in(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-21*I*c
os(f*x+e)^5*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)-21*I*EllipticE(I*(-1+cos(f
*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(
1/2)*cos(f*x+e)^4*sin(f*x+e)-21*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
^(1/2)+42*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1
/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^3*sin(f*x+e)+21*I*cos(f*x+
e)^5*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*
EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-42*I*EllipticF(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*co
s(f*x+e)^3*sin(f*x+e)+21*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(co
s(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^4*sin(f*x+e
)-42*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(
cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*cos(f*x+e)^2*sin(f*x+e)+21*cos(f*x+e)^5-14
*cos(f*x+e)^4-42*cos(f*x+e)^3+26*cos(f*x+e)^2+21*cos(f*x+e))*(cos(f*x+e)+1)
^2*(b/cos(f*x+e))^(1/2)/b/sin(f*x+e)^9
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^6}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^6/sqrt(b*sec(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^6\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

3.424 $\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{2b^7}{17f(b\sec(e+fx))^{17/2}}-\frac{6b^5}{13f(b\sec(e+fx))^{13/2}}+\frac{2b^3}{3f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[Out] 2/17*b^7/f/(b*sec(f*x+e))^(17/2)-6/13*b^5/f/(b*sec(f*x+e))^(13/2)+2/3*b^3/f
/(b*sec(f*x+e))^(9/2)-2/5*b/f/(b*sec(f*x+e))^(5/2)
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$\frac{2b^7}{17f(b\sec(e+fx))^{17/2}}-\frac{6b^5}{13f(b\sec(e+fx))^{13/2}}+\frac{2b^3}{3f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^7/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b^7)/(17*f*(b*Sec[e + f*x])^(17/2)) - (6*b^5)/(13*f*(b*Sec[e + f*x])^(13
/2)) + (2*b^3)/(3*f*(b*Sec[e + f*x])^(9/2)) - (2*b)/(5*f*(b*Sec[e + f*x])^(
5/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b^7\,\mathrm{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{19/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^7\,\mathrm{Subst}\left(\int \left(-\frac{1}{x^{19/2}}+\frac{3}{b^2x^{15/2}}-\frac{3}{b^4x^{11/2}}+\frac{1}{b^6x^{7/2}}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b^7}{17f(b\sec(e+fx))^{17/2}} - \frac{6b^5}{13f(b\sec(e+fx))^{13/2}} + \frac{2b^3}{3f(b\sec(e+fx))^{9/2}} - \frac{2b}{5f(b\sec(e+$$

Mathematica [A] time = 0.43, size = 52, normalized size = 0.60

$$\frac{b(8365\cos(2(e+fx))-1890\cos(4(e+fx))+195\cos(6(e+fx))-10766)}{53040f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^7/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (b*(-10766 + 8365*Cos[2*(e + f*x)] - 1890*Cos[4*(e + f*x)] + 195*Cos[6*(e +
 f*x)]))/(53040*f*(b*Sec[e + f*x])^(5/2))
```

fricas [A] time = 0.66, size = 61, normalized size = 0.70

$$\frac{2\left(195\,\cos\left(fx+e\right)^9-765\,\cos\left(fx+e\right)^7+1105\,\cos\left(fx+e\right)^5-663\,\cos\left(fx+e\right)^3\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{3315\,b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*cos(f*x + e)^9 - 765*cos(f*x + e)^7 + 1105*cos(f*x + e)^5 - 663
*cos(f*x + e)^3)*sqrt(b/cos(f*x + e))/(b^2*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/b/f*2/3315*(424320*b*sqrt(-b)*(-sq
rt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^13-254592*b^
2*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^12-203
6736*b^2*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))
/2)^4+b))^11+678912*b^3*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+e
xp(1))/2)^4+b))^10+3818880*b^3*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sq
rt(-b*tan((f*x+exp(1))/2)^4+b))^9-707200*b^4*(-sqrt(-b)*tan((f*x+exp(1))/2)
^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^8-3168256*b^4*sqrt(-b)*(-sqrt(-b)*tan(
(f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^7+452608*b^5*(-sqrt(-b)
*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^6+1046656*b^5*sqrt
(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^5-1
19680*b^6*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b)
)^4+2176*b^7*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp
(1))/2)^4+b))-87040*b^6*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*t
an((f*x+exp(1))/2)^4+b))^3-17408*b^7*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(
-b*tan((f*x+exp(1))/2)^4+b))^2+128*b^8)/(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sq
rt(-b*tan((f*x+exp(1))/2)^4+b)-sqrt(-b))^17/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [A] time = 0.21, size = 56, normalized size = 0.64

$$\frac{2\left(195\left(\cos^{6}\left(fx+e\right)\right)-765\left(\cos^{4}\left(fx+e\right)\right)+1105\left(\cos^{2}\left(fx+e\right)\right)-663\right)\cos\left(fx+e\right)}{3315f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/3315/f*(195*cos(f*x+e)^6-765*cos(f*x+e)^4+1105*cos(f*x+e)^2-663)*cos(f*x+
e)/(b/cos(f*x+e))^(3/2)
```

maxima [A] time = 0.44, size = 63, normalized size = 0.72

$$\frac{2\left(195\,b^{6}-\frac{765\,b^{6}}{\cos\left(fx+e\right)^{2}}+\frac{1105\,b^{6}}{\cos\left(fx+e\right)^{4}}-\frac{663\,b^{6}}{\cos\left(fx+e\right)^{6}}\right)b}{3315\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3315*(195*b^6 - 765*b^6/cos(f*x + e)^2 + 1105*b^6/cos(f*x + e)^4 - 663*b^
6/cos(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(17/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^7}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^7/(b/cos(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)^7/(b/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**7/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.425 $\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2b^5}{13f(b\sec(e+fx))^{13/2}}+\frac{4b^3}{9f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[Out] -2/13*b^5/f/(b*sec(f*x+e))^(13/2)+4/9*b^3/f/(b*sec(f*x+e))^(9/2)-2/5*b/f/(b
*sec(f*x+e))^(5/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$-\frac{2b^5}{13f(b\sec(e+fx))^{13/2}}+\frac{4b^3}{9f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b^5)/(13*f*(b*Sec[e + f*x])^(13/2)) + (4*b^3)/(9*f*(b*Sec[e + f*x])^(9/
2)) - (2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b^5 \operatorname{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{15/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^5 \operatorname{Subst}\left(\int \left(\frac{1}{x^{15/2}} - \frac{2}{b^2x^{11/2}} + \frac{1}{b^4x^{7/2}}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= -\frac{2b^5}{13f(b\sec(e+fx))^{13/2}} + \frac{4b^3}{9f(b\sec(e+fx))^{9/2}} - \frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Mathematica [A] time = 0.25, size = 42, normalized size = 0.65

$$\frac{b(340\cos(2(e+fx)) - 45\cos(4(e+fx)) - 551)}{2340f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
[Out] (b*(-551 + 340*Cos[2*(e + f*x)] - 45*Cos[4*(e + f*x)]))/(2340*f*(b*Sec[e +
f*x])^(5/2))
```

fricas [A] time = 0.76, size = 51, normalized size = 0.78

$$-\frac{2\left(45\cos\left(fx+e\right)^7 - 130\cos\left(fx+e\right)^5 + 117\cos\left(fx+e\right)^3\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{585\,b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] -2/585*(45*cos(f*x + e)^7 - 130*cos(f*x + e)^5 + 117*cos(f*x + e)^3)*sqrt(b
/cos(f*x + e))/(b^2*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/b/f*2/585*(-12480*b*(-sqrt(-b)*tan
((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^10-6240*b*sqrt(-b)*(-s
qrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^9+48672*b^2
*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^8+9984*
b^2*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4
+b))^7-64896*b^3*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/
2)^4+b))^6-7488*b^3*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((
f*x+exp(1))/2)^4+b))^5+29120*b^4*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*t
an((f*x+exp(1))/2)^4+b))^4+416*b^5*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^
2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))+3328*b^4*sqrt(-b)*(-sqrt(-b)*tan((f*x+e
xp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^3-2496*b^5*(-sqrt(-b)*tan((f*
x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^2+32*b^6)/(-sqrt(-b)*tan((
f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b)-sqrt(-b))^13/sign(tan((f*
x+exp(1))/2)^2-1)
```

maple [A] time = 0.16, size = 46, normalized size = 0.71

$$-\frac{2\left(45\left(\cos^{4}\left(fx+e\right)\right)-130\left(\cos^{2}\left(fx+e\right)\right)+117\right)\cos\left(fx+e\right)}{585f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -2/585/f*(45*cos(f*x+e)^4-130*cos(f*x+e)^2+117)*cos(f*x+e)/(b/cos(f*x+e))^(
3/2)
```

maxima [A] time = 0.48, size = 50, normalized size = 0.77

$$-\frac{2\left(45\,b^{4}-\frac{130\,b^{4}}{\cos\left(fx+e\right)^{2}}+\frac{117\,b^{4}}{\cos\left(fx+e\right)^{4}}\right)b}{585\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/585*(45*b^4 - 130*b^4/cos(f*x + e)^2 + 117*b^4/cos(f*x + e)^4)*b/(f*(b/c
os(f*x + e))^(13/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(e+f\,x\right)^5}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^5/(b/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

3.426 $\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2b^3}{9f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[Out] 2/9*b^3/f/(b*sec(f*x+e))^(9/2)-2/5*b/f/(b*sec(f*x+e))^(5/2)
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2b^3}{9f(b\sec(e+fx))^{9/2}}-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b^3)/(9*f*(b*Sec[e + f*x])^(9/2)) - (2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{11/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^3\,\text{Subst}\left(\int \left(-\frac{1}{x^{11/2}}+\frac{1}{b^2x^{7/2}}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b^3}{9f(b\sec(e+fx))^{9/2}} - \frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Mathematica [A] time = 0.17, size = 32, normalized size = 0.74

$$\frac{b(5\cos(2(e+fx))-13)}{45f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
[Out] (b*(-13 + 5*Cos[2*(e + f*x)]))/(45*f*(b*Sec[e + f*x])^(5/2))
```

fricas [A] time = 0.95, size = 41, normalized size = 0.95

$$\frac{2\left(5\cos\left(fx+e\right)^5-9\cos\left(fx+e\right)^3\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{45\,b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] 2/45*(5*cos(f*x + e)^5 - 9*cos(f*x + e)^3)*sqrt(b/cos(f*x + e))/(b^2*f)
```

giac [A] time = 1.72, size = 50, normalized size = 1.16

$$\frac{2\left(5\,b^5-\frac{9\,b^5}{\cos\left(fx+e\right)^2}\right)\cos\left(fx+e\right)^4}{45\,b^6f\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 2/45*(5*b^5 - 9*b^5/cos(f*x + e)^2)*cos(f*x + e)^4/(b^6*f*sqrt(b/cos(f*x +
e)))
```

maple [A] time = 0.15, size = 36, normalized size = 0.84

$$\frac{2\left(5\left(\cos^{2}\left(fx+e\right)\right)-9\right)\cos\left(fx+e\right)}{45f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/45/f*(5*cos(f*x+e)^2-9)*cos(f*x+e)/(b/cos(f*x+e))^(3/2)
```

maxima [A] time = 1.26, size = 37, normalized size = 0.86

$$\frac{2\left(5\,b^{2}-\frac{9\,b^{2}}{\cos\left(fx+e\right)^{2}}\right)b}{45\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45*(5*b^2 - 9*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(9/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(e+f\,x\right)^{3}}{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(b/cos(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)^3/(b/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.427 $\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

```
[Out] -2/5*b/f/(b*sec(f*x+e))^(5/2)
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{2b}{5f(b\sec(e+fx))^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 20, normalized size = 1.00

$$-\frac{2b}{5f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
[Out] (-2*b)/(5*f*(b*Sec[e + f*x])^(5/2))
```

fricas [A] time = 0.59, size = 28, normalized size = 1.40

$$-\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{3}}{5\,b^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] -2/5*sqrt(b/cos(f*x + e))*cos(f*x + e)^3/(b^2*f)
```

giac [A] time = 1.53, size = 30, normalized size = 1.50

$$-\frac{2\cos\left(fx+e\right)^{2}}{5\,bf\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] -2/5*cos(f*x + e)^2/(b*f*sqrt(b/cos(f*x + e)))
```

maple [A] time = 0.02, size = 17, normalized size = 0.85

$$-\frac{2b}{5f\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x)
[Out] -2/5*b/f/(b*sec(f*x+e))^(5/2)
```

maxima [A] time = 0.45, size = 23, normalized size = 1.15

$$-\frac{2\,\cos\left(fx+e\right)}{5\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/5*cos(f*x + e)/(f*(b/cos(f*x + e))^(3/2))
```

mupad [B] time = 0.57, size = 28, normalized size = 1.40

$$-\frac{2\cos\left(e+f\,x\right)^3\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{5\,b^2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(b/cos(e + f*x))^(3/2),x)
```

```
[Out] -(2*cos(e + f*x)^3*(b/cos(e + f*x))^(1/2))/(5*b^2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sin(e + f*x)/(b*sec(e + f*x))**(3/2), x)
```

3.428 $\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f}+\frac{2}{bf\sqrt{b\sec(e+fx)}}$$

```
[Out] arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f-arctanh((b*sec(f*x+e))^(1/2)
/b^(1/2))/b^(3/2)/f+2/b/f/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2622, 325, 329, 298, 203, 206}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f}+\frac{2}{bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(b^(3/2)*f) - ArcTanh[Sqrt[b*Sec[e + f
*x]]/Sqrt[b]]/(b^(3/2)*f) + 2/(b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{bf}$$
$$= \frac{2}{bf\sqrt{b\sec(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$
$$= \frac{2}{bf\sqrt{b\sec(e+fx)}} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{b^3 f}$$
$$= \frac{2}{bf\sqrt{b\sec(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{bf} + \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\right.}{bf}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f} + \frac{2}{bf\sqrt{b\sec(e+fx)}}$$

Mathematica [A] time = 2.00, size = 89, normalized size = 1.14

$$\frac{\sqrt{\sec(e+fx)}\left(\log\left(1-\sqrt{\sec(e+fx)}\right)-\log\left(\sqrt{\sec(e+fx)}+1\right)\right)+2\sqrt{\sec(e+fx)}\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)+4}{2bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4 + 2*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[Sec[e + f*x]] + (Log[1 - Sqrt[Sec[e
+ f*x]]] - Log[1 + Sqrt[Sec[e + f*x]]])*Sqrt[Sec[e + f*x]])/(2*b*f*Sqrt[b*S
ec[e + f*x]])
```

fricas [B] time = 1.02, size = 314, normalized size = 4.03

$$\left[\frac{2\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right)+8\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)-\sqrt{-b}\log\left(-\frac{b\cos(fx+e)^2+4\left(\cos(fx+e)^2-\cos(f\right.}{\cos(fx+e)^2\right.}\right.}{4b^2f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b)*arctan(2*sqrt(-b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos
(f*x + e) + b)) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e) - sqrt(-b)*log(-(b*co
s(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x +
e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/(b^2*f)
, 1/4*(2*sqrt(b)*arctan(2*sqrt(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(
f*x + e) - b)) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e) + sqrt(b)*log(-(b*cos(
f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e))
 + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/(b^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/b/f*2/(sqrt(-b)*tan((f*x+exp(1))/2
)^2-sqrt(-b*tan((f*x+exp(1))/2)^4+b)+sqrt(-b))/sign(tan((f*x+exp(1))/2)^2-1
)
```

maple [B] time = 0.17, size = 221, normalized size = 2.83

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\left(4\cos\left(fx+e\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}-\ln\left(-\frac{2\left(\cos^{2}\left(fx+e\right)\right)\sqrt{-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}}-\left(\cos^{2}\left(fx+e\right)\right)+2\cos\left(fx+e\right)-2\sqrt{\cdots}}{\sin\left(fx+e\right)^{2}}\cdots\right.\right.}{2f\cos\left(fx+e\right)\sin\left(fx+e\right)^{2}\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\sqrt{-\frac{\cos\left(f\cdots\right)}{\left(\cos\left(fx\cdots\right)\right.}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(-1+cos(f*x+e))*(4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-l
n(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(
f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+arctan(1/2/(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+4*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
)/cos(f*x+e)/sin(f*x+e)^2/(b/cos(f*x+e))^(3/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)
```

maxima [A] time = 0.62, size = 89, normalized size = 1.14

$$\frac{b\left(\frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}+\frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{5}{2}}}+\frac{4}{b^{2}\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(2*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/b^(5/2) + log(-(sqrt(b) - sqr
t(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/b^(5/2) + 4/(b^2*sqrt(
b/cos(f*x + e))))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(b/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(b/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(b*sec(e + f*x))**(3/2), x)
```

3.429 $\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f}-\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3f}$$

```
[Out] -1/4*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f+1/4*arctanh((b*sec(f*x+
e))^(1/2)/b^(1/2))/b^(3/2)/f-1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(3/2)/b^3/f
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2622, 290, 329, 298, 203, 206}

$$-\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3f}-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f}+\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(4*b^(3/2)*f) + ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(4*b^(3/2)*f) - (Cot[e + f*x]^2*(b*Sec[e + f*x])^(3/2))/(2
*b^3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3 f} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{4b^3 f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3 f} - \frac{\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2b^3 f}$$

$$= -\frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3 f} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4bf} - \frac{\text{Subst}\left(\int \frac{1}{b+}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2} f} + \frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{3/2} f} - \frac{\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{2b^3 f}$$

Mathematica [A] time = 0.46, size = 98, normalized size = 1.05

$$\frac{-4\csc^2(e+fx) + \sqrt{\sec(e+fx)}\left(\log\left(\sqrt{\sec(e+fx)}+1\right) - \log\left(1-\sqrt{\sec(e+fx)}\right)\right) - 2\sqrt{\sec(e+fx)}\tan^{-1}\left(\sqrt{\text{se}}\right.}{8bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-4*Csc[e + f*x]^2 - 2*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[Sec[e + f*x]] + (-Lo
g[1 - Sqrt[Sec[e + f*x]]] + Log[1 + Sqrt[Sec[e + f*x]]])*Sqrt[Sec[e + f*x]]
)/(8*b*f*Sqrt[b*Sec[e + f*x]])
```

fricas [B] time = 0.75, size = 364, normalized size = 3.91

$$\left[-\frac{2\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right) + \left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\log\left(\frac{b\cos(fx+e)^2-4\left(\cos(f\right.}{}\right.}{16\left(b^2 f\cos\left(fx+e\right)^2-b^2 f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x
+ e))*(cos(f*x + e) + 1)/b) + (cos(f*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x
+ e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) -
6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)) - 8*sqrt(b/cos
(f*x + e))*cos(f*x + e))/(b^2*f*cos(f*x + e)^2 - b^2*f), 1/16*(2*(cos(f*x +
 e)^2 - 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/sqrt(
b)) + (cos(f*x + e)^2 - 1)*sqrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^
2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos
(f*x + e)^2 - 2*cos(f*x + e) + 1)) + 8*sqrt(b/cos(f*x + e))*cos(f*x + e))/(
b^2*f*cos(f*x + e)^2 - b^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/b/f/16*(sqrt(-b*tan((
f*x+exp(1))/2)^4+b)/b+2*(sqrt(-b)/(-(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-
b*tan((f*x+exp(1))/2)^4+b))^2+b)+atan((-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt
(-b*tan((f*x+exp(1))/2)^4+b))/sqrt(-b))/sqrt(-b)-1/2*ln(abs(-sqrt(-b)*tan((
f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b)))/sqrt(-b)))/sign(tan((f*
x+exp(1))/2)^2-1)
```

maple [B] time = 0.18, size = 426, normalized size = 4.58

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\left(8\left(\cos^{2}\left(fx+e\right)\right)\left(-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)^{\frac{3}{2}}+16\cos\left(fx+e\right)\left(-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)^{\frac{3}{2}}+\left(\cos^{2}\left(fx+e\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))*(8*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
+16*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+cos(f*x+e)^2*arctan(1/2
/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/
(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(
3/2)+4*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-4*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+ln(-(2*
cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)
-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2))/cos(f*x+e)/sin(f*
x+e)^4/(b/cos(f*x+e))^(3/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

maxima [A] time = 1.96, size = 106, normalized size = 1.14

$$\frac{b\left(\frac{4\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}}{b^4-\frac{b^4}{\cos\left(fx+e\right)^2}}-\frac{2\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}-\frac{\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos\left(fx+e\right)}}}\right)}{b^{\frac{5}{2}}}\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*b*(4*(b/cos(f*x + e))^(3/2)/(b^4 - b^4/cos(f*x + e)^2) - 2*arctan(sqrt(
b/cos(f*x + e))/sqrt(b))/b^(5/2) - log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(s
qrt(b) + sqrt(b/cos(f*x + e))))/b^(5/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}{\left(e + f x \right)}}{\left(b \sec{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(b*sec(e + f*x))**(3/2), x)
```

3.430 $\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f}-\frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3f}-\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3f}$$

```
[Out] -3/32*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f+3/32*arctanh((b*sec(f*
x+e))^(1/2)/b^(1/2))/b^(3/2)/f-3/16*cot(f*x+e)^2*(b*sec(f*x+e))^(3/2)/b^3/f
-1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(3/2)/b^3/f
```

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2622, 288, 290, 329, 298, 203, 206}

$$-\frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3f}-\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3f}-\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-3*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*b^(3/2)*f) + (3*ArcTanh[Sqrt[
b*Sec[e + f*x]]/Sqrt[b]])/(32*b^(3/2)*f) - (3*Cot[e + f*x]^2*(b*Sec[e + f*x
])^(3/2))/(16*b^3*f) - (Cot[e + f*x]^4*(b*Sec[e + f*x])^(3/2))/(4*b^3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^{5/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{x}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3 f}$$

$$= -\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3 f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{x}}{-1+\frac{x^2}{b^2}}\right.}{32}$$

$$= -\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3 f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+\frac{x^4}{b^2}}\right.}{1}$$

$$= -\frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3 f} - \frac{\cot^4(e+fx)(b\sec(e+fx))^{3/2}}{4b^3 f} + \frac{3\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx\right.}{3}$$

$$= -\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2} f} + \frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{3/2} f} - \frac{3\cot^2(e+fx)(b\sec(e+fx))^{3/2}}{16b^3 f} - $$

Mathematica [A] time = 0.63, size = 109, normalized size = 0.89

$$\frac{-16\csc^4(e+fx) + 4\csc^2(e+fx) + 3\sqrt{\sec(e+fx)}\left(\log\left(\sqrt{\sec(e+fx)}+1\right) - \log\left(1-\sqrt{\sec(e+fx)}\right)\right) - 6\sqrt{\sec}}{64bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*Csc[e + f*x]^2 - 16*Csc[e + f*x]^4 - 6*ArcTan[Sqrt[Sec[e + f*x]]]*Sqrt[S
ec[e + f*x]] + 3*(-Log[1 - Sqrt[Sec[e + f*x]]] + Log[1 + Sqrt[Sec[e + f*x]]
])*Sqrt[Sec[e + f*x]])/(64*b*f*Sqrt[b*Sec[e + f*x]])
```

fricas [B] time = 0.78, size = 454, normalized size = 3.69

$$\left[-\frac{6\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)+3\left(\cos\left(fx+e\right)^{4}-2\cos(f \cdots\right.}{128\left(b^{2}f\cos(f \cdots\right.} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(1/2*sqrt
(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 3*(cos(f*x + e)^4 - 2*cos
(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f
*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e
)^2 + 2*cos(f*x + e) + 1)) + 8*(cos(f*x + e)^3 + 3*cos(f*x + e))*sqrt(b/cos
(f*x + e)))/(b^2*f*cos(f*x + e)^4 - 2*b^2*f*cos(f*x + e)^2 + b^2*f), 1/128*
(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*
x + e))*(cos(f*x + e) - 1)/sqrt(b)) + 3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2
+ 1)*sqrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt
(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*
x + e) + 1)) - 8*(cos(f*x + e)^3 + 3*cos(f*x + e))*sqrt(b/cos(f*x + e)))/(b
^2*f*cos(f*x + e)^4 - 2*b^2*f*cos(f*x + e)^2 + b^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/b/f/64*(2*(1/4*tan((f
*x+exp(1))/2)^2/b+1/2/b)*sqrt(-b*tan((f*x+exp(1))/2)^4+b)+2*(1/2*(-b*(-sqrt
(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))-(-sqrt(-b)*tan
((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^3-2*sqrt(-b)*(-sqrt(-b
)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^2+2*b*sqrt(-b))/(
-(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^2+b)^2+
3/2*atan((-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))
/sqrt(-b))/sqrt(-b)-3/4*ln(abs(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan(
(f*x+exp(1))/2)^4+b)))/sqrt(-b)))/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [B] time = 0.19, size = 729, normalized size = 5.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/64/f*(8*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-8*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-3*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*cos(f*x+e)^3*arctan(1/2/(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2))-40*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(3/2)+12*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(f*x+e)^2*
ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos
(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*cos(f*x+e
)^2*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-24*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(3/2)-24*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(
f*x+e)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^
2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3*co
s(f*x+e)*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))+12*(-cos(f*x+e)/(
cos(f*x+e)+1)^2)^(1/2)-3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)
^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/
sin(f*x+e)^2)+3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)))/cos(f*x+e
)/sin(f*x+e)^4/(b/cos(f*x+e))^(3/2)/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
```

maxima [A] time = 0.85, size = 137, normalized size = 1.11

$$-\frac{b\left(\frac{4\left(b^{2}\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}+3\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{7}{2}}\right)}{b^{6}-\frac{2b^{6}}{\cos\left(fx+e\right)^{2}}+\frac{b^{6}}{\cos\left(fx+e\right)^{4}}}+\frac{6\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}+\frac{3\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos\left(fx+e\right)}}}\right)}{b^{\frac{5}{2}}}\right)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/64*b*(4*(b^2*(b/cos(f*x + e))^(3/2) + 3*(b/cos(f*x + e))^(7/2))/(b^6 - 2
*b^6/cos(f*x + e)^2 + b^6/cos(f*x + e)^4) + 6*arctan(sqrt(b/cos(f*x + e))/s
qrt(b))/b^(5/2) + 3*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b
/cos(f*x + e))))/b^(5/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^5\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{5}\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(b*sec(f*x+e))**(3/2),x)
[Out] Integral(csc(e + f*x)**5/(b*sec(e + f*x))**(3/2), x)
```

3.431 $\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=126

$$\frac{8\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{77b^2f}-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}}-\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))}$$

```
[Out] -12/77*b*sin(f*x+e)/f/(b*sec(f*x+e))^(5/2)-2/11*b*sin(f*x+e)^3/f/(b*sec(f*x
+e))^(5/2)+8/77*sin(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)+8/77*(cos(1/2*f*x+1/2*e
)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x
+e)^(1/2)*(b*sec(f*x+e))^(1/2)/b^2/f
```

Rubi [A] time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2627, 3769, 3771, 2641}

$$\frac{8\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{77b^2f}-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}}-\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (8*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(77*b
^2*f) - (12*b*Sin[e + f*x])/(77*f*(b*Sec[e + f*x])^(5/2)) + (8*Sin[e + f*x]
)/(77*b*f*Sqrt[b*Sec[e + f*x]]) - (2*b*Sin[e + f*x]^3)/(11*f*(b*Sec[e + f*x
])^(5/2))
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx &= -\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{6}{11}\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx\\
&= -\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{12}{77}\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx\\
&= -\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}+\frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{4\int\sqrt{b\sec(}}{77}\\
&= -\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}+\frac{8\sin(e+fx)}{77bf\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{11f(b\sec(e+fx))^{5/2}}+\frac{\left(4\sqrt{\cos(e+}\right.}{}\\
&= \frac{8\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{77b^2f}-\frac{12b\sin(e+fx)}{77f(b\sec(e+fx))^{5/2}}+\frac{8\sin(e}{77bf\sqrt{b\,\mathrm{se}}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 81, normalized size = 0.64

$$\frac{\sec^2(e+fx)\left(-5\sin(2(e+fx))-24\sin(4(e+fx))+7\sin(6(e+fx))+128\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{1232f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(128*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] - 5*Sin[2
*(e + f*x)] - 24*Sin[4*(e + f*x)] + 7*Sin[6*(e + f*x)]))/(1232*f*(b*Sec[e +
 f*x])^(3/2))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos\left(fx+e\right)^{4}-2\,\cos\left(fx+e\right)^{2}+1\right)\sqrt{b\sec\left(fx+e\right)}}{b^{2}\sec\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b*sec(f*x + e))/(b^2*
sec(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

maple [C] time = 0.23, size = 173, normalized size = 1.37

$$-\frac{2\left(\cos\left(fx+e\right)+1\right)^{2}\left(-1+\cos\left(fx+e\right)\right)\left(-7\left(\cos^{6}\left(fx+e\right)\right)+4i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\ \text{EllipticF}\left(\frac{i\left(-1+\text{co}\right.}{\sin\left(\right.}\right.\right.}{77f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -2/77/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))*(-7*cos(f*x+e)^6+4*I*EllipticF(I*(
-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e
)/(cos(f*x+e)+1))^(1/2)+7*cos(f*x+e)^5+13*cos(f*x+e)^4-13*cos(f*x+e)^3-4*co
s(f*x+e)^2+4*cos(f*x+e))/cos(f*x+e)^2/sin(f*x+e)^3/(b/cos(f*x+e))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^4}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^4/(b/cos(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{4}{\left(e + f x \right)}}{\left(b \sec{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(b*sec(f*x+e))**(3/2),x)
[Out] Integral(sin(e + f*x)**4/(b*sec(e + f*x))**(3/2), x)
```

3.432 $\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{4\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{21b^2f}+\frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}$$

```
[Out] -2/7*b*sin(f*x+e)/f/(b*sec(f*x+e))^(5/2)+4/21*sin(f*x+e)/b/f/(b*sec(f*x+e))
^(1/2)+4/21*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1
/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/b^2/f
```

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2627, 3769, 3771, 2641}

$$\frac{4\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{21b^2f}+\frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(21*b
^2*f) - (2*b*Sin[e + f*x])/(7*f*(b*Sec[e + f*x])^(5/2)) + (4*Sin[e + f*x])/
(21*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}} + \frac{2}{7}\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx$$
$$= -\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}} + \frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}} + \frac{2\int \sqrt{b\sec(e+fx)}\,dx}{21b^2}$$
$$= -\frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}} + \frac{4\sin(e+fx)}{21bf\sqrt{b\sec(e+fx)}} + \frac{\left(2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int -$$
$$= \frac{4\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{21b^2f} - \frac{2b\sin(e+fx)}{7f(b\sec(e+fx))^{5/2}} + \frac{4\sin(e-$$

Mathematica [A] time = 0.13, size = 71, normalized size = 0.72

$$\frac{\sec^2(e+fx)\left(2\sin(2(e+fx)) - 3\sin(4(e+fx)) + 16\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{84f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*(16*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + 2*Sin[2*
(e + f*x)] - 3*Sin[4*(e + f*x)]))/(84*f*(b*Sec[e + f*x])^(3/2))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^2-1\right)\sqrt{b\sec\left(fx+e\right)}}{b^2\sec\left(fx+e\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(b*sec(f*x + e))/(b^2*sec(f*x + e)^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

maple [C] time = 0.19, size = 153, normalized size = 1.56

$$-\frac{2\left(\cos\left(fx+e\right)+1\right)^{2}\left(-1+\cos\left(fx+e\right)\right)\left(2i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right.\right.}{21f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)^{3}\left(\frac{b}{\cos\left(fx\right.}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -2/21/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))*(2*I*(1/(cos(f*x+e)+1))^(1/2)*(cos
(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin
(f*x+e)+3*cos(f*x+e)^4-3*cos(f*x+e)^3-2*cos(f*x+e)^2+2*cos(f*x+e))/cos(f*x+
e)^2/sin(f*x+e)^3/(b/cos(f*x+e))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(b/cos(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^2/(b/cos(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}{\left(e + f x \right)}}{\left(b \sec{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(b*sec(f*x+e))**(3/2),x)
[Out] Integral(sin(e + f*x)**2/(b*sec(e + f*x))**(3/2), x)
```

3.433 $\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3b^2f}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

```
[Out] 2/3*sin(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)+2/3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/co
s(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*
sec(f*x+e))^(1/2)/b^2/f
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3769, 3771, 2641}

$$\frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3b^2f}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(3*b^
2*f) + (2*Sin[e + f*x])/(3*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\sec(e+fx))^{3/2}}\,dx &= \frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}+\frac{\int\sqrt{b\sec(e+fx)}\,dx}{3b^2}\\ &= \frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}+\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{\cos(e+fx)}}\,dx}{3b^2}\\ &= \frac{2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{3b^2f}+\frac{2\sin(e+fx)}{3bf\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 59, normalized size = 0.82

$$\frac{\sec^2(e+fx)\left(\sin(2(e+fx))+2\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(-3/2),x]
[Out] (Sec[e + f*x]^2*(2*Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2] + Sin[2*(e
+ f*x)]))/(3*f*(b*Sec[e + f*x])^(3/2))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}}{b^2\sec\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))/(b^2*sec(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(-3/2), x)
```

maple [C] time = 0.18, size = 131, normalized size = 1.82

$$-\frac{2\left(\cos\left(fx+e\right)+1\right)^{2}\left(-1+\cos\left(fx+e\right)\right)\left(i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\operatorname{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\right.}{3f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\cos\left(fx+e\right)^{2}\sin\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -2/3/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*
x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*
x+e)-cos(f*x+e)^2+cos(f*x+e))/(b/cos(f*x+e))^(3/2)/cos(f*x+e)^2/sin(f*x+e)^
3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cos(e + f*x))^(3/2),x)
```

```
[Out] int(1/(b/cos(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((b*sec(e + f*x))**(-3/2), x)
```

3.434 $\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{b^2f}-\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}}$$

```
[Out] -csc(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)-(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f
*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x
+e))^(1/2)/b^2/f
```

Rubi [A] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2623, 3771, 2641}

$$-\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{b^2f}-\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(Csc[e + f*x]/(b*f*Sqrt[b*Sec[e + f*x]])) - (Sqrt[Cos[e + f*x]]*EllipticF[
(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(b^2*f)
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}} - \frac{\int \sqrt{b\sec(e+fx)}\,dx}{2b^2}$$
$$= -\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e+fx)}}\,dx}{2b^2}$$
$$= -\frac{\csc(e+fx)}{bf\sqrt{b\sec(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{b^2f}$$

Mathematica [A] time = 0.10, size = 58, normalized size = 0.85

$$\frac{-F\left(\frac{1}{2}(e+fx)\middle|2\right)-\sqrt{\cos(e+fx)}\csc(e+fx)}{f\cos^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-(Sqrt[Cos[e + f*x]]*Csc[e + f*x]) - EllipticF[(e + f*x)/2, 2])/(f*Cos[e + f*x]^(3/2)*(b*Sec[e + f*x])^(3/2))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^2}{b^2\sec\left(fx+e\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^2/(b^2*sec(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

maple [C] time = 0.18, size = 191, normalized size = 2.81

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)+i\sqrt{\frac{}{\mathrm{co}}}\right.}{f\cos\left(fx+e\right)^{2}\sin\left(fx+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e))^2*(I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+I
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1
+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)+cos(f*x+e))*(cos(f*x+e)+1)^2/cos(f*x+
e)^2/sin(f*x+e)^5/(b/cos(f*x+e))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^{2}\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(b*sec(f*x+e))**(3/2),x)
[Out] Integral(csc(e + f*x)**2/(b*sec(e + f*x))**(3/2), x)
```

3.435 $\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{6b^2f}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}+\frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}}$$

```
[Out] 1/6*csc(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)-1/3*csc(f*x+e)^3/b/f/(b*sec(f*x+e))
^(1/2)-1/6*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/
2*f*x+1/2*e),2^(1/2))*cos(f*x+e)^(1/2)*(b*sec(f*x+e))^(1/2)/b^2/f
```

Rubi [A] time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2623, 2625, 3771, 2641}

$$-\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{6b^2f}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}+\frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] Csc[e + f*x]/(6*b*f*Sqrt[b*Sec[e + f*x]]) - Csc[e + f*x]^3/(3*b*f*Sqrt[b*Se
c[e + f*x]]) - (Sqrt[Cos[e + f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e +
 f*x]])/(6*b^2*f)
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx &= -\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}-\frac{\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx}{6b^2}\\ &= \frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}-\frac{\int \sqrt{b\sec(e+fx)}\,dx}{12b^2}\\ &= \frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}-\frac{\left(\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{\cos(e-}}}{12b^2}\\ &= \frac{\csc(e+fx)}{6bf\sqrt{b\sec(e+fx)}}-\frac{\csc^3(e+fx)}{3bf\sqrt{b\sec(e+fx)}}-\frac{\sqrt{\cos(e+fx)}F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e-}}{6b^2f}\end{aligned}$$

Mathematica [A] time = 0.22, size = 62, normalized size = 0.61

$$\frac{-2\csc^3(e+fx)+\csc(e+fx)-\frac{F\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{6bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Csc[e + f*x] - 2*Csc[e + f*x]^3 - EllipticF[(e + f*x)/2, 2]/Sqrt[Cos[e + f
*x]])/(6*b*f*Sqrt[b*Sec[e + f*x]])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\,\csc\left(fx+e\right)^4}{b^2\sec\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^4/(b^2*sec(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

maple [C] time = 0.20, size = 343, normalized size = 3.36

$$\frac{\left(\cos\left(fx+e\right)+1\right)^{2}\left(-1+\cos\left(fx+e\right)\right)^{2}\left(i\left(\cos^{3}\left(fx+e\right)\right)\sin\left(fx+e\right)\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i(-}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/6/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)+I*cos(f*x+e)^2*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-I*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+co
s(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-I*(1/(cos(f*x+e)+1))^(1/2)*(c
os(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*s
in(f*x+e)-cos(f*x+e)^3-cos(f*x+e))/cos(f*x+e)^2/sin(f*x+e)^7/(b/cos(f*x+e))
^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^4\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{4}{\left(e + f x \right)}}{\left(b \sec{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(b*sec(f*x+e))**(3/2),x)
[Out] Integral(csc(e + f*x)**4/(b*sec(e + f*x))**(3/2), x)
```

3.436 $\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{12b^2f}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}+\frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}}+\frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}}$$

```
[Out] 1/12*csc(f*x+e)/b/f/(b*sec(f*x+e))^(1/2)+1/30*csc(f*x+e)^3/b/f/(b*sec(f*x+e
))^(1/2)-1/5*csc(f*x+e)^5/b/f/(b*sec(f*x+e))^(1/2)-1/12*(cos(1/2*f*x+1/2*e)
^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticF(sin(1/2*f*x+1/2*e),2^(1/2))*cos(f*x+
e)^(1/2)*(b*sec(f*x+e))^(1/2)/b^2/f
```

Rubi [A] time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2623, 2625, 3771, 2641}

$$-\frac{\sqrt{\cos(e+fx)}\,F\left(\frac{1}{2}(e+fx)\middle|2\right)\sqrt{b\sec(e+fx)}}{12b^2f}-\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}}+\frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}}+\frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] Csc[e + f*x]/(12*b*f*Sqrt[b*Sec[e + f*x]]) + Csc[e + f*x]^3/(30*b*f*Sqrt[b*
Sec[e + f*x]]) - Csc[e + f*x]^5/(5*b*f*Sqrt[b*Sec[e + f*x]]) - (Sqrt[Cos[e
+ f*x]]*EllipticF[(e + f*x)/2, 2]*Sqrt[b*Sec[e + f*x]])/(12*b^2*f)
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{3/2}}\,dx = -\frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}} - \frac{\int \csc^4(e+fx)\sqrt{b\sec(e+fx)}\,dx}{10b^2}$$

$$= \frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}} - \frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}} - \frac{\int \csc^2(e+fx)\sqrt{b\sec(e+fx)}\,dx}{12b^2}$$

$$= \frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}} + \frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}} - \frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}} - \frac{\int \sqrt{b\sec(e+f}}{24b^2}$$

$$= \frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}} + \frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}} - \frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}} - \frac{\left(\sqrt{\cos(e+fx)}\right.}{}$$

$$= \frac{\csc(e+fx)}{12bf\sqrt{b\sec(e+fx)}} + \frac{\csc^3(e+fx)}{30bf\sqrt{b\sec(e+fx)}} - \frac{\csc^5(e+fx)}{5bf\sqrt{b\sec(e+fx)}} - \frac{\sqrt{\cos(e+fx)}\,F}{}$$

Mathematica [A] time = 0.31, size = 74, normalized size = 0.56

$$\frac{-12\csc^5(e+fx)+2\csc^3(e+fx)+5\csc(e+fx)-\frac{5F\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{60bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (5*Csc[e + f*x] + 2*Csc[e + f*x]^3 - 12*Csc[e + f*x]^5 - (5*EllipticF[(e +
f*x)/2, 2])/Sqrt[Cos[e + f*x]])/(60*b*f*Sqrt[b*Sec[e + f*x]])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^{6}}{b^{2}\sec\left(fx+e\right)^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^6/(b^2*sec(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(3/2), x)
```

maple [C] time = 0.24, size = 493, normalized size = 3.73

$$-\frac{\left(\cos\left(fx+e\right)+1\right)^{2}\left(-1+\cos\left(fx+e\right)\right)^{2}\left(5i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\left(\cos^{5}\left(fx+e\right)\right)\sin\left(fx+e\right)\text{EllipticF}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x)
[Out] -1/60/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(5*I*cos(f*x+e)^5*sin(f*x+e)*(1/
(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)+5*I*cos(f*x+e)^4*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)
*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I
)-10*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*
x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-10*I*cos(f*x+e)^2*
sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Ellip
ticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+5*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+
e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+
```

```
e)*cos(f*x+e)+5*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-5*cos(f*x+e)^5+12*co
s(f*x+e)^3+5*cos(f*x+e))/cos(f*x+e)^2/sin(f*x+e)^9/(b/cos(f*x+e))^(3/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(3/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^{6}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.437 $\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=87

$$\frac{2b^7}{19f(b\sec(e+fx))^{19/2}}-\frac{2b^5}{5f(b\sec(e+fx))^{15/2}}+\frac{6b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[Out] 2/19*b^7/f/(b*sec(f*x+e))^(19/2)-2/5*b^5/f/(b*sec(f*x+e))^(15/2)+6/11*b^3/f
/(b*sec(f*x+e))^(11/2)-2/7*b/f/(b*sec(f*x+e))^(7/2)
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$\frac{2b^7}{19f(b\sec(e+fx))^{19/2}}-\frac{2b^5}{5f(b\sec(e+fx))^{15/2}}+\frac{6b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^7/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*b^7)/(19*f*(b*Sec[e + f*x])^(19/2)) - (2*b^5)/(5*f*(b*Sec[e + f*x])^(15/
2)) + (6*b^3)/(11*f*(b*Sec[e + f*x])^(11/2)) - (2*b)/(7*f*(b*Sec[e + f*x])^
(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\sin^7(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{b^7\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^3}{x^{21/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{b^7\,\text{Subst}\left(\int \left(-\frac{1}{x^{21/2}}+\frac{3}{b^2x^{17/2}}-\frac{3}{b^4x^{13/2}}+\frac{1}{b^6x^{9/2}}\right)dx, x, b\sec(e+fx)\right)}{f}$$

$$= \frac{2b^7}{19f(b\sec(e+fx))^{19/2}} - \frac{2b^5}{5f(b\sec(e+fx))^{15/2}} + \frac{6b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e}$$

Mathematica [A] time = 0.35, size = 62, normalized size = 0.71

$$\frac{\cos^4(e+fx)(14287\cos(2(e+fx))-3542\cos(4(e+fx))+385\cos(6(e+fx))-15226)\sqrt{b\sec(e+fx)}}{117040b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^7/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]^4*(-15226 + 14287*Cos[2*(e + f*x)] - 3542*Cos[4*(e + f*x)] +
385*Cos[6*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(117040*b^3*f)
```

fricas [A] time = 0.79, size = 61, normalized size = 0.70

$$\frac{2\left(385\cos\left(fx+e\right)^{10}-1463\cos\left(fx+e\right)^{8}+1995\cos\left(fx+e\right)^{6}-1045\cos\left(fx+e\right)^{4}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{7315\,b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*cos(f*x + e)^10 - 1463*cos(f*x + e)^8 + 1995*cos(f*x + e)^6 - 1
045*cos(f*x + e)^4)*sqrt(b/cos(f*x + e))/(b^3*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2/b^2/7315*(-936320*b*sqrt(-b)*(
-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^15+168537
6*b^2*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^14
+7303296*b^2*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp
(1))/2)^4+b))^13-6768256*b^3*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((
f*x+exp(1))/2)^4+b))^12-19341696*b^3*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2
)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^11+11316096*b^4*(-sqrt(-b)*tan((f*x+e
xp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^10+23140480*b^4*sqrt(-b)*(-sq
rt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^9-9667200*b^
5*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^8-1321
7920*b^5*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))
/2)^4+b))^7+3830400*b^6*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+e
xp(1))/2)^4+b))^6+3173760*b^6*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqr
t(-b*tan((f*x+exp(1))/2)^4+b))^5-440192*b^7*(-sqrt(-b)*tan((f*x+exp(1))/2)^
2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^4+2432*b^8*sqrt(-b)*(-sqrt(-b)*tan((f*x
+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))-124032*b^7*sqrt(-b)*(-sqrt(
-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^3-21888*b^8*(-s
qrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^2+128*b^9)/
(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b)-sqrt(-b))
^19/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [A] time = 0.22, size = 56, normalized size = 0.64

$$\frac{2\left(385\left(\cos^{6}\left(fx+e\right)\right)-1463\left(\cos^{4}\left(fx+e\right)\right)+1995\left(\cos^{2}\left(fx+e\right)\right)-1045\right)\cos\left(fx+e\right)}{7315f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/7315/f*(385*cos(f*x+e)^6-1463*cos(f*x+e)^4+1995*cos(f*x+e)^2-1045)*cos(f*
x+e)/(b/cos(f*x+e))^(5/2)
```

maxima [A] time = 0.39, size = 63, normalized size = 0.72

$$\frac{2\left(385\,b^{6}-\frac{1463\,b^{6}}{\cos\left(fx+e\right)^{2}}+\frac{1995\,b^{6}}{\cos\left(fx+e\right)^{4}}-\frac{1045\,b^{6}}{\cos\left(fx+e\right)^{6}}\right)b}{7315\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{19}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^7/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7315*(385*b^6 - 1463*b^6/cos(f*x + e)^2 + 1995*b^6/cos(f*x + e)^4 - 1045*
b^6/cos(f*x + e)^6)*b/(f*(b/cos(f*x + e))^(19/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^7}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^7/(b/cos(e + f*x))^(5/2),x)
```

```
[Out] int(sin(e + f*x)^7/(b/cos(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**7/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.438 $\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2b^5}{15f(b\sec(e+fx))^{15/2}}+\frac{4b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[Out] -2/15*b^5/f/(b*sec(f*x+e))^(15/2)+4/11*b^3/f/(b*sec(f*x+e))^(11/2)-2/7*b/f/
(b*sec(f*x+e))^(7/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 270}

$$-\frac{2b^5}{15f(b\sec(e+fx))^{15/2}}+\frac{4b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*b^5)/(15*f*(b*Sec[e + f*x])^(15/2)) + (4*b^3)/(11*f*(b*Sec[e + f*x])^(1
1/2)) - (2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{b^5\,\text{Subst}\left(\int \frac{\left(-1+\frac{x^2}{b^2}\right)^2}{x^{17/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^5\,\text{Subst}\left(\int \left(\frac{1}{x^{17/2}} - \frac{2}{b^2x^{13/2}} + \frac{1}{b^4x^{9/2}}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= -\frac{2b^5}{15f(b\sec(e+fx))^{15/2}} + \frac{4b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Mathematica [A] time = 0.21, size = 52, normalized size = 0.80

$$\frac{\cos^4(e+fx)(532\cos(2(e+fx)) - 77\cos(4(e+fx)) - 711)\sqrt{b\sec(e+fx)}}{4620b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (Cos[e + f*x]^4*(-711 + 532*Cos[2*(e + f*x)] - 77*Cos[4*(e + f*x)])*Sqrt[b*
Sec[e + f*x]])/(4620*b^3*f)
```

fricas [A] time = 0.75, size = 51, normalized size = 0.78

$$-\frac{2\left(77\cos\left(fx+e\right)^8 - 210\cos\left(fx+e\right)^6 + 165\cos\left(fx+e\right)^4\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{1155\,b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/1155*(77*cos(f*x + e)^8 - 210*cos(f*x + e)^6 + 165*cos(f*x + e)^4)*sqrt(
b/cos(f*x + e))/(b^3*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2/b^2/1155*(24640*b*(-sqrt(-b)*t
an((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^12+36960*b*sqrt(-b)*
(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^11-17001
6*b^2*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^10
-110880*b^2*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(
1))/2)^4+b))^9+343200*b^3*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x
+exp(1))/2)^4+b))^8+137280*b^3*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sq
rt(-b*tan((f*x+exp(1))/2)^4+b))^7-271040*b^4*(-sqrt(-b)*tan((f*x+exp(1))/2)
^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^6-73920*b^4*sqrt(-b)*(-sqrt(-b)*tan((f
*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^5+80640*b^5*(-sqrt(-b)*ta
n((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^4+480*b^6*sqrt(-b)*(-
sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))+10080*b^5*
sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))
^3-3360*b^6*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+
b))^2+32*b^7)/(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^
4+b)-sqrt(-b))^15/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [A] time = 0.17, size = 46, normalized size = 0.71

$$-\frac{2\left(77\left(\cos^{4}\left(fx+e\right)\right)-210\left(\cos^{2}\left(fx+e\right)\right)+165\right)\cos\left(fx+e\right)}{1155f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -2/1155/f*(77*cos(f*x+e)^4-210*cos(f*x+e)^2+165)*cos(f*x+e)/(b/cos(f*x+e))^
(5/2)
```

maxima [A] time = 0.53, size = 50, normalized size = 0.77

$$-\frac{2\left(77\,b^{4}-\frac{210\,b^{4}}{\cos\left(fx+e\right)^{2}}+\frac{165\,b^{4}}{\cos\left(fx+e\right)^{4}}\right)b}{1155\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/1155*(77*b^4 - 210*b^4/cos(f*x + e)^2 + 165*b^4/cos(f*x + e)^4)*b/(f*(b/
cos(f*x + e))^(15/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(e+f\,x\right)^5}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(b/cos(e + f*x))^(5/2),x)
[Out] int(sin(e + f*x)^5/(b/cos(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(b*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

3.439 $\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[Out] 2/11*b^3/f/(b*sec(f*x+e))^(11/2)-2/7*b/f/(b*sec(f*x+e))^(7/2)
```

Rubi [A] time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2622, 14}

$$\frac{2b^3}{11f(b\sec(e+fx))^{11/2}}-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (2*b^3)/(11*f*(b*Sec[e + f*x])^(11/2)) - (2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{b^3\,\text{Subst}\left(\int \frac{-1+\frac{x^2}{b^2}}{x^{13/2}}\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^3\,\text{Subst}\left(\int \left(-\frac{1}{x^{13/2}}+\frac{1}{b^2x^{9/2}}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{2b^3}{11f(b\sec(e+fx))^{11/2}} - \frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Mathematica [A] time = 0.16, size = 42, normalized size = 0.98

$$\frac{\cos^4(e+fx)(7\cos(2(e+fx))-15)\sqrt{b\sec(e+fx)}}{77b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
[Out] (Cos[e + f*x]^4*(-15 + 7*Cos[2*(e + f*x)])*Sqrt[b*Sec[e + f*x]])/(77*b^3*f)
```

fricas [A] time = 0.73, size = 41, normalized size = 0.95

$$\frac{2\left(7\cos\left(fx+e\right)^6-11\cos\left(fx+e\right)^4\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{77\,b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
[Out] 2/77*(7*cos(f*x + e)^6 - 11*cos(f*x + e)^4)*sqrt(b/cos(f*x + e))/(b^3*f)
```

giac [A] time = 1.67, size = 50, normalized size = 1.16

$$\frac{2\left(7\,b^4-\frac{11\,b^4}{\cos\left(fx+e\right)^2}\right)\cos\left(fx+e\right)^5}{77\,b^6f\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 2/77*(7*b^4 - 11*b^4/cos(f*x + e)^2)*cos(f*x + e)^5/(b^6*f*sqrt(b/cos(f*x +
 e)))
```

maple [A] time = 0.15, size = 36, normalized size = 0.84

$$\frac{2\left(7\left(\cos^{2}\left(fx+e\right)\right)-11\right)\cos\left(fx+e\right)}{77f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/77/f*(7*cos(f*x+e)^2-11)*cos(f*x+e)/(b/cos(f*x+e))^(5/2)
```

maxima [A] time = 0.45, size = 37, normalized size = 0.86

$$\frac{2\left(7\,b^{2}-\frac{11\,b^{2}}{\cos\left(fx+e\right)^{2}}\right)b}{77\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/77*(7*b^2 - 11*b^2/cos(f*x + e)^2)*b/(f*(b/cos(f*x + e))^(11/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(e+f\,x\right)^3}{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(b/cos(e + f*x))^(5/2),x)
```

```
[Out] int(sin(e + f*x)^3/(b/cos(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.440 $\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

```
[Out] -2/7*b/f/(b*sec(f*x+e))^(7/2)
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 30}

$$-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx &= \frac{b\,\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{2b}{7f(b\sec(e+fx))^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 20, normalized size = 1.00

$$-\frac{2b}{7f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
[Out] (-2*b)/(7*f*(b*Sec[e + f*x])^(7/2))
```

fricas [A] time = 0.74, size = 28, normalized size = 1.40

$$-\frac{2\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{4}}{7\,b^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
[Out] -2/7*sqrt(b/cos(f*x + e))*cos(f*x + e)^4/(b^3*f)
```

giac [A] time = 2.11, size = 30, normalized size = 1.50

$$-\frac{2\cos\left(fx+e\right)^{3}}{7\,b^{2}f\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
[Out] -2/7*cos(f*x + e)^3/(b^2*f*sqrt(b/cos(f*x + e)))
```

maple [A] time = 0.02, size = 17, normalized size = 0.85

$$-\frac{2b}{7f\left(b\sec\left(fx+e\right)\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x)
[Out] -2/7*b/f/(b*sec(f*x+e))^(7/2)
```

maxima [A] time = 0.43, size = 23, normalized size = 1.15

$$-\frac{2\,\cos\left(fx+e\right)}{7\,f\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/7*cos(f*x + e)/(f*(b/cos(f*x + e))^(5/2))
```

mupad [B] time = 0.57, size = 28, normalized size = 1.40

$$-\frac{2\cos\left(e+f\,x\right)^4\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{7\,b^3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(b/cos(e + f*x))^(5/2),x)
```

```
[Out] -(2*cos(e + f*x)^4*(b/cos(e + f*x))^(1/2))/(7*b^3*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(e+fx\right)}{\left(b\sec\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Integral(sin(e + f*x)/(b*sec(e + f*x))**(5/2), x)
```

3.441 $\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f}+\frac{2}{3bf(b\sec(e+fx))^{3/2}}$$

```
[Out] -arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(5/2)/f-arctanh((b*sec(f*x+e))^(1/2
)/b^(1/2))/b^(5/2)/f+2/3/b/f/(b*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2622, 325, 329, 212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f}-\frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f}+\frac{2}{3bf(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]]/(b^(5/2)*f)) - ArcTanh[Sqrt[b*Sec[e
+ f*x]]/Sqrt[b]]/(b^(5/2)*f) + 2/(3*b*f*(b*Sec[e + f*x])^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^{5/2}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{bf}$$

$$= \frac{2}{3bf(b\sec(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= \frac{2}{3bf(b\sec(e+fx))^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{b^3 f}$$

$$= \frac{2}{3bf(b\sec(e+fx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{b^2 f} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \cdot\right.}{b^2 f}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f} - \frac{\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{b^{5/2}f} + \frac{2}{3bf(b\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.20, size = 90, normalized size = 1.11

$$\frac{\sqrt{\sec(e+fx)}\left(\frac{4}{\sec^{\frac{3}{2}}(e+fx)} + 3\log\left(1-\sqrt{\sec(e+fx)}\right) - 3\log\left(\sqrt{\sec(e+fx)}+1\right) - 6\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{6b^2 f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-6*ArcTan[Sqrt[Sec[e + f*x]]] + 3*Log[1 - Sqrt[Sec[e + f*x]]] - 3*Log[1 +
 Sqrt[Sec[e + f*x]]] + 4/Sec[e + f*x]^(3/2))*Sqrt[Sec[e + f*x]])/(6*b^2*f*S
qrt[b*Sec[e + f*x]])
```

fricas [B] time = 1.04, size = 319, normalized size = 3.94

$$\left[\frac{8\sqrt{\frac{b}{\cos(fx+e)}}\cos\left(fx+e\right)^2 + 6\sqrt{-b}\arctan\left(\frac{2\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\cos(fx+e)}{b\cos(fx+e)+b}\right) - 3\sqrt{-b}\log\left(-\frac{b\cos(fx+e)^2-4\left(\cos(fx+e)^2-\cos\right.}{\cos(fx+\right.}\right.}{12\,b^3 f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 + 6*sqrt(-b)*arctan(2*sqrt(-b)
*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) + b)) - 3*sqrt(-b)*log(-
(b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 - cos(f*x + e))*sqrt(-b)*sqrt(b/cos(f
*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 + 2*cos(f*x + e) + 1)))/(b
^3*f), 1/12*(8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 - 6*sqrt(b)*arctan(2*sqr
t(b)*sqrt(b/cos(f*x + e))*cos(f*x + e)/(b*cos(f*x + e) - b)) + 3*sqrt(b)*lo
g(-(b*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sqrt(b/cos
(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e) + 1)))/
(b^3*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/b^2/2*
2*(1/3*(-6*sqrt(-b)*(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1
))/2)^4+b))^2+2*b*sqrt(-b))/sqrt(-b)/(sqrt(-b)*tan((f*x+exp(1))/2)^2-sqrt(-
b*tan((f*x+exp(1))/2)^4+b)+sqrt(-b))^3/sign(tan((f*x+exp(1))/2)^2-1)-1/2*at
an((-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))/sqrt(
-b))/sqrt(-b)/sign(tan((f*x+exp(1))/2)^2-1)-1/4*ln(abs(-sqrt(-b)*tan((f*x+e
xp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b)))/sqrt(-b)/sign(tan((f*x+exp(1
))/2)^2-1))
```

maple [B] time = 0.18, size = 377, normalized size = 4.65

$$\frac{\left(\cos \left(f x+e\right)+1\right)^{2} \left(-1+\cos \left(f x+e\right)\right)^{2} \left(-3 \cos \left(f x+e\right) \sqrt{-\frac{\cos \left(f x+e\right)}{\left(\cos \left(f x+e\right)+1\right)^{2}}}\, \ln \left(-\frac{2 \left(\cos^{2}\left(f x+e\right)\right) \sqrt{-\frac{\cos \left(f x+e\right)}{\left(\cos \left(f x+e\right)+1\right)^{2}}}-\left(\cos^{2}\left(\right.\right.}{\sin\left(\right.}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/6/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(-3*cos(f*x+e)*(-cos(f*x+e)/(cos(f
*x+e)+1)^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-
cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x
+e)^2)-3*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)*arctan(1/2/(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2))+4*cos(f*x+e)^2-3*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+
e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)-3
*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))*(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2))/cos(f*x+e)^3/sin(f*x+e)^4/(b/cos(f*x+e))^(5/2)
```

maxima [A] time = 0.43, size = 90, normalized size = 1.11

$$-\frac{b\left(\frac{6\,\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}-\frac{3\,\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{7}{2}}}-\frac{4}{b^{2}\left(\frac{b}{\cos(fx+e)}\right)^{\frac{3}{2}}}\right)}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*b*(6*arctan(sqrt(b/cos(f*x + e))/sqrt(b))/b^(7/2) - 3*log(-(sqrt(b) -
sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/cos(f*x + e))))/b^(7/2) - 4/(b^2*(b
/cos(f*x + e))^(3/2)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(b/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(b/cos(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.442 $\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=93

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f}-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3f}$$

```
[Out] 3/4*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(5/2)/f+3/4*arctanh((b*sec(f*x+e
))^(1/2)/b^(1/2))/b^(5/2)/f-1/2*cot(f*x+e)^2*(b*sec(f*x+e))^(1/2)/b^3/f
```

Rubi [A] time = 0.07, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2622, 290, 329, 212, 206, 203}

$$-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3f}+\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (3*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(4*b^(5/2)*f) + (3*ArcTanh[Sqrt[b*
Sec[e + f*x]]/Sqrt[b]])/(4*b^(5/2)*f) - (Cot[e + f*x]^2*Sqrt[b*Sec[e + f*x]
])/(2*b^3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3 f} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\,dx, x, b\sec(e+fx)\right)}{4b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3 f} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{2b^3 f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3 f} + \frac{3\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\sec(e+fx)}\right)}{4b^2 f} + \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2} f} + \frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{4b^{5/2} f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{2b^3 f}$$

Mathematica [A] time = 2.43, size = 98, normalized size = 1.05

$$\frac{\sqrt{\sec(e+fx)}\left(-3\log\left(1-\sqrt{\sec(e+fx)}\right)+3\log\left(\sqrt{\sec(e+fx)}+1\right)-\frac{4\csc^2(e+fx)}{\sec^{\frac{3}{2}}(e+fx)}+6\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)\right)}{8b^2 f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((6*ArcTan[Sqrt[Sec[e + f*x]]] - 3*Log[1 - Sqrt[Sec[e + f*x]]] + 3*Log[1 +
Sqrt[Sec[e + f*x]]] - (4*Csc[e + f*x]^2)/Sec[e + f*x]^(3/2))*Sqrt[Sec[e + f
*x]])/(8*b^2*f*Sqrt[b*Sec[e + f*x]])
```

fricas [B] time = 0.83, size = 370, normalized size = 3.98

$$\left[-\frac{6\left(\cos\left(fx+e\right)^2-1\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2\,b}\right)-8\,\sqrt{\frac{b}{\cos(fx+e)}}\,\cos\left(fx+e\right)^2+3\left(\cos\left(fx+e\right)^2\right.}{16\left(b^3 f\cos\left(fx+e\right)^2-b^3 f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(6*(cos(f*x + e)^2 - 1)*sqrt(-b)*arctan(1/2*sqrt(-b)*sqrt(b/cos(f*x
+ e))*(cos(f*x + e) + 1)/b) - 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 + 3*(co
s(f*x + e)^2 - 1)*sqrt(-b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(
f*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x +
e)^2 + 2*cos(f*x + e) + 1)))/(b^3*f*cos(f*x + e)^2 - b^3*f), -1/16*(6*(cos(
f*x + e)^2 - 1)*sqrt(b)*arctan(1/2*sqrt(b/cos(f*x + e))*(cos(f*x + e) - 1)/
sqrt(b)) - 8*sqrt(b/cos(f*x + e))*cos(f*x + e)^2 - 3*(cos(f*x + e)^2 - 1)*s
qrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + cos(f*x + e))*sqrt(b)*sq
rt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos(f*x + e)
 + 1)))/(b^3*f*cos(f*x + e)^2 - b^3*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/b^2/16*(-sqrt(-b*ta
n((f*x+exp(1))/2)^4+b)/b+2*(sqrt(-b)/(-(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqr
t(-b*tan((f*x+exp(1))/2)^4+b))^2+b)+3*atan((-sqrt(-b)*tan((f*x+exp(1))/2)^2
+sqrt(-b*tan((f*x+exp(1))/2)^4+b))/sqrt(-b))/sqrt(-b)+3/2*ln(abs(-sqrt(-b)*
tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b)))/sqrt(-b)))/sign(ta
n((f*x+exp(1))/2)^2-1)
```

maple [B] time = 0.19, size = 437, normalized size = 4.70

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)\left(8\left(\cos^{2}\left(fx+e\right)\right)\left(-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)^{\frac{3}{2}}+16\cos\left(fx+e\right)\left(-\frac{\cos\left(fx+e\right)}{\left(\cos\left(fx+e\right)+1\right)^{2}}\right)^{\frac{3}{2}}-4\left(\cos^{2}\left(fx+e\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/8/f*(-1+cos(f*x+e))*(8*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)
+16*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-4*cos(f*x+e)^2*(-cos(f*
x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e
)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x
+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*cos(f*x+e)^2*arctan(1/2/(-cos(f*x+e)/(co
s(f*x+e)+1)^2)^(1/2))+8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+4*cos(f*x+e)*(
-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos
(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)
^2)^(1/2)-1)/sin(f*x+e)^2)-3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2
)))/cos(f*x+e)^2/sin(f*x+e)^4/(b/cos(f*x+e))^(5/2)/(-cos(f*x+e)/(cos(f*x+e)
+1)^2)^(1/2)
```

maxima [A] time = 0.50, size = 106, normalized size = 1.14

$$\frac{b\left(\frac{4\sqrt{\frac{b}{\cos(fx+e)}}}{b^4-\frac{b^4}{\cos(fx+e)^2}}+\frac{6\arctan\left(\frac{\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}-\frac{3\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos(fx+e)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos(fx+e)}}}\right)}{b^{\frac{7}{2}}}\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/8*b*(4*sqrt(b/cos(f*x + e))/(b^4 - b^4/cos(f*x + e)^2) + 6*arctan(sqrt(b/
cos(f*x + e))/sqrt(b))/b^(7/2) - 3*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(s
qrt(b) + sqrt(b/cos(f*x + e))))/b^(7/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(b/cos(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.443 $\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=123

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f}-\frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3f}-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3f}$$

```
[Out] 3/32*arctan((b*sec(f*x+e))^(1/2)/b^(1/2))/b^(5/2)/f+3/32*arctanh((b*sec(f*x
+e))^(1/2)/b^(1/2))/b^(5/2)/f-1/16*cot(f*x+e)^2*(b*sec(f*x+e))^(1/2)/b^3/f-
1/4*cot(f*x+e)^4*(b*sec(f*x+e))^(1/2)/b^3/f
```

Rubi [A] time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2622, 288, 290, 329, 212, 206, 203}

$$-\frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3f}-\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3f}+\frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f}+\frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (3*ArcTan[Sqrt[b*Sec[e + f*x]]/Sqrt[b]])/(32*b^(5/2)*f) + (3*ArcTanh[Sqrt[b
*Sec[e + f*x]]/Sqrt[b]])/(32*b^(5/2)*f) - (Cot[e + f*x]^2*Sqrt[b*Sec[e + f*
x]])/(16*b^3*f) - (Cot[e + f*x]^4*Sqrt[b*Sec[e + f*x]])/(4*b^3*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^{3/2}}{\left(-1+\frac{x^2}{b^2}\right)^3}\,dx, x, b\sec(e+fx)\right)}{b^5 f}$$

$$= -\frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3f} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{8b^3f}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3f} - \frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3f} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-1+\frac{x^2}{b^2}\right)}\right.}{32b}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3f} - \frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3f} - \frac{3\,\text{Subst}\left(\int \frac{1}{-1+\frac{x^4}{b^2}}\,dx,\right.}{16b^3}$$

$$= -\frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3f} - \frac{\cot^4(e+fx)\sqrt{b\sec(e+fx)}}{4b^3f} + \frac{3\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x,\right.}{32b^2}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f} + \frac{3\tanh^{-1}\left(\frac{\sqrt{b\sec(e+fx)}}{\sqrt{b}}\right)}{32b^{5/2}f} - \frac{\cot^2(e+fx)\sqrt{b\sec(e+fx)}}{16b^3f} - \cot^4$$

Mathematica [A] time = 2.40, size = 110, normalized size = 0.89

$$\frac{\sqrt{\sec(e+fx)}\left(-3\log\left(1-\sqrt{\sec(e+fx)}\right)+3\log\left(\sqrt{\sec(e+fx)}+1\right)+6\tan^{-1}\left(\sqrt{\sec(e+fx)}\right)-\frac{2(3\cos(2(e+fx))}{\sec^{\frac{3}{2}}(}\right.}{64b^2f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((6*ArcTan[Sqrt[Sec[e + f*x]]] - 3*Log[1 - Sqrt[Sec[e + f*x]]] + 3*Log[1 +
Sqrt[Sec[e + f*x]]] - (2*(5 + 3*Cos[2*(e + f*x)])*Csc[e + f*x]^4)/Sec[e + f
*x]^(3/2))*Sqrt[Sec[e + f*x]])/(64*b^2*f*Sqrt[b*Sec[e + f*x]])
```

fricas [B] time = 0.85, size = 458, normalized size = 3.72

$$\left[-\frac{6\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\sqrt{\frac{b}{\cos(fx+e)}}\left(\cos(fx+e)+1\right)}{2b}\right)+3\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx\right.\right.}{128\left(b^{3}f\cos\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b)*arctan(1/2*sqrt
(-b)*sqrt(b/cos(f*x + e))*(cos(f*x + e) + 1)/b) + 3*(cos(f*x + e)^4 - 2*cos
(f*x + e)^2 + 1)*sqrt(-b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 - cos(f
*x + e))*sqrt(-b)*sqrt(b/cos(f*x + e)) - 6*b*cos(f*x + e) + b)/(cos(f*x + e
)^2 + 2*cos(f*x + e) + 1)) + 8*(3*cos(f*x + e)^4 + cos(f*x + e)^2)*sqrt(b/c
os(f*x + e)))/(b^3*f*cos(f*x + e)^4 - 2*b^3*f*cos(f*x + e)^2 + b^3*f), -1/1
28*(6*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b)*arctan(1/2*sqrt(b/cos
(f*x + e))*(cos(f*x + e) - 1)/sqrt(b)) - 3*(cos(f*x + e)^4 - 2*cos(f*x + e)
^2 + 1)*sqrt(b)*log((b*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + cos(f*x + e))*s
qrt(b)*sqrt(b/cos(f*x + e)) + 6*b*cos(f*x + e) + b)/(cos(f*x + e)^2 - 2*cos
(f*x + e) + 1)) + 8*(3*cos(f*x + e)^4 + cos(f*x + e)^2)*sqrt(b/cos(f*x + e)
))/(b^3*f*cos(f*x + e)^4 - 2*b^3*f*cos(f*x + e)^2 + b^3*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/b^2/64*(2*sqrt(-b*t
an((f*x+exp(1))/2)^4+b)*(-1/4*tan((f*x+exp(1))/2)^2/b+1/2/b)+2*(1/2*(-b*(-s
qrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))-(-sqrt(-b)*
tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^3+2*sqrt(-b)*(-sqrt
(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^2-2*b*sqrt(-b)
)/(-(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+b))^2+b)
^2+3/2*atan((-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*tan((f*x+exp(1))/2)^4+
b))/sqrt(-b))/sqrt(-b)+3/4*ln(abs(-sqrt(-b)*tan((f*x+exp(1))/2)^2+sqrt(-b*t
an((f*x+exp(1))/2)^4+b)))/sqrt(-b)))/sign(tan((f*x+exp(1))/2)^2-1)
```

maple [B] time = 0.20, size = 737, normalized size = 5.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/64/f*(24*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+40*cos(f*x+e)^
2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-12*cos(f*x+e)^3*(-cos(f*x+e)/(cos(f*
x+e)+1)^2)^(1/2)-3*cos(f*x+e)^3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(f*x+e
)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1
/2)-1)/sin(f*x+e)^2)-3*cos(f*x+e)^3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2))+8*cos(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)+24*cos(f*x+e)^2
*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(f*x+e)^2*ln(-(2*cos(f*x+e)^2*(-
cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)
/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*cos(f*x+e)^2*arctan(1/2/(-cos(f
*x+e)/(cos(f*x+e)+1)^2)^(1/2))-8*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(3/2)-12*co
s(f*x+e)*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)+3*cos(f*x+e)*ln(-(2*cos(f*x+e
)^2*(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(
f*x+e)/(cos(f*x+e)+1)^2)^(1/2)-1)/sin(f*x+e)^2)+3*cos(f*x+e)*arctan(1/2/(-c
os(f*x+e)/(cos(f*x+e)+1)^2)^(1/2))-3*ln(-(2*cos(f*x+e)^2*(-cos(f*x+e)/(cos(
f*x+e)+1)^2)^(1/2)-cos(f*x+e)^2+2*cos(f*x+e)-2*(-cos(f*x+e)/(cos(f*x+e)+1)^
2)^(1/2)-1)/sin(f*x+e)^2)-3*arctan(1/2/(-cos(f*x+e)/(cos(f*x+e)+1)^2)^(1/2)
))/cos(f*x+e)^2/sin(f*x+e)^4/(b/cos(f*x+e))^(5/2)/(-cos(f*x+e)/(cos(f*x+e)+
1)^2)^(1/2)
```

maxima [A] time = 0.90, size = 136, normalized size = 1.11

$$-\frac{b\left(\frac{4\left(3\,b^{2}\sqrt{\frac{b}{\cos\left(fx+e\right)}}+\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{5}{2}}\right)}{b^{6}-\frac{2\,b^{6}}{\cos\left(fx+e\right)^{2}}+\frac{b^{6}}{\cos\left(fx+e\right)^{4}}}-\frac{6\,\arctan\left(\frac{\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}+\frac{3\,\log\left(-\frac{\sqrt{b}-\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{\sqrt{b}+\sqrt{\frac{b}{\cos\left(fx+e\right)}}}\right)}{b^{\frac{7}{2}}}\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/64*b*(4*(3*b^2*sqrt(b/cos(f*x + e)) + (b/cos(f*x + e))^(5/2))/(b^6 - 2*b
^6/cos(f*x + e)^2 + b^6/cos(f*x + e)^4) - 6*arctan(sqrt(b/cos(f*x + e))/sqr
t(b))/b^(7/2) + 3*log(-(sqrt(b) - sqrt(b/cos(f*x + e)))/(sqrt(b) + sqrt(b/c
os(f*x + e))))/b^(7/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^5\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(5/2)),x)
[Out] int(1/(sin(e + f*x)^5*(b/cos(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(b*sec(f*x+e))**(5/2),x)
[Out] Timed out
```

3.444 $\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{65b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}+\frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}}-\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}}$$

```
[Out] -4/39*b*sin(f*x+e)/f/(b*sec(f*x+e))^(7/2)+8/195*sin(f*x+e)/b/f/(b*sec(f*x+e
))^(3/2)-2/13*b*sin(f*x+e)^3/f/(b*sec(f*x+e))^(7/2)+8/65*(cos(1/2*f*x+1/2*e
)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/b^2/f/c
os(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2627, 3769, 3771, 2639}

$$\frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{65b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}}+\frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}}-\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (8*EllipticE[(e + f*x)/2, 2])/(65*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) - (4*b*Sin[e + f*x])/(39*f*(b*Sec[e + f*x])^(7/2)) + (8*Sin[e + f*x])
/(195*b*f*(b*Sec[e + f*x])^(3/2)) - (2*b*Sin[e + f*x]^3)/(13*f*(b*Sec[e + f
*x])^(7/2))
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}} + \frac{6}{13}\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$$

$$= -\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}} - \frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}} + \frac{4}{39}\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx$$

$$= -\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}} + \frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}} - \frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}} + \frac{4\int \frac{1}{\sqrt{b\sec(}}}{65b}$$

$$= -\frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}} + \frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}} - \frac{2b\sin^3(e+fx)}{13f(b\sec(e+fx))^{7/2}} + \frac{4\int}{65b^2\sqrt{\cos}}$$

$$= \frac{8E\left(\frac{1}{2}(e+fx)\middle|2\right)}{65b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{4b\sin(e+fx)}{39f(b\sec(e+fx))^{7/2}} + \frac{8\sin(e+fx)}{195bf(b\sec(e+fx))^{3/2}}$$

Mathematica [A] time = 0.50, size = 83, normalized size = 0.66

$$\frac{192E\left(\frac{1}{2}(e+fx)\middle|2\right) + (-6\sin(e+fx) - 55\sin(3(e+fx)) + 15\sin(5(e+fx)))\cos^{\frac{3}{2}}(e+fx)}{1560f\cos^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (192*EllipticE[(e + f*x)/2, 2] + Cos[e + f*x]^(3/2)*(-6*Sin[e + f*x] - 55*S
in[3*(e + f*x)] + 15*Sin[5*(e + f*x)]))/(1560*f*Cos[e + f*x]^(5/2)*(b*Sec[e
 + f*x])^(5/2))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1\right)\sqrt{b\sec\left(fx+e\right)}}{b^{3}\sec\left(fx+e\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b*sec(f*x + e))/(b^3*
sec(f*x + e)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

maple [C] time = 0.21, size = 343, normalized size = 2.72

$$-\frac{2\left(15\left(\cos^{8}\left(fx+e\right)\right)-40\left(\cos^{6}\left(fx+e\right)\right)+12i\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\ \text{Elliptic}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -2/195/f*(15*cos(f*x+e)^8-40*cos(f*x+e)^6+12*I*EllipticE(I*(-1+cos(f*x+e))/
sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)-12*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1
))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+12
*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-12*I*sin(f*x+e)*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))
```

```
^(1/2)+29*cos(f*x+e)^4+8*cos(f*x+e)^2-12*cos(f*x+e))/cos(f*x+e)^3/sin(f*x+e
)/(b/cos(f*x+e))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^4}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(b/cos(e + f*x))^(5/2),x)
```

```
[Out] int(sin(e + f*x)^4/(b/cos(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.445 $\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}$$

```
[Out] -2/9*b*sin(f*x+e)/f/(b*sec(f*x+e))^(7/2)+4/45*sin(f*x+e)/b/f/(b*sec(f*x+e))
^(3/2)+4/15*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1
/2*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2627, 3769, 3771, 2639}

$$\frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (4*EllipticE[(e + f*x)/2, 2])/(15*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f
*x]]) - (2*b*Sin[e + f*x])/(9*f*(b*Sec[e + f*x])^(7/2)) + (4*Sin[e + f*x])/
(45*b*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2627

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[(b*(a*Csc[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1)
)/(a*f*(m + n)), x] + Dist[(m + 1)/(a^2*(m + n)), Int[(a*Csc[e + f*x])^(m +
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1] &
& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx &= -\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}+\frac{2}{9}\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx\\ &= -\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}+\frac{2\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{15b^2}\\ &= -\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}+\frac{2\int \sqrt{\cos(e+fx)}\,dx}{15b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= \frac{4E\left(\frac{1}{2}(e+fx)\middle|2\right)}{15b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b\sin(e+fx)}{9f(b\sec(e+fx))^{7/2}}+\frac{4\sin(e+fx)}{45bf(b\sec(e+fx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.40, size = 66, normalized size = 0.67

$$\frac{-4\sin(2(e+fx))-10\sin(4(e+fx))+\frac{96E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{360b^2f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((96*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]] - 4*Sin[2*(e + f*x)] - 1
0*Sin[4*(e + f*x)])/(360*b^2*f*Sqrt[b*Sec[e + f*x]])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^2-1\right)\sqrt{b\sec\left(fx+e\right)}}{b^3\sec\left(fx+e\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(b*sec(f*x + e))/(b^3*sec(f*x + e)^3), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

maple [C] time = 0.18, size = 333, normalized size = 3.40

$$\frac{\frac{4i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticF}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)\sin\left(fx+e\right)\cos\left(fx+e\right)}{15}-\frac{4i\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticE}}{15}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 2/45/f*(6*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elli
pticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-6*I*EllipticE(I
*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/
2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+5*cos(f*x+e)^6+6*I*EllipticF(I*(-1+cos
(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos
(f*x+e)+1))^(1/2)-6*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*
(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-7*cos(f*x+e)^4-4
*cos(f*x+e)^2+6*cos(f*x+e))/cos(f*x+e)^3/sin(f*x+e)/(b/cos(f*x+e))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(b/cos(e + f*x))^(5/2),x)
```

```
[Out] int(sin(e + f*x)^2/(b/cos(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.446 $\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

```
[Out] 2/5*sin(f*x+e)/b/f/(b*sec(f*x+e))^(3/2)+6/5*(cos(1/2*f*x+1/2*e)^2)^(1/2)/co
s(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/
2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3769, 3771, 2639}

$$\frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}+\frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^(-5/2),x]
```

```
[Out] (6*EllipticE[(e + f*x)/2, 2])/(5*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*
x]]) + (2*Sin[e + f*x])/(5*b*f*(b*Sec[e + f*x])^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(b\sec(e+fx))^{5/2}}\,dx &= \frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{5b^2}\\
&= \frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3\int \sqrt{\cos(e+fx)}\,dx}{5b^2\sqrt{\cos(e+fx)}\,\sqrt{b\sec(e+fx)}}\\
&= \frac{6E\left(\frac{1}{2}(e+fx)\middle|2\right)}{5b^2f\sqrt{\cos(e+fx)}\,\sqrt{b\sec(e+fx)}} + \frac{2\sin(e+fx)}{5bf(b\sec(e+fx))^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 60, normalized size = 0.83

$$
\frac{\sqrt{b\sec(e+fx)}\left(\sin(e+fx)+\sin(3(e+fx))+12\sqrt{\cos(e+fx)}\,E\left(\frac{1}{2}(e+fx)\middle|2\right)\right)}{10b^3f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^(-5/2),x]
```

```
[Out] (Sqrt[b*Sec[e + f*x]]*(12*Sqrt[Cos[e + f*x]]*EllipticE[(e + f*x)/2, 2] + Si
n[e + f*x] + Sin[3*(e + f*x)]))/(10*b^3*f)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}}{b^3\sec\left(fx+e\right)^3}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))/(b^3*sec(f*x + e)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^(-5/2), x)
```

maple [C] time = 0.22, size = 321, normalized size = 4.46

$$-\frac{2\left(3i\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\mathrm{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)-3i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos}{\cos}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -2/5/f*(3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)
*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*(1/(cos(f*x
+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))
/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)-3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+
1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+cos(f*x+e)^4+2*cos(f*x+e)^2-3*c
os(f*x+e))/(b/cos(f*x+e))^(5/2)/cos(f*x+e)^3/sin(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/cos(e + f*x))^(5/2),x)
```

```
[Out] int(1/(b/cos(e + f*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sec \left(e+fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(5/2),x)
[Out] Integral((b*sec(e + f*x))**(-5/2), x)
```

3.447 $\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}$$

```
[Out] -csc(f*x+e)/b/f/(b*sec(f*x+e))^(3/2)-3*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2
*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/2)/(b
*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2623, 3771, 2639}

$$-\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] -(Csc[e + f*x]/(b*f*(b*Sec[e + f*x])^(3/2))) - (3*EllipticE[(e + f*x)/2, 2]
)/(b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}} - \frac{3\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{2b^2}$$
$$= -\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}} - \frac{3\int \sqrt{\cos(e+fx)}\,dx}{2b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= -\frac{\csc(e+fx)}{bf(b\sec(e+fx))^{3/2}} - \frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Mathematica [A] time = 0.16, size = 51, normalized size = 0.75

$$\frac{-\cot(e+fx) - \frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{\sqrt{\cos(e+fx)}}}{b^2f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (-Cot[e + f*x] - (3*EllipticE[(e + f*x)/2, 2])/Sqrt[Cos[e + f*x]])/(b^2*f*S
qrt[b*Sec[e + f*x]])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^2}{b^3\sec\left(fx+e\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^2/(b^3*sec(f*x + e)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

maple [C] time = 0.22, size = 312, normalized size = 4.59

$$\frac{3i\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\,\operatorname{EllipticE}\left(\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)},i\right)-3i\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1
/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*(1/(cos(f*x+e)
+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/si
n(f*x+e),I)*sin(f*x+e)*cos(f*x+e)+3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e
),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-
3*I*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+2*cos(f*x+e)^2-3*cos(f*x+e))/cos(f
*x+e)^3/sin(f*x+e)/(b/cos(f*x+e))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^{2}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(b/cos(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.448 $\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=102

$$\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}+\frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}}$$

```
[Out] 1/2*csc(f*x+e)/b/f/(b*sec(f*x+e))^(3/2)-1/3*csc(f*x+e)^3/b/f/(b*sec(f*x+e))
^(3/2)+1/2*(cos(1/2*f*x+1/2*e)^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/
2*f*x+1/2*e),2^(1/2))/b^2/f/cos(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2623, 2625, 3771, 2639}

$$\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}+\frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] Csc[e + f*x]/(2*b*f*(b*Sec[e + f*x])^(3/2)) - Csc[e + f*x]^3/(3*b*f*(b*Sec[
e + f*x])^(3/2)) + EllipticE[(e + f*x)/2, 2]/(2*b^2*f*Sqrt[Cos[e + f*x]]*Sq
rt[b*Sec[e + f*x]])
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx &= -\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}-\frac{\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx}{2b^2}\\ &= \frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}+\frac{\int \frac{1}{\sqrt{b\sec(e+fx)}}\,dx}{4b^2}\\ &= \frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}+\frac{\int \sqrt{\cos(e+fx)}\,dx}{4b^2\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= \frac{\csc(e+fx)}{2bf(b\sec(e+fx))^{3/2}}-\frac{\csc^3(e+fx)}{3bf(b\sec(e+fx))^{3/2}}+\frac{E\left(\frac{1}{2}(e+fx)\middle|2\right)}{2b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.24, size = 79, normalized size = 0.77

$$\frac{\sin(e+fx)\sqrt{b\sec(e+fx)}\left(-2\csc^4(e+fx)+5\csc^2(e+fx)+3\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e+fx)\middle|2\right)-3\right)}{6b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-3 + 5*Csc[e + f*x]^2 - 2*Csc[e + f*x]^4 + 3*Sqrt[Cos[e + f*x]]*Csc[e + f
*x]*EllipticE[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]]*Sin[e + f*x])/(6*b^3*f)
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^4}{b^3\sec\left(fx+e\right)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^4/(b^3*sec(f*x + e)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^4}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

maple [C] time = 0.23, size = 623, normalized size = 6.11

$$-\frac{\left(\cos\left(fx+e\right)+1\right)^2\left(-1+\cos\left(fx+e\right)\right)^2\left(3i\left(\cos^3\left(fx+e\right)\right)\sin\left(fx+e\right)\sqrt{\frac{1}{\cos\left(fx+e\right)+1}}\,\sqrt{\frac{\cos\left(fx+e\right)}{\cos\left(fx+e\right)+1}}\ \mathrm{EllipticF}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] -1/6/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(3*I*cos(f*x+e)^3*sin(f*x+e)*(1/(
cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(
f*x+e))/sin(f*x+e),I)-3*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)
+3*I*cos(f*x+e)^2*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+
e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*cos(f*x+e)^2*sin
(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*Elliptic
E(I*(-1+cos(f*x+e))/sin(f*x+e),I)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*
cos(f*x+e)+3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)*sin(f*x
+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)-3*I*Elliptic
F(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(
f*x+e)/(cos(f*x+e)+1))^(1/2)+3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*
sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+3*cos
(f*x+e)^3+2*cos(f*x+e)^2-3*cos(f*x+e))/cos(f*x+e)^3/sin(f*x+e)^7/(b/cos(f*x
+e))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{4}}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^4/(b*sec(f*x + e))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^4\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^4*(b/cos(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.449 $\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=132

$$\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}+\frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}}+\frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}}$$

```
[Out] 3/20*csc(f*x+e)/b/f/(b*sec(f*x+e))^(3/2)+1/10*csc(f*x+e)^3/b/f/(b*sec(f*x+e
))^(3/2)-1/5*csc(f*x+e)^5/b/f/(b*sec(f*x+e))^(3/2)+3/20*(cos(1/2*f*x+1/2*e)
^2)^(1/2)/cos(1/2*f*x+1/2*e)*EllipticE(sin(1/2*f*x+1/2*e),2^(1/2))/b^2/f/co
s(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2623, 2625, 3771, 2639}

$$\frac{3E\left(\frac{1}{2}(e+fx)\middle|2\right)}{20b^2f\sqrt{\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}}+\frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}}+\frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^6/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] (3*Csc[e + f*x])/(20*b*f*(b*Sec[e + f*x])^(3/2)) + Csc[e + f*x]^3/(10*b*f*(
b*Sec[e + f*x])^(3/2)) - Csc[e + f*x]^5/(5*b*f*(b*Sec[e + f*x])^(3/2)) + (3
*EllipticE[(e + f*x)/2, 2])/(20*b^2*f*Sqrt[Cos[e + f*x]]*Sqrt[b*Sec[e + f*x
]])
```

Rule 2623

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(a*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n + 1)
)/(f*b*(m - 1)), x] + Dist[(a^2*(n + 1))/(b^2*(m - 1)), Int[(a*Csc[e + f*x]
)^(m - 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && IntegersQ[2*m, 2*n]
```

Rule 2625

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> -Simp[(a*b*(a*Csc[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
 1))/(f*(m - 1)), x] + Dist[(a^2*(m + n - 2))/(m - 1), Int[(a*Csc[e + f*x])
^(m - 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m,
```

```
1] && IntegersQ[2*m, 2*n] &&  !GtQ[n, m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\csc^6(e+fx)}{(b\sec(e+fx))^{5/2}}\,dx = -\frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} - \frac{3\int \frac{\csc^4(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx}{10b^2}$$

$$= \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}} - \frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} - \frac{3\int \frac{\csc^2(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx}{20b^2}$$

$$= \frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}} + \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}} - \frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{b\sec(e+}}}{40b^2}$$

$$= \frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}} + \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}} - \frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3\int}{40b^2\sqrt{\cos(}}$$

$$= \frac{3\csc(e+fx)}{20bf(b\sec(e+fx))^{3/2}} + \frac{\csc^3(e+fx)}{10bf(b\sec(e+fx))^{3/2}} - \frac{\csc^5(e+fx)}{5bf(b\sec(e+fx))^{3/2}} + \frac{3E}{20b^2f\sqrt{\cos}}$$

Mathematica [A] time = 0.18, size = 87, normalized size = 0.66

$$\frac{\sin(e+fx)\sqrt{b\sec(e+fx)}\left(-4\csc^6(e+fx)+6\csc^4(e+fx)+\csc^2(e+fx)+3\sqrt{\cos(e+fx)}\csc(e+fx)E\left(\frac{1}{2}(e\right.\right.}{20b^3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^6/(b*Sec[e + f*x])^(5/2),x]
```

```
[Out] ((-3 + Csc[e + f*x]^2 + 6*Csc[e + f*x]^4 - 4*Csc[e + f*x]^6 + 3*Sqrt[Cos[e
+ f*x]]*Csc[e + f*x]*EllipticE[(e + f*x)/2, 2])*Sqrt[b*Sec[e + f*x]]*Sin[e
+ f*x])/(20*b^3*f)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\csc\left(fx+e\right)^{6}}{b^{3}\sec\left(fx+e\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*csc(f*x + e)^6/(b^3*sec(f*x + e)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(5/2), x)
```

maple [C] time = 0.25, size = 923, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x)
```

```
[Out] 1/20/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(6*I*(1/(cos(f*x+e)+1))^(1/2)*(co
s(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*co
s(f*x+e)^3*sin(f*x+e)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*cos(f*x+e)-6
*I*cos(f*x+e)^3*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)
+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)+3*I*(1/(cos(f*x+e)+1))
^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(f*
x+e),I)*cos(f*x+e)^4*sin(f*x+e)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(c
os(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^4*
sin(f*x+e)-3*I*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*sin(f*x+e)*(1/(cos
```

```
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)+6*I*(1/(cos(f*x+e)+1))^
(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x
+e),I)*cos(f*x+e)^2*sin(f*x+e)-6*I*cos(f*x+e)^2*sin(f*x+e)*(1/(cos(f*x+e)+1
))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x+e))/sin(
f*x+e),I)+3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*El
lipticF(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^5*sin(f*x+e)+3*I*(1/(cos
(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*EllipticF(I*(-1+cos(f*x
+e))/sin(f*x+e),I)*sin(f*x+e)+3*cos(f*x+e)^5-3*I*EllipticE(I*(-1+cos(f*x+e)
)/sin(f*x+e),I)*cos(f*x+e)*sin(f*x+e)*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/
(cos(f*x+e)+1))^(1/2)-3*I*(1/(cos(f*x+e)+1))^(1/2)*(cos(f*x+e)/(cos(f*x+e)+
1))^(1/2)*EllipticE(I*(-1+cos(f*x+e))/sin(f*x+e),I)*cos(f*x+e)^5*sin(f*x+e)
-2*cos(f*x+e)^4-6*cos(f*x+e)^3-2*cos(f*x+e)^2+3*cos(f*x+e))/cos(f*x+e)^3/si
n(f*x+e)^9/(b/cos(f*x+e))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{6}}{\left(b\sec\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^6/(b*sec(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^6/(b*sec(f*x + e))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^{6}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(5/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^6*(b/cos(e + f*x))^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**6/(b*sec(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.450 $\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{9/2}\,dx$

Optimal. Leaf size=449

$$-\frac{21a^{9/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{a\sin(e+fx)}}{\sqrt{a}\,\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}\,\sqrt{b}\,f}+\frac{21a^{9/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}}{32\sqrt{2}\,\sqrt{b}\,f}$$

```
[Out] -7/16*a^3*b*(a*sin(f*x+e))^(3/2)/f/(b*sec(f*x+e))^(1/2)-1/4*a*b*(a*sin(f*x+
e))^(7/2)/f/(b*sec(f*x+e))^(1/2)-21/64*a^(9/2)*arctan(1-2^(1/2)*b^(1/2)*(a*
sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec
(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)+21/64*a^(9/2)*arctan(1+2^(1/2)*b^(1/2)*(a*
sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec
(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)+21/128*a^(9/2)*ln(a^(1/2)-2^(1/2)*b^(1/2)*
(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))*(b*cos(f*x+e)
)^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)-21/128*a^(9/2)*ln(a^(1/2)+2^
(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))
*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2583, 2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}}-\frac{21a^{9/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{a\sin(e+fx)}}{\sqrt{a}\,\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}\,\sqrt{b}\,f}+\frac{21a^{9/2}\sqrt{b\cos(e+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-21*a^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqr
t[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(32*Sqrt[2]*
Sqrt[b]*f) + (21*a^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/
(Sqrt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/
(32*Sqrt[2]*Sqrt[b]*f) + (21*a^(9/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sq
rt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e +
f*x]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*Sqrt[b]*f) - (21*a^(9/2)*Sqrt[b*Cos
[e + f*x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[
e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*Sqrt[b]
*f) - (7*a^3*b*(a*Sin[e + f*x])^(3/2))/(16*f*Sqrt[b*Sec[e + f*x]]) - (a*b*(
a*Sin[e + f*x])^(7/2))/(4*f*Sqrt[b*Sec[e + f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
```

```
& LtQ[m, 1]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{9/2}\,dx = -\frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}} + \frac{1}{8}\left(7a^2\right)\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{5/2}\,dx$$

$$= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}} + \frac{1}{32}\left(21a^4\right)\int \sqrt{b\sec(e$$

$$= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}} + \frac{1}{32}\left(21a^4\sqrt{b\cos(e+fx\right.$$

$$= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}} + \frac{\left(21a^5b\sqrt{b\cos(e+fx)}\right.}{}$$

$$= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}} - \frac{\left(21a^5\sqrt{b\cos(e+fx)}\right.}{}$$

$$= -\frac{7a^3b(a\sin(e+fx))^{3/2}}{16f\sqrt{b\sec(e+fx)}} - \frac{ab(a\sin(e+fx))^{7/2}}{4f\sqrt{b\sec(e+fx)}} + \frac{\left(21a^5\sqrt{b\cos(e+fx)}\right.}{}$$

$$= \frac{21a^{9/2}\sqrt{b\cos(e+fx)}\,\log\left(\sqrt{a} - \frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}} + \sqrt{a}\tan(e+fx)\right)\sqrt{}}{64\sqrt{2}\sqrt{b}\,f}$$

$$= -\frac{21a^{9/2}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{32\sqrt{2}\sqrt{b}\,f} + $$

Mathematica [C] time = 0.37, size = 80, normalized size = 0.18

$$\frac{a^4\tan(e+fx)\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}\left(14\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right) - 7\cos(2(e+fx)) + \cos(4(e+fx))\right.}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2),x]
```

```
[Out] (a^4*(-8 - 7*Cos[2*(e + f*x)] + Cos[4*(e + f*x)] + 14*Hypergeometric2F1[3/4
, 1, 7/4, -Tan[e + f*x]^2])*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]]*Tan[e
 + f*x])/(32*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(9/2), x)
```

maple [C] time = 0.23, size = 538, normalized size = 1.20

$$\left(8\left(\cos^4\left(fx+e\right)\right)\sqrt{2}-21i\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos\left(fx-\right.}{\sin}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(9/2)*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/64/f*(8*cos(f*x+e)^4*2^(1/2)-21*I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2
*I,1/2*2^(1/2))+21*I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))
-8*cos(f*x+e)^3*2^(1/2)+21*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^
(1/2))+21*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-22*cos(f*x
+e)^2*2^(1/2)+22*cos(f*x+e)*2^(1/2))*(a*sin(f*x+e))^(9/2)*(b/cos(f*x+e))^(1
/2)/sin(f*x+e)^3/(-1+cos(f*x+e))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)}\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a \sin\left(e+f x\right)\right)^{9/2} \sqrt{\frac{b}{\cos\left(e+f x\right)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(9/2)*(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a*sin(e + f*x))^(9/2)*(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(9/2)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.451 $\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=414

$$-\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{a\sin(e+fx)}}{\sqrt{a}\,\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}\,\sqrt{b}\,f}+\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}\left(\right.}{4\sqrt{2}\,\sqrt{b}\,f}$$

```
[Out] -1/2*a*b*(a*sin(f*x+e))^(3/2)/f/(b*sec(f*x+e))^(1/2)-3/8*a^(5/2)*arctan(1-2
^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*
x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)+3/8*a^(5/2)*arctan(1+2^(
1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+
e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)+3/16*a^(5/2)*ln(a^(1/2)-2^
(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))
*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)-3/16*a^(5/2)*l
n(a^(1/2)+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)
*tan(f*x+e))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2583, 2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{a\sin(e+fx)}}{\sqrt{a}\,\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}\,\sqrt{b}\,f}+\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}\left(\right.}{4\sqrt{2}\,\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-3*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt
[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*Sqrt[2]*Sq
rt[b]*f) + (3*a^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sq
rt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*
Sqrt[2]*Sqrt[b]*f) + (3*a^(5/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]
*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]
*Sqrt[b*Sec[e + f*x]])/(8*Sqrt[2]*Sqrt[b]*f) - (3*a^(5/2)*Sqrt[b*Cos[e + f*
x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x
]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(8*Sqrt[2]*Sqrt[b]*f) - (a
*b*(a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[b*Sec[e + f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{5/2}\,dx = -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} + \frac{1}{4}\left(3a^2\right)\int \sqrt{b\sec(e+fx)}\,\sqrt{a\sin(e+fx)}\,dx$$

$$= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} + \frac{1}{4}\left(3a^2\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\int \frac{\sqrt{a\ldots}}{\sqrt{b\ldots}}$$

$$= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} + \frac{\left(3a^3b\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\ldots\right.}{2f}$$

$$= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} - \frac{\left(3a^3\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\ldots\right.}{4f}$$

$$= -\frac{ab(a\sin(e+fx))^{3/2}}{2f\sqrt{b\sec(e+fx)}} + \frac{\left(3a^3\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int\ldots\right.}{8bf}$$

$$= \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\,\log\left(\sqrt{a}-\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\,\tan(e+fx)\right)\sqrt{\ldots}}{8\sqrt{2}\,\sqrt{b}\,f}$$

$$= -\frac{3a^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{a\sin(e+fx)}}{\sqrt{a}\,\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}}{4\sqrt{2}\,\sqrt{b}\,f} + \ldots$$

Mathematica [C] time = 0.26, size = 65, normalized size = 0.16

$$-\frac{a(a\sin(e+fx))^{3/2}(b\sec(e+fx))^{3/2}\left(-2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)+\cos(2(e+fx))+1\right)}{4bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -1/4*(a*(1 + Cos[2*(e + f*x)] - 2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f
*x]^2])*(b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(3/2))/(b*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(fx+e\right)}\,\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(5/2), x)
```

maple [C] time = 0.18, size = 512, normalized size = 1.24

$$-\frac{\left(-3i\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(5/2)*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/8/f*(-3*I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(
((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*((1
```

```
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*((1-cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2
*I,1/2*2^(1/2))+2*cos(f*x+e)^2*2^(1/2)-2*cos(f*x+e)*2^(1/2))*(a*sin(f*x+e))
^(5/2)*(b/cos(f*x+e))^(1/2)/(-1+cos(f*x+e))/sin(f*x+e)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)} \left(a \sin\left(fx+e\right)\right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(5/2)*(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a*sin(e + f*x))^(5/2)*(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(5/2)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.452 $\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}\,dx$

Optimal. Leaf size=376

$$-\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{b}f}+\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{a}\cdots}\right.}{\sqrt{2}\sqrt{b}f}$$

```
[Out] -1/2*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(
1/2))*a^(1/2)*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)+1
/2*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/
2))*a^(1/2)*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/b^(1/2)+1/4
*ln(a^(1/2)-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/
2)*tan(f*x+e))*a^(1/2)*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/f*2^(1/2)/
b^(1/2)-1/4*ln(a^(1/2)+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^
(1/2)+a^(1/2)*tan(f*x+e))*a^(1/2)*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)
/f*2^(1/2)/b^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}\sqrt{b}f}+\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{a}\cdots}\right.}{\sqrt{2}\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[
b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*Sqrt[
b]*f)) + (Sqrt[a]*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a
]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2
]*Sqrt[b]*f) + (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]*Sqrt[b]
*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*
Sec[e + f*x]])/(2*Sqrt[2]*Sqrt[b]*f) - (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[Sq
rt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[
a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(2*Sqrt[2]*Sqrt[b]*f)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\,\sqrt{a\sin(e+fx)}\,dx = \left(\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\,dx$$

$$= \frac{\left(2ab\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{f}$$

$$= -\frac{\left(a\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{a-bx^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right)}{f} +$$

$$= \frac{\left(a\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx, x, \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\right.}{2bf}$$

$$= \frac{\sqrt{a}\,\sqrt{b\cos(e+fx)}\,\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\,\tan(e+fx)\right)\sqrt{b\sec}}{2\sqrt{2}\,\sqrt{b}\,f}$$

$$= -\frac{\sqrt{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}}{\sqrt{2}\,\sqrt{b}\,f} + \frac{\sqrt{a}\,\text{t}}{}$$

Mathematica [C] time = 0.13, size = 55, normalized size = 0.15

$$\frac{2\tan(e+fx)\sqrt{a\sin(e+fx)}\,\sqrt{b\sec(e+fx)}\;{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*Sqrt[b*Sec[e + f*x]]*Sqr
t[a*Sin[e + f*x]]*Tan[e + f*x])/(3*f)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e)), x)
```

maple [C] time = 0.17, size = 273, normalized size = 0.73

$$-\frac{\sqrt{a\sin\left(fx+e\right)}\,\sqrt{\frac{b}{\cos\left(fx+e\right)}}\,\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\mathrm{co}}{}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2),x)
[Out] -1/2/f*(a*sin(f*x+e))^(1/2)*(b/cos(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*(I*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-EllipticPi(((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2)))*sin(f*x+e)/(-1+cos(f*x+e))*2
^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a\,\sin\left(e+f\,x\right)}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^(1/2)*(b/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin\left(e+fx\right)}\,\sqrt{b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)*(b*sec(f*x+e))**(1/2),x)
[Out] Integral(sqrt(a*sin(e + f*x))*sqrt(b*sec(e + f*x)), x)
```

3.453 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=33

$$-\frac{2b}{af\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

```
[Out] -2*b/a/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2578}

$$-\frac{2b}{af\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-2*b)/(a*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx=-\frac{2b}{af\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}}$$

Mathematica [A] time = 0.08, size = 37, normalized size = 1.12

$$-\frac{\sin(2(e+fx))\sqrt{b\sec(e+fx)}}{f(a\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(3/2),x]
```

[Out] -((Sqrt[b*Sec[e + f*x]]*Sin[2*(e + f*x)])/(f*(a*Sin[e + f*x])^(3/2)))

fricas [A] time = 1.13, size = 44, normalized size = 1.33

$$-\frac{2\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)}{a^2 f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] -2*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*cos(f*x + e)/(a^2*f*sin(f*x + e))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(3/2), x)

maple [A] time = 0.17, size = 40, normalized size = 1.21

$$-\frac{2\sin\left(fx+e\right)\cos\left(fx+e\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{f\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x)

[Out] -2/f*sin(f*x+e)*cos(f*x+e)*(b/cos(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(3/2), x)
```

mupad [B] time = 0.87, size = 36, normalized size = 1.09

$$-\frac{2\cos\left(e+fx\right)\sqrt{\frac{b}{\cos\left(e+fx\right)}}}{a\,f\,\sqrt{a\,\sin\left(e+fx\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(3/2),x)
[Out] -(2*cos(e + f*x)*(b/cos(e + f*x))^(1/2))/(a*f*(a*sin(e + f*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(e+fx\right)}}{\left(a\sin\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(3/2),x)
[Out] Integral(sqrt(b*sec(e + f*x))/(a*sin(e + f*x))**(3/2), x)
```

3.454 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{8b}{5a^3f\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{5af(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}$$

```
[Out] -2/5*b/a/f/(a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(1/2)-8/5*b/a^3/f/(b*sec(f*x
+e))^(1/2)/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2584, 2578}

$$-\frac{8b}{5a^3f\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{2b}{5af(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-2*b)/(5*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2)) - (8*b)/(5*a^3*f
*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx = -\frac{2b}{5af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{5/2}} + \frac{4\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{3/2}}\,dx}{5a^2}$$
$$= -\frac{2b}{5af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{5/2}} - \frac{8b}{5a^3f\sqrt{b\sec(e+fx)}\,\sqrt{a\sin(e+fx)}}$$

Mathematica [A] time = 0.20, size = 52, normalized size = 0.73

$$\frac{2(2\cos(2(e+fx))-3)\cot(e+fx)\sqrt{b\sec(e+fx)}}{5a^2f(a\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(7/2),x]
```

```
[Out] (2*(-3 + 2*Cos[2*(e + f*x)])*Cot[e + f*x]*Sqrt[b*Sec[e + f*x]])/(5*a^2*f*(a
*Sin[e + f*x])^(3/2))
```

fricas [A] time = 1.13, size = 73, normalized size = 1.03

$$-\frac{2\left(4\cos\left(fx+e\right)^3-5\cos\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{5\left(a^4f\cos\left(fx+e\right)^2-a^4f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(4*cos(f*x + e)^3 - 5*cos(f*x + e))*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*
x + e))/((a^4*f*cos(f*x + e)^2 - a^4*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(7/2), x)
```

maple [A] time = 0.18, size = 52, normalized size = 0.73

$$\frac{2\left(4\left(\cos^{2}\left(fx+e\right)\right)-5\right)\cos\left(fx+e\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}\,\sin\left(fx+e\right)}{5f\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/5/f*(4*cos(f*x+e)^2-5)*cos(f*x+e)*(b/cos(f*x+e))^(1/2)*sin(f*x+e)/(a*sin(
f*x+e))^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(7/2), x)
```

mupad [B] time = 1.81, size = 83, normalized size = 1.17

$$-\frac{4\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,\left(3\,\cos\left(e+f\,x\right)-4\,\cos\left(3\,e+3\,f\,x\right)+\cos\left(5\,e+5\,f\,x\right)\right)}{5\,a^3\,f\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\left(\cos\left(4\,e+4\,f\,x\right)-4\,\cos\left(2\,e+2\,f\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(7/2),x)
```

```
[Out] -(4*(b/cos(e + f*x))^(1/2)*(3*cos(e + f*x) - 4*cos(3*e + 3*f*x) + cos(5*e +
 5*f*x)))/(5*a^3*f*(a*sin(e + f*x))^(1/2)*(cos(4*e + 4*f*x) - 4*cos(2*e + 2
*f*x) + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.455 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{64b}{45a^5f\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{16b}{45a^3f(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}-\frac{2b}{9af(a\sin(e+fx))^{9/2}\sqrt{b\sec(e+fx)}}$$

```
[Out] -2/9*b/a/f/(a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(1/2)-16/45*b/a^3/f/(a*sin(f
*x+e))^(5/2)/(b*sec(f*x+e))^(1/2)-64/45*b/a^5/f/(b*sec(f*x+e))^(1/2)/(a*sin
(f*x+e))^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2584, 2578}

$$-\frac{64b}{45a^5f\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{16b}{45a^3f(a\sin(e+fx))^{5/2}\sqrt{b\sec(e+fx)}}-\frac{2b}{9af(a\sin(e+fx))^{9/2}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(11/2),x]
```

```
[Out] (-2*b)/(9*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(9/2)) - (16*b)/(45*a^3
*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(5/2)) - (64*b)/(45*a^5*f*Sqrt[b*S
ec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{11/2}}\,dx = -\frac{2b}{9af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{9/2}} + \frac{8\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{7/2}}\,dx}{9a^2}$$

$$= -\frac{2b}{9af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{9/2}} - \frac{16b}{45a^3f\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{5/2}} + -$$

$$= -\frac{2b}{9af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{9/2}} - \frac{16b}{45a^3f\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{5/2}} - \frac{\;}{4}$$

Mathematica [A] time = 0.21, size = 65, normalized size = 0.61

$$\frac{2b(20\cos(2(e+fx))-4\cos(4(e+fx))-21)\csc^5(e+fx)\sqrt{a\sin(e+fx)}}{45a^6f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(11/2),x]
```

```
[Out] (2*b*(-21 + 20*Cos[2*(e + f*x)] - 4*Cos[4*(e + f*x)])*Csc[e + f*x]^5*Sqrt[a
*Sin[e + f*x]])/(45*a^6*f*Sqrt[b*Sec[e + f*x]])
```

fricas [A] time = 0.97, size = 96, normalized size = 0.91

$$-\frac{2\left(32\cos\left(fx+e\right)^5-72\cos\left(fx+e\right)^3+45\cos\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)}\sqrt{\frac{b}{\cos\left(fx+e\right)}}}{45\left(a^6f\cos\left(fx+e\right)^4-2\,a^6f\cos\left(fx+e\right)^2+a^6f\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x, algorithm="fricas")
```

```
[Out] -2/45*(32*cos(f*x + e)^5 - 72*cos(f*x + e)^3 + 45*cos(f*x + e))*sqrt(a*sin(
f*x + e))*sqrt(b/cos(f*x + e))/((a^6*f*cos(f*x + e)^4 - 2*a^6*f*cos(f*x + e
)^2 + a^6*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(11/2), x)
```

maple [A] time = 0.20, size = 62, normalized size = 0.58

$$-\frac{2\left(32\left(\cos^{4}\left(fx+e\right)\right)-72\left(\cos^{2}\left(fx+e\right)\right)+45\right)\cos\left(fx+e\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}\ \sin\left(fx+e\right)}{45f\left(a\sin\left(fx+e\right)\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x)
```

```
[Out] -2/45/f*(32*cos(f*x+e)^4-72*cos(f*x+e)^2+45)*cos(f*x+e)*(b/cos(f*x+e))^(1/2
)*sin(f*x+e)/(a*sin(f*x+e))^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(11/2), x)
```

mupad [B] time = 5.69, size = 169, normalized size = 1.59

$$-\frac{\mathrm{e}^{-e\,5\mathrm{i}-f\,x\,5\mathrm{i}}\,\sqrt{\frac{b}{\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{352\,\cos\left(e+f\,x\right)\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}}{45\,a^5\,f}-\frac{256\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(3\,e+3\,f\,x\right)}{45\,a^5\,f}+\frac{64\,\mathrm{e}^{e\,5\mathrm{i}+f\,x\,5\mathrm{i}}\,\cos\left(5\,e+5\,f\,x\right)}{45\,a^5\,f}\right)}{16\,\sin\left(e+f\,x\right)^4\,\sqrt{a\,\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(11/2),x)
```

```
[Out] -(exp(- e*5i - f*x*5i)*(b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^
(1/2)*((352*cos(e + f*x)*exp(e*5i + f*x*5i))/(45*a^5*f) - (256*exp(e*5i + f
*x*5i)*cos(3*e + 3*f*x))/(45*a^5*f) + (64*exp(e*5i + f*x*5i)*cos(5*e + 5*f*
```

```
x))/(45*a^5*f)))/(16*sin(e + f*x)^4*(a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(
e*1i + f*x*1i)*1i)/2))^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(11/2),x)
```

```
[Out] Timed out
```

3.456 $\int \sqrt{b \sec(e+fx)}\, (a \sin(e+fx))^{7/2}\, dx$

Optimal. Leaf size=128

$$\frac{5a^4\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{12f\sqrt{a\sin(e+fx)}}-\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

```
[Out] -1/3*a*b*(a*sin(f*x+e))^(5/2)/f/(b*sec(f*x+e))^(1/2)-5/6*a^3*b*(a*sin(f*x+e
))^(1/2)/f/(b*sec(f*x+e))^(1/2)-5/12*a^4*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+
1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2
*f*x+2*e)^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2583, 2585, 2573, 2641}

$$-\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}+\frac{5a^4\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{12f\sqrt{a\sin(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2),x]
```

```
[Out] (-5*a^3*b*Sqrt[a*Sin[e + f*x]])/(6*f*Sqrt[b*Sec[e + f*x]]) - (a*b*(a*Sin[e
+ f*x])^(5/2))/(3*f*Sqrt[b*Sec[e + f*x]]) + (5*a^4*EllipticF[e - Pi/4 + f*x
, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(12*f*Sqrt[a*Sin[e + f*x]
])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{7/2}\,dx &= -\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{1}{6}\left(5a^2\right)\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}\,dx\\ &= -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{1}{12}\left(5a^4\right)\int \frac{\sqrt{b\sec(e+}}{\sqrt{a\sin(e+}}\\ &= -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{1}{12}\left(5a^4\sqrt{b\cos(e+fx)}\right.\\ &= -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{\left(5a^4\sqrt{b\sec(e+fx)}\sqrt{s}\right.}{12\sqrt{}}\\ &= -\frac{5a^3b\sqrt{a\sin(e+fx)}}{6f\sqrt{b\sec(e+fx)}}-\frac{ab(a\sin(e+fx))^{5/2}}{3f\sqrt{b\sec(e+fx)}}+\frac{5a^4F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{}}{12f\sqrt{}}\end{aligned}$$

Mathematica [C] time = 0.87, size = 90, normalized size = 0.70

$$\frac{a^3b\sqrt{a\sin(e+fx)}\left(5\left(-\tan^2(e+fx)\right)^{3/4}\csc^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)+2(\cos(2(e+fx))-6)\right)}{12f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*b*Sqrt[a*Sin[e + f*x]]*(2*(-6 + Cos[2*(e + f*x)]) + 5*Csc[e + f*x]^2*H
ypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(
12*f*Sqrt[b*Sec[e + f*x]])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^3\cos\left(fx+e\right)^2-a^3\right)\sqrt{b\sec\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)}\,\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^3*cos(f*x + e)^2 - a^3)*sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x +
e))*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)}\,\left(a \sin\left(fx+e\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(7/2), x)
```

maple [A] time = 0.24, size = 212, normalized size = 1.66

$$-\frac{\left(5\sin\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\right.\right.}{12 f \sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(7/2)*(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/12/f*(5*sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*El
lipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2*cos(f*x
+e)^4*2^(1/2)+2*cos(f*x+e)^3*2^(1/2)+7*cos(f*x+e)^2*2^(1/2)-7*cos(f*x+e)*2^
(1/2))*(a*sin(f*x+e))^(7/2)*(b/cos(f*x+e))^(1/2)/sin(f*x+e)^3/(-1+cos(f*x+e
))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)}\,\left(a \sin\left(fx+e\right)\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left (a\,\sin \left (e+f\,x\right)\right)}^{7/2}\,\sqrt {\frac {b}{\cos \left (e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(7/2)*(b/cos(e + f*x))^(1/2),x)
[Out] int((a*sin(e + f*x))^(7/2)*(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(7/2)*(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

3.457 $\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=91

$$\frac{a^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{2f\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}$$

```
[Out] -a*b*(a*sin(f*x+e))^(1/2)/f/(b*sec(f*x+e))^(1/2)-1/2*a^2*(sin(e+1/4*Pi+f*x)
^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x
+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2583, 2585, 2573, 2641}

$$\frac{a^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{2f\sqrt{a\sin(e+fx)}}-\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -((a*b*Sqrt[a*Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]])) + (a^2*EllipticF[e -
 Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*f*Sqrt[a*Si
n[e + f*x]])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
```

```
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}\,dx = -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}} + \frac{1}{2}a^2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$$

$$= -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}} + \frac{1}{2}\left(a^2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{}{\sqrt{b\cos}}$$

$$= -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}} + \frac{\left(a^2\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2f}}}{2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{ab\sqrt{a\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}} + \frac{a^2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2}}{2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] time = 1.28, size = 66, normalized size = 0.73

$$\frac{(a\sin(e+fx))^{5/2}(b\sec(e+fx))^{3/2}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{1}{2};\sec^2(e+fx)\right)}{abf\left(-\tan^2(e+fx)\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Hypergeometric2F1[-1/2, -1/4, 1/2, Sec[e + f*x]^2]*(b*Sec[e + f*x])^(3/2)*
(a*Sin[e + f*x])^(5/2))/(a*b*f*(-Tan[e + f*x]^2)^(5/4))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}\,a\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))*a*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(fx + e\right)} \left(a \sin \left(fx + e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(3/2), x)
```

maple [A] time = 0.19, size = 184, normalized size = 2.02

$$-\frac{\left(\sin \left(fx+e\right) \sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\, \sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\, \sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\, \mathrm{EllipticF}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}, \frac{\sqrt{2}}{2}\right.\right.}{2f\left(-1+\cos\left(fx+e\right)\right)\sin\left(fx+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)*(b*sec(f*x+e))^(1/2),x)
[Out] -1/2/f*(sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+cos(f*x+e)^2
*2^(1/2)-cos(f*x+e)*2^(1/2))*(a*sin(f*x+e))^(3/2)*(b/cos(f*x+e))^(1/2)/(-1+
cos(f*x+e))/sin(f*x+e)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec \left(fx + e\right)} \left(a \sin \left(fx + e\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)*(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))*(a*sin(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,\sin\left(e+f\,x\right)\right)}^{3/2}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int((a*sin(e + f*x))^(3/2)*(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)*(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.458 $\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

```
[Out] -(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),
2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2585, 2573, 2641}

$$\frac{\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/
(f*Sqrt[a*Sin[e + f*x]])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx = \left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int \frac{1}{\sqrt{b\cos(e+fx)}\sqrt{a\sin(e+fx)}}\,dx$$
$$= \frac{\left(\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e+2fx)}}\,dx}{\sqrt{a\sin(e+fx)}}$$
$$= \frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}{f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] time = 0.38, size = 66, normalized size = 1.25

$$\frac{\left(-\tan^2(e+fx)\right)^{3/4}\cot(e+fx)\sqrt{b\sec(e+fx)}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)}{f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/Sqrt[a*Sin[e + f*x]],x]
```

```
[Out] (Cot[e + f*x]*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*Sqrt[b*Sec[e
 + f*x]]*(-Tan[e + f*x]^2)^(3/4))/(f*Sqrt[a*Sin[e + f*x]])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}}{a\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/(a*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

maple [B] time = 0.16, size = 153, normalized size = 2.89

$$-\frac{\sqrt{\frac{b}{\cos(fx+e)}}\left(\sin^2\left(fx+e\right)\right)\sqrt{\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos(fx+e}{\sin(}}\right.}{f\sqrt{a\sin\left(fx+e\right)}\left(-1+\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(b/cos(f*x+e))^(1/2)*sin(f*x+e)^2*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin
(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2
^(1/2))/(a*sin(f*x+e))^(1/2)/(-1+cos(f*x+e))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/sqrt(a*sin(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{b}{\cos(e+f\,x)}}}{\sqrt{a\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2),x)
```

```
[Out] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(e+fx\right)}}{\sqrt{a\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*sec(e + f*x))/sqrt(a*sin(e + f*x)), x)
```

3.459 $\int \frac{\sqrt{b \sec(e+fx)}}{(a \sin(e+fx))^{5/2}} \, dx$

Optimal. Leaf size=95

$$\frac{2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{3a^2f\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

```
[Out] -2/3*b/a/f/(a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/2)-2/3*(sin(e+1/4*Pi+f*x)
^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x
+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/a^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2584, 2585, 2573, 2641}

$$\frac{2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{3a^2f\sqrt{a\sin(e+fx)}}-\frac{2b}{3af(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-2*b)/(3*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) + (2*EllipticF[e
 - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*f*Sqr
t[a*Sin[e + f*x]])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx = -\frac{2b}{3af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}} + \frac{2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{3a^2}$$

$$= -\frac{2b}{3af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}} + \frac{\left(2\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\int \frac{\quad}{\sqrt{b\cos(e}}}{3a^2}$$

$$= -\frac{2b}{3af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}} + \frac{\left(2\sqrt{b\sec(e+fx)}\,\sqrt{\sin(2e+2fx)}\right)\int \frac{1}{\sqrt{\sin(2e}}}{3a^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{2b}{3af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}} + \frac{2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(e+fx)}\,\sqrt{\sin(2e+}}{3a^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] time = 0.43, size = 75, normalized size = 0.79

$$\frac{2\cot(e+fx)\sqrt{b\sec(e+fx)}\,\left(\left(-\tan^2(e+fx)\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)-1\right)}{3a^2f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (2*Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*(-1 + Hypergeometric2F1[1/2, 3/4, 3/2,
 Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(3*a^2*f*Sqrt[a*Sin[e + f*x]])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{b\sec\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)}}{\left(a^3\cos\left(fx+e\right)^2-a^3\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/((a^3*cos(f*x + e)^2 -
a^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec\left(fx+e\right)}}{\left(a \sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

maple [B] time = 0.18, size = 279, normalized size = 2.94

$$\frac{\left(2\sin\left(fx+e\right)\cos\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{1-\cos\left(fx+e\right)}{\sin\left(f\right.}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/3/f*(2*sin(f*x+e)*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+
2*sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((
(1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)*2^(1/2)
)*(b/cos(f*x+e))^(1/2)*sin(f*x+e)/(a*sin(f*x+e))^(5/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \sec\left(fx+e\right)}}{\left(a \sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{\left(a\,\sin\left(e+f\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2),x)
[Out] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(5/2),x)
[Out] Timed out
```

3.460 $\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=130

$$\frac{4\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{7a^4f\sqrt{a\sin(e+fx)}}-\frac{4b}{7a^3f(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}-\frac{2b}{7af(a\sin(e+fx))^{7/2}}$$

```
[Out] -2/7*b/a/f/(a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(1/2)-4/7*b/a^3/f/(a*sin(f*x
+e))^(3/2)/(b*sec(f*x+e))^(1/2)-4/7*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*P
i+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+
2*e)^(1/2)/a^4/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2584, 2585, 2573, 2641}

$$-\frac{4b}{7a^3f(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}+\frac{4\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{7a^4f\sqrt{a\sin(e+fx)}}-\frac{2b}{7af(a\sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-2*b)/(7*a*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) - (4*b)/(7*a^3*f
*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) + (4*EllipticF[e - Pi/4 + f*x
, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(7*a^4*f*Sqrt[a*Sin[e + f
*x]])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx = -\frac{2b}{7af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{7/2}} + \frac{6\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx}{7a^2}$$

$$= -\frac{2b}{7af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}} + \frac{4\int$$

$$= -\frac{2b}{7af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}} + \frac{(4\sqrt{}$$

$$= -\frac{2b}{7af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}} + \frac{(4\sqrt{}$$

$$= -\frac{2b}{7af\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{7/2}} - \frac{4b}{7a^3f\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}} + \frac{4F(}$$

Mathematica [C] time = 1.02, size = 111, normalized size = 0.85

$$-\frac{2\cos(2(e+fx))(b\sec(e+fx))^{3/2}\left(2\left(-\tan^2(e+fx)\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)+(\cos(2(e+fx))-2)\csc^2}{7a^3bf\left(\sec^2(e+fx)-2\right)(a\sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[e + f*x]]/(a*Sin[e + f*x])^(9/2),x]
```

```
[Out] (-2*Cos[2*(e + f*x)]*(b*Sec[e + f*x])^(3/2)*((-2 + Cos[2*(e + f*x)])*Csc[e
+ f*x]^2 + 2*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x
]^2)^(3/4)))/(7*a^3*b*f*(-2 + Sec[e + f*x]^2)*(a*Sin[e + f*x])^(3/2))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}}{\left(a^{5}\cos\left(fx+e\right)^{4}-2\,a^{5}\cos\left(fx+e\right)^{2}+a^{5}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/((a^5*cos(f*x + e)^4 - 2
*a^5*cos(f*x + e)^2 + a^5)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(9/2), x)
```

maple [B] time = 0.21, size = 532, normalized size = 4.09

$$-\frac{\left(4\sin\left(fx+e\right)\left(\cos^{3}\left(fx+e\right)\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\text{EllipticF}\left(\sqrt{\frac{1-\cos(}{}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x)
```

```
[Out] -1/7/f*(4*sin(f*x+e)*cos(f*x+e)^3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2
))+4*sin(f*x+e)*cos(f*x+e)^2*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-4*
sin(f*x+e)*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*El
lipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-4*sin(f*x
```

```
+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2*cos(f*x+e)^3*2^(1/2)+3*co
s(f*x+e)*2^(1/2))*(b/cos(f*x+e))^(1/2)*sin(f*x+e)/(a*sin(f*x+e))^(9/2)*2^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\sec\left(fx+e\right)}}{\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e))/(a*sin(f*x + e))^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}{{\left(a\,\sin\left(e+f\,x\right)\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(9/2),x)
```

```
[Out] int((b/cos(e + f*x))^(1/2)/(a*sin(e + f*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**(1/2)/(a*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.461 $\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}+\frac{7\sqrt{\sin(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{20f\sqrt{\sin(2e+2fx)}\,\sqrt{b\sec(e+fx)}}$$

```
[Out] -7/30*b*sin(f*x+e)^(3/2)/f/(b*sec(f*x+e))^(3/2)-1/5*b*sin(f*x+e)^(7/2)/f/(b
*sec(f*x+e))^(3/2)-7/20*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ellip
ticE(cos(e+1/4*Pi+f*x),2^(1/2))*sin(f*x+e)^(1/2)/f/(b*sec(f*x+e))^(1/2)/sin
(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2583, 2585, 2572, 2639}

$$-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}-\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}+\frac{7\sqrt{\sin(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{20f\sqrt{\sin(2e+2fx)}\,\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^(9/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (-7*b*Sin[e + f*x]^(3/2))/(30*f*(b*Sec[e + f*x])^(3/2)) - (b*Sin[e + f*x]^(
7/2))/(5*f*(b*Sec[e + f*x])^(3/2)) + (7*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[S
in[e + f*x]])/(20*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{\frac{9}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= -\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7}{10}\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7}{20}\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx\\
&= -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{20\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\
&= -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{\left(7\sqrt{\sin(e+fx)}\right)\int \sqrt{\sin(2e+2fx)}\,dx}{20\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}\\
&= -\frac{7b\sin^{\frac{3}{2}}(e+fx)}{30f(b\sec(e+fx))^{3/2}}-\frac{b\sin^{\frac{7}{2}}(e+fx)}{5f(b\sec(e+fx))^{3/2}}+\frac{7E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{20f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}
\end{aligned}$$

Mathematica [C] time = 0.53, size = 86, normalized size = 0.75

$$-\frac{b\left(42\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\sec^2(e+fx)\right)-26\cos(2(e+fx))+3\cos(4(e+fx))+23\right)}{120f\sqrt{\sin(e+fx)}\,(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^(9/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/120*(b*(23 - 26*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] + 42*Hypergeometri
c2F1[-1/2, 1/4, 1/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(1/4)))/(f*(b*Sec[e
+ f*x])^(3/2)*Sqrt[Sin[e + f*x]])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}+1\right)\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(b*sec(f*x + e))*sqrt(
sin(f*x + e))/(b*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{\frac{9}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^(9/2)/sqrt(b*sec(f*x + e)), x)
```

maple [B] time = 0.16, size = 524, normalized size = 4.56

$$-\frac{\left(12\left(\cos^{6}\left(fx+e\right)\right)\sqrt{2}-38\left(\cos^{4}\left(fx+e\right)\right)\sqrt{2}-21\cos\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/120/f*(12*cos(f*x+e)^6*2^(1/2)-38*cos(f*x+e)^4*2^(1/2)-21*cos(f*x+e)*((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+42*cos(f*x+e)*((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2),1/2*2^(1/2))-21*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
```

```
2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+42*(
(1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin
(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+47*cos(f*x+e)^2*2^(1/2)-21*cos(f
*x+e)*2^(1/2))/cos(f*x+e)/sin(f*x+e)^(1/2)/(b/cos(f*x+e))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{\frac{9}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^(9/2)/sqrt(b*sec(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^{9/2}}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^(9/2)/(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^(9/2)/(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**(9/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.462 $\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{\sin(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2f\sqrt{\sin(2e+2fx)}\,\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

```
[Out] -1/3*b*sin(f*x+e)^(3/2)/f/(b*sec(f*x+e))^(3/2)-1/2*(sin(e+1/4*Pi+f*x)^2)^(1
/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*sin(f*x+e)^(1/2)
/f/(b*sec(f*x+e))^(1/2)/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2583, 2585, 2572, 2639}

$$\frac{\sqrt{\sin(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{2f\sqrt{\sin(2e+2fx)}\,\sqrt{b\sec(e+fx)}}-\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^(5/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -(b*Sin[e + f*x]^(3/2))/(3*f*(b*Sec[e + f*x])^(3/2)) + (EllipticE[e - Pi/4
+ f*x, 2]*Sqrt[Sin[e + f*x]])/(2*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*
x]])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^{\frac{5}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx &= -\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}+\frac{1}{2}\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx\\ &= -\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}+\frac{\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}\\ &= -\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}+\frac{\sqrt{\sin(e+fx)}\int \sqrt{\sin(2e+2fx)}\,dx}{2\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}\\ &= -\frac{b\sin^{\frac{3}{2}}(e+fx)}{3f(b\sec(e+fx))^{3/2}}+\frac{E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{2f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}\end{aligned}$$

Mathematica [C] time = 0.30, size = 74, normalized size = 0.87

$$\frac{b\left(-3\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\sec^2(e+fx)\right)+\cos(2(e+fx))-1\right)}{6f\sqrt{\sin(e+fx)}\,(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^(5/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (b*(-1 + Cos[2*(e + f*x)] - 3*Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f*x
]^2]*(-Tan[e + f*x]^2)^(1/4)))/(6*f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x
]])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^2-1\right)\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}{b\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))/(b*s
ec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^(5/2)/sqrt(b*sec(f*x + e)), x)
```

maple [B] time = 0.18, size = 511, normalized size = 6.01

$$\left(2\left(\cos^{4}\left(fx+e\right)\right)\sqrt{2}+3\cos\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\mathrm{EllipticF}\left(\sqrt{\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/12/f*(2*cos(f*x+e)^4*2^(1/2)+3*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),
1/2*2^(1/2))-6*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+3*((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-6*((1-cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))
/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1
/2*2^(1/2))-5*cos(f*x+e)^2*2^(1/2)+3*cos(f*x+e)*2^(1/2))/cos(f*x+e)/sin(f*x
+e)^(1/2)/(b/cos(f*x+e))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{\frac{5}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^(5/2)/sqrt(b*sec(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^{5/2}}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^(5/2)/(b/cos(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^(5/2)/(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**(5/2)/(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

3.463 $\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{\sin(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\,\sqrt{b\sec(e+fx)}}$$

```
[Out] -(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),
2^(1/2))*sin(f*x+e)^(1/2)/f/(b*sec(f*x+e))^(1/2)/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2585, 2572, 2639}

$$\frac{\sqrt{\sin(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\,\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[e + f*x]]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (EllipticE[e - Pi/4 + f*x, 2]*Sqrt[Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]]*S
qrt[Sin[2*e + 2*f*x]])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx = \frac{\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= \frac{\sqrt{\sin(e+fx)}\int \sqrt{\sin(2e+2fx)}\,dx}{\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$
$$= \frac{E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

Mathematica [C] time = 1.10, size = 60, normalized size = 1.18

$$-\frac{b\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\sec^2(e+fx)\right)}{f\sqrt{\sin(e+fx)}\,(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[e + f*x]]/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -((b*Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(1
/4))/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}{b\sec\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))/(b*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin\left(fx+e\right)}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(f*x + e))/sqrt(b*sec(f*x + e)), x)
```

maple [B] time = 0.15, size = 497, normalized size = 9.75

$$-\frac{\left(2\cos\left(fx+e\right)\sqrt{\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\,\sqrt{\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}},\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(2*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)
*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2),1/2*2^(1/2))-((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elliptic
F(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+cos(f*x+e)^2*2^
(1/2)-cos(f*x+e)*2^(1/2))/cos(f*x+e)/sin(f*x+e)^(1/2)/(b/cos(f*x+e))^(1/2)*
2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin\left(fx+e\right)}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(f*x + e))/sqrt(b*sec(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sin\left(e+f\,x\right)}}{\sqrt{\frac{b}{\cos(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^(1/2)/(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^(1/2)/(b/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sin\left(e+fx\right)}}{\sqrt{b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**(1/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(e + f*x))/sqrt(b*sec(e + f*x)), x)
```

3.464 $\int \frac{1}{\sqrt{b\sec(e+fx)}\ \sin^{\frac{3}{2}}(e+fx)}\,dx$

Optimal. Leaf size=81

$$-\frac{2b}{f\sqrt{\sin(e+fx)}\,(b\sec(e+fx))^{3/2}}-\frac{2\sqrt{\sin(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\,\sqrt{b\sec(e+fx)}}$$

```
[Out] -2*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(1/2)+2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/
sin(e+1/4*Pi+f*x)*EllipticE(cos(e+1/4*Pi+f*x),2^(1/2))*sin(f*x+e)^(1/2)/f/(
b*sec(f*x+e))^(1/2)/sin(2*f*x+2*e)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2584, 2585, 2572, 2639}

$$-\frac{2b}{f\sqrt{\sin(e+fx)}\,(b\sec(e+fx))^{3/2}}-\frac{2\sqrt{\sin(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{f\sqrt{\sin(2e+2fx)}\,\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(3/2)),x]
```

```
[Out] (-2*b)/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]) - (2*EllipticE[e - Pi/
4 + f*x, 2]*Sqrt[Sin[e + f*x]])/(f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*
x]])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
```

```
f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\,\sin^{\frac{3}{2}}(e+fx)}\,dx = -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - 2\int \frac{\sqrt{\sin(e+fx)}}{\sqrt{b\sec(e+fx)}}\,dx$$

$$= -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{2\int \sqrt{b\cos(e+fx)}\sqrt{\sin(e+fx)}\,dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{\left(2\sqrt{\sin(e+fx)}\right)\int \sqrt{\sin(2e+2fx)}}{\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

$$= -\frac{2b}{f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} - \frac{2E\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{\sin(e+fx)}}{f\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}}$$

Mathematica [C] time = 0.25, size = 63, normalized size = 0.78

$$\frac{2b\left(\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\sec^2(e+fx)\right)-1\right)}{f\sqrt{\sin(e+fx)}\,(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(3/2)),x]
```

```
[Out] (2*b*(-1 + Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f*x]^2]*(-Tan[e + f*x]
^2)^(1/4)))/(f*(b*Sec[e + f*x])^(3/2)*Sqrt[Sin[e + f*x]])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}{\left(b\cos\left(fx+e\right)^2-b\right)\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))/((b*cos(f*x + e)^2 - b)*s
ec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sec\left(fx+e\right)} \sin\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(3/2)), x)
```

maple [B] time = 0.15, size = 484, normalized size = 5.98

$$\left(2\cos\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\ \ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(2*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)*((
1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),
1/2*2^(1/2))-((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((
(1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)*2^(1/2)
)/cos(f*x+e)/sin(f*x+e)^(1/2)/(b/cos(f*x+e))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sec\left(fx+e\right)} \sin\left(fx+e\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^{3/2}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^(3/2)*(b/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^(3/2)*(b/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(e+fx\right)}\,\sin^{\frac{3}{2}}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(3/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*sec(e + f*x))*sin(e + f*x)**(3/2)), x)
```

3.465 $\int \frac{1}{\sqrt{b \sec(e+fx)} \sin^{\frac{7}{2}}(e+fx)} dx$

Optimal. Leaf size=115

$$-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{4b}{5f\sqrt{\sin(e+fx)}\,(b\sec(e+fx))^{3/2}}-\frac{4\sqrt{\sin(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{5f\sqrt{\sin(2e+2fx)}\,\sqrt{b\sec(e+fx)}}$$

```
[Out] -2/5*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(5/2)-4/5*b/f/(b*sec(f*x+e))^(3/2)
/sin(f*x+e)^(1/2)+4/5*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*Ellipti
cE(cos(e+1/4*Pi+f*x),2^(1/2))*sin(f*x+e)^(1/2)/f/(b*sec(f*x+e))^(1/2)/sin(2
*f*x+2*e)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2584, 2585, 2572, 2639}

$$-\frac{2b}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{4b}{5f\sqrt{\sin(e+fx)}\,(b\sec(e+fx))^{3/2}}-\frac{4\sqrt{\sin(e+fx)}\,E\left(e+fx-\frac{\pi}{4}\middle|2\right)}{5f\sqrt{\sin(2e+2fx)}\,\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(7/2)),x]
```

```
[Out] (-2*b)/(5*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(5/2)) - (4*b)/(5*f*(b*Sec[
e + f*x])^(3/2)*Sqrt[Sin[e + f*x]]) - (4*EllipticE[e - Pi/4 + f*x, 2]*Sqrt[
Sin[e + f*x]])/(5*f*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])
```

Rule 2572

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_)]]
, x_Symbol] :> Dist[(Sqrt[a*Sin[e + f*x]]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[2*
e + 2*f*x]], Int[Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\,\sin^{\frac{7}{2}}(e+fx)}\,dx = -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} + \frac{2}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}\,\sin^{\frac{3}{2}}(e+fx)}\,d$$

$$= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} - \frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} -$$

$$= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} - \frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} -$$

$$= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} - \frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} -$$

$$= -\frac{2b}{5f(b\sec(e+fx))^{3/2}\sin^{\frac{5}{2}}(e+fx)} - \frac{4b}{5f(b\sec(e+fx))^{3/2}\sqrt{\sin(e+fx)}} -$$

Mathematica [C] time = 0.47, size = 82, normalized size = 0.71

$$\frac{2b\left(2\sin^2(e+fx)\sqrt[4]{-\tan^2(e+fx)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{1}{2};\sec^2(e+fx)\right)+\cos(2(e+fx))-2\right)}{5f\sin^{\frac{5}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(7/2)),x]
```

```
[Out] (2*b*(-2 + Cos[2*(e + f*x)] + 2*Hypergeometric2F1[-1/2, 1/4, 1/2, Sec[e + f
*x]^2]*Sin[e + f*x]^2*(-Tan[e + f*x]^2)^(1/4)))/(5*f*(b*Sec[e + f*x])^(3/2)
*Sin[e + f*x]^(5/2))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}{\left(b\cos\left(fx+e\right)^{4}-2\,b\cos\left(fx+e\right)^{2}+b\right)\sec\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))/((b*cos(f*x + e)^4 - 2*b*c
os(f*x + e)^2 + b)*sec(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(7/2)), x)
```

maple [B] time = 0.18, size = 1030, normalized size = 8.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -16/5/f*(-1+cos(f*x+e))^4*(4*cos(f*x+e)^3*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)-2*cos(f*x+e)^3*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((1-cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2
)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+4*cos
(f*x+e)^2*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)-2*cos(f*x+e)^2*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-4*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
```

```
(f*x+e))/sin(f*x+e))^(1/2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2),1/2*2^(1/2))+2*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e
))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)
)-2*cos(f*x+e)^3*2^(1/2)-4*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-
1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*EllipticE(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*((
1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+cos(f*x+e)^2*2^(1/2)+2*cos(f*x+e)
*2^(1/2))/sin(f*x+e)^(5/2)/(b/cos(f*x+e))^(1/2)/(-1+cos(f*x+e)+sin(f*x+e))/
(1-cos(f*x+e)+sin(f*x+e))/(sin(f*x+e)^2+cos(f*x+e)^2-2*cos(f*x+e)+1)^3*2^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(7/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^{7/2}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^(7/2)*(b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^(7/2)*(b/cos(e + f*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(7/2)/(b*sec(f*x+e))**(1/2),x)
[Out] Timed out
```

3.466 $\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx$

Optimal. Leaf size=363

$$-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}+\frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\log\left(\sqrt{\right.}}{8\sqrt{2}f}$$

```
[Out] 1/8*arctan(1-2^(1/2)*(b*cos(f*x+e))^(1/2)/b^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)
/f*2^(1/2)/(b*cos(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)-1/8*arctan(1+2^(1/2)*(
b*cos(f*x+e))^(1/2)/b^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)/f*2^(1/2)/(b*cos(f*x+
e))^(1/2)/(b*sec(f*x+e))^(1/2)-1/16*ln(b^(1/2)+cot(f*x+e)*b^(1/2)-2^(1/2)*(
b*cos(f*x+e))^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)/f*2^(1/2)/(b*cos(f*x+e))^(1/2
)/(b*sec(f*x+e))^(1/2)+1/16*ln(b^(1/2)+cot(f*x+e)*b^(1/2)+2^(1/2)*(b*cos(f*
x+e))^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)/f*2^(1/2)/(b*cos(f*x+e))^(1/2)/(b*sec
(f*x+e))^(1/2)-1/2*b*sin(f*x+e)^(1/2)/f/(b*sec(f*x+e))^(3/2)
```

Rubi [A] time = 0.27, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2583, 2585, 2575, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}}+\frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\log\left(\sqrt{\right.}}{8\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^(3/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[b]*ArcTan[1 - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*
x]])])/(4*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*A
rcTan[1 + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*x]])])/(4*
Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*Log[Sqrt[b]
 + Sqrt[b]*Cot[e + f*x] - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x]]
])/(8*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) + (Sqrt[b]*Log[S
qrt[b] + Sqrt[b]*Cot[e + f*x] + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e +
 f*x]]])/(8*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (b*Sqrt[
Sin[e + f*x]])/(2*f*(b*Sec[e + f*x])^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\sin^{\frac{3}{2}}(e+fx)}{\sqrt{b\sec(e+fx)}}\,dx = -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}} + \frac{1}{4}\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx$$

$$= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}} + \frac{\int \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\,dx}{4\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}} - \frac{b\,\text{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

$$= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}} + \frac{b\,\text{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{4f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\,\text{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b}\cdots}{\sqrt{s}\cdots}\right.}{4f\sqrt{b\cos(e+fx)}\sqrt{b\sec\cdots}}$$

$$= -\frac{b\sqrt{\sin(e+fx)}}{2f(b\sec(e+fx))^{3/2}} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{\cdots}{-b\cdots}\right.}{8\sqrt{2}f\sqrt{b\,\text{co}\cdots}}$$

$$= -\frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)+\frac{\sqrt{2}\cdots}{\cdots}\right.}{8\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(\cdots}}$$

$$= \frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{4\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\log\cdots}{8\cdots}$$

Mathematica [A] time = 1.56, size = 218, normalized size = 0.60

$$-\frac{\sqrt{\sin(e+fx)}\sqrt{b\sec(e+fx)}\left(4\sqrt[4]{\tan^2(e+fx)}+2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{\tan^2(e+fx)}\right)-2\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt[4]{\tan\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^(3/2)/Sqrt[b*Sec[e + f*x]],x]
```

```
[Out] -1/16*(Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]*(2*Sqrt[2]*ArcTan[1 - Sqrt[2
]*(Tan[e + f*x]^2)^(1/4)] - 2*Sqrt[2]*ArcTan[1 + Sqrt[2]*(Tan[e + f*x]^2)^(
1/4)] + Sqrt[2]*Log[1 - Sqrt[2]*(Tan[e + f*x]^2)^(1/4) + Sqrt[Tan[e + f*x]^
2]] - Sqrt[2]*Log[1 + Sqrt[2]*(Tan[e + f*x]^2)^(1/4) + Sqrt[Tan[e + f*x]^2]
] + 4*(Tan[e + f*x]^2)^(1/4) + 4*Cos[2*(e + f*x)]*(Tan[e + f*x]^2)^(1/4)))/
(b*f*(Tan[e + f*x]^2)^(1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^(3/2)/sqrt(b*sec(f*x + e)), x)
```

maple [C] time = 0.15, size = 646, normalized size = 1.78

$$-\frac{\left(i\sqrt{\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}}\sqrt{\frac{-1+\cos(fx+e)}{\sin(fx+e)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/8/f*(I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(f*x+e)
-I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))
/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(f*x+e)+((1-co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(f*x+e)+((1-cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(f*x+e)-2*sin(f*x+e)*((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*cos(f*x+e)^3*2^(1/2)-2*cos(f*x+e)^2*2^(1
/2))*sin(f*x+e)^(1/2)/(-1+cos(f*x+e))/(b/cos(f*x+e))^(1/2)/cos(f*x+e)*2^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{\frac{3}{2}}}{\sqrt{b\sec\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^(3/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^(3/2)/sqrt(b*sec(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^{3/2}}{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^(3/2)/(b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^(3/2)/(b/cos(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**(3/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.467 $\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx$

Optimal. Leaf size=328

$$\frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\log\left(\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+}}{\sqrt{\sin(e+f}}\right.}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e-}}$$

```
[Out] 1/2*arctan(1-2^(1/2)*(b*cos(f*x+e))^(1/2)/b^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)
/f*2^(1/2)/(b*cos(f*x+e))^(1/2)/(b*sec(f*x+e))^(1/2)-1/2*arctan(1+2^(1/2)*(
b*cos(f*x+e))^(1/2)/b^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)/f*2^(1/2)/(b*cos(f*x+
e))^(1/2)/(b*sec(f*x+e))^(1/2)-1/4*ln(b^(1/2)+cot(f*x+e)*b^(1/2)-2^(1/2)*(b
*cos(f*x+e))^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)/f*2^(1/2)/(b*cos(f*x+e))^(1/2)
/(b*sec(f*x+e))^(1/2)+1/4*ln(b^(1/2)+cot(f*x+e)*b^(1/2)+2^(1/2)*(b*cos(f*x+
e))^(1/2)/sin(f*x+e)^(1/2))*b^(1/2)/f*2^(1/2)/(b*cos(f*x+e))^(1/2)/(b*sec(f
*x+e))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2585, 2575, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}+1\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}-\frac{\sqrt{b}\log\left(\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+}}{\sqrt{\sin(e+f}}\right.}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e-}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]),x]
```

```
[Out] (Sqrt[b]*ArcTan[1 - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*
x]])])/(Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*Arc
Tan[1 + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/(Sqrt[b]*Sqrt[Sin[e + f*x]])])/(Sqrt
[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) - (Sqrt[b]*Log[Sqrt[b] + S
qrt[b]*Cot[e + f*x] - (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x]]])/(
2*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]]) + (Sqrt[b]*Log[Sqrt[
b] + Sqrt[b]*Cot[e + f*x] + (Sqrt[2]*Sqrt[b*Cos[e + f*x]])/Sqrt[Sin[e + f*x
]]])/(2*Sqrt[2]*f*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2575

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> With[{k = Denominator[m]}, -Dist[(k*a*b)/f, Subst[Int[x^(k
*(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Cos[e + f*x])^(1/k)/(b*Sin[e +
 f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0]
&& LtQ[m, 1]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sqrt{\sin(e+fx)}}\,dx = \frac{\int \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\,dx}{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= -\frac{(2b)\,\mathrm{Subst}\left(\int \frac{x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= \frac{b\,\mathrm{Subst}\left(\int \frac{b-x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{b\,\mathrm{Subst}\left(\int \frac{b+x^2}{b^2+x^4}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$
$$= -\frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}+2x}{-b-\sqrt{2}\sqrt{b}x-x^2}\,dx, x, \frac{\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\,\mathrm{Subst}\left(\int \frac{\sqrt{2}\sqrt{b}-2x}{-b+\sqrt{2}\sqrt{b}x-\ldots}\right.}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\ldots}$$
$$= -\frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\cot(e+fx)-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{\sin(e+fx)}}\right)}{2\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} + \frac{\sqrt{b}\log\left(\sqrt{b}+\sqrt{b}\,\mathrm{co}\ldots\right.}{2\sqrt{2}f\sqrt{b\cos(e\ldots}}$$
$$= \frac{\sqrt{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}} - \frac{\sqrt{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{b\cos(e+fx)}}{\sqrt{b}\sqrt{\sin(e+fx)}}\right)}{\sqrt{2}f\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}$$

Mathematica [A] time = 0.86, size = 166, normalized size = 0.51

$$\frac{\sqrt{\sin(e+fx)}\sqrt{b\sec(e+fx)}\left(-2\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{\tan^2(e+fx)}\right)+2\tan^{-1}\left(\sqrt{2}\sqrt[4]{\tan^2(e+fx)}+1\right)-\log\left(\sqrt{\tan\ldots}\right.\right.}{2\sqrt{2}bf\sqrt[4]{\tan^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[e + f*x]]),x]
```

```
[Out] ((-2*ArcTan[1 - Sqrt[2]*(Tan[e + f*x]^2)^(1/4)] + 2*ArcTan[1 + Sqrt[2]*(Tan
[e + f*x]^2)^(1/4)] - Log[1 - Sqrt[2]*(Tan[e + f*x]^2)^(1/4) + Sqrt[Tan[e +
```

```
 f*x]^2]] + Log[1 + Sqrt[2]*(Tan[e + f*x]^2)^(1/4) + Sqrt[Tan[e + f*x]^2]])
*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[e + f*x]])/(2*Sqrt[2]*b*f*(Tan[e + f*x]^2)^(
1/4))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\,\sqrt{\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))), x)
```

maple [C] time = 0.14, size = 304, normalized size = 0.93

$$-\frac{\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\left(i\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(I*EllipticPi(((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*Elliptic
Pi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+Elli
pticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+
EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/
2))-2*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)))*
sin(f*x+e)^(3/2)/(b/cos(f*x+e))^(1/2)/cos(f*x+e)/(-1+cos(f*x+e))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sqrt{\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(1/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sqrt(sin(f*x + e))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\sin\left(e+f\,x\right)}\,\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^(1/2)*(b/cos(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^(1/2)*(b/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(e+fx\right)}\sqrt{\sin\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(1/2)/(b*sec(f*x+e))**(1/2),x)
[Out] Integral(1/(sqrt(b*sec(e + f*x))*sqrt(sin(e + f*x))), x)
```

3.468 $\int \frac{1}{\sqrt{b\sec(e+fx)}\,\sin^{\frac{5}{2}}(e+fx)}\,dx$

Optimal. Leaf size=30

$$-\frac{2b}{3f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

```
[Out] -2/3*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2578}

$$-\frac{2b}{3f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(5/2)),x]
```

```
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\,\sin^{\frac{5}{2}}(e+fx)}\,dx = -\frac{2b}{3f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

Mathematica [A] time = 0.10, size = 30, normalized size = 1.00

$$-\frac{2b}{3f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(5/2)),x]
[Out] (-2*b)/(3*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

fricas [A] time = 0.89, size = 48, normalized size = 1.60

$$\frac{2\,\sqrt{\frac{b}{\cos\left(fx+e\right)}}\cos\left(fx+e\right)^{2}\sqrt{\sin\left(fx+e\right)}}{3\,\left(bf\cos\left(fx+e\right)^{2}-bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b/cos(f*x + e))*cos(f*x + e)^2*sqrt(sin(f*x + e))/(b*f*cos(f*x + e
)^2 - b*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(5/2)), x)
```

maple [B] time = 0.14, size = 70, normalized size = 2.33

$$-\frac{8\cos\left(fx+e\right)\left(-1+\cos\left(fx+e\right)\right)^{2}}{3f\sin\left(fx+e\right)^{\frac{3}{2}}\left(\sin^{2}\left(fx+e\right)+\cos^{2}\left(fx+e\right)-2\cos\left(fx+e\right)+1\right)^{2}\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x)
[Out] -8/3/f*cos(f*x+e)*(-1+cos(f*x+e))^2/sin(f*x+e)^(3/2)/(sin(f*x+e)^2+cos(f*x+
e)^2-2*cos(f*x+e)+1)^2/(b/cos(f*x+e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(5/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(5/2)), x)
```

mupad [B] time = 1.25, size = 57, normalized size = 1.90

$$\frac{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,\left(\sin\left(e+f\,x\right)+\sin\left(3\,e+3\,f\,x\right)\right)}{3\,b\,f\,\sqrt{\sin\left(e+f\,x\right)}\,\left(\cos\left(2\,e+2\,f\,x\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^(5/2)*(b/cos(e + f*x))^(1/2)),x)
```

```
[Out] ((b/cos(e + f*x))^(1/2)*(sin(e + f*x) + sin(3*e + 3*f*x)))/(3*b*f*sin(e + f
*x)^(1/2)*(cos(2*e + 2*f*x) - 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(5/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.469 $\int \frac{1}{\sqrt{b\sec(e+fx)}\,\sin^{\frac{9}{2}}(e+fx)}\,dx$

Optimal. Leaf size=61

$$-\frac{8b}{21f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

```
[Out] -2/7*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(7/2)-8/21*b/f/(b*sec(f*x+e))^(3/2
)/sin(f*x+e)^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2584, 2578}

$$-\frac{8b}{21f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{2b}{7f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(9/2)),x]
```

```
[Out] (-2*b)/(7*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)) - (8*b)/(21*f*(b*Sec
[e + f*x])^(3/2)*Sin[e + f*x]^(3/2))
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\,\sin^{\frac{9}{2}}(e+fx)}\,dx = -\frac{2b}{7f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)} + \frac{4}{7}\int \frac{1}{\sqrt{b\sec(e+fx)}\,\sin^{\frac{5}{2}}(e+fx)}\,dx$$
$$= -\frac{2b}{7f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)} - \frac{8b}{21f(b\sec(e+fx))^{3/2}\sin^{\frac{3}{2}}(e+fx)}$$

Mathematica [A] time = 0.14, size = 42, normalized size = 0.69

$$\frac{2b(2\cos(2(e+fx))-5)}{21f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(9/2)),x]
```

```
[Out] (2*b*(-5 + 2*Cos[2*(e + f*x)]))/(21*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(
7/2))
```

fricas [A] time = 0.64, size = 72, normalized size = 1.18

$$\frac{2\left(4\cos\left(fx+e\right)^{4}-7\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}\sqrt{\sin\left(fx+e\right)}}{21\left(bf\cos\left(fx+e\right)^{4}-2\,bf\cos\left(fx+e\right)^{2}+bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(4*cos(f*x + e)^4 - 7*cos(f*x + e)^2)*sqrt(b/cos(f*x + e))*sqrt(sin(f*
x + e))/(b*f*cos(f*x + e)^4 - 2*b*f*cos(f*x + e)^2 + b*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(9/2)), x)
```

maple [A] time = 0.15, size = 82, normalized size = 1.34

$$\frac{32\cos\left(fx+e\right)\left(4\left(\cos^2\left(fx+e\right)\right)-7\right)\left(-1+\cos\left(fx+e\right)\right)^4}{21f\sin\left(fx+e\right)^{\frac{7}{2}}\left(\sin^2\left(fx+e\right)+\cos^2\left(fx+e\right)-2\cos\left(fx+e\right)+1\right)^4\sqrt{\frac{b}{\cos(fx+e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 32/21/f*cos(f*x+e)*(4*cos(f*x+e)^2-7)*(-1+cos(f*x+e))^4/sin(f*x+e)^(7/2)/(s
in(f*x+e)^2+cos(f*x+e)^2-2*cos(f*x+e)+1)^4/(b/cos(f*x+e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(9/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(9/2)), x)
```

mupad [B] time = 2.66, size = 103, normalized size = 1.69

$$\frac{4\sqrt{\frac{b}{\cos(e+f\,x)}}\,\left(11\,\sin\left(e+f\,x\right)+4\,\sin\left(3\,e+3\,f\,x\right)-6\,\sin\left(5\,e+5\,f\,x\right)+\sin\left(7\,e+7\,f\,x\right)\right)}{21\,b\,f\,\sqrt{\sin\left(e+f\,x\right)}\,\left(15\,\cos\left(2\,e+2\,f\,x\right)-6\,\cos\left(4\,e+4\,f\,x\right)+\cos\left(6\,e+6\,f\,x\right)-10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^(9/2)*(b/cos(e + f*x))^(1/2)),x)
```

```
[Out] (4*(b/cos(e + f*x))^(1/2)*(11*sin(e + f*x) + 4*sin(3*e + 3*f*x) - 6*sin(5*e
 + 5*f*x) + sin(7*e + 7*f*x)))/(21*b*f*sin(e + f*x)^(1/2)*(15*cos(2*e + 2*f
*x) - 6*cos(4*e + 4*f*x) + cos(6*e + 6*f*x) - 10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(9/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.470 $\int \frac{1}{\sqrt{b\sec(e+fx)}\,\sin^{\frac{13}{2}}(e+fx)}\,dx$

Optimal. Leaf size=91

$$-\frac{64b}{231f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{16b}{77f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

```
[Out] -2/11*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(11/2)-16/77*b/f/(b*sec(f*x+e))^(
3/2)/sin(f*x+e)^(7/2)-64/231*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2584, 2578}

$$-\frac{64b}{231f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{16b}{77f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{2b}{11f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(13/2)),x]
```

```
[Out] (-2*b)/(11*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(11/2)) - (16*b)/(77*f*(b*
Sec[e + f*x])^(3/2)*Sin[e + f*x]^(7/2)) - (64*b)/(231*f*(b*Sec[e + f*x])^(3
/2)*Sin[e + f*x]^(3/2))
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}dx = -\frac{2b}{11f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} + \frac{8}{11}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{9}{2}}(e+fx)}$$
$$= -\frac{2b}{11f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} - \frac{16b}{77f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)}$$
$$= -\frac{2b}{11f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)} - \frac{16b}{77f(b\sec(e+fx))^{3/2}\sin^{\frac{7}{2}}(e+fx)}$$

Mathematica [A] time = 0.21, size = 52, normalized size = 0.57

$$\frac{2b(28\cos(2(e+fx)) - 4\cos(4(e+fx)) - 45)}{231f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(13/2)),x]
```

```
[Out] (2*b*(-45 + 28*Cos[2*(e + f*x)] - 4*Cos[4*(e + f*x)]))/(231*f*(b*Sec[e + f*
x])^(3/2)*Sin[e + f*x]^(11/2))
```

fricas [A] time = 0.50, size = 95, normalized size = 1.04

$$\frac{2\left(32\cos\left(fx+e\right)^{6} - 88\cos\left(fx+e\right)^{4} + 77\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}\sqrt{\sin\left(fx+e\right)}}{231\left(bf\cos\left(fx+e\right)^{6} - 3bf\cos\left(fx+e\right)^{4} + 3bf\cos\left(fx+e\right)^{2} - bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(13/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/231*(32*cos(f*x + e)^6 - 88*cos(f*x + e)^4 + 77*cos(f*x + e)^2)*sqrt(b/co
s(f*x + e))*sqrt(sin(f*x + e))/(b*f*cos(f*x + e)^6 - 3*b*f*cos(f*x + e)^4 +
 3*b*f*cos(f*x + e)^2 - b*f)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(13/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 0.18, size = 92, normalized size = 1.01

$$-\frac{128\cos\left(fx+e\right)\left(32\left(\cos^4\left(fx+e\right)\right)-88\left(\cos^2\left(fx+e\right)\right)+77\right)\left(-1+\cos\left(fx+e\right)\right)^6}{231f\sin\left(fx+e\right)^{\frac{11}{2}}\left(\sin^2\left(fx+e\right)+\cos^2\left(fx+e\right)-2\cos\left(fx+e\right)+1\right)^6\sqrt{\frac{b}{\cos(fx+e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sin(f*x+e)^(13/2)/(b*sec(f*x+e))^(1/2),x)

[Out] -128/231/f*cos(f*x+e)*(32*cos(f*x+e)^4-88*cos(f*x+e)^2+77)*(-1+cos(f*x+e))^6/sin(f*x+e)^(11/2)/(sin(f*x+e)^2+cos(f*x+e)^2-2*cos(f*x+e)+1)^6/(b/cos(f*x+e))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sin(f*x+e)^(13/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(13/2)), x)

mupad [B] time = 6.24, size = 163, normalized size = 1.79

$$\frac{e^{-e\,6\mathrm{i}-f\,x\,6\mathrm{i}}\,\sqrt{\frac{b}{\frac{e^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{e^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\,\left(\frac{e^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,992\mathrm{i}}{231\,b\,f}+\frac{e^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)\,608\mathrm{i}}{231\,b\,f}-\frac{e^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)\,320\mathrm{i}}{231\,b\,f}+\frac{e^{e\,6\mathrm{i}+f\,x\,6\mathrm{i}}\,\cos\left(6\,e+\right.}{231\,b\,f}}{32\sin\left(e+f\,x\right)^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(e + f*x)^(13/2)*(b/cos(e + f*x))^(1/2)),x)

[Out] (exp(- e*6i - f*x*6i)*(b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(1/2)*((exp(e*6i + f*x*6i)*992i)/(231*b*f) + (exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*608i)/(231*b*f) - (exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*320i)/(231*b*f) + (exp(e*6i + f*x*6i)*cos(6*e + 6*f*x)*64i)/(231*b*f))*1i)/(32*sin(e + f*x)^(11/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(13/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.471 $\int \frac{1}{\sqrt{b\sec(e+fx)}\,\sin^{\frac{17}{2}}(e+fx)}\,dx$

Optimal. Leaf size=121

$$-\frac{256b}{1155f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{64b}{385f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{8b}{55f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

```
[Out] -2/15*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(15/2)-8/55*b/f/(b*sec(f*x+e))^(3
/2)/sin(f*x+e)^(11/2)-64/385*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(7/2)-256/
1155*b/f/(b*sec(f*x+e))^(3/2)/sin(f*x+e)^(3/2)
```

Rubi [A] time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2584, 2578}

$$-\frac{256b}{1155f\sin^{\frac{3}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{64b}{385f\sin^{\frac{7}{2}}(e+fx)(b\sec(e+fx))^{3/2}}-\frac{8b}{55f\sin^{\frac{11}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(17/2)),x]
```

```
[Out] (-2*b)/(15*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(15/2)) - (8*b)/(55*f*(b*S
ec[e + f*x])^(3/2)*Sin[e + f*x]^(11/2)) - (64*b)/(385*f*(b*Sec[e + f*x])^(3
/2)*Sin[e + f*x]^(7/2)) - (256*b)/(1155*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*
x]^(3/2))
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{17}{2}}(e+fx)}\,dx = -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} + \frac{4}{5}\int \frac{1}{\sqrt{b\sec(e+fx)}\sin^{\frac{13}{2}}(e+fx)}$$
$$= -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} - \frac{8b}{55f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)}$$
$$= -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} - \frac{8b}{55f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)}$$
$$= -\frac{2b}{15f(b\sec(e+fx))^{3/2}\sin^{\frac{15}{2}}(e+fx)} - \frac{8b}{55f(b\sec(e+fx))^{3/2}\sin^{\frac{11}{2}}(e+fx)}$$

Mathematica [A] time = 0.26, size = 62, normalized size = 0.51

$$\frac{2b(150\cos(2(e+fx)) - 36\cos(4(e+fx)) + 4\cos(6(e+fx)) - 195)}{1155f\sin^{\frac{15}{2}}(e+fx)(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*Sec[e + f*x]]*Sin[e + f*x]^(17/2)),x]
```

```
[Out] (2*b*(-195 + 150*Cos[2*(e + f*x)] - 36*Cos[4*(e + f*x)] + 4*Cos[6*(e + f*x)
]))/(1155*f*(b*Sec[e + f*x])^(3/2)*Sin[e + f*x]^(15/2))
```

fricas [A] time = 0.55, size = 116, normalized size = 0.96

$$\frac{2\left(128\cos\left(fx+e\right)^{8} - 480\cos\left(fx+e\right)^{6} + 660\cos\left(fx+e\right)^{4} - 385\cos\left(fx+e\right)^{2}\right)\sqrt{\frac{b}{\cos\left(fx+e\right)}}\sqrt{\sin\left(fx+e\right)}}{1155\left(bf\cos\left(fx+e\right)^{8} - 4bf\cos\left(fx+e\right)^{6} + 6bf\cos\left(fx+e\right)^{4} - 4bf\cos\left(fx+e\right)^{2} + bf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(17/2)/(b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(128*cos(f*x + e)^8 - 480*cos(f*x + e)^6 + 660*cos(f*x + e)^4 - 385*
cos(f*x + e)^2)*sqrt(b/cos(f*x + e))*sqrt(sin(f*x + e))/(b*f*cos(f*x + e)^8
 - 4*b*f*cos(f*x + e)^6 + 6*b*f*cos(f*x + e)^4 - 4*b*f*cos(f*x + e)^2 + b*f
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\sin\left(fx+e\right)^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(17/2)/(b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(17/2)), x)
```

maple [A] time = 0.21, size = 102, normalized size = 0.84

$$\frac{512\cos\left(fx+e\right)\left(128\left(\cos^6\left(fx+e\right)\right)-480\left(\cos^4\left(fx+e\right)\right)+660\left(\cos^2\left(fx+e\right)\right)-385\right)\left(-1+\cos\left(fx+e\right)\right)^8}{1155f\sin\left(fx+e\right)^{\frac{15}{2}}\left(\sin^2\left(fx+e\right)+\cos^2\left(fx+e\right)-2\cos\left(fx+e\right)+1\right)^8\sqrt{\frac{b}{\cos\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(f*x+e)^(17/2)/(b*sec(f*x+e))^(1/2),x)
```

```
[Out] 512/1155/f*cos(f*x+e)*(128*cos(f*x+e)^6-480*cos(f*x+e)^4+660*cos(f*x+e)^2-3
85)*(-1+cos(f*x+e))^8/sin(f*x+e)^(15/2)/(sin(f*x+e)^2+cos(f*x+e)^2-2*cos(f*
x+e)+1)^8/(b/cos(f*x+e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)}\,\sin\left(fx+e\right)^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)^(17/2)/(b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*sec(f*x + e))*sin(f*x + e)^(17/2)), x)
```

mupad [B] time = 6.41, size = 192, normalized size = 1.59

$$\frac{e^{-e\,8\mathrm{i}-f\,x\,8\mathrm{i}}\sqrt{\frac{b}{\frac{e^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}}{2}+\frac{e^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}}{2}}}\left(\frac{e^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,1024\mathrm{i}}{77\,b\,f}+\frac{e^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos\left(2\,e+2\,f\,x\right)384\mathrm{i}}{55\,b\,f}-\frac{e^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos\left(4\,e+4\,f\,x\right)5248\mathrm{i}}{1155\,b\,f}+\frac{e^{e\,8\mathrm{i}+f\,x\,8\mathrm{i}}\,\cos(6}{165\,b}\right.}{128\sin\left(e+f\,x\right)^{15/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^(17/2)*(b/cos(e + f*x))^(1/2)),x)
```

```
[Out] (exp(- e*8i - f*x*8i)*(b/(exp(- e*1i - f*x*1i)/2 + exp(e*1i + f*x*1i)/2))^(
1/2)*((exp(e*8i + f*x*8i)*1024i)/(77*b*f) + (exp(e*8i + f*x*8i)*cos(2*e + 2
*f*x)*384i)/(55*b*f) - (exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*5248i)/(1155*b*
```

```
f) + (exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*256i)/(165*b*f) - (exp(e*8i + f*x
*8i)*cos(8*e + 8*f*x)*256i)/(1155*b*f))*1i)/(128*sin(e + f*x)^(15/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(f*x+e)**(17/2)/(b*sec(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

3.472 $\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=490

$$-\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{a\sin(e+fx)}}{\sqrt{a}\,\sqrt{b\cos(e+fx)}}\right)}{128\sqrt{2}\,b^{5/2}f}+\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}\left(\right.}{128\sqrt{2}\,b^{5/2}f}$$

```
[Out] -7/192*a^3*(a*sin(f*x+e))^(3/2)/b/f/(b*sec(f*x+e))^(1/2)-1/48*a*(a*sin(f*x+
e))^(7/2)/b/f/(b*sec(f*x+e))^(1/2)+1/6*(a*sin(f*x+e))^(11/2)/a/b/f/(b*sec(f
*x+e))^(1/2)-7/256*a^(9/2)*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^
(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/
2)/f*2^(1/2)+7/256*a^(9/2)*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^
(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/
2)/f*2^(1/2)+7/512*a^(9/2)*ln(a^(1/2)-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/
(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e)
)^(1/2)/b^(5/2)/f*2^(1/2)-7/512*a^(9/2)*ln(a^(1/2)+2^(1/2)*b^(1/2)*(a*sin(f
*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))*(b*cos(f*x+e))^(1/2)*
(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2582, 2583, 2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{a\sin(e+fx)}}{\sqrt{a}\,\sqrt{b\cos(e+fx)}}\right)}{128\sqrt{2}\,b^{5/2}f}+\frac{7a^{9/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}\left(\right.}{128\sqrt{2}\,b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(9/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-7*a^(9/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt
[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(128*Sqrt[2]*
b^(5/2)*f) + (7*a^(9/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(
Sqrt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(
128*Sqrt[2]*b^(5/2)*f) + (7*a^(9/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqr
t[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f
*x]]*Sqrt[b*Sec[e + f*x]])/(256*Sqrt[2]*b^(5/2)*f) - (7*a^(9/2)*Sqrt[b*Cos[
e + f*x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e
 + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(256*Sqrt[2]*b^(5/2)
*f) - (7*a^3*(a*Sin[e + f*x])^(3/2))/(192*b*f*Sqrt[b*Sec[e + f*x]]) - (a*(a
*Sin[e + f*x])^(7/2))/(48*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x])^(11/
2)/(6*a*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
```

```
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2582

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m - n)), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{9/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{9/2}\,dx}{12b^2}$$

$$= -\frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{(7a^2)\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx)}{96b^2}$$

$$= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{(7a^4)\int \sqrt{b\,s}}{}$$

$$= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{(7a^4\sqrt{b\cos(e}}{}$$

$$= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{(7a^5\sqrt{b\cos(e}}{}$$

$$= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} - \frac{(7a^5\sqrt{b\cos(e}}{}$$

$$= -\frac{7a^3(a\sin(e+fx))^{3/2}}{192bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{7/2}}{48bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{11/2}}{6abf\sqrt{b\sec(e+fx)}} + \frac{(7a^5\sqrt{b\cos(e}}{}$$

$$= \frac{7a^{9/2}\sqrt{b\cos(e+fx)}\,\log\left(\sqrt{a} - \frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}} + \sqrt{a}\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{256\sqrt{2}\,b^{5/2}f} -$$

$$= -\frac{7a^{9/2}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{128\sqrt{2}\,b^{5/2}f} + \frac{7a^{9/2}\tan^{-1}\left(1+\right.}{}$$

Mathematica [C] time = 0.43, size = 97, normalized size = 0.20

$$-\frac{a^5\tan^2(e+fx)\left(-14\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right) + 9\cos(2(e+fx)) + 3\cos(4(e+fx)) - 2\cos(6(e+fx)) + 4\right)}{384bf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(9/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/384*(a^5*(4 + 9*Cos[2*(e + f*x)] + 3*Cos[4*(e + f*x)] - 2*Cos[6*(e + f*x
)] - 14*Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2])*Tan[e + f*x]^2)/(b
*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sin\left(fx+e\right)\right)^{\frac{9}{2}}}{\left(b \sec\left(fx+e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(9/2)/(b*sec(f*x + e))^(3/2), x)
```

maple [C] time = 0.21, size = 572, normalized size = 1.17

$$\left(64 \left(\cos^{6}\left(fx+e\right)\right) \sqrt{2} - 64 \left(\cos^{5}\left(fx+e\right)\right) \sqrt{2} - 120 \left(\cos^{4}\left(fx+e\right)\right) \sqrt{2} - 21 i \sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\, \sqrt{\frac{-1+\cos}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/768/f*(64*cos(f*x+e)^6*2^(1/2)-64*cos(f*x+e)^5*2^(1/2)-120*cos(f*x+e)^4*2
^(1/2)-21*I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi((
(1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+21*I*((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+120*cos(f*x+e)^3*2^(1/2
)+21*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e
```

```
))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+21*((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+42*cos(f*x+e)^2*2^(1/2)-42*cos(f
*x+e)*2^(1/2))*(a*sin(f*x+e))^(9/2)/(-1+cos(f*x+e))/sin(f*x+e)^3/cos(f*x+e)
^2/(b/cos(f*x+e))^(3/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sin\left(fx+e\right)\right)^{\frac{9}{2}}}{\left(b \sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(9/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(9/2)/(b*sec(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,\sin\left(e+f\,x\right)\right)}^{9/2}}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(9/2)/(b/cos(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(9/2)/(b/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(9/2)/(b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

3.473 $\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=453

$$-\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}\,b^{5/2}f}+\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}(}{32\sqrt{2}\,b^{5/2}f}$$

```
[Out] -1/16*a*(a*sin(f*x+e))^(3/2)/b/f/(b*sec(f*x+e))^(1/2)+1/4*(a*sin(f*x+e))^(7
/2)/a/b/f/(b*sec(f*x+e))^(1/2)-3/64*a^(5/2)*arctan(1-2^(1/2)*b^(1/2)*(a*sin
(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*
x+e))^(1/2)/b^(5/2)/f*2^(1/2)+3/64*a^(5/2)*arctan(1+2^(1/2)*b^(1/2)*(a*sin(
f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x
+e))^(1/2)/b^(5/2)/f*2^(1/2)+3/128*a^(5/2)*ln(a^(1/2)-2^(1/2)*b^(1/2)*(a*si
n(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))*(b*cos(f*x+e))^(1/
2)*(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)-3/128*a^(5/2)*ln(a^(1/2)+2^(1/2)*
b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))*(b*co
s(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2582, 2583, 2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{32\sqrt{2}\,b^{5/2}f}+\frac{3a^{5/2}\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\,\tan^{-1}(}{32\sqrt{2}\,b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(5/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-3*a^(5/2)*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt
[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(32*Sqrt[2]*b
^(5/2)*f) + (3*a^(5/2)*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(S
qrt[a]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(3
2*Sqrt[2]*b^(5/2)*f) + (3*a^(5/2)*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[
2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x
]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*b^(5/2)*f) - (3*a^(5/2)*Sqrt[b*Cos[e +
 f*x]]*Log[Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e +
f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(64*Sqrt[2]*b^(5/2)*f)
- (a*(a*Sin[e + f*x])^(3/2))/(16*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x
])^(7/2)/(4*a*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
```

```
& LtQ[m, 1]
```

Rule 2582

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m - n)), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{5/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} + \frac{\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{5/2}\,dx}{8b^2}$$

$$= -\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} + \frac{\left(3a^2\right)\int \sqrt{b\sec(e+fx)}\,\sqrt{a\sin(e+fx}}{32b^2}$$

$$= -\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} + \frac{\left(3a^2\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)}{32b^2}$$

$$= -\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} + \frac{\left(3a^3\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)}{16b}$$

$$= -\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} - \frac{\left(3a^3\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)}{32b^2}$$

$$= -\frac{a(a\sin(e+fx))^{3/2}}{16bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{7/2}}{4abf\sqrt{b\sec(e+fx)}} + \frac{\left(3a^3\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)}{64}$$

$$= \frac{3a^{5/2}\sqrt{b\cos(e+fx)}\,\log\left(\sqrt{a} - \frac{\sqrt{2}\,\sqrt{b}\,\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}} + \sqrt{a}\,\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{64\sqrt{2}\,b^{5/2}f} -$$

$$= -\frac{3a^{5/2}\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt{b}\,\sqrt{a\sin(e+fx)}}{\sqrt{a}\,\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}}{32\sqrt{2}\,b^{5/2}f} + \frac{3a^{5/2}\tan^{-1}\left(1+\right.}{}$$

Mathematica [C] time = 0.32, size = 82, normalized size = 0.18

$$-\frac{a^3\tan^2(e+fx)\left(-2\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)+\cos(2(e+fx))+\cos(4(e+fx))\right)}{32bf\sqrt{a\sin(e+fx)}\,\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(5/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/32*(a^3*(Cos[2*(e + f*x)] + Cos[4*(e + f*x)] - 2*Hypergeometric2F1[3/4,
1, 7/4, -Tan[e + f*x]^2])*Tan[e + f*x]^2)/(b*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*
Sin[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(f*x + e))^(5/2)/(b*sec(f*x + e))^(3/2), x)
```

maple [C] time = 0.20, size = 546, normalized size = 1.21

$$-\frac{\left(8\left(\cos^4\left(fx+e\right)\right)\sqrt{2}\;+3i\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\;\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\;\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\;\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos(fx}{\sin}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x)
[Out] -1/64/f*(8*cos(f*x+e)^4*2^(1/2)+3*I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^
(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*
x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2
*I,1/2*2^(1/2))-3*I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-
8*cos(f*x+e)^3*2^(1/2)-3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1
/2))-3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-co
```

```
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*cos(f*x+e)^
2*2^(1/2)+6*cos(f*x+e)*2^(1/2))*(a*sin(f*x+e))^(5/2)/(-1+cos(f*x+e))/sin(f*
x+e)/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a \sin\left(fx+e\right)\right)^{\frac{5}{2}}}{\left(b \sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(5/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(5/2)/(b*sec(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,\sin\left(e+f\,x\right)\right)^{5/2}}{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(5/2)/(b/cos(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(5/2)/(b/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(5/2)/(b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

3.474 $\int \frac{\sqrt{a \sin(e+fx)}}{(b \sec(e+fx))^{3/2}} \, dx$

Optimal. Leaf size=418

$$-\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}b^{5/2}f}+\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{}}\right.}{4\sqrt{2}b^{5/2}f}$$

```
[Out] 1/2*(a*sin(f*x+e))^(3/2)/a/b/f/(b*sec(f*x+e))^(1/2)-1/8*arctan(1-2^(1/2)*b^
(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*a^(1/2)*(b*cos(f*x
+e))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)+1/8*arctan(1+2^(1/2)*b^(1
/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2))*a^(1/2)*(b*cos(f*x+e
))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)+1/16*ln(a^(1/2)-2^(1/2)*b^(
1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+e))*a^(1/2)*
(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)-1/16*ln(a^(1/2)
+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2)*tan(f*x+
e))*a^(1/2)*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/b^(5/2)/f*2^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2582, 2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{4\sqrt{2}b^{5/2}f}+\frac{\sqrt{a}\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{}}\right.}{4\sqrt{2}b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[e + f*x]]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(Sqrt[a]*ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b
*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*Sqrt[2]*b^(5
/2)*f) + (Sqrt[a]*ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a
]*Sqrt[b*Cos[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(4*Sqrt
[2]*b^(5/2)*f) + (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]*Sqrt[
b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[
b*Sec[e + f*x]])/(8*Sqrt[2]*b^(5/2)*f) - (Sqrt[a]*Sqrt[b*Cos[e + f*x]]*Log[
Sqrt[a] + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqr
t[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(8*Sqrt[2]*b^(5/2)*f) + (a*Sin[e +
 f*x])^(3/2)/(2*a*b*f*Sqrt[b*Sec[e + f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2582

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m - n)), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a\sin(e+fx)}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\int \sqrt{b\sec(e+fx)}\,\sqrt{a\sin(e+fx)}\,dx}{4b^2}$$

$$= \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\left(\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\int \frac{\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}\,dx}{4b^2}$$

$$= \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\left(a\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{x^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\,\text{s}}}{\sqrt{b\,\text{c}}}\right.}{2bf}$$

$$= \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} - \frac{\left(a\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{a-bx^2}{a^2+b^2x^4}\,dx, x, \frac{\sqrt{a\,\text{s}}}{\sqrt{b\,\text{c}}}\right.}{4b^2f}$$

$$= \frac{(a\sin(e+fx))^{3/2}}{2abf\sqrt{b\sec(e+fx)}} + \frac{\left(a\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}-\frac{\sqrt{2}\sqrt{a}x}{\sqrt{b}}+x^2}\,dx, x\right.}{8b^3f}$$

$$= \frac{\sqrt{a}\,\sqrt{b\cos(e+fx)}\,\log\left(\sqrt{a}-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}}+\sqrt{a}\,\tan(e+fx)\right)\sqrt{b\sec(e+fx)}}{8\sqrt{2}\,b^{5/2}f} -$$

$$= -\frac{\sqrt{a}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}}{4\sqrt{2}\,b^{5/2}f} + \frac{\sqrt{a}\,\tan^{-1}\left(1+\right.}{}$$

Mathematica [C] time = 0.23, size = 76, normalized size = 0.18

$$\frac{\sec^2(e+fx)\sqrt{a\sin(e+fx)}\left(2\tan(e+fx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)+3\sin(2(e+fx))\right)}{12f(b\sec(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[e + f*x]]/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Sec[e + f*x]^2*Sqrt[a*Sin[e + f*x]]*(3*Sin[2*(e + f*x)] + 2*Hypergeometric
2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*Tan[e + f*x]))/(12*f*(b*Sec[e + f*x])^(3/
2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))/(b*sec(f*x + e))^(3/2), x)
```

maple [C] time = 0.21, size = 516, normalized size = 1.23

$$\left(i\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{1}{2}+\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/8/f*(I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f
*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-I*((1-cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))+((1-cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/
2),1/2+1/2*I,1/2*2^(1/2))+((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(
1/2))+2*cos(f*x+e)^2*2^(1/2)-2*cos(f*x+e)*2^(1/2))*(a*sin(f*x+e))^(1/2)*sin
(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(fx+e\right)}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(1/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e))/(b*sec(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,\sin\left(e+f\,x\right)}}{{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(1/2)/(b/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a*sin(e + f*x))^(1/2)/(b/cos(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\sin\left(e+fx\right)}}{\left(b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(1/2)/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(sqrt(a*sin(e + f*x))/(b*sec(e + f*x))**(3/2), x)
```

3.475 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=411

$$\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}a^{3/2}b^{5/2}f}-\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(}{\sqrt{a}\sqrt{b\cos(e}}\right.}{\sqrt{2}a^{3/2}b^{5/2}f}$$

```
[Out] 1/2*arctan(1-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1
/2))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/a^(3/2)/b^(5/2)/f*2^(1/2)-1/
2*arctan(1+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/a^(1/2)/(b*cos(f*x+e))^(1/2
))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/a^(3/2)/b^(5/2)/f*2^(1/2)-1/4*
ln(a^(1/2)-2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(1/2)+a^(1/2
)*tan(f*x+e))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/a^(3/2)/b^(5/2)/f*2
^(1/2)+1/4*ln(a^(1/2)+2^(1/2)*b^(1/2)*(a*sin(f*x+e))^(1/2)/(b*cos(f*x+e))^(
1/2)+a^(1/2)*tan(f*x+e))*(b*cos(f*x+e))^(1/2)*(b*sec(f*x+e))^(1/2)/a^(3/2)/
b^(5/2)/f*2^(1/2)-2/a/b/f/(b*sec(f*x+e))^(1/2)/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {2581, 2585, 2574, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)}{\sqrt{2}a^{3/2}b^{5/2}f}-\frac{\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(}{\sqrt{a}\sqrt{b\cos(e}}\right.}{\sqrt{2}a^{3/2}b^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (ArcTan[1 - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Cos[e +
f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*a^(3/2)*b^(5/2)
*f) - (ArcTan[1 + (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/(Sqrt[a]*Sqrt[b*Co
s[e + f*x]])]*Sqrt[b*Cos[e + f*x]]*Sqrt[b*Sec[e + f*x]])/(Sqrt[2]*a^(3/2)*b
^(5/2)*f) - (Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] - (Sqrt[2]*Sqrt[b]*Sqrt[a*Sin
[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e + f*x]]*Sqrt[b*Sec[e + f*x
]])/(2*Sqrt[2]*a^(3/2)*b^(5/2)*f) + (Sqrt[b*Cos[e + f*x]]*Log[Sqrt[a] + (Sq
rt[2]*Sqrt[b]*Sqrt[a*Sin[e + f*x]])/Sqrt[b*Cos[e + f*x]] + Sqrt[a]*Tan[e +
f*x]]*Sqrt[b*Sec[e + f*x]])/(2*Sqrt[2]*a^(3/2)*b^(5/2)*f) - 2/(a*b*f*Sqrt[b
*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2574

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> With[{k = Denominator[m]}, Dist[(k*a*b)/f, Subst[Int[x^(k*
(m + 1) - 1)/(a^2 + b^2*x^(2*k)), x], x, (a*Sin[e + f*x])^(1/k)/(b*Cos[e +
f*x])^(1/k)], x]] /; FreeQ[{a, b, e, f}, x] && EqQ[m + n, 0] && GtQ[m, 0] &
& LtQ[m, 1]
```

Rule 2581

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m + 1)), x] - Dist[(n + 1)/(a^2*b^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n,
 -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rubi steps

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{3/2}}\,dx = -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} - \frac{\int \sqrt{b\sec(e+fx)}\sqrt{a\sin(e+}}{a^2b^2}$$

$$= -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} - \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+}\right.}{a^2b^2}$$

$$= -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} - \frac{\left(2\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+}\right.}{}$$

$$= -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} + \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+}\right.}{}$$

$$= -\frac{2}{abf\sqrt{b\sec(e+fx)}\sqrt{a\sin(e+fx)}} - \frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+}\right.}{}$$

$$= -\frac{\sqrt{b\cos(e+fx)}\log\left(\sqrt{a} - \frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{b\cos(e+fx)}} + \sqrt{a}\tan(e+fx)\right)\sqrt{b}}{2\sqrt{2}a^{3/2}b^{5/2}f}$$

$$= \frac{\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt{b}\sqrt{a\sin(e+fx)}}{\sqrt{a}\sqrt{b\cos(e+fx)}}\right)\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}}{\sqrt{2}a^{3/2}b^{5/2}f} - \frac{\tan^{-}}{}$$

Mathematica [C] time = 0.19, size = 66, normalized size = 0.16

$$-\frac{2\left(\tan^2(e+fx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(e+fx)\right)+3\right)}{3abf\sqrt{a\sin(e+fx)}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(3/2)),x]
```

```
[Out] (-2*(3 + Hypergeometric2F1[3/4, 1, 7/4, -Tan[e + f*x]^2]*Tan[e + f*x]^2))/(
3*a*b*f*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(3/2)), x)
```

maple [C] time = 0.17, size = 957, normalized size = 2.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(I*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ell
ipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))
-I*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+
```

```
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi
(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))-((1-co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))*cos(f*x+e)-((1-cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))*cos(f*x+e)+I*((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+c
os(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*
I,1/2*2^(1/2))-((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)
+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticP
i(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2-1/2*I,1/2*2^(1/2))-((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticPi(((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*cos(f*x+e)*2^(1/2))*sin
(f*x+e)/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a\,\sin\left(e+f\,x\right)\right)^{3/2}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(3/2)*(b/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a*sin(e + f*x))^(3/2)*(b/cos(e + f*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.476 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx$

Optimal. Leaf size=35

$$-\frac{2b}{5af(a\sin(e+fx))^{5/2}(b\sec(e+fx))^{5/2}}$$

```
[Out] -2/5*b/a/f/(b*sec(f*x+e))^(5/2)/(a*sin(f*x+e))^(5/2)
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2578}

$$-\frac{2b}{5af(a\sin(e+fx))^{5/2}(b\sec(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(7/2)),x]
```

```
[Out] (-2*b)/(5*a*f*(b*Sec[e + f*x])^(5/2)*(a*Sin[e + f*x])^(5/2))
```

Rule 2578

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1
))/(a*f*(m + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m - n + 2, 0] &
& NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{7/2}}\,dx = -\frac{2b}{5af(b\sec(e+fx))^{5/2}(a\sin(e+fx))^{5/2}}$$

Mathematica [A] time = 0.11, size = 45, normalized size = 1.29

$$-\frac{2\cot^3(e+fx)\sqrt{a\sin(e+fx)}\,\sqrt{b\sec(e+fx)}}{5a^4b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(7/2)),x]
```

```
[Out] (-2*Cot[e + f*x]^3*Sqrt[b*Sec[e + f*x]]*Sqrt[a*Sin[e + f*x]])/(5*a^4*b^2*f)
```

fricas [B] time = 1.10, size = 68, normalized size = 1.94

$$\frac{2 \sqrt{a \sin\left(fx+e\right)} \sqrt{\frac{b}{\cos\left(fx+e\right)}} \cos\left(fx+e\right)^{3}}{5 \left(a^{4} b^{2} f \cos\left(fx+e\right)^{2} - a^{4} b^{2} f\right) \sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x, algorithm="fricas"
)
```

```
[Out] 2/5*sqrt(a*sin(f*x + e))*sqrt(b/cos(f*x + e))*cos(f*x + e)^3/((a^4*b^2*f*co
s(f*x + e)^2 - a^4*b^2*f)*sin(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sec\left(fx+e\right)\right)^{\frac{3}{2}} \left(a \sin\left(fx+e\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(7/2)), x)
```

maple [A] time = 0.14, size = 40, normalized size = 1.14

$$-\frac{2 \sin\left(fx+e\right) \cos\left(fx+e\right)}{5 f \left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}} \left(a \sin\left(fx+e\right)\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x)
```

```
[Out] -2/5/f*sin(f*x+e)*cos(f*x+e)/(b/cos(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \sec\left(fx+e\right)\right)^{\frac{3}{2}} \left(a \sin\left(fx+e\right)\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(7/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(7/2)), x)
```

mupad [B] time = 1.86, size = 84, normalized size = 2.40

$$\frac{\sqrt{\frac{b}{\cos\left(e+f\,x\right)}}\,\left(\cos\left(3\,e+3\,f\,x\right)-2\,\cos\left(e+f\,x\right)+\cos\left(5\,e+5\,f\,x\right)\right)}{5\,a^3\,b^2\,f\,\sqrt{a\,\sin\left(e+f\,x\right)}\,\left(\cos\left(4\,e+4\,f\,x\right)-4\,\cos\left(2\,e+2\,f\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(7/2)*(b/cos(e + f*x))^(3/2)),x)
```

```
[Out] ((b/cos(e + f*x))^(1/2)*(cos(3*e + 3*f*x) - 2*cos(e + f*x) + cos(5*e + 5*f*
x)))/(5*a^3*b^2*f*(a*sin(e + f*x))^(1/2)*(cos(4*e + 4*f*x) - 4*cos(2*e + 2*
f*x) + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

3.477 $\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=172

$$\frac{a^4\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{24b^2f\sqrt{a\sin(e+fx)}}-\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}-\frac{a(a\sin(e+fx}{30bf\sqrt{b\sec(e}$$

```
[Out] -1/30*a*(a*sin(f*x+e))^(5/2)/b/f/(b*sec(f*x+e))^(1/2)+1/5*(a*sin(f*x+e))^(9
/2)/a/b/f/(b*sec(f*x+e))^(1/2)-1/12*a^3*(a*sin(f*x+e))^(1/2)/b/f/(b*sec(f*x
+e))^(1/2)-1/24*a^4*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF
(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/b^2/f
/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2582, 2583, 2585, 2573, 2641}

$$\frac{a^4\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{24b^2f\sqrt{a\sin(e+fx)}}-\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}}+\frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}}-\frac{a(a\sin(e+fx}{30bf\sqrt{b\sec(e}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(7/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(a^3*Sqrt[a*Sin[e + f*x]])/(12*b*f*Sqrt[b*Sec[e + f*x]]) - (a*(a*Sin[e + f
*x])^(5/2))/(30*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x])^(9/2)/(5*a*b*f
*Sqrt[b*Sec[e + f*x]]) + (a^4*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec[e + f
*x]]*Sqrt[Sin[2*e + 2*f*x]])/(24*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2582

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m - n)), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{7/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{7/2}\,dx}{10b^2}$$

$$= -\frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{a^2\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/}}{12b^2}$$

$$= -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{a^4\int \frac{\sqrt{b\sec(e+}}{\sqrt{a\sin(e+}}}{24b^2}$$

$$= -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{\left(a^4\sqrt{b\cos(e\right.}}{}$$

$$= -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{\left(a^4\sqrt{b\sec(e\right.}}{}$$

$$= -\frac{a^3\sqrt{a\sin(e+fx)}}{12bf\sqrt{b\sec(e+fx)}} - \frac{a(a\sin(e+fx))^{5/2}}{30bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{9/2}}{5abf\sqrt{b\sec(e+fx)}} + \frac{a^4F\left(e-\frac{\pi}{4}+\right.}{}$$

Mathematica [C] time = 0.85, size = 103, normalized size = 0.60

$$-\frac{a^5\left(-20\left(-\tan^2(e+fx)\right)^{3/4}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)+17\cos(2(e+fx))-16\cos(4(e+fx))+3\cos(6(e+fx$$
$$480bf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(7/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -1/480*(a^5*(-4 + 17*Cos[2*(e + f*x)] - 16*Cos[4*(e + f*x)] + 3*Cos[6*(e +
f*x)] - 20*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^
2)^(3/4)))/(b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^3\cos\left(fx+e\right)^2-a^3\right)\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}\sin\left(fx+e\right)}{b^2\sec\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^3*cos(f*x + e)^2 - a^3)*sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x +
e))*sin(f*x + e)/(b^2*sec(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(7/2)/(b*sec(f*x + e))^(3/2), x)
```

maple [A] time = 0.21, size = 246, normalized size = 1.43

$$-\frac{\left(-12\left(\cos^6\left(fx+e\right)\right)\sqrt{2}+12\left(\cos^5\left(fx+e\right)\right)\sqrt{2}+5\sin\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right.}{\sin\left(fx+e\right)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/120/f*(-12*cos(f*x+e)^6*2^(1/2)+12*cos(f*x+e)^5*2^(1/2)+5*sin(f*x+e)*((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f
*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+22*cos(f*x+e)^4*2^(1/2)-22*cos(f*x
+e)^3*2^(1/2)-5*cos(f*x+e)^2*2^(1/2)+5*cos(f*x+e)*2^(1/2))*(a*sin(f*x+e))^(
7/2)/(-1+cos(f*x+e))/sin(f*x+e)^3/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{7}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e))^(7/2)/(b*sec(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,\sin\left(e+f\,x\right)\right)^{7/2}}{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(7/2)/(b/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a*sin(e + f*x))^(7/2)/(b/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(7/2)/(b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.478 $\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=135

$$\frac{a^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{12b^2f\sqrt{a\sin(e+fx)}}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}-\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}}$$

```
[Out] 1/3*(a*sin(f*x+e))^(5/2)/a/b/f/(b*sec(f*x+e))^(1/2)-1/6*a*(a*sin(f*x+e))^(1
/2)/b/f/(b*sec(f*x+e))^(1/2)-1/12*a^2*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4
*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*
x+2*e)^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2582, 2583, 2585, 2573, 2641}

$$\frac{a^2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{12b^2f\sqrt{a\sin(e+fx)}}+\frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}}-\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[e + f*x])^(3/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] -(a*Sqrt[a*Sin[e + f*x]])/(6*b*f*Sqrt[b*Sec[e + f*x]]) + (a*Sin[e + f*x])^(
5/2)/(3*a*b*f*Sqrt[b*Sec[e + f*x]]) + (a^2*EllipticF[e - Pi/4 + f*x, 2]*Sqr
t[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(12*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2582

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m - n)), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2583

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*b*(a*Sin[e + f*x])^(m - 1)*(b*Sec[e + f*x])^(n -
1))/(f*(m - n)), x] + Dist[(a^2*(m - 1))/(m - n), Int[(a*Sin[e + f*x])^(m -
 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(a\sin(e+fx))^{3/2}}{(b\sec(e+fx))^{3/2}}\,dx = \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{\int \sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}\,dx}{6b^2}$$

$$= -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{a^2\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{12b^2}$$

$$= -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{\left(a^2\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e+fx)}\right)\int}{12b^2}$$

$$= -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{\left(a^2\sqrt{b\sec(e+fx)}\,\sqrt{\sin(2e+2fx)}\right)\int}{12b^2\sqrt{a\sin(e+fx)}}$$

$$= -\frac{a\sqrt{a\sin(e+fx)}}{6bf\sqrt{b\sec(e+fx)}} + \frac{(a\sin(e+fx))^{5/2}}{3abf\sqrt{b\sec(e+fx)}} + \frac{a^2F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(e+fx)}\,\sqrt{}}{12b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] time = 0.41, size = 87, normalized size = 0.64

$$\frac{a\sqrt{a\sin(e+fx)}\left(\left(-\tan^2(e+fx)\right)^{3/4}\csc^2(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)-2\cos(2(e+fx))\right)}{12bf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[e + f*x])^(3/2)/(b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a*Sin[e + f*x]]*(-2*Cos[2*(e + f*x)] + Csc[e + f*x]^2*Hypergeometri
c2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(12*b*f*Sqrt[
b*Sec[e + f*x]])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}\,a\sin\left(fx+e\right)}{b^2\sec\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))*a*sin(f*x + e)/(b^2*sec(
f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^(3/2)/(b*sec(f*x + e))^(3/2), x)
```

maple [A] time = 0.20, size = 218, normalized size = 1.61

$$-\frac{\left(\sin\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\text{EllipticF}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right.\right.}{12f\left(-1+\cos\left(fx+e\right)\right)\sin\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/12/f*(sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
```

```
pticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*cos(f*x+e
)^4*2^(1/2)-2*cos(f*x+e)^3*2^(1/2)-cos(f*x+e)^2*2^(1/2)+cos(f*x+e)*2^(1/2))
*(a*sin(f*x+e))^(3/2)/(-1+cos(f*x+e))/sin(f*x+e)/cos(f*x+e)^2/(b/cos(f*x+e)
)^(3/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sin\left(fx+e\right)\right)^{\frac{3}{2}}}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^(3/2)/(b*sec(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,\sin\left(e+f\,x\right)\right)^{3/2}}{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^(3/2)/(b/cos(e + f*x))^(3/2),x)
[Out] int((a*sin(e + f*x))^(3/2)/(b/cos(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(f*x+e))**(3/2)/(b*sec(f*x+e))**(3/2),x)
[Out] Timed out
```

3.479 $\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx$

Optimal. Leaf size=94

$$\frac{\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{2b^2f\sqrt{a\sin(e+fx)}}+\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}$$

```
[Out] (a*sin(f*x+e))^(1/2)/a/b/f/(b*sec(f*x+e))^(1/2)-1/2*(sin(e+1/4*Pi+f*x)^2)^(
1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^
(1/2)*sin(2*f*x+2*e)^(1/2)/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2582, 2585, 2573, 2641}

$$\frac{\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{2b^2f\sqrt{a\sin(e+fx)}}+\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*Sqrt[a*Sin[e + f*x]]),x]
```

```
[Out] Sqrt[a*Sin[e + f*x]]/(a*b*f*Sqrt[b*Sec[e + f*x]]) + (EllipticF[e - Pi/4 + f
*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(2*b^2*f*Sqrt[a*Sin[e +
 f*x]])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2582

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m - n)), x] - Dist[(n + 1)/(b^2*(m - n)), Int[(a*Sin[e + f*x])^m*(b*
Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1] &&
 NeQ[m - n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\sec(e+fx))^{3/2}\sqrt{a\sin(e+fx)}}\,dx=\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}+\frac{\int\frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{2b^2}$$

$$=\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}+\frac{\left(\sqrt{b\cos(e+fx)}\sqrt{b\sec(e+fx)}\right)\int\frac{1}{\sqrt{b\cos(e+fx)}\ \cdots}}{2b^2}$$

$$=\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}+\frac{\left(\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2fx)}\right)\int\frac{1}{\sqrt{\sin(2e+2fx)}}}{2b^2\sqrt{a\sin(e+fx)}}$$

$$=\frac{\sqrt{a\sin(e+fx)}}{abf\sqrt{b\sec(e+fx)}}+\frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(e+fx)}\sqrt{\sin(2e+2f\cdots}}{2b^2f\sqrt{a\sin(e+fx)}}$$

Mathematica [C] time = 0.53, size = 84, normalized size = 0.89

$$-\frac{\cot(e+fx)\sqrt{b\sec(e+fx)}\left(-\left(-\tan^2(e+fx)\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)+\cos(2(e+fx))-1\right)}{2b^2f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*Sqrt[a*Sin[e + f*x]]),x]
```

```
[Out] -1/2*(Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*(-1 + Cos[2*(e + f*x)] - Hypergeome
tric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(b^2*f*Sqr
t[a*Sin[e + f*x]])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}}{ab^2\sec\left(fx+e\right)^2\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/(a*b^2*sec(f*x + e)^2*si
n(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*sqrt(a*sin(f*x + e))), x)
```

maple [A] time = 0.17, size = 190, normalized size = 2.02

$$-\frac{\left(\sin\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\cdots\right.\right.}{2f\left(-1+\cos\left(fx+e\right)\right)\cos\left(fx+e\right)^{2}\left(\frac{b}{\cos\left(fx+e\right)}\right)^{\frac{3}{2}}\sqrt{a\sin\left(\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/2/f*(sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellip
ticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)^2
*2^(1/2)+cos(f*x+e)*2^(1/2))*sin(f*x+e)/(-1+cos(f*x+e))/cos(f*x+e)^2/(b/cos
(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\sqrt{a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*sqrt(a*sin(f*x + e))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a\sin\left(e+fx\right)}\left(\frac{b}{\cos\left(e+fx\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(1/2)*(b/cos(e + f*x))^(3/2)),x)
[Out] int(1/((a*sin(e + f*x))^(1/2)*(b/cos(e + f*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(1/2),x)
[Out] Timed out
```

3.480 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{3a^2b^2f\sqrt{a\sin(e+fx)}}-\frac{2}{3abf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

```
[Out] -2/3/a/b/f/(a*sin(f*x+e))^(3/2)/(b*sec(f*x+e))^(1/2)+1/3*(sin(e+1/4*Pi+f*x)
^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x
+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/a^2/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2581, 2585, 2573, 2641}

$$-\frac{\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{3a^2b^2f\sqrt{a\sin(e+fx)}}-\frac{2}{3abf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -2/(3*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) - (EllipticF[e - P
i/4 + f*x, 2]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(3*a^2*b^2*f*Sqr
t[a*Sin[e + f*x]])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2581

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m + 1)), x] - Dist[(n + 1)/(a^2*b^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n,
 -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
```

```
f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{5/2}}\,dx = -\frac{2}{3abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}} - \frac{\int \frac{\sqrt{b\sec(e+fx)}}{\sqrt{a\sin(e+fx)}}\,dx}{3a^2b^2}$$

$$= -\frac{2}{3abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}} - \frac{\left(\sqrt{b\cos(e+fx)}\,\sqrt{b\sec(e$$

$$= -\frac{2}{3abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}} - \frac{\left(\sqrt{b\sec(e+fx)}\,\sqrt{\sin(2e}{3a^2b^2\sqrt{a\,\mathrm{si}}}$$

$$= -\frac{2}{3abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{3/2}} - \frac{F\left(e-\frac{\pi}{4}+fx\middle|2\right)\sqrt{b\sec(}{3a^2b^2f\sqrt{a\,\mathrm{si}}}$$

Mathematica [C] time = 0.48, size = 78, normalized size = 0.78

$$-\frac{\cot(e+fx)\sqrt{b\sec(e+fx)}\,\left(\left(-\tan^2(e+fx)\right)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)+2\right)}{3a^2b^2f\sqrt{a\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(5/2)),x]
```

```
[Out] -1/3*(Cot[e + f*x]*Sqrt[b*Sec[e + f*x]]*(2 + Hypergeometric2F1[1/2, 3/4, 3/
2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(a^2*b^2*f*Sqrt[a*Sin[e + f*x]
])
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{b\sec\left(fx+e\right)}\,\sqrt{a\sin\left(fx+e\right)}}{\left(a^3b^2\cos\left(fx+e\right)^2-a^3b^2\right)\sec\left(fx+e\right)^2\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/((a^3*b^2*cos(f*x + e)^
2 - a^3*b^2)*sec(f*x + e)^2*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(5/2)), x)
```

maple [B] time = 0.16, size = 284, normalized size = 2.84

$$-\frac{\left(\sin\left(fx+e\right)\cos\left(fx+e\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{1-\cos\left(fx+e\right)}{\sin\left(f\cdots\right.}}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/3/f*(sin(f*x+e)*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+s
in(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+cos(f*x+e)*2^(1/2))*s
in(f*x+e)/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,\sin\left(e+f\,x\right)\right)^{5/2}\,\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(5/2)*(b/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a*sin(e + f*x))^(5/2)*(b/cos(e + f*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

3.481 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{21a^4b^2f\sqrt{a\sin(e+fx)}}+\frac{2}{21a^3bf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}-\frac{}{7abf(a\sin(e+f}$$

```
[Out] -2/7/a/b/f/(a*sin(f*x+e))^(7/2)/(b*sec(f*x+e))^(1/2)+2/21/a^3/b/f/(a*sin(f*
x+e))^(3/2)/(b*sec(f*x+e))^(1/2)+2/21*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4
*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*
x+2*e)^(1/2)/a^4/b^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2581, 2584, 2585, 2573, 2641}

$$-\frac{2\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{21a^4b^2f\sqrt{a\sin(e+fx)}}+\frac{2}{21a^3bf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}-\frac{}{7abf(a\sin(e+f}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(9/2)),x]
```

```
[Out] -2/(7*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) + 2/(21*a^3*b*f*Sq
rt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(3/2)) - (2*EllipticF[e - Pi/4 + f*x, 2
]*Sqrt[b*Sec[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(21*a^4*b^2*f*Sqrt[a*Sin[e +
 f*x]])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2581

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m + 1)), x] - Dist[(n + 1)/(a^2*b^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n,
 -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{9/2}}\,dx = -\frac{2}{7abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{7/2}} - \frac{\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{5/2}}\,dx}{7a^2b^2}$$

$$= -\frac{2}{7abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}\,(a\,s}$$

$$= -\frac{2}{7abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}\,(a\,s}$$

$$= -\frac{2}{7abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}\,(a\,s}$$

$$= -\frac{2}{7abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{7/2}} + \frac{2}{21a^3bf\sqrt{b\sec(e+fx)}\,(a\,s}$$

Mathematica [C] time = 0.84, size = 119, normalized size = 0.87

$$\frac{\cos(2(e+fx))\csc^4(e+fx)\sqrt{a\sin(e+fx)}\left((\cos(2(e+fx))+5)\sec^2(e+fx)-2\left(-\tan^2(e+fx)\right)^{7/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\right.\right.}{21a^5bf\left(\sec^2(e+fx)-2\right)\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(9/2)),x]
```

```
[Out] (Cos[2*(e + f*x)]*Csc[e + f*x]^4*Sqrt[a*Sin[e + f*x]]*((5 + Cos[2*(e + f*x)
])*Sec[e + f*x]^2 - 2*Hypergeometric2F1[1/2, 3/4, 3/2, Sec[e + f*x]^2]*(-Ta
n[e + f*x]^2)^(7/4)))/(21*a^5*b*f*Sqrt[b*Sec[e + f*x]]*(-2 + Sec[e + f*x]^2
))
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}}{\left(a^5b^2\cos\left(fx+e\right)^4-2\,a^5b^2\cos\left(fx+e\right)^2+a^5b^2\right)\sec\left(fx+e\right)^2\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/((a^5*b^2*cos(f*x + e)^4
 - 2*a^5*b^2*cos(f*x + e)^2 + a^5*b^2)*sec(f*x + e)^2*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(9/2)), x)
```

maple [B] time = 0.18, size = 540, normalized size = 3.94

$$\left(2\sin\left(fx+e\right)\left(\cos^3\left(fx+e\right)\right)\sqrt{\frac{1-\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\ \text{EllipticF}\left(\sqrt{\frac{1-\cos(fx}{\text{si}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/21/f*(2*sin(f*x+e)*cos(f*x+e)^3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2
))+2*sin(f*x+e)*cos(f*x+e)^2*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*(
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2*
sin(f*x+e)*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos
(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*El
lipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2*sin(f*x
+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)^3*2^(1/2)-2*cos(
f*x+e)*2^(1/2))*sin(f*x+e)/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2)/(a*sin(f*x+e))
^(9/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(9/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(9/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,\sin\left(e+f\,x\right)\right)^{9/2}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(9/2)*(b/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a*sin(e + f*x))^(9/2)*(b/cos(e + f*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

3.482 $\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{4\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{77a^6b^2f\sqrt{a\sin(e+fx)}}+\frac{4}{77a^5bf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}+\frac{}{77a^3bf(a\sin(e+}$$

```
[Out] -2/11/a/b/f/(a*sin(f*x+e))^(11/2)/(b*sec(f*x+e))^(1/2)+2/77/a^3/b/f/(a*sin(
f*x+e))^(7/2)/(b*sec(f*x+e))^(1/2)+4/77/a^5/b/f/(a*sin(f*x+e))^(3/2)/(b*sec
(f*x+e))^(1/2)+4/77*(sin(e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF
(cos(e+1/4*Pi+f*x),2^(1/2))*(b*sec(f*x+e))^(1/2)*sin(2*f*x+2*e)^(1/2)/a^6/b
^2/f/(a*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2581, 2584, 2585, 2573, 2641}

$$-\frac{4\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)\sqrt{b\sec(e+fx)}}{77a^6b^2f\sqrt{a\sin(e+fx)}}+\frac{4}{77a^5bf(a\sin(e+fx))^{3/2}\sqrt{b\sec(e+fx)}}+\frac{}{77a^3bf(a\sin(e+}$$

Antiderivative was successfully verified.

```
[In] Int[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(13/2)),x]
```

```
[Out] -2/(11*a*b*f*Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(11/2)) + 2/(77*a^3*b*f*
Sqrt[b*Sec[e + f*x]]*(a*Sin[e + f*x])^(7/2)) + 4/(77*a^5*b*f*Sqrt[b*Sec[e +
 f*x]]*(a*Sin[e + f*x])^(3/2)) - (4*EllipticF[e - Pi/4 + f*x, 2]*Sqrt[b*Sec
[e + f*x]]*Sqrt[Sin[2*e + 2*f*x]])/(77*a^6*b^2*f*Sqrt[a*Sin[e + f*x]])
```

Rule 2573

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(b_.)]*Sqrt[(a_.)*sin[(e_.) + (f_.)*(x_
)]]), x_Symbol] :> Dist[Sqrt[Sin[2*e + 2*f*x]]/(Sqrt[a*Sin[e + f*x]]*Sqrt[b
*Cos[e + f*x]]), Int[1/Sqrt[Sin[2*e + 2*f*x]], x], x] /; FreeQ[{a, b, e, f}
, x]
```

Rule 2581

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n + 1))/(
a*b*f*(m + 1)), x] - Dist[(n + 1)/(a^2*b^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && LtQ[n,
 -1] && LtQ[m, -1] && IntegersQ[2*m, 2*n]
```

Rule 2584

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b*(a*Sin[e + f*x])^(m + 1)*(b*Sec[e + f*x])^(n - 1))
/(a*f*(m + 1)), x] + Dist[(m - n + 2)/(a^2*(m + 1)), Int[(a*Sin[e + f*x])^(
m + 2)*(b*Sec[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2585

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(b*Cos[e + f*x])^n*(b*Sec[e + f*x])^n, Int[(a*Sin[e +
 f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m, n}, x] && Inte
gerQ[m - 1/2] && IntegerQ[n - 1/2]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(b\sec(e+fx))^{3/2}(a\sin(e+fx))^{13/2}}\,dx = -\frac{2}{11abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{11/2}} - \frac{\int \frac{\sqrt{b\sec(e+fx)}}{(a\sin(e+fx))^{9/2}}\,dx}{11a^2b^2}$$

$$= -\frac{2}{11abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{11/2}} + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}}$$

$$= -\frac{2}{11abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{11/2}} + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}}$$

$$= -\frac{2}{11abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{11/2}} + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}}$$

$$= -\frac{2}{11abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{11/2}} + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}}$$

$$= -\frac{2}{11abf\sqrt{b\sec(e+fx)}\,(a\sin(e+fx))^{11/2}} + \frac{2}{77a^3bf\sqrt{b\sec(e+fx)}}$$

Mathematica [C] time = 1.21, size = 131, normalized size = 0.75

$$\frac{2\cot(2(e+fx))\csc(2(e+fx))\sqrt{a\sin(e+fx)}\left(8\left(-\tan^2(e+fx)\right)^{3/4}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{3}{2};\sec^2(e+fx)\right)+(6\cos(2(e+fx}{77a^7bf\left(\sec^2(e+fx)-2\right)\sqrt{b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((b*Sec[e + f*x])^(3/2)*(a*Sin[e + f*x])^(13/2)),x]
```

```
[Out] (2*Cot[2*(e + f*x)]*Csc[2*(e + f*x)]*Sqrt[a*Sin[e + f*x]]*((23 + 6*Cos[2*(e
 + f*x)] - Cos[4*(e + f*x)])*Csc[e + f*x]^4 + 8*Hypergeometric2F1[1/2, 3/4,
 3/2, Sec[e + f*x]^2]*(-Tan[e + f*x]^2)^(3/4)))/(77*a^7*b*f*Sqrt[b*Sec[e +
f*x]]*(-2 + Sec[e + f*x]^2))
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{b\sec\left(fx+e\right)}\sqrt{a\sin\left(fx+e\right)}}{\left(a^7b^2\cos\left(fx+e\right)^6-3\,a^7b^2\cos\left(fx+e\right)^4+3\,a^7b^2\cos\left(fx+e\right)^2-a^7b^2\right)\sec\left(fx+e\right)^2\sin\left(fx+e\right)},x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="fricas
")
```

```
[Out] integral(-sqrt(b*sec(f*x + e))*sqrt(a*sin(f*x + e))/((a^7*b^2*cos(f*x + e)^
6 - 3*a^7*b^2*cos(f*x + e)^4 + 3*a^7*b^2*cos(f*x + e)^2 - a^7*b^2)*sec(f*x
+ e)^2*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(13/2)), x)
```

maple [B] time = 0.22, size = 793, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x)
```

```
[Out] -1/77/f*(4*sin(f*x+e)*cos(f*x+e)^5*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(
1/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x
+e))^(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/
2))+4*sin(f*x+e)*cos(f*x+e)^4*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*
((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^
(1/2)*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-8
*sin(f*x+e)*cos(f*x+e)^3*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+
cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)
*EllipticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-8*sin(
f*x+e)*cos(f*x+e)^2*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f
*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Elli
pticF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+4*sin(f*x+e
)*cos(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF((
(1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))-2*cos(f*x+e)^5*2^(
1/2)+4*sin(f*x+e)*((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))^(1/2)*Ellipt
icF(((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2))+5*cos(f*x+e)^
3*2^(1/2)+4*cos(f*x+e)*2^(1/2))*sin(f*x+e)/cos(f*x+e)^2/(b/cos(f*x+e))^(3/2
)/(a*sin(f*x+e))^(13/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)\right)^{\frac{3}{2}}\left(a\sin\left(fx+e\right)\right)^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))^(3/2)/(a*sin(f*x+e))^(13/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*sec(f*x + e))^(3/2)*(a*sin(f*x + e))^(13/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\,\sin\left(e+f\,x\right)\right)^{13/2}\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*sin(e + f*x))^(13/2)*(b/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/((a*sin(e + f*x))^(13/2)*(b/cos(e + f*x))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*sec(f*x+e))**(3/2)/(a*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

3.483 $\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=75

$$\frac{d\cos^2(a+bx)^{3/4}(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[Out] d*(cos(b*x+a)^2)^(3/4)*hypergeom([7/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)
*(d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^(1+m)/b/c/(1+m)
```

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2577}

$$\frac{d\cos^2(a+bx)^{3/4}(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[a + b*x])^(5/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[7/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int (d\sec(a+bx))^{5/2}(c\sin(a+bx))^m\,dx = \left(d^2(d\cos(a+bx))^{3/2}(d\sec(a+bx))^{3/2}\right)\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{5/2}}\,dx$$
$$= \frac{d\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{7}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(d\sec(a+bx))^{3/2}(c\sin}{bc(1+m)}$$

Mathematica [A] time = 8.68, size = 96, normalized size = 1.28

$$-\frac{2\cot(a+bx)(d\sec(a+bx))^{5/2}\left(-\tan^2(a+bx)\right)^{\frac{1-m}{2}}(c\sin(a+bx))^m\,{}_2F_1\left(\frac{1}{4}(5-2m),\frac{1-m}{2};\frac{1}{4}(9-2m);\sec^2(a+bx\right.}{b(2m-5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(5/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (-2*Cot[a + b*x]*Hypergeometric2F1[(5 - 2*m)/4, (1 - m)/2, (9 - 2*m)/4, Sec
[a + b*x]^2]*(d*Sec[a + b*x])^(5/2)*(c*Sin[a + b*x])^m*(-Tan[a + b*x]^2)^((
1 - m)/2))/(b*(-5 + 2*m))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\sec(bx+a)}\,(c\sin(bx+a))^m d^2\sec(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m*d^2*sec(b*x + a)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((d*sec(b*x + a))^(5/2)*(c*sin(b*x + a))^m, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int (d\sec(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x)
```

```
[Out] int((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(bx+a))^{\frac{5}{2}}(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(5/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(b*x + a))^(5/2)*(c*sin(b*x + a))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,\sin(a+b\,x))^m\left(\frac{d}{\cos(a+b\,x)}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(5/2),x)
```

```
[Out] int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(5/2)*(c*sin(b*x+a))**m,x)
```

```
[Out] Timed out
```

3.484 $\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=75

$$\frac{d\sqrt[4]{\cos^2(a+bx)}\,\sqrt{d\sec(a+bx)}\,(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

```
[Out] d*(cos(b*x+a)^2)^(1/4)*hypergeom([5/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)
*(c*sin(b*x+a))^(1+m)*(d*sec(b*x+a))^(1/2)/b/c/(1+m)
```

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2577}

$$\frac{d\sqrt[4]{\cos^2(a+bx)}\,\sqrt{d\sec(a+bx)}\,(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[5/4, (1 + m)/2, (3 + m)/2, Sin[
a + b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^(1 + m))/(b*c*(1 + m))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int (d\sec(a+bx))^{3/2}(c\sin(a+bx))^m\,dx = \left(d^2\sqrt{d\cos(a+bx)}\sqrt{d\sec(a+bx)}\right)\int \frac{(c\sin(a+bx))^m}{(d\cos(a+bx))^{3/2}}\,dx$$
$$= \frac{d\sqrt[4]{\cos^2(a+bx)}\ {}_2F_1\left(\frac{5}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)\sqrt{d\sec(a+bx)}\,(c\sin(}{bc(1+m)}$$

Mathematica [A] time = 1.36, size = 96, normalized size = 1.28

$$-\frac{2\cot(a+bx)(d\sec(a+bx))^{3/2}\left(-\tan^2(a+bx)\right)^{\frac{1-m}{2}}(c\sin(a+bx))^m\,{}_2F_1\left(\frac{1}{4}(3-2m),\frac{1-m}{2};\frac{1}{4}(7-2m);\sec^2(a+b\right.}{b(2m-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^m,x]
```

```
[Out] (-2*Cot[a + b*x]*Hypergeometric2F1[(3 - 2*m)/4, (1 - m)/2, (7 - 2*m)/4, Sec
[a + b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^m*(-Tan[a + b*x]^2)^((
1 - m)/2))/(b*(-3 + 2*m))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\sec(bx+a)}\,(c\sin(bx+a))^m\,d\sec(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m*d*sec(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(bx+a))^{\frac{3}{2}}\,(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate((d*sec(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int (d\sec(bx+a))^{\frac{3}{2}}\,(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

```
[Out] int((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d\sec(bx+a))^{\frac{3}{2}}(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate((d*sec(b*x + a))^(3/2)*(c*sin(b*x + a))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,\sin(a+b\,x))^m\left(\frac{d}{\cos(a+b\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(3/2),x)
```

```
[Out] int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(3/2)*(c*sin(b*x+a))**m,x)
```

```
[Out] Timed out
```

3.485 $\int \sqrt{d \sec(a+bx)}\,(c\sin(a+bx))^m\,dx$

Optimal. Leaf size=77

$$\frac{\cos^2(a+bx)^{3/4}(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)}$$

```
[Out] (cos(b*x+a)^2)^(3/4)*hypergeom([3/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(
d*sec(b*x+a))^(3/2)*(c*sin(b*x+a))^(1+m)/b/c/d/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2586, 2577}

$$\frac{\cos^2(a+bx)^{3/4}(d\sec(a+bx))^{3/2}(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Hypergeometric2F1[3/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*(d*Sec[a + b*x])^(3/2)*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2586

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n + 1))
/b^2, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && LtQ[n, 1]
```

Rubi steps

$$\int \sqrt{d\sec(a+bx)}\,(c\sin(a+bx))^m\,dx = \frac{\left((d\cos(a+bx))^{3/2}(d\sec(a+bx))^{3/2}\right)\int \frac{(c\sin(a+bx))^m}{\sqrt{d\cos(a+bx)}}\,dx}{d^2}$$
$$= \frac{\cos^2(a+bx)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(d\sec(a+bx))^{3/2}(c\sin(a\ }{bcd(1+m)}$$

Mathematica [A] time = 1.37, size = 106, normalized size = 1.38

$$-\frac{\sin(2(a+bx))\csc^2(a+bx)\sqrt{d\sec(a+bx)}\left(-\tan^2(a+bx)\right)^{\frac{1-m}{2}}(c\sin(a+bx))^m\,{}_2F_1\left(\frac{1}{4}(1-2m),\frac{1-m}{2};\frac{1}{4}(5-2m);\right.}{b(2m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^m,x]
```

```
[Out] -((Csc[a + b*x]^2*Hypergeometric2F1[(1 - 2*m)/4, (1 - m)/2, (5 - 2*m)/4, Se
c[a + b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^m*Sin[2*(a + b*x)]*(-Ta
n[a + b*x]^2)^((1 - m)/2))/(b*(-1 + 2*m)))
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\sec(bx+a)}\,(c\sin(bx+a))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\sec(bx+a)}\,(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \sqrt{d\sec(bx+a)}\,(c\sin(bx+a))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

```
[Out] int((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \sec(bx+a)} \, (c \sin(bx+a))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))^(1/2)*(c*sin(b*x+a))^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c \sin(a+b\,x))^m \sqrt{\frac{d}{\cos(a+b\,x)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(1/2),x)
```

```
[Out] int((c*sin(a + b*x))^m*(d/cos(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c \sin(a+bx))^m \sqrt{d \sec(a+bx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sec(b*x+a))**(1/2)*(c*sin(b*x+a))**m,x)
```

```
[Out] Integral((c*sin(a + b*x))**m*sqrt(d*sec(a + b*x)), x)
```

3.486 $\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt[4]{\cos^2(a+bx)}\,\sqrt{d\sec(a+bx)}\,(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)}$$

```
[Out] (cos(b*x+a)^2)^(1/4)*hypergeom([1/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(
c*sin(b*x+a))^(1+m)*(d*sec(b*x+a))^(1/2)/b/c/d/(1+m)
```

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2586, 2577}

$$\frac{\sqrt[4]{\cos^2(a+bx)}\,\sqrt{d\sec(a+bx)}\,(c\sin(a+bx))^{m+1}\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^m/Sqrt[d*Sec[a + b*x]],x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Hypergeometric2F1[1/4, (1 + m)/2, (3 + m)/2, Sin[a
+ b*x]^2]*Sqrt[d*Sec[a + b*x]]*(c*Sin[a + b*x])^(1 + m))/(b*c*d*(1 + m))
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2586

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n + 1))
/b^2, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && LtQ[n, 1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx = \frac{\left(\sqrt{d\cos(a+bx)}\,\sqrt{d\sec(a+bx)}\right)\int \sqrt{d\cos(a+bx)}\,(c\sin(a+bx))^m\,dx}{d^2}$$

$$= \frac{\sqrt[4]{\cos^2(a+bx)}\;{}_2F_1\left(\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)\sqrt{d\sec(a+bx)}\,(c\sin(a+bx))^{1+m}}{bcd(1+m)}$$

Mathematica [C] time = 1.75, size = 289, normalized size = 3.75

$$\frac{8c(m+3)\sin^2\left(\frac{1}{2}(a+bx)\right)\cos^4\left(\frac{1}{2}(a-\right.}{b(m+1)\sqrt{d\sec(a+bx)}\left((\cos(a+bx)-1)\left((2m+3)F_1\left(\frac{m+3}{2};-\frac{1}{2},m+\frac{5}{2};\frac{m+5}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+b\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Sin[a + b*x])^m/Sqrt[d*Sec[a + b*x]],x]
```

```
[Out] (8*c*(3 + m)*AppellF1[(1 + m)/2, -1/2, 3/2 + m, (3 + m)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^4*Sin[(a + b*x)/2]^2*(c*Sin[a + b
*x])^(-1 + m))/(b*(1 + m)*(((3 + 2*m)*AppellF1[(3 + m)/2, -1/2, 5/2 + m, (5
 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + AppellF1[(3 + m)/2, 1/2
, 3/2 + m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a
 + b*x]) + (3 + m)*AppellF1[(1 + m)/2, -1/2, 3/2 + m, (3 + m)/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x]))*Sqrt[d*Sec[a + b*x]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{d\sec\left(bx+a\right)}\,\left(c\sin\left(bx+a\right)\right)^m}{d\sec\left(bx+a\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m/(d*sec(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sin\left(bx+a\right)\right)^m}{\sqrt{d\sec\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((c*sin(b*x + a))^m/sqrt(d*sec(b*x + a)), x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^m}{\sqrt{d\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x)
[Out] int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^m}{\sqrt{d\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m/sqrt(d*sec(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,\sin(a+b\,x))^m}{\sqrt{\frac{d}{\cos(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m/(d/cos(a + b*x))^(1/2),x)
[Out] int((c*sin(a + b*x))^m/(d/cos(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(a+bx))^m}{\sqrt{d\sec(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**m/(d*sec(b*x+a))**(1/2),x)
[Out] Integral((c*sin(a + b*x))**m/sqrt(d*sec(a + b*x)), x)
```

3.487 $\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)\sqrt[4]{\cos^2(a+bx)}\,\sqrt{d\sec(a+bx)}}$$

```
[Out] hypergeom([-1/4, 1/2+1/2*m],[3/2+1/2*m],sin(b*x+a)^2)*(c*sin(b*x+a))^(1+m)/
b/c/d/(1+m)/(cos(b*x+a)^2)^(1/4)/(d*sec(b*x+a))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2586, 2577}

$$\frac{(c\sin(a+bx))^{m+1}\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\sin^2(a+bx)\right)}{bcd(m+1)\sqrt[4]{\cos^2(a+bx)}\,\sqrt{d\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x])^m/(d*Sec[a + b*x])^(3/2),x]
```

```
[Out] (Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, Sin[a + b*x]^2]*(c*Sin[a + b
*x])^(1 + m))/(b*c*d*(1 + m)*(Cos[a + b*x]^2)^(1/4)*Sqrt[d*Sec[a + b*x]])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2586

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n + 1))
/b^2, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && LtQ[n, 1]
```

Rubi steps

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{3/2}}\,dx = \frac{\int (d\cos(a+bx))^{3/2}(c\sin(a+bx))^m\,dx}{d^2\sqrt{d\cos(a+bx)}\,\sqrt{d\sec(a+bx)}}$$
$$= \frac{{}_2F_1\left(-\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\sin^2(a+bx)\right)(c\sin(a+bx))^{1+m}}{bcd(1+m)\sqrt[4]{\cos^2(a+bx)}\,\sqrt{d\sec(a+bx)}}$$

Mathematica [A] time = 4.03, size = 116, normalized size = 1.51

$$\frac{2c\cos(2(a+bx))\left(-\tan^2(a+bx)\right)^{\frac{1-m}{2}}(c\sin(a+bx))^{m-1}\,{}_2F_1\left(\frac{1}{4}(-2m-3),\frac{1-m}{2};\frac{1}{4}(1-2m);\sec^2(a+bx)\right)}{bd(2m+3)\left(\sec^2(a+bx)-2\right)\sqrt{d\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x])^m/(d*Sec[a + b*x])^(3/2),x]
```

```
[Out] (2*c*Cos[2*(a + b*x)]*Hypergeometric2F1[(-3 - 2*m)/4, (1 - m)/2, (1 - 2*m)/
4, Sec[a + b*x]^2]*(c*Sin[a + b*x])^(-1 + m)*(-Tan[a + b*x]^2)^((1 - m)/2))
/(b*d*(3 + 2*m)*Sqrt[d*Sec[a + b*x]]*(-2 + Sec[a + b*x]^2))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\sec(bx+a)}\,(c\sin(bx+a))^m}{d^2\sec(bx+a)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*sec(b*x + a))*(c*sin(b*x + a))^m/(d^2*sec(b*x + a)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^m}{(d\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^m/(d*sec(b*x + a))^(3/2), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^m}{(d\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x)
[Out] int((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(bx+a))^m}{(d\sec(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))^m/(d*sec(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*sin(b*x + a))^m/(d*sec(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\ \sin(a+b\,x))^m}{\left(\frac{d}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^m/(d/cos(a + b*x))^(3/2),x)
[Out] int((c*sin(a + b*x))^m/(d/cos(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sin(a+bx))^m}{(d\sec(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a))**m/(d*sec(b*x+a))**(3/2),x)
[Out] Integral((c*sin(a + b*x))**m/(d*sec(a + b*x))**(3/2), x)
```

3.488 $\int \sec^n(e+fx)\sin^m(e+fx)\,dx$

Optimal. Leaf size=86

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -hypergeom([1/2-1/2*m, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-1+
n)*sin(f*x+e)^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1-n)
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2587, 2576}

$$-\frac{\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}\sec^{n-1}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*Sin[e + f*x]^m,x]
```

```
[Out] -((Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec[e
 + f*x]^(-1 + n)*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1
- n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int \sec^n(e+fx)\sin^m(e+fx)\,dx = \left(\cos^n(e+fx)\sec^n(e+fx)\right)\int \cos^{-n}(e+fx)\sin^m(e+fx)\,dx$$
$$= -\frac{{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)\sin^{-1+m}(e+fx)\sin^2(e+}{f(1-n)}$$

Mathematica [C] time = 1.47, size = 285, normalized size = 3.31

$$\frac{4(m+3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)}{f(m+1)\left((m+3)(\cos(e+fx)+1)F_1\left(\frac{m+1}{2};n,m-n+1;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4\sin^2\left(\frac{1}{2}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*Sin[e + f*x]^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sec[e + f*x]^n*Sin[(e + f*x)/2]*Si
n[e + f*x]^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)
/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4*((1 + m
 - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sec\left(fx+e\right)^n\sin\left(fx+e\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*sin(f*x+e)^m,x, algorithm="fricas")
```

```
[Out] integral(sec(f*x + e)^n*sin(f*x + e)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)^n\sin\left(fx+e\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*sin(f*x+e)^m,x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^n*sin(f*x + e)^m, x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(\sin^{m}\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*sin(f*x+e)^m,x)
[Out] int(sec(f*x+e)^n*sin(f*x+e)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)^{n}\sin\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*sin(f*x+e)^m,x, algorithm="maxima")
[Out] integrate(sec(f*x + e)^n*sin(f*x + e)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^{m}\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^m*(1/cos(e + f*x))^n,x)
[Out] int(sin(e + f*x)^m*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{m}{\left(e+fx\right)}\sec^{n}{\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*sin(f*x+e)**m,x)
[Out] Integral(sin(e + f*x)**m*sec(e + f*x)**n, x)
```

3.489 $\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=89

$$-\frac{a\sin^2(e+fx)^{\frac{1-m}{2}}\sec^{n-1}(e+fx)(a\sin(e+fx))^{m-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -a*hypergeom([1/2-1/2*m, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*sec(f*x+e)^(-
1+n)*(a*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1-n)
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2587, 2576}

$$-\frac{a\sin^2(e+fx)^{\frac{1-m}{2}}\sec^{n-1}(e+fx)(a\sin(e+fx))^{m-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*Sec
[e + f*x]^(-1 + n)*(a*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/
(f*(1 - n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int \sec^n(e+fx)(a\sin(e+fx))^m\,dx = \left(\cos^n(e+fx)\sec^n(e+fx)\right)\int \cos^{-n}(e+fx)(a\sin(e+fx))^m\,dx$$
$$= -\frac{a\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)\sec^{-1+n}(e+fx)(a\sin(e+fx))^{-1+m}\sin}{f(1-n)}$$

Mathematica [C] time = 0.15, size = 287, normalized size = 3.22

$$\frac{4(m+3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)\mathrm{se}}{f(m+1)\left((m+3)(\cos(e+fx)+1)F_1\left(\frac{m+1}{2};n,m-n+1;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4\sin^2\left(\frac{1}{2}(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Sec[e + f*x]^n*Sin[(e + f*x)/2]*(a
*Sin[e + f*x])^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3
+ m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4*((1
 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)\right)^m\sec\left(fx+e\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e))^m*sec(f*x + e)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)\right)^m\sec\left(fx+e\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e))^m*sec(f*x + e)^n, x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \left(\sec^{n}\left(fx+e\right)\right)\left(a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^n*(a*sin(f*x+e))^m,x)
[Out] int(sec(f*x+e)^n*(a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(fx+e\right)\right)^{m}\sec\left(fx+e\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^n*(a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((a*sin(f*x + e))^m*sec(f*x + e)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(\frac{1}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^m*(1/cos(e + f*x))^n,x)
[Out] int((a*sin(e + f*x))^m*(1/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(e+fx\right)\right)^{m}\sec^{n}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**n*(a*sin(f*x+e))**m,x)
[Out] Integral((a*sin(e + f*x))**m*sec(e + f*x)**n, x)
```

3.490 $\int (b\sec(e+fx))^n \sin^m(e+fx)\,dx$

Optimal. Leaf size=89

$$-\frac{b\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -b*hypergeom([1/2-1/2*m, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e)
)^(-1+n)*sin(f*x+e)^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1-n)
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2587, 2576}

$$-\frac{b\sin^{m-1}(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^m,x]
```

```
[Out] -((b*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*
Sec[e + f*x])^(-1 + n)*Sin[e + f*x]^(-1 + m)*(Sin[e + f*x]^2)^((1 - m)/2))/
(f*(1 - n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int (b\sec(e+fx))^n \sin^m(e+fx)\,dx = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\sin^m(e+f$$

$$= -\frac{b\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin^{-1+m}(e+fx)\,\mathrm{si}}{f(1-n)}$$

Mathematica [C] time = 0.14, size = 287, normalized size = 3.22

$$\frac{4(m+3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)}{f(m+1)\left((m+3)(\cos(e+fx)+1)F_1\left(\frac{m+1}{2};n,m-n+1;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4\sin^2\left(\frac{1}{2}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(b*Sec[e + f*x])^n*Sin[(e + f*x)/2
]*Sin[e + f*x]^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3
+ m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4*((1
 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)\right)^n\sin\left(fx+e\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^n*sin(f*x + e)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^n\sin\left(fx+e\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^m,x, algorithm="giac")
```

[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^m, x)

maple **[F]** time = 0.62, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(f x+e\right)\right)^{n} \left(\sin^{m}\left(f x+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*sec(f*x+e))^n*sin(f*x+e)^m,x)

[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^m,x)

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(f x+e\right)\right)^{n} \sin \left(f x+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^m,x, algorithm="maxima")

[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^m, x)

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int {\sin\left(e+f\,x\right)}^m\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f*x)^m*(b/cos(e + f*x))^n,x)

[Out] int(sin(e + f*x)^m*(b/cos(e + f*x))^n, x)

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec{\left(e + f x \right)}\right)^{n} \sin^{m}{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**m,x)

[Out] Integral((b*sec(e + f*x))**n*sin(e + f*x)**m, x)

3.491 $\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx$

Optimal. Leaf size=92

$$-\frac{ab\sin^2(e+fx)^{\frac{1-m}{2}}(a\sin(e+fx))^{m-1}(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -a*b*hypergeom([1/2-1/2*m, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+
e))^(-1+n)*(a*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)/f/(1-n)
```

Rubi [A] time = 0.10, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2587, 2576}

$$-\frac{ab\sin^2(e+fx)^{\frac{1-m}{2}}(a\sin(e+fx))^{m-1}(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] -((a*b*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(
b*Sec[e + f*x])^(-1 + n)*(a*Sin[e + f*x])^(-1 + m)*(Sin[e + f*x]^2)^((1 - m
)/2))/(f*(1 - n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int (b\sec(e+fx))^n(a\sin(e+fx))^m\,dx = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}(a\sin(e+$$

$$= -\frac{ab\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}(a\sin(e+fx))^{-}}{f(1-n)}$$

Mathematica [C] time = 0.13, size = 289, normalized size = 3.14

$$\frac{4(m+3)\sin\left(\frac{1}{2}(e+fx)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)(a\,s}{f(m+1)\left((m+3)(\cos(e+fx)+1)F_1\left(\frac{m+1}{2};n,m-n+1;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-4\sin^2\left(\frac{1}{2}(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n, (3 + m)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(b*Sec[e + f*x])^n*Sin[(e + f*x)/2
]*(a*Sin[e + f*x])^m)/(f*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m - n,
 (3 + m)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(1 + Cos[e + f*x]) - 4
*((1 + m - n)*AppellF1[(3 + m)/2, n, 2 + m - n, (5 + m)/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m - n, (5 + m)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sec\left(fx+e\right)\right)^n\left(a\sin\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^n\left(a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\left(a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x)
[Out] int((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\left(a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,\sin\left(e+f\,x\right)\right)}^m\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^m*(b/cos(e + f*x))^n,x)
[Out] int((a*sin(e + f*x))^m*(b/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(e+fx\right)\right)^{m}\left(b\sec\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*(a*sin(f*x+e))**m,x)
[Out] Integral((a*sin(e + f*x))**m*(b*sec(e + f*x))**n, x)
```

3.492 $\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx$

Optimal. Leaf size=80

$$-\frac{b^5(b\sec(e+fx))^{n-5}}{f(5-n)}+\frac{2b^3(b\sec(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

```
[Out] -b^5*(b*sec(f*x+e))^(-5+n)/f/(5-n)+2*b^3*(b*sec(f*x+e))^(-3+n)/f/(3-n)-b*(b
*sec(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 270}

$$-\frac{b^5(b\sec(e+fx))^{n-5}}{f(5-n)}+\frac{2b^3(b\sec(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^5,x]
```

```
[Out] -((b^5*(b*Sec[e + f*x])^(-5 + n))/(f*(5 - n))) + (2*b^3*(b*Sec[e + f*x])^(-
3 + n))/(f*(3 - n)) - (b*(b*Sec[e + f*x])^(-1 + n))/(f*(1 - n))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int (b\sec(e+fx))^n \sin^5(e+fx)\,dx = \frac{b^5\,\text{Subst}\left(\int x^{-6+n}\left(-1+\frac{x^2}{b^2}\right)^2\,dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^5\,\text{Subst}\left(\int \left(x^{-6+n}-\frac{2x^{-4+n}}{b^2}+\frac{x^{-2+n}}{b^4}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= -\frac{b^5(b\sec(e+fx))^{-5+n}}{f(5-n)}+\frac{2b^3(b\sec(e+fx))^{-3+n}}{f(3-n)}-\frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}$$

Mathematica [A] time = 0.37, size = 80, normalized size = 1.00

$$\frac{b\left(-4\left(n^2-8n+7\right)\cos(2(e+fx))+\left(n^2-4n+3\right)\cos(4(e+fx))+3n^2-28n+89\right)(b\sec(e+fx))^{n-1}}{8f(n-5)(n-3)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^5,x]
```

```
[Out] (b*(89 - 28*n + 3*n^2 - 4*(7 - 8*n + n^2)*Cos[2*(e + f*x)] + (3 - 4*n + n^2
)*Cos[4*(e + f*x)])*(b*Sec[e + f*x])^(-1 + n))/(8*f*(-5 + n)*(-3 + n)*(-1 +
 n))
```

fricas [A] time = 0.62, size = 85, normalized size = 1.06

$$\frac{\left(\left(n^2-4n+3\right)\cos\left(fx+e\right)^5-2\left(n^2-6n+5\right)\cos\left(fx+e\right)^3+\left(n^2-8n+15\right)\cos\left(fx+e\right)\right)\left(\frac{b}{\cos\left(fx+e\right)}\right)^n}{fn^3-9fn^2+23fn-15f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] ((n^2 - 4*n + 3)*cos(f*x + e)^5 - 2*(n^2 - 6*n + 5)*cos(f*x + e)^3 + (n^2 -
 8*n + 15)*cos(f*x + e))*(b/cos(f*x + e))^n/(f*n^3 - 9*f*n^2 + 23*f*n - 15*
f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^n \sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^5,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^5, x)
```

maple [F] time = 1.77, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(f x+e\right)\right)^{n} \left(\sin^{5}\left(f x+e\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^5,x)
[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^5,x)
```

maxima [A] time = 0.35, size = 85, normalized size = 1.06

$$\frac{\frac{b^{n} \cos\left(f x+e\right)^{-n} \cos\left(f x+e\right)^{5}}{n-5}-\frac{2 b^{n} \cos\left(f x+e\right)^{-n} \cos\left(f x+e\right)^{3}}{n-3}+\frac{b^{n} \cos\left(f x+e\right)^{-n} \cos\left(f x+e\right)}{n-1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^5,x, algorithm="maxima")
[Out] (b^n*cos(f*x + e)^(-n)*cos(f*x + e)^5/(n - 5) - 2*b^n*cos(f*x + e)^(-n)*cos
(f*x + e)^3/(n - 3) + b^n*cos(f*x + e)^(-n)*cos(f*x + e)/(n - 1))/f
```

mupad [B] time = 1.60, size = 134, normalized size = 1.68

$$\frac{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{n}\,\left(150\,\cos\left(e+f\,x\right)-25\,\cos\left(3\,e+3\,f\,x\right)+3\,\cos\left(5\,e+5\,f\,x\right)-24\,n\,\cos\left(e+f\,x\right)+28\,n\,\cos\left(3\,e+\right.\right.}{16\,f\,\left(n^{3}-9\,n^{2}+23\,n\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(b/cos(e + f*x))^n,x)
[Out] ((b/cos(e + f*x))^n*(150*cos(e + f*x) - 25*cos(3*e + 3*f*x) + 3*cos(5*e + 5
*f*x) - 24*n*cos(e + f*x) + 28*n*cos(3*e + 3*f*x) - 4*n*cos(5*e + 5*f*x) +
2*n^2*cos(e + f*x) - 3*n^2*cos(3*e + 3*f*x) + n^2*cos(5*e + 5*f*x)))/(16*f*
(23*n - 9*n^2 + n^3 - 15))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

3.493 $\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx$

Optimal. Leaf size=52

$$\frac{b^3(b\sec(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

```
[Out] b^3*(b*sec(f*x+e))^(-3+n)/f/(3-n)-b*(b*sec(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 14}

$$\frac{b^3(b\sec(e+fx))^{n-3}}{f(3-n)}-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^3,x]
```

```
[Out] (b^3*(b*Sec[e + f*x])^(-3 + n))/(f*(3 - n)) - (b*(b*Sec[e + f*x])^(-1 + n))
/(f*(1 - n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int (b\sec(e+fx))^n \sin^3(e+fx)\,dx = \frac{b^3 \operatorname{Subst}\left(\int x^{-4+n}\left(-1+\frac{x^2}{b^2}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^3 \operatorname{Subst}\left(\int \left(-x^{-4+n}+\frac{x^{-2+n}}{b^2}\right)dx, x, b\sec(e+fx)\right)}{f}$$
$$= \frac{b^3(b\sec(e+fx))^{-3+n}}{f(3-n)} - \frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}$$

Mathematica [A] time = 0.15, size = 47, normalized size = 0.90

$$-\frac{b((n-1)\cos(2(e+fx))-n+5)(b\sec(e+fx))^{n-1}}{2f(n-3)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^3,x]
```

```
[Out] -1/2*(b*(5 - n + (-1 + n)*Cos[2*(e + f*x)])*(b*Sec[e + f*x])^(-1 + n))/(f*(
-3 + n)*(-1 + n))
```

fricas [A] time = 0.68, size = 53, normalized size = 1.02

$$-\frac{\left((n-1)\cos\left(fx+e\right)^3-(n-3)\cos\left(fx+e\right)\right)\left(\frac{b}{\cos\left(fx+e\right)}\right)^n}{fn^2-4fn+3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -((n - 1)*cos(f*x + e)^3 - (n - 3)*cos(f*x + e))*(b/cos(f*x + e))^n/(f*n^2
- 4*f*n + 3*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^n \sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^3, x)
```

maple [C] time = 1.86, size = 1732, normalized size = 33.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^3,x)

[Out] -1/8/(f*n-3*f)*b^n*2^n*(exp(2*I*(f*x+e))+1)^(-n)*exp(I*(f*x+e))^n*exp(-1/2*
I*(Pi*n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f
*x+e))/(exp(2*I*(f*x+e))+1))-Pi*n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I
*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-Pi*n*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*
(f*x+e))/(exp(2*I*(f*x+e))+1))^2+Pi*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e
))+1))^3-Pi*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f
*x+e))/(exp(2*I*(f*x+e))+1))^2+Pi*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))
+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)+Pi*n*csgn(I*b*
exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3-Pi*n*csgn(I*b*exp(I*(f*x+e))/(exp(2*
I*(f*x+e))+1))^2*csgn(I*b)+6*f*x+6*e))-1/8*exp(I*(f*x+e))^n*(exp(2*I*(f*x+e
))+1)^(-n)*2^n*b^n/(f*n-3*f)*exp(1/2*I*(-Pi*n*csgn(I/(exp(2*I*(f*x+e))+1))*
csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))+Pi*n*csg
n(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2+Pi*
n*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-Pi*n
*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3+Pi*n*csgn(I*exp(I*(f*x+e))/(
exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-Pi*n*c
sgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I
*(f*x+e))+1))*csgn(I*b)-Pi*n*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^
3+Pi*n*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*b)+6*f*x+6*e)
)+1/8*exp(I*(f*x+e))^n*(exp(2*I*(f*x+e))+1)^(-n)*2^n*b^n/(-3+n)/(-1+n)/f*(n
-9)*exp(-1/2*I*(Pi*n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*cs
gn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))-Pi*n*csgn(I/(exp(2*I*(f*x+e))+1))
*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2-Pi*n*csgn(I*exp(I*(f*x+e)))*
csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2+Pi*n*csgn(I*exp(I*(f*x+e))/(e
xp(2*I*(f*x+e))+1))^3-Pi*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn
(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^2+Pi*n*csgn(I*exp(I*(f*x+e))/(exp
(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)+P
i*n*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3-Pi*n*csgn(I*b*exp(I*(f*
x+e))/(exp(2*I*(f*x+e))+1))^2*csgn(I*b)+2*f*x+2*e))+1/8*exp(I*(f*x+e))^n*(e
xp(2*I*(f*x+e))+1)^(-n)*2^n*b^n/(-3+n)/(-1+n)/f*(n-9)*exp(1/2*I*(-Pi*n*csgn
(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(exp(
2*I*(f*x+e))+1))+Pi*n*csgn(I/(exp(2*I*(f*x+e))+1))*csgn(I*exp(I*(f*x+e))/(e
xp(2*I*(f*x+e))+1))^2+Pi*n*csgn(I*exp(I*(f*x+e)))*csgn(I*exp(I*(f*x+e))/(ex
p(2*I*(f*x+e))+1))^2-Pi*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))^3+Pi*
n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b*exp(I*(f*x+e))/(exp(
2*I*(f*x+e))+1))^2-Pi*n*csgn(I*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*
b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1))*csgn(I*b)-Pi*n*csgn(I*b*exp(I*(f*x+e
```

```
))/(exp(2*I*(f*x+e))+1))^3+Pi*n*csgn(I*b*exp(I*(f*x+e))/(exp(2*I*(f*x+e))+1
))^2*csgn(I*b)+2*f*x+2*e))
```

maxima [A] time = 1.33, size = 59, normalized size = 1.13

$$-\frac{\frac{b^n \cos\left(fx+e\right)^{-n} \cos\left(fx+e\right)^3}{n-3} - \frac{b^n \cos\left(fx+e\right)^{-n} \cos\left(fx+e\right)}{n-1}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] -(b^n*cos(f*x + e)^(-n)*cos(f*x + e)^3/(n - 3) - b^n*cos(f*x + e)^(-n)*cos(
f*x + e)/(n - 1))/f
```

mupad [B] time = 0.94, size = 67, normalized size = 1.29

$$-\frac{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^n \left(9\,\cos\left(e+f\,x\right)-\cos\left(3\,e+3\,f\,x\right)-n\,\cos\left(e+f\,x\right)+n\,\cos\left(3\,e+3\,f\,x\right)\right)}{4\,f\,\left(n^2-4\,n+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(b/cos(e + f*x))^n,x)
```

```
[Out] -((b/cos(e + f*x))^n*(9*cos(e + f*x) - cos(3*e + 3*f*x) - n*cos(e + f*x) +
n*cos(3*e + 3*f*x)))/(4*f*(n^2 - 4*n + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**3,x)
```

```
[Out] Timed out
```

3.494 $\int (b\sec(e+fx))^n \sin(e+fx)\,dx$

Optimal. Leaf size=25

$$-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

```
[Out] -b*(b*sec(f*x+e))^(-1+n)/f/(1-n)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2622, 30}

$$-\frac{b(b\sec(e+fx))^{n-1}}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x],x]
```

```
[Out] -((b*(b*Sec[e + f*x])^(-1 + n))/(f*(1 - n)))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int (b\sec(e+fx))^n \sin(e+fx)\,dx &= \frac{b\,\text{Subst}\left(\int x^{-2+n}\,dx, x, b\sec(e+fx)\right)}{f}\\ &= -\frac{b(b\sec(e+fx))^{-1+n}}{f(1-n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.88

$$\frac{b(b\sec(e+fx))^{n-1}}{f(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x],x]
```

```
[Out] (b*(b*Sec[e + f*x])^(-1 + n))/(f*(-1 + n))
```

fricas [A] time = 0.50, size = 28, normalized size = 1.12

$$\frac{\left(\frac{b}{\cos\left(fx+e\right)}\right)^{n}\cos\left(fx+e\right)}{fn-f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] (b/cos(f*x + e))^n*cos(f*x + e)/(f*n - f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e), x)
```

maple [B] time = 0.04, size = 120, normalized size = 4.80

$$\frac{\frac{\mathrm{e}^{n\ln\left(\frac{b\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{1-\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}\right)}}{f(-1+n)}-\frac{\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)\mathrm{e}^{n\ln\left(\frac{b\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{1-\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}\right)}}{f(-1+n)}}{1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e),x)
```

```
[Out] (1/f/(-1+n)*exp(n*ln(b*(1+tan(1/2*f*x+1/2*e)^2)/(1-tan(1/2*f*x+1/2*e)^2)))-
1/f/(-1+n)*tan(1/2*f*x+1/2*e)^2*exp(n*ln(b*(1+tan(1/2*f*x+1/2*e)^2)/(1-tan(
1/2*f*x+1/2*e)^2))))/(1+tan(1/2*f*x+1/2*e)^2)
```

maxima [A] time = 0.36, size = 28, normalized size = 1.12

$$\frac{b^{n}\cos\left(fx+e\right)^{-n}\cos\left(fx+e\right)}{f(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] b^n*cos(f*x + e)^(-n)*cos(f*x + e)/(f*(n - 1))
```

mupad [B] time = 0.19, size = 27, normalized size = 1.08

$$\frac{\cos\left(e+f\,x\right)\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^n}{f\,\left(n-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(b/cos(e + f*x))^n,x)
```

```
[Out] (cos(e + f*x)*(b/cos(e + f*x))^n)/(f*(n - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(e+fx\right)\right)^{n}\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e),x)
```

```
[Out] Integral((b*sec(e + f*x))**n*sin(e + f*x), x)
```

3.495 $\int \csc(e+fx)(b\sec(e+fx))^n\,dx$

Optimal. Leaf size=49

$$-\frac{(b\sec(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sec^2(e+fx)\right)}{bf(n+1)}$$

```
[Out] -hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],sec(f*x+e)^2)*(b*sec(f*x+e))^(1+n)/b/
f/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2622, 364}

$$-\frac{(b\sec(e+fx))^{n+1}\,{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\sec^2(e+fx)\right)}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(b*Sec[e + f*x])^n,x]
```

```
[Out] -((Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Sec[e + f*x]^2]*(b*Sec[e + f*
x])^(1 + n))/(b*f*(1 + n)))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc(e+fx)(b\sec(e+fx))^n\,dx = \frac{\text{Subst}\left(\int \frac{x^n}{-1+\frac{x^2}{b^2}}\,dx, x, b\sec(e+fx)\right)}{bf}$$
$$= -\frac{{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\sec^2(e+fx)\right)(b\sec(e+fx))^{1+n}}{bf(1+n)}$$

Mathematica [A] time = 0.33, size = 92, normalized size = 1.88

$$\frac{(b\sec(e+fx))^n\left({}_2F_1(1,-n;1-n;\cos(e+fx)) - 2^n\sec^2\left(\frac{1}{2}(e+fx)\right)^{-n}\,{}_2F_1\left(-n,-n;1-n;\frac{1}{2}\cos(e+fx)\sec^2\left(\frac{1}{2}(\right.\right.\right.}{2fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(b*Sec[e + f*x])^n,x]
```

```
[Out] ((Hypergeometric2F1[1, -n, 1 - n, Cos[e + f*x]] - (2^n*Hypergeometric2F1[-n
, -n, 1 - n, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2])/(Sec[(e + f*x)/2]^2)^n)*
(b*Sec[e + f*x])^n)/(2*f*n)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)\right)^n\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^n*csc(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^n\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \csc\left(fx+e\right)\left(b\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(b*sec(f*x+e))^n,x)
```

```
[Out] int(csc(f*x+e)*(b*sec(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{n}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^n/sin(e + f*x),x)
```

```
[Out] int((b/cos(e + f*x))^n/sin(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(e+fx\right)\right)^{n}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(b*sec(f*x+e))**n,x)
```

```
[Out] Integral((b*sec(e + f*x))**n*csc(e + f*x), x)
```

3.496 $\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx$

Optimal. Leaf size=48

$$\frac{(b\sec(e+fx))^{n+3}\,{}_2F_1\left(2,\frac{n+3}{2};\frac{n+5}{2};\sec^2(e+fx)\right)}{b^3f(n+3)}$$

```
[Out] hypergeom([2, 3/2+1/2*n],[5/2+1/2*n],sec(f*x+e)^2)*(b*sec(f*x+e))^(3+n)/b^3
/f/(3+n)
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2622, 364}

$$\frac{(b\sec(e+fx))^{n+3}\,{}_2F_1\left(2,\frac{n+3}{2};\frac{n+5}{2};\sec^2(e+fx)\right)}{b^3f(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(b*Sec[e + f*x])^n,x]
```

```
[Out] (Hypergeometric2F1[2, (3 + n)/2, (5 + n)/2, Sec[e + f*x]^2]*(b*Sec[e + f*x]
)^(3 + n))/(b^3*f*(3 + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \csc^3(e+fx)(b\sec(e+fx))^n\,dx = \frac{\text{Subst}\left(\int \frac{x^{2+n}}{\left(-1+\frac{x^2}{b^2}\right)^2}\,dx, x, b\sec(e+fx)\right)}{b^3 f}$$
$$= \frac{{}_2F_1\left(2,\frac{3+n}{2};\frac{5+n}{2};\sec^2(e+fx)\right)(b\sec(e+fx))^{3+n}}{b^3 f(3+n)}$$

Mathematica [B] time = 4.52, size = 201, normalized size = 4.19

$$\frac{b(b\sec(e+fx))^{n-1}\left(2\,{}_2F_1(1,1-n;2-n;\cos(e+fx))+2\,{}_2F_1(2,1-n;2-n;\cos(e+fx))+2^n\sec^{1-n}(e+fx)\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^3*(b*Sec[e + f*x])^n,x]
```

```
[Out] (b*(b*Sec[e + f*x])^(-1 + n)*(2*Hypergeometric2F1[1, 1 - n, 2 - n, Cos[e +
f*x]] + 2*Hypergeometric2F1[2, 1 - n, 2 - n, Cos[e + f*x]] + 2^n*Hypergeome
tric2F1[1 - n, -n, 2 - n, (Cos[e + f*x]*Sec[(e + f*x)/2]^2)/2]*(Sec[(e + f*
x)/2]^2)^(1 - n) + 2^n*Hypergeometric2F1[1 - n, 1 - n, 2 - n, (Cos[e + f*x]
*Sec[(e + f*x)/2]^2)/2]*Sec[e + f*x]^(1 - n)*(Cos[(e + f*x)/2]^2*Sec[e + f*
x])^(-1 + n)))/(8*f*(-1 + n))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)\right)^n\csc\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^n*csc(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^n\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^3, x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \left(\csc^{3}\left(fx+e\right)\right)\left(b\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(b*sec(f*x+e))^n,x)
```

```
[Out] int(csc(f*x+e)^3*(b*sec(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(b*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{n}}{\sin\left(e+f\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^n/sin(e + f*x)^3,x)
```

```
[Out] int((b/cos(e + f*x))^n/sin(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(e+fx\right)\right)^{n}\csc^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(b*sec(f*x+e))**n,x)
```

```
[Out] Integral((b*sec(e + f*x))**n*csc(e + f*x)**3, x)
```

3.497 $\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx$

Optimal. Leaf size=73

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(-\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -b*hypergeom([-5/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e))^(-1
+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(-\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^6,x]
```

```
[Out] -((b*Hypergeometric2F1[-5/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\sec(e+fx))^n \sin^6(e+fx)\,dx = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\sin^6(e+fx)\,d$$

$$= -\frac{b\,{}_2F_1\left(-\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [C] time = 25.46, size = 8327, normalized size = 114.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^6,x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^6 - 3\cos\left(fx+e\right)^4 + 3\cos\left(fx+e\right)^2 - 1\right)\left(b\sec\left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*(b*sec
(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^n \sin\left(fx+e\right)^6\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^6,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^6, x)
```

maple [F] time = 1.30, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^n \left(\sin^6\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^6,x)
```

```
[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^6,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx+e\right)\right)^{n} \sin\left(fx+e\right)^{6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^6\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6*(b/cos(e + f*x))^n,x)
```

```
[Out] int(sin(e + f*x)^6*(b/cos(e + f*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**6,x)
```

```
[Out] Timed out
```

3.498 $\int (b\sec(e+fx))^n \sin^4(e+fx)\,dx$

Optimal. Leaf size=73

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(-\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -b*hypergeom([-3/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e))^(-1
+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(-\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^4,x]
```

```
[Out] -((b*Hypergeometric2F1[-3/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\sec(e+fx))^n \sin^4(e+fx)\,dx = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\sin^4(e+fx$$
$$= -\frac{b\,{}_2F_1\left(-\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [C] time = 24.18, size = 6192, normalized size = 84.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^4,x]
[Out] Result too large to show
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos\left(fx+e\right)^4 - 2\cos\left(fx+e\right)^2 + 1\right)\left(b\sec\left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^4,x, algorithm="fricas")
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(b*sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^n \sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^4,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^4, x)
```

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^n \left(\sin^4\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^4,x)
```

```
[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^{4}\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(b/cos(e + f*x))^n,x)
```

```
[Out] int(sin(e + f*x)^4*(b/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(e+fx\right)\right)^{n}\sin^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**4,x)
```

```
[Out] Integral((b*sec(e + f*x))**n*sin(e + f*x)**4, x)
```

3.499 $\int (b\sec(e+fx))^n \sin^2(e+fx)\,dx$

Optimal. Leaf size=73

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -b*hypergeom([-1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e))^(-1
+n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(-\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n*Sin[e + f*x]^2,x]
```

```
[Out] -((b*Hypergeometric2F1[-1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
 + f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (b\sec(e+fx))^n\sin^2(e+fx)\,dx = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\sin^2(e+fx)\,dx$$

$$= -\frac{b\,{}_2F_1\left(-\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [C] time = 18.63, size = 4143, normalized size = 56.75

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b*Sec[e + f*x])^n*Sin[e + f*x]^2,x]
```

```
[Out] (24*(Sec[(e + f*x)/2]^2)^(-3 + n)*(b*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Se
c[e + f*x])^n*Sin[e + f*x]^2*Tan[(e + f*x)/2]*((AppellF1[1/2, n, 2 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[
1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-2 + n)*
AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*A
ppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*T
an[(e + f*x)/2]^2) - AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]/(3*AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + 2*((-3 + n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3 - n, 5/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))/(f*(12*(Sec[(e + f*x)/2]^2
)^(-2 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*((AppellF1[1/2, n, 2 - n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1
[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-2 + n)
*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*
AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2) - AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]/(3*AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2] + 2*((-3 + n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3 - n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + 24*(-3 + n)*(Sec[(e + f*
x)/2]^2)^(-3 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2]^2*((
AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2)/(3*AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2] + 2*((-2 + n)*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - AppellF1[1/2, n, 3 - n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, n, 3 - n, 3/2,
```

Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-3 + n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)) + 24*(Sec[(e + f*x)/2]^2)^(-3 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*Tan[(e + f*x)/2]*((AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3*AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-2 + n)*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (Sec[(e + f*x)/2]^2*(-1/3*((2 - n)*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (n*AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-2 + n)*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (-1/3*((3 - n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (n*AppellF1[3/2, 1 + n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)/(3*AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-3 + n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(2*((-2 + n)*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/3*((2 - n)*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (n*AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-2 + n)*((-3*(3 - n)*AppellF1[5/2, n, 4 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*n*AppellF1[5/2, 1 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + n*((-3*(2 - n)*AppellF1[5/2, 1 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 + n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-2 + n)*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2 + (AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(2*((-3 + n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*(-1/3*((3 - n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +

```
f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (n*AppellF1[3/2, 1 + n, 3
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[
(e + f*x)/2])/3) + 2*Tan[(e + f*x)/2]^2*((-3 + n)*((-3*(4 - n)*AppellF1[5/2
, n, 5 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/5 + (3*n*AppellF1[5/2, 1 + n, 4 - n, 7/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) + n*((-
3*(3 - n)*AppellF1[5/2, 1 + n, 4 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + n)*AppellF1[5/2,
2 + n, 3 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2
]^2*Tan[(e + f*x)/2])/5))))/(3*AppellF1[1/2, n, 3 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 2*((-3 + n)*AppellF1[3/2, n, 4 - n, 5/2, Tan[(e
 + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2) + 24*n*(Sec[(
e + f*x)/2]^2)^(-3 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(-1 + n)*Tan[(e +
 f*x)/2]*((AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Sec[(e + f*x)/2]^2)/(3*AppellF1[1/2, n, 2 - n, 3/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] + 2*((-2 + n)*AppellF1[3/2, n, 3 - n, 5/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - AppellF1[1/2, n,
 3 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]/(3*AppellF1[1/2, n, 3
 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-3 + n)*AppellF1[
3/2, n, 4 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + n*AppellF1[3
/2, 1 + n, 3 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f
*x)/2]^2))*(-(Cos[(e + f*x)/2]*Sec[e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*
x)/2]^2*Sec[e + f*x]*Tan[e + f*x])))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^{2}-1\right)\left(b\sec\left(fx+e\right)\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(b*sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^2, x)
```

maple [F] time = 1.34, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\left(\sin^{2}\left(fx+e\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n*sin(f*x+e)^2,x)
[Out] int((b*sec(f*x+e))^n*sin(f*x+e)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\sin\left(fx+e\right)^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n*sin(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^n*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^{2}\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(b/cos(e + f*x))^n,x)
[Out] int(sin(e + f*x)^2*(b/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(e+fx\right)\right)^{n}\sin^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n*sin(f*x+e)**2,x)
[Out] Integral((b*sec(e + f*x))**n*sin(e + f*x)**2, x)
```

3.500 $\int (b\sec(e+fx))^n\,dx$

Optimal. Leaf size=73

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -b*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f*x+e))^(-1+
n)*sin(f*x+e)/f/(1-n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3772, 2643}

$$-\frac{b\sin(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[e + f*x])^n,x]
```

```
[Out] -((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]^2]*(b*Sec[e
+ f*x])^(-1 + n)*Sin[e + f*x])/(f*(1 - n)*Sqrt[Sin[e + f*x]^2]))
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3772

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rubi steps

$$\int (b\sec(e+fx))^n\,dx = \left(\frac{\cos(e+fx)}{b}\right)^n (b\sec(e+fx))^n \int \left(\frac{\cos(e+fx)}{b}\right)^{-n} dx$$
$$= -\frac{\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^n\sin(e+fx)}{f(1-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 0.84

$$\frac{\sqrt{-\tan^2(e+fx)}\,\cot(e+fx)(b\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\sec^2(e+fx)\right)}{fn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[e + f*x])^n,x]
[Out] (Cot[e + f*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[e + f*x]^2]*(b*Sec
[e + f*x])^n*Sqrt[-Tan[e + f*x]^2])/(f*n)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n, x)
```

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(f*x+e))^n,x)
```

```
[Out] int((b*sec(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^n,x)
```

```
[Out] int((b/cos(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(e+fx\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(f*x+e))**n,x)
```

```
[Out] Integral((b*sec(e + f*x))**n, x)
```

3.501 $\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx$

Optimal. Leaf size=73

$$-\frac{b\sqrt{\sin^2(e+fx)}\,\csc(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -b*csc(f*x+e)*hypergeom([3/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f
*x+e))^(-1+n)*(sin(f*x+e)^2)^(1/2)/f/(1-n)
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$-\frac{b\sqrt{\sin^2(e+fx)}\,\csc(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(b*Sec[e + f*x])^n,x]
```

```
[Out] -((b*Csc[e + f*x]*Hypergeometric2F1[3/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]
^2]*(b*Sec[e + f*x])^(-1 + n)*Sqrt[Sin[e + f*x]^2])/(f*(1 - n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \csc^2(e+fx)(b\sec(e+fx))^n\,dx = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\csc^2(e+fx)$$

$$= -\frac{b\csc(e+fx)\,{}_2F_1\left(\frac{3}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sqrt{\sin^2(e+}}{f(1-n)}$$

Mathematica [C] time = 15.22, size = 2638, normalized size = 36.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2*(b*Sec[e + f*x])^n,x]
```

```
[Out] (Cot[(e + f*x)/2]*Csc[e + f*x]^2*(b*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*Sec
[e + f*x])^n*(-(AppellF1[-1/2, n, -n, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n) + (3*AppellF1[1/2, n, -n, 3/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Tan[(e +
f*x)/2]^2)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2])*Tan[(e + f*x)/2]^2)))/(2*f*(-1/4*(Csc[(e + f*x)/2]^2*(Cos[(e + f*x)/
2]^2*Sec[e + f*x])^n*(-(AppellF1[-1/2, n, -n, 1/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n) + (3*AppellF1[1/2, n,
 -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*T
an[(e + f*x)/2]^2)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2]^2))) + (Cot[(e + f*x)/2]*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^n*(-((Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(-(n*AppellF1[1/2,
 n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2
*Tan[(e + f*x)/2]) - n*AppellF1[1/2, 1 + n, -n, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])) - n*AppellF1[-1/2,
 n, -n, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e
+ f*x)/2]^2)^(-1 + n)*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Se
c[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (3*AppellF1[1/2, n, -n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(1 + n)*Tan[(e + f*x)/
2])/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2) + (3*n*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^3)/(3*AppellF1[1
```

```
/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/
2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2,
1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^
2) + (3*(Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^2*((n*AppellF1[3/2, n, 1 -
n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e
+ f*x)/2])/3 + (n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, n,
-n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n,
-n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (3
*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e
 + f*x)/2]^2)^n*Tan[(e + f*x)/2]^2*(2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*
((n*AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*S
ec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (n*AppellF1[3/2, 1 + n, -n, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])
/3) + 2*n*Tan[(e + f*x)/2]^2*((-3*(1 - n)*AppellF1[5/2, n, 2 - n, 7/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5
 + (6*n*AppellF1[5/2, 1 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2
+ n, -n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*T
an[(e + f*x)/2])/5)))/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)^2))/2 + (n*Cot[(e + f*x)/2]*(Cos[(e +
 f*x)/2]^2*Sec[e + f*x])^(-1 + n)*(-(AppellF1[-1/2, n, -n, 1/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n) + (3*App
ellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f
*x)/2]^2)^n*Tan[(e + f*x)/2]^2)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/
2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))*(-(Cos[(e + f*x)/2]*Sec[e
+ f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))/2
))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(f*x + e))^n*csc(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^2, x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \left(\csc^{2}\left(fx+e\right)\right)\left(b\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(b*sec(f*x+e))^n,x)
```

```
[Out] int(csc(f*x+e)^2*(b*sec(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(b*sec(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{n}}{\sin\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^n/sin(e + f*x)^2,x)
```

```
[Out] int((b/cos(e + f*x))^n/sin(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(e+fx\right)\right)^{n}\csc^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(b*sec(f*x+e))**n,x)
```

```
[Out] Integral((b*sec(e + f*x))**n*csc(e + f*x)**2, x)
```

3.502 $\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx$

Optimal. Leaf size=73

$$-\frac{b\sqrt{\sin^2(e+fx)}\,\csc(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

```
[Out] -b*csc(f*x+e)*hypergeom([5/2, 1/2-1/2*n],[3/2-1/2*n],cos(f*x+e)^2)*(b*sec(f
*x+e))^(-1+n)*(sin(f*x+e)^2)^(1/2)/f/(1-n)
```

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2632, 2576}

$$-\frac{b\sqrt{\sin^2(e+fx)}\,\csc(e+fx)(b\sec(e+fx))^{n-1}\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(b*Sec[e + f*x])^n,x]
```

```
[Out] -((b*Csc[e + f*x]*Hypergeometric2F1[5/2, (1 - n)/2, (3 - n)/2, Cos[e + f*x]
^2]*(b*Sec[e + f*x])^(-1 + n)*Sqrt[Sin[e + f*x]^2])/(f*(1 - n)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2632

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(a^2*(a*Sec[e + f*x])^(m - 1)*(b*Csc[e + f*x])^(n + 1
)*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))/b^2, Int[1/((a*Cos[e +
 f*x])^m*(b*Sin[e + f*x])^n), x], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \csc^4(e+fx)(b\sec(e+fx))^n\,dx = \left(b^2(b\cos(e+fx))^{-1+n}(b\sec(e+fx))^{-1+n}\right)\int (b\cos(e+fx))^{-n}\csc^4(e+fx$$

$$= -\frac{b\csc(e+fx)\,{}_2F_1\left(\frac{5}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(e+fx)\right)(b\sec(e+fx))^{-1+n}\sqrt{\sin^2(e+}}{f(1-n)}$$

Mathematica [C] time = 17.55, size = 3833, normalized size = 52.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^4*(b*Sec[e + f*x])^n,x]
```

```
[Out] (Cot[(e + f*x)/2]^3*Csc[e + f*x]^4*(b*Sec[e + f*x])^n*(Cos[(e + f*x)/2]^2*S
ec[e + f*x])^n*(-(AppellF1[-3/2, n, -n, -1/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n) - 9*AppellF1[-1/2, n, -n,
1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2
]^2)^n*Tan[(e + f*x)/2]^2 + AppellF1[3/2, n, -n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^6
+ (27*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(S
ec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^4)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)))/(24*f*(-1/16*(Co
t[(e + f*x)/2]^2*Csc[(e + f*x)/2]^2*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^n*(-(
AppellF1[-3/2, n, -n, -1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e
 + f*x]*Sec[(e + f*x)/2]^2)^n) - 9*AppellF1[-1/2, n, -n, 1/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*
x)/2]^2 + AppellF1[3/2, n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^6 + (27*AppellF1[1/2
, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)
^n*Tan[(e + f*x)/2]^4)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))) + (Cot[(e + f*x)/2]^3*(Cos[(e + f*
x)/2]^2*Sec[e + f*x])^n*(-9*AppellF1[-1/2, n, -n, 1/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n
*Tan[(e + f*x)/2] + 3*AppellF1[3/2, n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e
 + f*x)/2]^5 - (Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*(3*n*AppellF1[-1/2, n, 1
 - n, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[
```

(e + f*x)/2] + 3*n*AppellF1[-1/2, 1 + n, -n, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - 9*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^2*(-(n*AppellF1[1/2, n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - n*AppellF1[1/2, 1 + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^6*((3*n*AppellF1[5/2, n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (3*n*AppellF1[5/2, 1 + n, -n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5) - n*AppellF1[-3/2, n, -n, -1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + n)*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) - 9*n*AppellF1[-1/2, n, -n, 1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + n)*Tan[(e + f*x)/2]^2*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + n*AppellF1[3/2, n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + n)*Tan[(e + f*x)/2]^6*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]) + (54*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(1 + n)*Tan[(e + f*x)/2]^3)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (27*n*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^5)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) + (27*(Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^4*((n*AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (27*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^4*(2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2] + 3*((n*AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3 + (n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3) + 2*n*Tan[(e + f*x)/2]^2*((-3*(1 - n)*AppellF1[5/2, n, 2 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5 + (6*n*AppellF1[5/2, 1 + n, 1 - n, 7/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan

```
[(e + f*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 + n, -n, 7/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/5)))/(3*Appe
llF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(Appel
lF1[3/2, n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1
[3/2, 1 + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*
x)/2]^2)^2))/24 + (n*Cot[(e + f*x)/2]^3*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(
-1 + n)*(-(AppellF1[-3/2, n, -n, -1/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n) - 9*AppellF1[-1/2, n, -n, 1/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*
Tan[(e + f*x)/2]^2 + AppellF1[3/2, n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^n*Tan[(e + f*x)/2]^6 + (27*A
ppellF1[1/2, n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*(Sec[(e +
 f*x)/2]^2)^n*Tan[(e + f*x)/2]^4)/(3*AppellF1[1/2, n, -n, 3/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + 2*n*(AppellF1[3/2, n, 1 - n, 5/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] + AppellF1[3/2, 1 + n, -n, 5/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2))*(-(Cos[(e + f*x)/2]*Sec[
e + f*x]*Sin[(e + f*x)/2]) + Cos[(e + f*x)/2]^2*Sec[e + f*x]*Tan[e + f*x]))
/24))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sec\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((b*sec(f*x + e))^n*csc(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sec\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^4, x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int\left(\csc^{4}\left(fx+e\right)\right)\left(b\sec\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(b*sec(f*x+e))^n,x)
[Out] int(csc(f*x+e)^4*(b*sec(f*x+e))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)\right)^{n}\csc\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(b*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((b*sec(f*x + e))^n*csc(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)^{n}}{\sin\left(e+f\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(e + f*x))^n/sin(e + f*x)^4,x)
[Out] int((b/cos(e + f*x))^n/sin(e + f*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(e+fx\right)\right)^{n}\csc^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(b*sec(f*x+e))**n,x)
[Out] Integral((b*sec(e + f*x))**n*csc(e + f*x)**4, x)
```

3.503 $\int (b\sec(a+bx))^n (c\sin(a+bx))^{3/2}\,dx$

Optimal. Leaf size=76

$$-\frac{c\sqrt{c\sin(a+bx)}\,(b\sec(a+bx))^{n-1}\,{}_2F_1\left(-\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{(1-n)\sqrt[4]{\sin^2(a+bx)}}$$

```
[Out] -c*hypergeom([-1/4, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(b*sec(b*x+a))^(-1
+n)*(c*sin(b*x+a))^(1/2)/(1-n)/(sin(b*x+a)^2)^(1/4)
```

Rubi [A] time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2576}

$$-\frac{c\sqrt{c\sin(a+bx)}\,(b\sec(a+bx))^{n-1}\,{}_2F_1\left(-\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{(1-n)\sqrt[4]{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -((c*Hypergeometric2F1[-1/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
 + b*x])^(-1 + n)*Sqrt[c*Sin[a + b*x]])/((1 - n)*(Sin[a + b*x]^2)^(1/4)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int (b\sec(a+bx))^n(c\sin(a+bx))^{3/2}\,dx = \left(b^2(b\cos(a+bx))^{-1+n}(b\sec(a+bx))^{-1+n}\right)\int (b\cos(a+bx))^{-n}(c\sin(a+$$

$$= -\frac{c\,{}_2F_1\left(-\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt{c\sin(a+bx)}}{(1-n)\sqrt[4]{\sin^2(a+bx)}}$$

Mathematica [A] time = 0.81, size = 104, normalized size = 1.37

$$\frac{2(c\sin(a+bx))^{5/2}\cos^2(a+bx)^{\frac{n-1}{2}}(b\sec(a+bx))^{n-1}\left(5\sin^2(a+bx)\,{}_2F_1\left(\frac{9}{4},\frac{n+1}{2};\frac{13}{4};\sin^2(a+bx)\right)+9\,{}_2F_1\left(\frac{5}{4},\frac{n-1}{2};\right.\right.}{45c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[a + b*x])^n*(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] (2*(Cos[a + b*x]^2)^((-1 + n)/2)*(b*Sec[a + b*x])^(-1 + n)*(c*Sin[a + b*x])
^(5/2)*(9*Hypergeometric2F1[5/4, (-1 + n)/2, 9/4, Sin[a + b*x]^2] + 5*Hyper
geometric2F1[9/4, (1 + n)/2, 13/4, Sin[a + b*x]^2]*Sin[a + b*x]^2))/(45*c)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{c\sin(bx+a)}\,(b\sec(bx+a))^n\,c\sin(bx+a)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n*c*sin(b*x + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^{\frac{3}{2}}(b\sec(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)*(b*sec(b*x + a))^n, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int (b\sec(bx+a))^n(c\sin(bx+a))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

```
[Out] int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sin(bx+a))^{\frac{3}{2}}(b\sec(bx+a))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a))^(3/2)*(b*sec(b*x + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,\sin(a+b\,x))^{3/2}\left(\frac{b}{\cos(a+b\,x)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(3/2)*(b/cos(a + b*x))^n,x)
```

```
[Out] int((c*sin(a + b*x))^(3/2)*(b/cos(a + b*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))**n*(c*sin(b*x+a))**(3/2),x)
```

```
[Out] Timed out
```

3.504 $\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx$

Optimal. Leaf size=76

$$-\frac{c\sqrt[4]{\sin^2(a+bx)}\,(b\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{(1-n)\sqrt{c\sin(a+bx)}}$$

```
[Out] -c*hypergeom([1/4, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(b*sec(b*x+a))^(-1+
n)*(sin(b*x+a)^2)^(1/4)/(1-n)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2576}

$$-\frac{c\sqrt[4]{\sin^2(a+bx)}\,(b\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{(1-n)\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((c*Hypergeometric2F1[1/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
+ b*x])^(-1 + n)*(Sin[a + b*x]^2)^(1/4))/((1 - n)*Sqrt[c*Sin[a + b*x]]))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int (b\sec(a+bx))^n\sqrt{c\sin(a+bx)}\,dx = \left(b^2(b\cos(a+bx))^{-1+n}(b\sec(a+bx))^{-1+n}\right)\int (b\cos(a+bx))^{-n}\sqrt{c\sin(a-$$

$$= -\frac{c\,{}_2F_1\left(\frac{1}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt[4]{\sin^2(a+bx)}}{(1-n)\sqrt{c\sin(a+bx)}}$$

Mathematica [A] time = 0.12, size = 75, normalized size = 0.99

$$\frac{\sin(2(a+bx))\sqrt{c\sin(a+bx)}\,\cos^2(a+bx)^{\frac{n-1}{2}}(b\sec(a+bx))^n\,{}_2F_1\left(\frac{3}{4},\frac{n+1}{2};\frac{7}{4};\sin^2(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[a + b*x])^n*Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] ((Cos[a + b*x]^2)^((-1 + n)/2)*Hypergeometric2F1[3/4, (1 + n)/2, 7/4, Sin[a
 + b*x]^2]*(b*Sec[a + b*x])^n*Sqrt[c*Sin[a + b*x]]*Sin[2*(a + b*x)])/(3*b)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{c\sin\left(bx+a\right)}\,\left(b\sec\left(bx+a\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\sin\left(bx+a\right)}\,\left(b\sec\left(bx+a\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n, x)
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(bx+a\right)\right)^n\sqrt{c\sin\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
[Out] int((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c \sin(bx+a)}\,(b\sec(bx+a))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n*(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,\sin(a+b\,x)}\,\left(\frac{b}{\cos(a+b\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x))^(1/2)*(b/cos(a + b*x))^n,x)
[Out] int((c*sin(a + b*x))^(1/2)*(b/cos(a + b*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(a+bx))^n\,\sqrt{c\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))**n*(c*sin(b*x+a))**(1/2),x)
[Out] Integral((b*sec(a + b*x))**n*sqrt(c*sin(a + b*x)), x)
```

3.505 $\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$

Optimal. Leaf size=76

$$-\frac{c\sin^2(a+bx)^{3/4}(b\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{3}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{(1-n)(c\sin(a+bx))^{3/2}}$$

```
[Out] -c*hypergeom([3/4, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(b*sec(b*x+a))^(-1+
n)*(sin(b*x+a)^2)^(3/4)/(1-n)/(c*sin(b*x+a))^(3/2)
```

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2576}

$$-\frac{c\sin^2(a+bx)^{3/4}(b\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{3}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{(1-n)(c\sin(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
```

```
[Out] -((c*Hypergeometric2F1[3/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a
+ b*x])^(-1 + n)*(Sin[a + b*x]^2)^(3/4))/((1 - n)*(c*Sin[a + b*x])^(3/2)))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx = \left(b^2(b\cos(a+bx))^{-1+n}(b\sec(a+bx))^{-1+n}\right)\int \frac{(b\cos(a+bx))^{-n}}{\sqrt{c\sin(a+bx)}}\,dx$$
$$= -\frac{c\,{}_2F_1\left(\frac{3}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sin^2(a+bx)^{3/4}}{(1-n)(c\sin(a+bx))^{3/2}}$$

Mathematica [A] time = 0.14, size = 72, normalized size = 0.95

$$\frac{\sin(2(a+bx))\cos^2(a+bx)^{\frac{n-1}{2}}(b\sec(a+bx))^n\,{}_2F_1\left(\frac{1}{4},\frac{n+1}{2};\frac{5}{4};\sin^2(a+bx)\right)}{b\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[a + b*x])^n/Sqrt[c*Sin[a + b*x]],x]
[Out] ((Cos[a + b*x]^2)^((-1 + n)/2)*Hypergeometric2F1[1/4, (1 + n)/2, 5/4, Sin[a
 + b*x]^2]*(b*Sec[a + b*x])^n*Sin[2*(a + b*x)])/(b*Sqrt[c*Sin[a + b*x]])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{c\sin(bx+a)}\,(b\sec(bx+a))^n}{c\sin(bx+a)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n/(c*sin(b*x + a)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(bx+a))^n}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(bx+a))^n}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

```
[Out] int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(bx+a))^n}{\sqrt{c\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(b*x + a))^n/sqrt(c*sin(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos(a+b\,x)}\right)^n}{\sqrt{c\,\sin(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(a + b*x))^n/(c*sin(a + b*x))^(1/2),x)
```

```
[Out] int((b/cos(a + b*x))^n/(c*sin(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(a+bx))^n}{\sqrt{c\sin(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))**n/(c*sin(b*x+a))**(1/2),x)
```

```
[Out] Integral((b*sec(a + b*x))**n/sqrt(c*sin(a + b*x)), x)
```

3.506 $\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{\sqrt[4]{\sin^2(a+bx)}\,(b\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{5}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{c(1-n)\sqrt{c\sin(a+bx)}}$$

```
[Out] -hypergeom([5/4, 1/2-1/2*n],[3/2-1/2*n],cos(b*x+a)^2)*(b*sec(b*x+a))^(-1+n)
*(sin(b*x+a)^2)^(1/4)/c/(1-n)/(c*sin(b*x+a))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2587, 2576}

$$-\frac{\sqrt[4]{\sin^2(a+bx)}\,(b\sec(a+bx))^{n-1}\,{}_2F_1\left(\frac{5}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)}{c(1-n)\sqrt{c\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -((Hypergeometric2F1[5/4, (1 - n)/2, (3 - n)/2, Cos[a + b*x]^2]*(b*Sec[a +
b*x])^(-1 + n)*(Sin[a + b*x]^2)^(1/4))/(c*(1 - n)*Sqrt[c*Sin[a + b*x]]))
```

Rule 2576

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Sin[e + f*x])^(2*Fr
acPart[(n - 1)/2])*(a*Cos[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
 - n)/2, (3 + m)/2, Cos[e + f*x]^2])/(a*f*(m + 1)*(Sin[e + f*x]^2)^FracPart
[(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2587

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int \frac{(b\sec(a+bx))^n}{(c\sin(a+bx))^{3/2}}\,dx = \left(b^2(b\cos(a+bx))^{-1+n}(b\sec(a+bx))^{-1+n}\right)\int \frac{(b\cos(a+bx))^{-n}}{(c\sin(a+bx))^{3/2}}\,dx$$
$$= -\frac{{}_2F_1\left(\frac{5}{4},\frac{1-n}{2};\frac{3-n}{2};\cos^2(a+bx)\right)(b\sec(a+bx))^{-1+n}\sqrt[4]{\sin^2(a+bx)}}{c(1-n)\sqrt{c\sin(a+bx)}}$$

Mathematica [A] time = 0.15, size = 73, normalized size = 0.94

$$-\frac{\sin(2(a+bx))\cos^2(a+bx)^{\frac{n-1}{2}}(b\sec(a+bx))^n\,{}_2F_1\left(-\frac{1}{4},\frac{n+1}{2};\frac{3}{4};\sin^2(a+bx)\right)}{b(c\sin(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[a + b*x])^n/(c*Sin[a + b*x])^(3/2),x]
```

```
[Out] -(((Cos[a + b*x]^2)^((-1 + n)/2)*Hypergeometric2F1[-1/4, (1 + n)/2, 3/4, Si
n[a + b*x]^2]*(b*Sec[a + b*x])^n*Sin[2*(a + b*x)])/(b*(c*Sin[a + b*x])^(3/2
)))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{c\sin(bx+a)}\,(b\sec(bx+a))^n}{c^2\cos(bx+a)^2-c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*sin(b*x + a))*(b*sec(b*x + a))^n/(c^2*cos(b*x + a)^2 - c^2
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(bx+a))^n}{(c\sin(bx+a))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(b \sec (bx + a))^n}{(c \sin (bx + a))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
[Out] int((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec (bx + a))^n}{(c \sin (bx + a))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))^n/(c*sin(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(b*x + a))^n/(c*sin(b*x + a))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{b}{\cos(a+b\,x)}\right)^n}{(c\,\sin(a+b\,x))^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/cos(a + b*x))^n/(c*sin(a + b*x))^(3/2),x)
[Out] int((b/cos(a + b*x))^n/(c*sin(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec (a + bx))^n}{(c \sin (a + bx))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(b*x+a))**n/(c*sin(b*x+a))**(3/2),x)
[Out] Integral((b*sec(a + b*x))**n/(c*sin(a + b*x))**(3/2), x)
```

3.507 $\int \sqrt{d\csc(e+fx)}\sin^4(e+fx)\,dx$

Optimal. Leaf size=100

$$-\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21f}$$

```
[Out] -2/7*d^3*cos(f*x+e)/f/(d*csc(f*x+e))^(5/2)-10/21*d*cos(f*x+e)/f/(d*csc(f*x+
e))^(1/2)-10/21*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*
x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*
x+e)^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3769, 3771, 2641}

$$-\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^4,x]
```

```
[Out] (-2*d^3*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*d*Cos[e + f*x])/(2
1*f*Sqrt[d*Csc[e + f*x]]) + (10*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 +
f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{d\csc(e+fx)}\,\sin^4(e+fx)\,dx &= d^4\int \frac{1}{(d\csc(e+fx))^{7/2}}\,dx\\
&= -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}+\frac{1}{7}\left(5d^2\right)\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx\\
&= -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{5}{21}\int \sqrt{d\csc(e+fx)}\,dx\\
&= -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{1}{21}\left(5\sqrt{d\csc(e+fx)}\,\sqrt{\sin(e-\right.\\
&= -\frac{2d^3\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}\right.\right.}{21f}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 67, normalized size = 0.67

$$
-\frac{\sqrt{d\csc(e+fx)}\left(26\sin(2(e+fx))-3\sin(4(e+fx))+40\sqrt{\sin(e+fx)}\,F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{84f}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^4,x]
```

```
[Out] -1/84*(Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Si
n[e + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/f
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$
\text{integral}\left(\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\sqrt{d\csc\left(fx+e\right)},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(d*csc(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \csc\left(fx+e\right)} \sin\left(fx+e\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^4, x)
```

maple [C] time = 0.35, size = 214, normalized size = 2.14

$$\frac{\sin\left(fx+e\right)\sqrt{\frac{d}{\sin\left(fx+e\right)}}\left(-5i\sin\left(fx+e\right)\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}, \frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i(\cdots}{}}}{2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x)
[Out] 1/21/f*sin(f*x+e)*(d/sin(f*x+e))^(1/2)*(-5*I*sin(f*x+e)*(-(I*cos(f*x+e)-sin
(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x
+e))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e
)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)+3*cos(f*x+e)^4*2^(1/2)-3*cos(f*x+e)^3*2^(
1/2)-8*cos(f*x+e)^2*2^(1/2)+8*cos(f*x+e)*2^(1/2))/(-1+cos(f*x+e))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \csc\left(fx+e\right)} \sin\left(fx+e\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^4\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(d/sin(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^4*(d/sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \csc \left(e + fx\right)} \sin^{4}\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(e + f*x))*sin(e + f*x)**4, x)
```

3.508 $\int \sqrt{d\csc(e+fx)}\sin^3(e+fx)\,dx$

Optimal. Leaf size=75

$$\frac{6dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

```
[Out] -2/5*d^2*cos(f*x+e)/f/(d*csc(f*x+e))^(3/2)-6/5*d*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3769, 3771, 2639}

$$\frac{6dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^3,x]
```

```
[Out] (-2*d^2*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*d*EllipticE[(e - Pi
/2 + f*x)/2, 2])/(5*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\csc(e+fx)}\,\sin^3(e+fx)\,dx &= d^3\int \frac{1}{(d\csc(e+fx))^{5/2}}\,dx\\ &= -\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{1}{5}(3d)\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\ &= -\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{(3d)\int \sqrt{\sin(e+fx)}\,dx}{5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= -\frac{2d^2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{6dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 62, normalized size = 0.83

$$-\frac{2\sqrt{d\csc(e+fx)}\left(\sin^2(e+fx)\cos(e+fx)+3\sqrt{\sin(e+fx)}\,E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^3,x]
```

```
[Out] (-2*Sqrt[d*Csc[e + f*x]]*(3*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e
+ f*x]] + Cos[e + f*x]*Sin[e + f*x]^2))/(5*f)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\sqrt{d\csc\left(fx+e\right)}\,\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(d*csc(f*x + e))*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\csc\left(fx+e\right)}\,\sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^3, x)
```

maple [C] time = 0.28, size = 538, normalized size = 7.17

$$\frac{\sqrt{\frac{d}{\sin(fx+e)}}\left(-6\cos\left(fx+e\right)\sqrt{-\frac{i(-1+\cos(fx+e))}{\sin(fx+e)}}\sqrt{\frac{i\cos(fx+e)+\sin(fx+e)-i}{\sin(fx+e)}}\sqrt{-\frac{i\cos(fx+e)-\sin(fx+e)-i}{\sin(fx+e)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos}{}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] 1/5/f*(d/sin(f*x+e))^(1/2)*(-6*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x
+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))
^(1/2),1/2*2^(1/2))+3*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*
cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/si
n(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/
2*2^(1/2))+cos(f*x+e)^3*2^(1/2)-6*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I
*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/s
in(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1
/2*2^(1/2))+3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+
e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*Ell
ipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-4*cos(f*
x+e)*2^(1/2)+3*2^(1/2))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\csc\left(fx+e\right)}\,\sin\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\sin\left(e+f\,x\right)}^3\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(d/sin(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^3*(d/sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(d*csc(f*x+e))**(1/2),x)
[Out] Timed out
```

3.509 $\int \sqrt{d \csc (e+f x)} \sin ^2(e+f x) \, dx$

Optimal. Leaf size=72

$$\frac{2 \sqrt{\sin (e+f x)} F\left(\left.\frac{1}{2} \left(e+f x-\frac{\pi }{2}\right)\right|2\right) \sqrt{d \csc (e+f x)}}{3 f}-\frac{2 d \cos (e+f x)}{3 f \sqrt{d \csc (e+f x)}}$$

```
[Out] -2/3*d*cos(f*x+e)/f/(d*csc(f*x+e))^(1/2)-2/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
)*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3769, 3771, 2641}

$$\frac{2 \sqrt{\sin (e+f x)} F\left(\left.\frac{1}{2} \left(e+f x-\frac{\pi }{2}\right)\right|2\right) \sqrt{d \csc (e+f x)}}{3 f}-\frac{2 d \cos (e+f x)}{3 f \sqrt{d \csc (e+f x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^2,x]
```

```
[Out] (-2*d*Cos[e + f*x])/(3*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*El
lipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{d\csc(e+fx)}\,\sin^2(e+fx)\,dx &= d^2\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx\\
&= -\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}+\frac{1}{3}\int \sqrt{d\csc(e+fx)}\,dx\\
&= -\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}+\frac{1}{3}\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx\\
&= -\frac{2d\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}+\frac{2\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{3f}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 55, normalized size = 0.76

$$-\frac{\sqrt{d\csc(e+fx)}\left(\sin(2(e+fx))+2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x]^2,x]
```

```
[Out] -1/3*(Sqrt[d*Csc[e + f*x]]*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[
e + f*x]] + Sin[2*(e + f*x)]))/f
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\sqrt{d\csc\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(d*csc(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\csc\left(fx+e\right)}\,\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^2, x)
```

maple [C] time = 0.18, size = 187, normalized size = 2.60

$$-\frac{\sin\left(fx+e\right)\sqrt{\frac{d}{\sin\left(fx+e\right)}}\left(i\sin\left(fx+e\right)\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i(-1}{}}}{3f\left(-1+\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*sin(f*x+e)*(d/sin(f*x+e))^(1/2)*(I*sin(f*x+e)*(-I*(-1+cos(f*x+e))/si
n(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x
+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/
sin(f*x+e))^(1/2),1/2*2^(1/2))+cos(f*x+e)^2*2^(1/2)-cos(f*x+e)*2^(1/2))/(-1
+cos(f*x+e))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\csc\left(fx+e\right)}\,\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(e+f\,x\right)^2\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(d/sin(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^2*(d/sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\csc\left(e+fx\right)}\,\sin^{2}\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(e + f*x))*sin(e + f*x)**2, x)
```

3.510 $\int \sqrt{d \csc(e+fx)} \sin(e+fx)\,dx$

Optimal. Leaf size=44

$$\frac{2dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] -2*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
E(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2
)
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3771, 2639}

$$\frac{2dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x],x]
```

```
[Out] (2*d*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e +
 f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sqrt{d\csc(e+fx)}\,\sin(e+fx)\,dx = d\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx$$
$$= \frac{d\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\,\sqrt{\sin(e+fx)}}$$
$$= \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\,\sqrt{\sin(e+fx)}}$$

Mathematica [A] time = 0.05, size = 43, normalized size = 0.98

$$-\frac{2dE\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{f\sqrt{\sin(e+fx)}\,\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[e + f*x]]*Sin[e + f*x],x]
[Out] (-2*d*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin
[e + f*x]])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\csc\left(fx+e\right)}\,\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*csc(f*x + e))*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\csc\left(fx+e\right)}\,\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e), x)
```

maple [C] time = 0.16, size = 525, normalized size = 11.93

$$-\frac{\left(2\cos\left(fx+e\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\;\mathrm{EllipticE}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx\right.}{\sin\left(fx+e\right)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin
(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2
)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos
(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/
sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF
(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*(-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-
(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(
f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+
e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^
(1/2),1/2*2^(1/2))+cos(f*x+e)*2^(1/2)-2^(1/2))*(d/sin(f*x+e))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\csc\left(fx+e\right)}\,\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*sin(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(e+f\,x\right)\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(d/sin(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)*(d/sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \csc \left(e+fx\right)} \sin \left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(d*csc(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*csc(e + f*x))*sin(e + f*x), x)
```

3.511 $\int \sqrt{d\csc(e+fx)}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{f}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]]
)/f
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\csc(e+fx)}\,dx &= \left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx\\ &= \frac{2\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{f}\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.98

$$-\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\sqrt{d\csc(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f
*x]])/f
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\csc\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{d\csc\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(f*x + e)), x)
```

maple [C] time = 0.13, size = 165, normalized size = 3.84

$$-\frac{i\sqrt{2}\,\sqrt{\frac{d}{\sin\left(fx+e\right)}}\,\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\,\text{EllipticF}}{f\sin\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(1/2),x)
```

```
[Out] -I/f*2^(1/2)*(d/sin(f*x+e))^(1/2)*(-1+cos(f*x+e))*((I*cos(f*x+e)+sin(f*x+e)
-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-I*(
```

```
-1+cos(f*x+e))/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin
(f*x+e))^(1/2),1/2*2^(1/2))/sin(f*x+e)^2*(cos(f*x+e)+1)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \csc\left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*csc(f*x + e)), x)
```

mupad [B] time = 0.62, size = 63, normalized size = 1.47

$$-\frac{2\sqrt{\sin\left(e+f\,x\right)}\;\mathrm{F}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{1-\sin\left(e+f\,x\right)}}{2}\right)\middle|\,2\right)\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}\,\sqrt{{\cos\left(e+f\,x\right)}^2}}{f\,\cos\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^(1/2),x)
[Out] -(2*sin(e + f*x)^(1/2)*ellipticF(asin((2^(1/2)*(1 - sin(e + f*x))^(1/2))/2)
, 2)*(d/sin(e + f*x))^(1/2)*(cos(e + f*x)^2)^(1/2))/(f*cos(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \csc\left(e+fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*csc(e + f*x)), x)
```

3.512 $\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx$

Optimal. Leaf size=68

$$-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}-\frac{2dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] -2*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/f+2*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2
)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/
(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {16, 3768, 3771, 2639}

$$-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}-\frac{2dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d*EllipticE[(e - Pi/2 + f*x)/
2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \csc(e+fx)\sqrt{d\csc(e+fx)}\,dx &= \frac{\int (d\csc(e+fx))^{3/2}\,dx}{d}\\ &= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - d\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\ &= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - \frac{d\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{f} - \frac{2dE\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.13, size = 57, normalized size = 0.84

$$\frac{(d\csc(e+fx))^{3/2}\left(2\sin^{\frac{3}{2}}(e+fx)E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)-\sin(2(e+fx))\right)}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] ((d*Csc[e + f*x])^(3/2)*(2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x]
^(3/2) - Sin[2*(e + f*x)]))/(d*f)
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e), x)
```

maple [C] time = 0.16, size = 514, normalized size = 7.56

$$\frac{\left(2\cos\left(fx+e\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\ \sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(
f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)
*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(
f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/s
in(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(
((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*(-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(
I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f
*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e
)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(
1/2),1/2*2^(1/2))-2^(1/2))*(d/sin(f*x+e))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\csc\left(fx+e\right)}\,\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^(1/2)/sin(e + f*x),x)
```

```
[Out] int((d/sin(e + f*x))^(1/2)/sin(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \csc \left(e + fx\right)} \csc \left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(d*csc(e + f*x))*csc(e + f*x), x)
```

3.513 $\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx$

Optimal. Leaf size=74

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df}$$

```
[Out] -2/3*cos(f*x+e)*(d*csc(f*x+e))^(3/2)/d/f-2/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
)*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*d*f) + (2*Sqrt[d*Csc[e + f*x]]*
EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)\sqrt{d\csc(e+fx)}\,dx &= \frac{\int (d\csc(e+fx))^{5/2}\,dx}{d^2}\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df}+\frac{1}{3}\int \sqrt{d\csc(e+fx)}\,dx\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df}+\frac{1}{3}\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{}{\sqrt{s}}\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3df}+\frac{2\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{}}{3f}\end{aligned}$$

Mathematica [A] time = 0.10, size = 55, normalized size = 0.74

$$-\frac{2(d\csc(e+fx))^{3/2}\left(\cos(e+fx)+\sin^{\frac{3}{2}}(e+fx)F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*(d*Csc[e + f*x])^(3/2)*(Cos[e + f*x] + EllipticF[(-2*e + Pi - 2*f*x)/4,
 2]*Sin[e + f*x]^(3/2)))/(3*d*f)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^2, x)
```

maple [C] time = 0.18, size = 319, normalized size = 4.31

$$\sqrt{\frac{d}{\sin\left(fx+e\right)}}\,\left(\cos\left(fx+e\right)+1\right)^{2}\left(-1+\cos\left(fx+e\right)\right)^{2}\left(i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+\right.}{\sin\left(fx+e\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*(d/sin(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(I*EllipticF(
((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)+
sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*
cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*sin(f*x+e)*cos(f*x+e)+I*sin(f*x+
e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f
*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*
cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)*2^(1/2))
/sin(f*x+e)^5*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\csc\left(fx+e\right)}\,\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}}{\sin\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^(1/2)/sin(e + f*x)^2,x)
```

```
[Out] int((d/sin(e + f*x))^(1/2)/sin(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \csc \left(e + f x\right)} \csc^{2}\left(e + f x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(d*csc(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*csc(e + f*x))*csc(e + f*x)**2, x)
```

3.514 $\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx$

Optimal. Leaf size=100

$$-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f}-\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}-\frac{6dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] -2/5*cos(f*x+e)*(d*csc(f*x+e))^(5/2)/d^2/f-6/5*cos(f*x+e)*(d*csc(f*x+e))^(1
/2)/f+6/5*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*E
llipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+
e)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3768, 3771, 2639}

$$-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f}-\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}-\frac{6dE\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-6*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(5*f) - (2*Cos[e + f*x]*(d*Csc[e + f
*x])^(5/2))/(5*d^2*f) - (6*d*EllipticE[(e - Pi/2 + f*x)/2, 2])/(5*f*Sqrt[d*
Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \csc^3(e+fx)\sqrt{d\csc(e+fx)}\,dx &= \frac{\int (d\csc(e+fx))^{7/2}\,dx}{d^3}\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f}+\frac{3\int (d\csc(e+fx))^{3/2}\,dx}{5d}\\ &= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f}-\frac{1}{5}(3d)\int\\ &= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f}-\frac{(3d)}{5\sqrt{d\csc}}\\ &= -\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^2f}-\frac{6dE}{5f\sqrt{d\,\mathrm{cs}}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 68, normalized size = 0.68

$$-\frac{2\sqrt{d\csc(e+fx)}\left(3\cos(e+fx)+\cot(e+fx)\csc(e+fx)-3\sqrt{\sin(e+fx)}\,E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Sqrt[d*Csc[e + f*x]]*(3*Cos[e + f*x] + Cot[e + f*x]*Csc[e + f*x] - 3*El
lipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f*x]]))/(5*f)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\csc\left(fx+e\right)}\,\csc\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)^3, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^3, x)
```

maple [C] time = 0.21, size = 1054, normalized size = 10.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/5/f*(6*cos(f*x+e)^3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)
+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^
(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))
-3*cos(f*x+e)^3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*
x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*E
llipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+6*cos(
f*x+e)^2*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)
/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*Elliptic
E(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*cos(f*x+e)^
2*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*
x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*c
os(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-6*cos(f*x+e)*(-I*(-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/
2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)
+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+3*cos(f*x+e)*(-I*(-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*c
os(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+
e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-6*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)
-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1
/2),1/2*2^(1/2))+3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin
(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2
)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*c
os(f*x+e)^2*2^(1/2)+cos(f*x+e)*2^(1/2)+3*2^(1/2))*(d/sin(f*x+e))^(1/2)/sin(
f*x+e)^2*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \csc\left(fx+e\right)} \csc\left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*csc(f*x + e))*csc(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}}{\sin\left(e+f\,x\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^(1/2)/sin(e + f*x)^3,x)
[Out] int((d/sin(e + f*x))^(1/2)/sin(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d \csc\left(e+fx\right)} \csc^{3}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(d*csc(f*x+e))**(1/2),x)
[Out] Integral(sqrt(d*csc(e + f*x))*csc(e + f*x)**3, x)
```

3.515 $\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx$

Optimal. Leaf size=103

$$-\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10d\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21f}$$

```
[Out] -2/7*d^4*cos(f*x+e)/f/(d*csc(f*x+e))^(5/2)-10/21*d^2*cos(f*x+e)/f/(d*csc(f*
x+e))^(1/2)-10/21*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/
2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*si
n(f*x+e)^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3769, 3771, 2641}

$$-\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10d\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] (-2*d^4*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*d^2*Cos[e + f*x])/
(21*f*Sqrt[d*Csc[e + f*x]]) + (10*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/
2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int (d\csc(e+fx))^{3/2}\sin^5(e+fx)\,dx &= d^5\int \frac{1}{(d\csc(e+fx))^{7/2}}\,dx\\ &= -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}+\frac{1}{7}\left(5d^3\right)\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx\\ &= -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{1}{21}(5d)\int \sqrt{d\csc(e+fx)}\\ &= -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{1}{21}\left(5d\sqrt{d\csc(e+fx)}\sqrt{\sin}\right.\\ &= -\frac{2d^4\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10d^2\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10d\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e\right.\right.}{2}\end{aligned}$$

Mathematica [A] time = 0.14, size = 68, normalized size = 0.66

$$-\frac{d\sqrt{d\csc(e+fx)}\left(26\sin(2(e+fx))-3\sin(4(e+fx))+40\sqrt{\sin(e+fx)}\,F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{84f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^5,x]
```

```
[Out] -1/84*(d*Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[
Sin[e + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/f
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\cos\left(fx+e\right)^4-2\,d\cos\left(fx+e\right)^2+d\right)\sqrt{d\csc\left(fx+e\right)}\,\csc\left(fx+e\right)\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="fricas")
```

[Out] integral((d*cos(f*x + e)^4 - 2*d*cos(f*x + e)^2 + d)*sqrt(d*csc(f*x + e))*c
sc(f*x + e)*sin(f*x + e), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \csc\left(fx+e\right)\right)^{\frac{3}{2}} \sin\left(fx+e\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="giac")

[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^5, x)

maple [C] time = 0.16, size = 216, normalized size = 2.10

$$-\frac{\left(5i \sin\left(fx+e\right) \sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}, \frac{\sqrt{2}}{2}\right) \sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\ \sqrt{\frac{i\cos\left(fx+e\right)+\mathrm{s}}{\sin\left(fx\right.}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x)

[Out] -1/21/f*(5*I*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*((I*cos(f*x+e
)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))
^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2)
)-3*cos(f*x+e)^4*2^(1/2)+3*cos(f*x+e)^3*2^(1/2)+8*cos(f*x+e)^2*2^(1/2)-8*co
s(f*x+e)*2^(1/2))*(d/sin(f*x+e))^(3/2)*sin(f*x+e)^2/(-1+cos(f*x+e))*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \csc\left(fx+e\right)\right)^{\frac{3}{2}} \sin\left(fx+e\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^5,x, algorithm="maxima")

[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^5, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^5\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(d/sin(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)^5*(d/sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**5,x)
```

```
[Out] Timed out
```

3.516 $\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx$

Optimal. Leaf size=77

$$\frac{6d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

```
[Out] -2/5*d^3*cos(f*x+e)/f/(d*csc(f*x+e))^(3/2)-6/5*d^2*(sin(1/2*e+1/4*Pi+1/2*f*
x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2
^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3769, 3771, 2639}

$$\frac{6d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (-2*d^3*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*d^2*EllipticE[(e -
Pi/2 + f*x)/2, 2])/(5*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int (d\csc(e+fx))^{3/2}\sin^4(e+fx)\,dx &= d^4\int \frac{1}{(d\csc(e+fx))^{5/2}}\,dx\\ &= -\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{1}{5}\left(3d^2\right)\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\ &= -\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{\left(3d^2\right)\int \sqrt{\sin(e+fx)}\,dx}{5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= -\frac{2d^3\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{6d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.19, size = 62, normalized size = 0.81

$$-\frac{2(d\csc(e+fx))^{3/2}\left(\sin^3(e+fx)\cos(e+fx)+3\sin^{\frac{3}{2}}(e+fx)E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^4,x]
```

```
[Out] (-2*(d*Csc[e + f*x])^(3/2)*(3*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f
*x]^(3/2) + Cos[e + f*x]*Sin[e + f*x]^3))/(5*f)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d\cos\left(fx+e\right)^4-2\,d\cos\left(fx+e\right)^2+d\right)\sqrt{d\csc\left(fx+e\right)}\,\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((d*cos(f*x + e)^4 - 2*d*cos(f*x + e)^2 + d)*sqrt(d*csc(f*x + e))*c
sc(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \csc \left(fx + e\right)\right)^{\frac{3}{2}} \sin \left(fx + e\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

maple [C] time = 0.18, size = 545, normalized size = 7.08

$$-\frac{\left(6 \cos \left(fx+e\right) \sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\, \sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\, \sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+\right.}{\sin\left(fx+e\right)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x)
```

```
[Out] -1/5/f*(6*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1
/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-3
*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)
-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*Ellip
ticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+6*(-I*(-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2
)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+
sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*(-I*(-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-si
n(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*
x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)^3*2^(1/2)+4*cos(f*x+e)*2^(1/2)-3*2^(1/2
))*(d/sin(f*x+e))^(3/2)*sin(f*x+e)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \csc \left(fx + e\right)\right)^{\frac{3}{2}} \sin \left(fx + e\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^4\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(d/sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^4*(d/sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**4,x)
[Out] Timed out
```

3.517 $\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx$

Optimal. Leaf size=75

$$\frac{2d\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

```
[Out] -2/3*d^2*cos(f*x+e)/f/(d*csc(f*x+e))^(1/2)-2/3*d*(sin(1/2*e+1/4*Pi+1/2*f*x)
^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(
1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3769, 3771, 2641}

$$\frac{2d\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] (-2*d^2*Cos[e + f*x])/(3*f*Sqrt[d*Csc[e + f*x]]) + (2*d*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (d\csc(e+fx))^{3/2}\sin^3(e+fx)\,dx = d^3\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$$
$$= -\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{1}{3}d\int \sqrt{d\csc(e+fx)}\,dx$$
$$= -\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{1}{3}\left(d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx}}$$
$$= -\frac{2d^2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{2d\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+f}}{3f}$$

Mathematica [A] time = 0.06, size = 56, normalized size = 0.75

$$-\frac{d\sqrt{d\csc(e+fx)}\left(\sin(2(e+fx))+2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^3,x]
```

```
[Out] -1/3*(d*Sqrt[d*Csc[e + f*x]]*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Si
n[e + f*x]] + Sin[2*(e + f*x)]))/f
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(d\cos\left(fx+e\right)^2-d\right)\sqrt{d\csc\left(fx+e\right)}\,\csc\left(fx+e\right)\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral(-(d*cos(f*x + e)^2 - d)*sqrt(d*csc(f*x + e))*csc(f*x + e)*sin(f*x
+ e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \csc \left(fx+e\right)\right)^{\frac{3}{2}} \sin \left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^3, x)
```

maple [C] time = 0.15, size = 189, normalized size = 2.52

$$-\frac{\left(i \sin \left(fx+e\right) \sqrt{-\frac{i \cos \left(fx+e\right)-\sin \left(fx+e\right)-i}{\sin \left(fx+e\right)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i \cos \left(fx+e\right)+\sin \left(fx+e\right)-i}{\sin \left(fx+e\right)}}, \frac{\sqrt{2}}{2}\right) \sqrt{-\frac{i \left(-1+\cos \left(fx+e\right)\right)}{\sin \left(fx+e\right)}}\ \sqrt{\frac{i \cos \left(fx+e\right)+\sin}{\sin \left(fx+\right.}}}{3f \left(-1+\cos \left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x)
```

```
[Out] -1/3/f*(I*sin(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1
/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+c
os(f*x+e)^2*2^(1/2)-cos(f*x+e)*2^(1/2))*(d/sin(f*x+e))^(3/2)*sin(f*x+e)^2/(
-1+cos(f*x+e))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d \csc \left(fx+e\right)\right)^{\frac{3}{2}} \sin \left(fx+e\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\sin\left(e+f\,x\right)}^3\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(d/sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^3*(d/sin(e + f*x))^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**3,x)
[Out] Timed out
```

3.518 $\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx$

Optimal. Leaf size=46

$$\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] -2*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Ellipt
icE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1
/2)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3771, 2639}

$$\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (2*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e
 + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (d\csc(e+fx))^{3/2}\sin^2(e+fx)\,dx = d^2\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx$$
$$= \frac{d^2\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$
$$= \frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.98

$$-\frac{2d^2E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x]^2,x]
```

```
[Out] (-2*d^2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[S
in[e + f*x]])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(d\cos\left(fx+e\right)^2-d\right)\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(d*cos(f*x + e)^2 - d)*sqrt(d*csc(f*x + e))*csc(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

maple [C] time = 0.15, size = 531, normalized size = 11.54

$$-\frac{\left(2\cos\left(fx+e\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+\right.}{\sin\left(fx+e\right)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x)
```

```
[Out] -1/f*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin
(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2
)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos
(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/
sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF
(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*(-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-
(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(
f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+
e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^
(1/2),1/2*2^(1/2))+cos(f*x+e)*2^(1/2)-2^(1/2))*(d/sin(f*x+e))^(3/2)*sin(f*x
+e)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sin\left(e+f\,x\right)^2\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(d/sin(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)^2*(d/sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e)**2,x)
[Out] Timed out
```

3.519 $\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx$

Optimal. Leaf size=44

$$\frac{2d\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{f}$$

```
[Out] -2*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*Elliptic
F(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/
f
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3771, 2641}

$$\frac{2d\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x],x]
```

```
[Out] (2*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x
]])/f
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int (d\csc(e+fx))^{3/2}\sin(e+fx)\,dx = d\int \sqrt{d\csc(e+fx)}\,dx$$
$$= \left(d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx$$
$$= \frac{2d\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{f}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.98

$$-\frac{2d\sqrt{\sin(e+fx)}\,F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\sqrt{d\csc(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(3/2)*Sin[e + f*x],x]
```

```
[Out] (-2*d*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
 f*x]])/f
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{d\csc\left(fx+e\right)}\,d\csc\left(fx+e\right)\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*d*csc(f*x + e)*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e), x)
```

maple [C] time = 0.17, size = 165, normalized size = 3.75

$$-\frac{i\sqrt{2}\,\left(\cos\left(fx+e\right)+1\right)^{2}\mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{\frac{i\cos}{}}}{f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(3/2)*sin(f*x+e),x)
```

```
[Out] -I/f*2^(1/2)*(cos(f*x+e)+1)^2*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*
x+e))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x
+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e)
)^(1/2)*(-1+cos(f*x+e))*(d/sin(f*x+e))^(3/2)/sin(f*x+e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2)*sin(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*sin(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(e+f\,x\right)\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(d/sin(e + f*x))^(3/2),x)
```

```
[Out] int(sin(e + f*x)*(d/sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}\sin\left(e+fx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(3/2)*sin(f*x+e),x)
```

```
[Out] Integral((d*csc(e + f*x))**(3/2)*sin(e + f*x), x)
```

3.520 $\int (d\csc(e+fx))^{3/2}\,dx$

Optimal. Leaf size=71

$$-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}$$

```
[Out] -2*d*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/f+2*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
)/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3768, 3771, 2639}

$$-\frac{2d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*d*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/f - (2*d^2*EllipticE[(e - Pi/2 + f
*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int (d\csc(e+fx))^{3/2}\,dx &= -\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}-d^2\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\ &= -\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}-\frac{d^2\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= -\frac{2d\cos(e+fx)\sqrt{d\csc(e+fx)}}{f}-\frac{2d^2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.76

$$\frac{(d\csc(e+fx))^{3/2}\left(2\sin^{\frac{3}{2}}(e+fx)E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)-\sin(2(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] ((d*Csc[e + f*x])^(3/2)*(2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x]
^(3/2) - Sin[2*(e + f*x)]))/f
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{d\csc\left(fx+e\right)}\,d\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*d*csc(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2), x)
```

maple [C] time = 0.14, size = 520, normalized size = 7.32

$$\frac{\left(2\cos\left(fx+e\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+\right.}{\sin\left(fx+e\right)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*csc(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(
f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)
*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(
f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/s
in(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(
((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*(-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(
I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f
*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e
)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(
1/2),1/2*2^(1/2))-2^(1/2))*(d/sin(f*x+e))^(3/2)*sin(f*x+e)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^(3/2),x)
```

```
[Out] int((d/sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*csc(e + f*x))**(3/2), x)
```

3.521 $\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx$

Optimal. Leaf size=72

$$\frac{2d\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}$$

```
[Out] -2/3*cos(f*x+e)*(d*csc(f*x+e))^(3/2)/f-2/3*d*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
)*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/f
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {16, 3768, 3771, 2641}

$$\frac{2d\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*f) + (2*d*Sqrt[d*Csc[e + f*x]]*
EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \csc(e+fx)(d\csc(e+fx))^{3/2}\,dx &= \frac{\int (d\csc(e+fx))^{5/2}\,dx}{d}\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}+\frac{1}{3}d\int \sqrt{d\csc(e+fx)}\,dx\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}+\frac{1}{3}\left(d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{}{\sqrt{s}}\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3f}+\frac{2d\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{3f}\end{aligned}$$

Mathematica [A] time = 0.19, size = 58, normalized size = 0.81

$$-\frac{(d\csc(e+fx))^{5/2}\left(\sin(2(e+fx))+2\sin^{\frac{5}{2}}(e+fx)F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] -1/3*((d*Csc[e + f*x])^(5/2)*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e +
 f*x]^(5/2) + Sin[2*(e + f*x)]))/(d*f)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{d\csc\left(fx+e\right)}\,d\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*d*csc(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e), x)
```

maple [C] time = 0.14, size = 319, normalized size = 4.43

$$\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\cos}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x)
```

```
[Out] 1/3/f*(-1+cos(f*x+e))^2*(I*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e
))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-I*(-
1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(
1/2)*sin(f*x+e)*cos(f*x+e)+I*sin(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e
)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(
1/2),1/2*2^(1/2))-cos(f*x+e)*2^(1/2))*(cos(f*x+e)+1)^2*(d/sin(f*x+e))^(3/2)
/sin(f*x+e)^4*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^(3/2)/sin(e + f*x),x)
```

```
[Out] int((d/sin(e + f*x))^(3/2)/sin(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*csc(e + f*x))**(3/2)*csc(e + f*x), x)
```

3.522 $\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx$

Optimal. Leaf size=103

$$-\frac{6d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df}-\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}$$

```
[Out] -2/5*cos(f*x+e)*(d*csc(f*x+e))^(5/2)/d/f-6/5*d*cos(f*x+e)*(d*csc(f*x+e))^(1
/2)/f+6/5*d^2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)
*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*
x+e)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3768, 3771, 2639}

$$-\frac{6d^2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df}-\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-6*d*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(5*f) - (2*Cos[e + f*x]*(d*Csc[e +
 f*x])^(5/2))/(5*d*f) - (6*d^2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(5*f*Sqrt[
d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \csc^2(e+fx)(d\csc(e+fx))^{3/2}\,dx = \frac{\int (d\csc(e+fx))^{7/2}\,dx}{d^2}$$

$$= -\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} + \frac{3}{5}\int (d\csc(e+fx))^{3/2}\,dx$$

$$= -\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} - \frac{1}{5}\left(3d^2\right.$$

$$= -\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} - \frac{\left(3d\right.}{5\sqrt{d\,c}}$$

$$= -\frac{6d\cos(e+fx)\sqrt{d\csc(e+fx)}}{5f} - \frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5df} - \frac{6d}{5f\sqrt{d}}$$

Mathematica [A] time = 0.26, size = 68, normalized size = 0.66

$$\frac{(d\csc(e+fx))^{5/2}\left(-7\cos(e+fx)+3\cos(3(e+fx))+12\sin^{\frac{5}{2}}(e+fx)E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{10df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] ((d*Csc[e + f*x])^(5/2)*(-7*Cos[e + f*x] + 3*Cos[3*(e + f*x)] + 12*Elliptic
E[(-2*e + Pi - 2*f*x)/4, 2]*Sin[e + f*x]^(5/2)))/(10*d*f)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{d\csc\left(fx+e\right)}\,d\csc\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*d*csc(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(3/2)*csc(f*x + e)^2, x)
```

maple [C] time = 0.17, size = 1054, normalized size = 10.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -1/5/f*(d/sin(f*x+e))^(3/2)*(6*cos(f*x+e)^3*(-I*(-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f
*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e
))^(1/2),1/2*2^(1/2))-3*cos(f*x+e)^3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*
((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I
)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2
),1/2*2^(1/2))+6*cos(f*x+e)^2*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos
(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f
*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2
^(1/2))-3*cos(f*x+e)^2*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)
+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^
(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))
-6*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+
e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*Ell
ipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+3*cos(f*
x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin
(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((
I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-6*(-I*(-1+cos(f*x
+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*
cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x
+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e
)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(
1/2),1/2*2^(1/2))-3*cos(f*x+e)^2*2^(1/2)+cos(f*x+e)*2^(1/2)+3*2^(1/2))/sin(
f*x+e)*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)^{3/2}}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d/sin(e + f*x))^(3/2)/sin(e + f*x)^2,x)
```

```
[Out] int((d/sin(e + f*x))^(3/2)/sin(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*csc(e + f*x))**(3/2)*csc(e + f*x)**2, x)
```

3.523 $\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=102

$$-\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21df}$$

```
[Out] -2/7*d^2*cos(f*x+e)/f/(d*csc(f*x+e))^(5/2)-10/21*cos(f*x+e)/f/(d*csc(f*x+e)
)^(1/2)-10/21*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)
*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+
e)^(1/2)/d/f
```

Rubi [A] time = 0.07, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3769, 3771, 2641}

$$-\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}-\frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}}+\frac{10\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21df}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*d^2*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*Cos[e + f*x])/(21*
f*Sqrt[d*Csc[e + f*x]]) + (10*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*
x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*d*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = d^3\int \frac{1}{(d\csc(e+fx))^{7/2}}\,dx$$
$$= -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} + \frac{1}{7}(5d)\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$$
$$= -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{5\int \sqrt{d\csc(e+fx)}\,dx}{21d}$$
$$= -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{\left(5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+}}}{21d}$$
$$= -\frac{2d^2\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21f\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{d\csc(e+fx)}F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{}}{21df}$$

Mathematica [A] time = 0.11, size = 70, normalized size = 0.69

$$-\frac{\sqrt{d\csc(e+fx)}\left(26\sin(2(e+fx))-3\sin(4(e+fx))+40\sqrt{\sin(e+fx)}F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{84df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] -1/84*(Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Si
n[e + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/(d*f)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^2-1\right)\sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)}{d\csc\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(d*csc(f*x + e))*sin(f*x + e)/(d*csc(f*x
 + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{3}}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

maple [C] time = 0.20, size = 208, normalized size = 2.04

$$-\frac{\left(5i\sin\left(fx+e\right)\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)\cdots}{\sin\left(\cdots\right.}}\cdots\right.}{21f\left(-1+\cos\left(fx\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/21/f*(5*I*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*((I*cos(f*x+e
)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))
^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2)
)-3*cos(f*x+e)^4*2^(1/2)+3*cos(f*x+e)^3*2^(1/2)+8*cos(f*x+e)^2*2^(1/2)-8*co
s(f*x+e)*2^(1/2))/(-1+cos(f*x+e))/(d/sin(f*x+e))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{3}}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(d/sin(e + f*x))^(1/2),x)
[Out] int(sin(e + f*x)^3/(d/sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(d*csc(f*x+e))**(1/2),x)
[Out] Timed out
```

3.524 $\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=72

$$\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

```
[Out] -2/5*d*cos(f*x+e)/f/(d*csc(f*x+e))^(3/2)-6/5*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
)/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3769, 3771, 2639}

$$\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*d*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*EllipticE[(e - Pi/2 +
 f*x)/2, 2])/(5*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx &= d^2\int \frac{1}{(d\csc(e+fx))^{5/2}}\,dx\\ &= -\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{3}{5}\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\ &= -\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{3\int \sqrt{\sin(e+fx)}\,dx}{5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= -\frac{2d\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.18, size = 57, normalized size = 0.79

$$\frac{-2\sin(2(e+fx))-\frac{12E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{\sqrt{\sin(e+fx)}}}{10f\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] ((-12*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x]] - 2*Sin[2*(e
+ f*x)])/(10*f*Sqrt[d*Csc[e + f*x]])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^2-1\right)\sqrt{d\csc\left(fx+e\right)}}{d\csc\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(d*csc(f*x + e))/(d*csc(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

maple [C] time = 0.19, size = 547, normalized size = 7.60

$$-\frac{\left(6\cos\left(fx+e\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx\right.}{\sin\left(fx+e\right)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/5/f*(6*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1
/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-3
*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)
-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*Ellip
ticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+6*(-I*(-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2
)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+
sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*(-I*(-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-si
n(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*
x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)^3*2^(1/2)+4*cos(f*x+e)*2^(1/2)-3*2^(1/2
))/(d/sin(f*x+e))^(1/2)/sin(f*x+e)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(d/sin(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)^2/(d/sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(e+fx\right)}{\sqrt{d\csc\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/sqrt(d*csc(e + f*x)), x)
```

3.525 $\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=74

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3df}-\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

```
[Out] -2/3*cos(f*x+e)/f/(d*csc(f*x+e))^(1/2)-2/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*
(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/d/f
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {16, 3769, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3df}-\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x])/(3*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*Elli
pticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx &= d\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx \\
&= -\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{\int \sqrt{d\csc(e+fx)}\,dx}{3d} \\
&= -\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d} \\
&= -\frac{2\cos(e+fx)}{3f\sqrt{d\csc(e+fx)}} + \frac{2\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{3df}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 64, normalized size = 0.86

$$-\frac{d\csc^2(e+fx)\left(\sin(2(e+fx))+2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] -1/3*(d*Csc[e + f*x]^2*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e +
f*x]] + Sin[2*(e + f*x)]))/(f*(d*Csc[e + f*x])^(3/2))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)}{d\csc\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*sin(f*x + e)/(d*csc(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

maple [C] time = 0.16, size = 181, normalized size = 2.45

$$-\frac{\left(i\sin\left(fx+e\right)\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\ \sqrt{\frac{i\cos\left(fx+e\right)+s}{\sin\left(fx\right.}}$$
$$\overline{3f\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{d}{\sin\left(fx+e\right)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*(I*sin(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1
/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+c
os(f*x+e)^2*2^(1/2)-cos(f*x+e)*2^(1/2))/(-1+cos(f*x+e))/(d/sin(f*x+e))^(1/2
)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)}{\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(d/sin(e + f*x))^(1/2),x)
```

```
[Out] int(sin(e + f*x)/(d/sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(e + fx\right)}{\sqrt{d \csc\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)/sqrt(d*csc(e + f*x)), x)
```

3.526 $\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=43

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3771, 2639}

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f
*x]])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx = \frac{\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$
$$= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.98

$$-\frac{2E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e
 + f*x]])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}}{d\csc\left(fx+e\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))/(d*csc(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(d*csc(f*x + e)), x)
```

maple [C] time = 0.15, size = 533, normalized size = 12.40

$$-\frac{\left(2\cos\left(fx+e\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx\right.}{\sin\left(fx+e\right)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin
(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2
)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos
(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/
sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF
(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*(-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-
(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(
f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+
e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^
(1/2),1/2*2^(1/2))+cos(f*x+e)*2^(1/2)-2^(1/2))/(d/sin(f*x+e))^(1/2)/sin(f*x
+e)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(d*csc(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d/sin(e + f*x))^(1/2),x)
```

```
[Out] int(1/(d/sin(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d\csc\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(d*csc(e + f*x)), x)
```

3.527 $\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{df}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/d/
f
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{df}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]]
)/(d*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx = \frac{\int \sqrt{d\csc(e+fx)}\,dx}{d}$$
$$= \frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{d}$$
$$= \frac{2\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{df}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.98

$$-\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\sqrt{d\csc(e+fx)}}{df}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f
*x]])/(d*f)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}}{d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))/d, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

maple [C] time = 0.14, size = 165, normalized size = 3.59

$$-\frac{i\left(-1+\cos\left(fx+e\right)\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\;\mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)\cdots}{\sin(\cdots}}\right.}{f\sqrt{\frac{d}{\sin\left(fx+e\right)}}\,\sin\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -I/f*(-1+cos(f*x+e))*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*co
s(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*(
cos(f*x+e)+1)^2*2^(1/2)/(d/sin(f*x+e))^(1/2)/sin(f*x+e)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/sqrt(d*csc(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(d/sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(d/sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{d\csc\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(d*csc(e + f*x)), x)
```

3.528 $\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=70

$$-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df}-\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] -2*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/d/f+2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2
)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/f/
(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3768, 3771, 2639}

$$-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df}-\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{f\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(d*f) - (2*EllipticE[(e - Pi/2 + f*x
)/2, 2])/(f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx &= \frac{\int (d\csc(e+fx))^{3/2}\,dx}{d^2}\\ &= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df}-\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx\\ &= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df}-\frac{\int \sqrt{\sin(e+fx)}\,dx}{\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{df}-\frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{f\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 52, normalized size = 0.74

$$\frac{\frac{2E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{\sqrt{\sin(e+fx)}}-2\cot(e+fx)}{f\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cot[e + f*x] + (2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x
]])/(f*Sqrt[d*Csc[e + f*x]])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)}{d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)/d, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

maple [C] time = 0.16, size = 522, normalized size = 7.46

$$\left(2\cos\left(fx+e\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+\right.}{\sin\left(fx+e\right)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] 1/f*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(
f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)
*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(
f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/s
in(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(
((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*(-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(
I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f
*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e
)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(
1/2),1/2*2^(1/2))-2^(1/2))/(d/sin(f*x+e))^(1/2)/sin(f*x+e)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(d*csc(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^2\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(d/sin(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(d/sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(e+fx\right)}{\sqrt{d\csc\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(d*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(d*csc(e + f*x)), x)
```

3.529 $\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3df}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f}$$

```
[Out] -2/3*cos(f*x+e)*(d*csc(f*x+e))^(3/2)/d^2/f-2/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2
)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/
2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/d/f
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3df}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*d^2*f) + (2*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(e+fx)}{\sqrt{d\csc(e+fx)}}\,dx &= \frac{\int (d\csc(e+fx))^{5/2}\,dx}{d^3}\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f}+\frac{\int \sqrt{d\csc(e+fx)}\,dx}{3d}\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f}+\frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d}\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^2f}+\frac{2\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{3df}\end{aligned}$$

Mathematica [A] time = 0.06, size = 60, normalized size = 0.78

$$-\frac{2\csc^2(e+fx)\left(\cos(e+fx)+\sin^{\frac{3}{2}}(e+fx)F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3f\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[d*Csc[e + f*x]],x]
```

```
[Out] (-2*Csc[e + f*x]^2*(Cos[e + f*x] + EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sin[
e + f*x]^(3/2)))/(3*f*Sqrt[d*Csc[e + f*x]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\,\csc\left(fx+e\right)^2}{d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)^2/d, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^3}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

maple [C] time = 0.18, size = 318, normalized size = 4.13

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^2\left(-i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{-\frac{i}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x)
```

```
[Out] -1/3/f*(-1+cos(f*x+e))^2*(-I*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x
+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-I*
(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))
^(1/2)*sin(f*x+e)*cos(f*x+e)-I*sin(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(
1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x
+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))
^(1/2),1/2*2^(1/2))+cos(f*x+e)*2^(1/2))*(cos(f*x+e)+1)^2/sin(f*x+e)^6/(d/si
n(f*x+e))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^3}{\sqrt{d\csc\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(d*csc(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,\sqrt{\frac{d}{\sin\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(d/sin(e + f*x))^(1/2)),x)
[Out] int(1/(sin(e + f*x)^3*(d/sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}\left(e+fx\right)}{\sqrt{d\csc\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(d*csc(f*x+e))**(1/2),x)
[Out] Integral(csc(e + f*x)**3/sqrt(d*csc(e + f*x)), x)
```

3.530 $\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{10\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21d^2f}-\frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}$$

```
[Out] -2/7*d*cos(f*x+e)/f/(d*csc(f*x+e))^(5/2)-10/21*cos(f*x+e)/d/f/(d*csc(f*x+e)
)^(1/2)-10/21*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)
*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+
e)^(1/2)/d^2/f
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3769, 3771, 2641}

$$\frac{10\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{21d^2f}-\frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}}-\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*d*Cos[e + f*x])/(7*f*(d*Csc[e + f*x])^(5/2)) - (10*Cos[e + f*x])/(21*d*
f*Sqrt[d*Csc[e + f*x]]) + (10*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*
x)/2, 2]*Sqrt[Sin[e + f*x]])/(21*d^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

]

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = d^2\int \frac{1}{(d\csc(e+fx))^{7/2}}\,dx$$
$$= -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} + \frac{5}{7}\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$$
$$= -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}} + \frac{5\int\sqrt{d\csc(e+fx)}\,dx}{21d^2}$$
$$= -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}} + \frac{\left(5\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{}{\sqrt{\mathrm{si}}}}{21d^2}$$
$$= -\frac{2d\cos(e+fx)}{7f(d\csc(e+fx))^{5/2}} - \frac{10\cos(e+fx)}{21df\sqrt{d\csc(e+fx)}} + \frac{10\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{21d^2f}$$

Mathematica [A] time = 0.12, size = 70, normalized size = 0.68

$$-\frac{\sqrt{d\csc(e+fx)}\left(26\sin(2(e+fx))-3\sin(4(e+fx))+40\sqrt{\sin(e+fx)}\,F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{84d^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] -1/84*(Sqrt[d*Csc[e + f*x]]*(40*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Si
n[e + f*x]] + 26*Sin[2*(e + f*x)] - 3*Sin[4*(e + f*x)]))/(d^2*f)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(\cos\left(fx+e\right)^2-1\right)\sqrt{d\csc\left(fx+e\right)}}{d^2\csc\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*sqrt(d*csc(f*x + e))/(d^2*csc(f*x + e)^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

maple [C] time = 0.18, size = 216, normalized size = 2.10

$$-\frac{\left(5i\sin\left(fx+e\right)\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\ \sqrt{\frac{i\cos\left(fx+e\right)\cdots}{\sin\left(\cdots\right.}}\cdots}{21f\left(-1+\cos\left(fx+e\right)\right)\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -1/21/f*(5*I*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*sin(f*x+e)*((I*cos(f*x+e
)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))
^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2)
)-3*cos(f*x+e)^4*2^(1/2)+3*cos(f*x+e)^3*2^(1/2)+8*cos(f*x+e)^2*2^(1/2)-8*co
s(f*x+e)*2^(1/2))/(-1+cos(f*x+e))/(d/sin(f*x+e))^(3/2)/sin(f*x+e)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e + f\,x\right)^2}{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(d/sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)^2/(d/sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(e + fx\right)}{\left(d\csc\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(d*csc(f*x+e))**(3/2),x)
[Out] Integral(sin(e + f*x)**2/(d*csc(e + f*x))**(3/2), x)
```

3.531 $\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

```
[Out] -2/5*cos(f*x+e)/f/(d*csc(f*x+e))^(3/2)-6/5*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/
d/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {16, 3769, 3771, 2639}

$$\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}-\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x])/(5*f*(d*Csc[e + f*x])^(3/2)) + (6*EllipticE[(e - Pi/2 + f
*x)/2, 2])/(5*d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx &= d\int \frac{1}{(d\csc(e+fx))^{5/2}}\,dx\\
&= -\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx}{5d}\\
&= -\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{3\int \sqrt{\sin(e+fx)}\,dx}{5d\sqrt{d\csc(e+fx)}\,\sqrt{\sin(e+fx)}}\\
&= -\frac{2\cos(e+fx)}{5f(d\csc(e+fx))^{3/2}}+\frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{5df\sqrt{d\csc(e+fx)}\,\sqrt{\sin(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.81

$$\frac{-2\sin(2(e+fx))-\frac{12E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{\sqrt{\sin(e+fx)}}}{10df\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] ((-12*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x]] - 2*Sin[2*(e
+ f*x)])/(10*d*f*Sqrt[d*Csc[e + f*x]])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\sin\left(fx+e\right)}{d^2\csc\left(fx+e\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*sin(f*x + e)/(d^2*csc(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

maple [C] time = 0.17, size = 547, normalized size = 7.39

$$-\frac{\left(6\cos\left(fx+e\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx\right.}{\sin\left(fx+e\right)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -1/5/f*(6*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1
/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-3
*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)
-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*Ellip
ticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+6*(-I*(-1+
cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2
)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+
sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*(-I*(-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-si
n(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*
x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)^3*2^(1/2)+4*cos(f*x+e)*2^(1/2)-3*2^(1/2
))/(d/sin(f*x+e))^(3/2)/sin(f*x+e)^2*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)}{{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(d/sin(e + f*x))^(3/2),x)
[Out] int(sin(e + f*x)/(d/sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(d*csc(f*x+e))**(3/2),x)
[Out] Integral(sin(e + f*x)/(d*csc(e + f*x))**(3/2), x)
```

3.532 $\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}$$

```
[Out] -2/3*cos(f*x+e)/d/f/(d*csc(f*x+e))^(1/2)-2/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^
(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2)
)*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/d^2/f
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3769, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Csc[e + f*x])^(-3/2),x]
```

```
[Out] (-2*Cos[e + f*x])/(3*d*f*Sqrt[d*Csc[e + f*x]]) + (2*Sqrt[d*Csc[e + f*x]]*El
lipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{1}{(d\csc(e+fx))^{3/2}}\,dx = -\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}} + \frac{\int \sqrt{d\csc(e+fx)}\,dx}{3d^2}$$
$$= -\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}} + \frac{\left(\sqrt{d\csc(e+fx)}\,\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d^2}$$
$$= -\frac{2\cos(e+fx)}{3df\sqrt{d\csc(e+fx)}} + \frac{2\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{3d^2f}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.82

$$-\frac{\csc^2(e+fx)\left(\sin(2(e+fx))+2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Csc[e + f*x])^(-3/2),x]
```

```
[Out] -1/3*(Csc[e + f*x]^2*(2*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f*
x]] + Sin[2*(e + f*x)]))/(f*(d*Csc[e + f*x])^(3/2))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}}{d^2\csc\left(fx+e\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))/(d^2*csc(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*csc(f*x + e))^(-3/2), x)
```

maple [C] time = 0.14, size = 189, normalized size = 2.45

$$-\frac{\left(i\sin\left(fx+e\right)\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\ \sqrt{\frac{i\cos\left(fx+e\right)+\mathrm{s}\cdots}{\sin\left(f\cdots\right.}}\cdots}{3f\left(-1+\cos\left(fx+e\right)\right)\left(\frac{d}{\sin\left(fx+e\right)}\right)^{\frac{3}{2}}\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -1/3/f*(I*sin(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+s
in(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1
/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+c
os(f*x+e)^2*2^(1/2)-cos(f*x+e)*2^(1/2))/(-1+cos(f*x+e))/(d/sin(f*x+e))^(3/2
)/sin(f*x+e)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*csc(f*x + e))^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d/sin(e + f*x))^(3/2),x)
```

```
[Out] int(1/(d/sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral((d*csc(e + f*x))**(-3/2), x)
```

3.533 $\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/d/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2
)
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {16, 3771, 2639}

$$\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (2*EllipticE[(e - Pi/2 + f*x)/2, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e +
 f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx}{d}$$
$$= \frac{\int \sqrt{\sin(e+fx)}\,dx}{d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$
$$= \frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.98

$$-\frac{2E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(d*Csc[e + f*x])^(3/2),x]
[Out] (-2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin
[e + f*x]])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\csc(fx+e)}}{d^2\csc(fx+e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*csc(f*x + e))/(d^2*csc(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(fx+e)}{(d\csc(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

maple [C] time = 0.14, size = 533, normalized size = 11.59

$$-\frac{\left(2\cos\left(fx+e\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\ \mathrm{EllipticE}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx\right.}{\sin\left(fx+e\right)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin
(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2
)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos
(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/
sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF
(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*(-1+cos(
f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-
(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(
f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1
/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+
e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^
(1/2),1/2*2^(1/2))+cos(f*x+e)*2^(1/2)-2^(1/2))/(d/sin(f*x+e))^(3/2)/sin(f*x
+e)^2*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)/(d*csc(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(d/sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(d/sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(d*csc(e + f*x))**(3/2), x)
```

3.534 $\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{d^2f}$$

```
[Out] -2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(
cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/d^
2/f
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {16, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (2*Sqrt[d*Csc[e + f*x]]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]]
)/(d^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx = \frac{\int \sqrt{d\csc(e+fx)}\,dx}{d^2}$$
$$= \frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{d^2}$$
$$= \frac{2\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{d^2 f}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.98

$$-\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\sqrt{d\csc(e+fx)}}{d^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Sqrt[d*Csc[e + f*x]]*EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sqrt[Sin[e + f
*x]])/(d^2*f)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}}{d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))/d^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

maple [C] time = 0.14, size = 165, normalized size = 3.59

$$-\frac{i\sqrt{2}\,\left(\cos\left(fx+e\right)+1\right)^{2}\operatorname{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{\cdots}}{f\left(\frac{d}{\sin\left(fx+e\right)}\right)^{\frac{3}{2}}\sin\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -I/f*2^(1/2)*(cos(f*x+e)+1)^2*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*
x+e))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x
+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e)
)^(1/2)*(-1+cos(f*x+e))/(d/sin(f*x+e))^(3/2)/sin(f*x+e)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(d*csc(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(e+fx\right)^{2}\left(\frac{d}{\sin\left(e+fx\right)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(d/sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(d/sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{2}\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(d*csc(e + f*x))**(3/2), x)
```

3.535 $\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f}-\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] -2*cos(f*x+e)*(d*csc(f*x+e))^(1/2)/d^2/f+2*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1
/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/
d/f/(d*csc(f*x+e))^(1/2)/sin(f*x+e)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3768, 3771, 2639}

$$-\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f}-\frac{2E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(d^2*f) - (2*EllipticE[(e - Pi/2 + f
*x)/2, 2])/(d*f*Sqrt[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx &= \frac{\int (d\csc(e+fx))^{3/2}\,dx}{d^3}\\ &= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f}-\frac{\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx}{d}\\ &= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f}-\frac{\int \sqrt{\sin(e+fx)}\,dx}{d\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\\ &= -\frac{2\cos(e+fx)\sqrt{d\csc(e+fx)}}{d^2f}-\frac{2E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)}{df\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.75

$$\frac{\frac{2E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)}{\sqrt{\sin(e+fx)}}-2\cot(e+fx)}{df\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cot[e + f*x] + (2*EllipticE[(-2*e + Pi - 2*f*x)/4, 2])/Sqrt[Sin[e + f*x
]])/(d*f*Sqrt[d*Csc[e + f*x]])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\csc\left(fx+e\right)}{d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)/d^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^3}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^3/(d*csc(f*x + e))^(3/2), x)
```

maple [C] time = 0.16, size = 522, normalized size = 7.15

$$\frac{\left(2\cos\left(fx+e\right)\sqrt{-\frac{i\left(-1+\cos\left(fx+e\right)\right)}{\sin\left(fx+e\right)}}\,\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i\cos\left(fx+e\right)-\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+\right.}{\sin\left(fx+e\right)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(2*cos(f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(
f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)
*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(
f*x+e)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/s
in(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(
((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+2*(-I*(-1+cos(f
*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(
I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f
*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/
2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e
)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(
1/2),1/2*2^(1/2))-2^(1/2))/(d/sin(f*x+e))^(3/2)/sin(f*x+e)^2*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^3}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^3/(d*csc(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(d/sin(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(d/sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}{\left(e + f x \right)}}{\left(d \csc{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(d*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(d*csc(e + f*x))**(3/2), x)
```

3.536 $\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f}$$

```
[Out] -2/3*cos(f*x+e)*(d*csc(f*x+e))^(3/2)/d^3/f-2/3*(sin(1/2*e+1/4*Pi+1/2*f*x)^2
)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)*EllipticF(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/
2))*(d*csc(f*x+e))^(1/2)*sin(f*x+e)^(1/2)/d^2/f
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3768, 3771, 2641}

$$\frac{2\sqrt{\sin(e+fx)}\,F\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{d\csc(e+fx)}}{3d^2f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Cos[e + f*x]*(d*Csc[e + f*x])^(3/2))/(3*d^3*f) + (2*Sqrt[d*Csc[e + f*x]
]*EllipticF[(e - Pi/2 + f*x)/2, 2]*Sqrt[Sin[e + f*x]])/(3*d^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx &= \frac{\int (d\csc(e+fx))^{5/2}\,dx}{d^4}\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f}+\frac{\int \sqrt{d\csc(e+fx)}\,dx}{3d^2}\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f}+\frac{\left(\sqrt{d\csc(e+fx)}\sqrt{\sin(e+fx)}\right)\int \frac{1}{\sqrt{\sin(e+fx)}}\,dx}{3d^2}\\ &= -\frac{2\cos(e+fx)(d\csc(e+fx))^{3/2}}{3d^3f}+\frac{2\sqrt{d\csc(e+fx)}\,F\left(\frac{1}{2}\left(e-\frac{\pi}{2}+fx\right)\middle|2\right)\sqrt{\sin(e+fx)}}{3d^2f}\end{aligned}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.78

$$-\frac{2\csc^3(e+fx)\left(\cos(e+fx)+\sin^{\frac{3}{2}}(e+fx)F\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{3f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-2*Csc[e + f*x]^3*(Cos[e + f*x] + EllipticF[(-2*e + Pi - 2*f*x)/4, 2]*Sin[
e + f*x]^(3/2)))/(3*f*(d*Csc[e + f*x])^(3/2))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\,\csc\left(fx+e\right)^2}{d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)^2/d^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^4}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/(d*csc(f*x + e))^(3/2), x)
```

maple [C] time = 0.17, size = 319, normalized size = 4.14

$$\frac{\left(\cos\left(fx+e\right)+1\right)^2\left(-1+\cos\left(fx+e\right)\right)^2\left(i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{i\cos\left(fx+e\right)+\sin\left(fx+e\right)-i}{\sin\left(fx+e\right)}}\,\sqrt{-\frac{i(-}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] 1/3/f*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))^2*(I*EllipticF(((I*cos(f*x+e)+sin(f*
x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x
+e))^(1/2)*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)
-I)/sin(f*x+e))^(1/2)*sin(f*x+e)*cos(f*x+e)+I*sin(f*x+e)*(-I*(-1+cos(f*x+e)
)/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos
(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)
-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-cos(f*x+e)*2^(1/2))/sin(f*x+e)^7/(d/sin(
f*x+e))^(3/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^4}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^4/(d*csc(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^4\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(d/sin(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^4*(d/sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{4}{\left(e + f x \right)}}{\left(d \csc{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(d*csc(f*x+e))**(3/2),x)
[Out] Integral(csc(e + f*x)**4/(d*csc(e + f*x))**(3/2), x)
```

3.537 $\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f}-\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f}-\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

```
[Out] -2/5*cos(f*x+e)*(d*csc(f*x+e))^(5/2)/d^4/f-6/5*cos(f*x+e)*(d*csc(f*x+e))^(1
/2)/d^2/f+6/5*(sin(1/2*e+1/4*Pi+1/2*f*x)^2)^(1/2)/sin(1/2*e+1/4*Pi+1/2*f*x)
*EllipticE(cos(1/2*e+1/4*Pi+1/2*f*x),2^(1/2))/d/f/(d*csc(f*x+e))^(1/2)/sin(
f*x+e)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 3768, 3771, 2639}

$$-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f}-\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f}-\frac{6E\left(\frac{1}{2}\left(e+fx-\frac{\pi}{2}\right)\middle|2\right)}{5df\sqrt{\sin(e+fx)}\sqrt{d\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (-6*Cos[e + f*x]*Sqrt[d*Csc[e + f*x]])/(5*d^2*f) - (2*Cos[e + f*x]*(d*Csc[e
 + f*x])^(5/2))/(5*d^4*f) - (6*EllipticE[(e - Pi/2 + f*x)/2, 2])/(5*d*f*Sqr
t[d*Csc[e + f*x]]*Sqrt[Sin[e + f*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\csc^5(e+fx)}{(d\csc(e+fx))^{3/2}}\,dx=\frac{\int (d\csc(e+fx))^{7/2}\,dx}{d^5}$$

$$=-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f}+\frac{3\int (d\csc(e+fx))^{3/2}\,dx}{5d^3}$$

$$=-\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f}-\frac{3\int \frac{1}{\sqrt{d\csc(e+fx)}}\,dx}{5d}$$

$$=-\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f}-\frac{3\int \sqrt{\sin(e+}}{5d\sqrt{d\csc(e+fx)}\sqrt{}}$$

$$=-\frac{6\cos(e+fx)\sqrt{d\csc(e+fx)}}{5d^2f}-\frac{2\cos(e+fx)(d\csc(e+fx))^{5/2}}{5d^4f}-\frac{6E\left(\frac{1}{2}\left(e-\frac{\pi}{2}+\right.\right.}{5df\sqrt{d\csc(e+fx)}}$$

Mathematica [A] time = 0.13, size = 73, normalized size = 0.70

$$\frac{\csc^4(e+fx)\left(-7\cos(e+fx)+3\cos(3(e+fx))+12\sin^{\frac{5}{2}}(e+fx)E\left(\frac{1}{4}(-2e-2fx+\pi)\middle|2\right)\right)}{10f(d\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5/(d*Csc[e + f*x])^(3/2),x]
```

```
[Out] (Csc[e + f*x]^4*(-7*Cos[e + f*x] + 3*Cos[3*(e + f*x)] + 12*EllipticE[(-2*e
+ Pi - 2*f*x)/4, 2]*Sin[e + f*x]^(5/2)))/(10*f*(d*Csc[e + f*x])^(3/2))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{d\csc\left(fx+e\right)}\,\csc\left(fx+e\right)^3}{d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*csc(f*x + e))*csc(f*x + e)^3/d^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{5}}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^5/(d*csc(f*x + e))^(3/2), x)
```

maple [C] time = 0.18, size = 1054, normalized size = 10.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x)
```

```
[Out] -1/5/f*(6*cos(f*x+e)^3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)
+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^
(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))
-3*cos(f*x+e)^3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*
x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*E
llipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+6*cos(
f*x+e)^2*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)
/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*Elliptic
E(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*cos(f*x+e)^
2*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*
x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*c
os(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-6*cos(f*x+e)*(-I*(-1
+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/
2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)
+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))+3*cos(f*x+e)*(-I*(-1+cos(f*x+
e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*c
os(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2)*EllipticF(((I*cos(f*x+e)+sin(f*x+
e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-6*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2
)*((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)
-I)/sin(f*x+e))^(1/2)*EllipticE(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1
```

```
/2),1/2*2^(1/2))+3*(-I*(-1+cos(f*x+e))/sin(f*x+e))^(1/2)*((I*cos(f*x+e)+sin
(f*x+e)-I)/sin(f*x+e))^(1/2)*(-(I*cos(f*x+e)-sin(f*x+e)-I)/sin(f*x+e))^(1/2
)*EllipticF(((I*cos(f*x+e)+sin(f*x+e)-I)/sin(f*x+e))^(1/2),1/2*2^(1/2))-3*c
os(f*x+e)^2*2^(1/2)+cos(f*x+e)*2^(1/2)+3*2^(1/2))/sin(f*x+e)^4/(d/sin(f*x+e
))^(3/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{5}}{\left(d\csc\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5/(d*csc(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(f*x + e)^5/(d*csc(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^5\,{\left(\frac{d}{\sin\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^5*(d/sin(e + f*x))^(3/2)),x)
[Out] int(1/(sin(e + f*x)^5*(d/sin(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{5}\left(e+fx\right)}{\left(d\csc\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5/(d*csc(f*x+e))**(3/2),x)
[Out] Integral(csc(e + f*x)**5/(d*csc(e + f*x))**(3/2), x)
```

3.538 $\int (b\csc(e+fx))^n (a\sin(e+fx))^m \, dx$

Optimal. Leaf size=87

$$\frac{\cos(e+fx)(a\sin(e+fx))^{m+1}(b\csc(e+fx))^n \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}(m-n+1);\frac{1}{2}(m-n+3);\sin^2(e+fx)\right)}{af(m-n+1)\sqrt{\cos^2(e+fx)}}$$

```
[Out] cos(f*x+e)*(b*csc(f*x+e))^n*hypergeom([1/2, 1/2+1/2*m-1/2*n],[3/2+1/2*m-1/2
*n],sin(f*x+e)^2)*(a*sin(f*x+e))^(1+m)/a/f/(1+m-n)/(cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2588, 2643}

$$\frac{\cos(e+fx)(a\sin(e+fx))^{m+1}(b\csc(e+fx))^n \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}(m-n+1);\frac{1}{2}(m-n+3);\sin^2(e+fx)\right)}{af(m-n+1)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Csc[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] (Cos[e + f*x]*(b*Csc[e + f*x])^n*Hypergeometric2F1[1/2, (1 + m - n)/2, (3 +
 m - n)/2, Sin[e + f*x]^2]*(a*Sin[e + f*x])^(1 + m))/(a*f*(1 + m - n)*Sqrt[
Cos[e + f*x]^2])
```

Rule 2588

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[(a*b)^IntPart[n]*(a*Sin[e + f*x])^FracPart[n]*(b*Csc
[e + f*x])^FracPart[n], Int[(a*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b
, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\csc(e+fx))^n(a\sin(e+fx))^m\,dx = \left((b\csc(e+fx))^n(a\sin(e+fx))^n\right)\int (a\sin(e+fx))^{m-n}\,dx$$
$$= \frac{\cos(e+fx)(b\csc(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m-n);\frac{1}{2}(3+m-n);\sin^2(e+\right.}{af(1+m-n)\sqrt{\cos^2(e+fx)}}$$

Mathematica [A] time = 10.47, size = 102, normalized size = 1.17

$$\frac{2\tan\left(\frac{1}{2}(e+fx)\right)(a\sin(e+fx))^m(b\csc(e+fx))^n\sec^2\left(\frac{1}{2}(e+fx)\right)^{m-n}\,{}_2F_1\left(\frac{1}{2}(m-n+1),m-n+1;\frac{1}{2}(m-n+3\right.}{f(m-n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Csc[e + f*x])^n*(a*Sin[e + f*x])^m,x]
```

```
[Out] (2*(b*Csc[e + f*x])^n*Hypergeometric2F1[(1 + m - n)/2, 1 + m - n, (3 + m -
n)/2, -Tan[(e + f*x)/2]^2]*(Sec[(e + f*x)/2]^2)^(m - n)*(a*Sin[e + f*x])^m*
Tan[(e + f*x)/2])/(f*(1 + m - n))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\csc\left(fx+e\right)\right)^n\left(a\sin\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\csc\left(fx+e\right)\right)^n\left(a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

maple [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \left(b\csc\left(fx+e\right)\right)^n\left(a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

```
[Out] int((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\csc\left(fx+e\right)\right)^{n}\left(a\sin\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))^n*(a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*(a*sin(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\,\sin\left(e+f\,x\right)\right)^{m}\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(e + f*x))^m*(b/sin(e + f*x))^n,x)
```

```
[Out] int((a*sin(e + f*x))^m*(b/sin(e + f*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(e+fx\right)\right)^{m}\left(b\csc\left(e+fx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*csc(f*x+e))**n*(a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x))**m*(b*csc(e + f*x))**n, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
   ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```